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QUALITATIVE FEATURE OF DYNAMIC CONCENTRATION OF STRESSES IN ELASTIC COMPOUND MOTOR-CAR DETAILS

According to a modified method of superposition the decision of a problem on harmonic vibrations of the non-uniform automobile details which consist of various number of joined elastic areas with different elastic properties is presented. Features of a wave field in a vicinity of singular border point on a joint of areas are investigated. Acquired results of numerical analysis may be used for optimal selection of mechanical properties of detail section.

Introduction

It is possible to approximate the decision of the elasticity theory problems in the best way and to construct effective numerical algorithm for its finding when you know character of behavior of components of the stressedly-deformed state (SDS) not far from special points and lines of a surface of a considered body. This problem is even more actual in problems of a construction details vibration loading when the tension can undergo qualitative changes according to external loading frequency. In accordance with scientific direction which is actively developed in many research establishments — the physical mesomechanics of materials — the behavior under loading of boundary areas of structurally-non-uniform medium is experimentally studied and determined that near internal borders of section arise oscillation of local stresses and the deformations which amplitude essentially exceeds their average values in volume of the material. They are defined by internal structure of the non-uniform medium and depend on elastic parameters of medium which they make contact with. At micro scale level such oscillation tension give rise to dispositions streams, at mesoscale level — to extended mesostrips of deformation, at macroscale level — to stationary distribution of macrostrips of the localized deformation and destruction of the material. The received experimental results have the important application in the mechanic of structurally-non-uniform environments, in microelectronics, geodynamics, materials technology. Thus, it is possible to consider as evidence that a local stress concentration (LSC) appears in a vicinity of interfaces in the loaded non-uniform body. The character and intensity of this concentration is defined both by character of external dynamic of the loading and by proportion of mechanical properties of contacting media.

Static tension distribution features in a vicinity of an angular point of a division line of areas of the body cross-section made from two various prism parts joined to each other on a side surface were considered earlier [1-3]. For example [1], an elastic-static flat problem about two diverse wedges which have random opening corners has been considered in the following way: the decision was plotted in Mellin transformants, having done this correspondent to conjugation conditions we can investigate local feature parameter (LFP) dependence on pressure in top of wedges from angles of an opening and combinations of elastic constants. The method [4] has been presented. It allows establishing the character of the specified features without the direct decision of a boundary problem. The dynamic aspects were considered [5-7] and, in particular, the concept of a "boundary" resonance — generalizations of well investigated edge resonance [8] has been introduced.

Some features of the SDS in a vicinity of top of the compound body which is a rigid connection of three diverse wedges with corners at the tops α, β and γ, are studied in [9]. The numerical analysis is done for a compound body, its total opening angle is equal $\alpha + \beta + \gamma = \pi$. The basic emphasis at research in [9] is made on studying the influence of LFP on value of wedges opening angles.

Object of work

Next, the problem of determination of qualitative and quantitative character of the wave field feature arising in a vicinity of an angular point of a joint of two, three and four diverse areas of a squared shape is stated. Similar problems are connected with calculation strength parameters of welded and soldered joints, including such ones which have angular seams [10]. The presented technique is universal enough and can be used at the analysis of intensity of LSC in compound bodies of any structure. With its help it is possible not only to research the intensity factors of LSC, but also to plot a decision in the whole area of a compound body section.

Main part

1. Interface of two different medias

1.1. Problem statement. We consider the wave movements completely characterized by a two-dimensional field in the plane $\alpha_1 O\alpha_2$ in infinite along an axis $O\alpha_3$ to a prism detail V. We assume that the prism section occupies region $D=\{(\alpha_1,\alpha_2): |\alpha_1|\le a, |\alpha_2|\le b\}$ in a plane $\alpha_1 O\alpha_2$. Suppose $S_{\pm}^{(m)}$ $(m=1,2,...,N)$ — the straight lines which have in chosen system of coordinates of the equation $\alpha_1=\pm a_m$ and divide section D into regions $D^{(1)}$ and $D^{(m)}=D_{-}^{(m)}\cup D_{+}^{(m)}$ $(m=2,3,...,N)$, such that $a_1<a_2<...<a_N=a$. Regions $D_{-}^{(m)}$ and $D_{+}^{(m)}$ also are arranged symmetric about the origin of coordinates, have an identical thickness $h^{(m)}=a_m-a_{m-1}$ and are defined as $D_{-}^{(m)}=\{(\alpha_1,\alpha_2): \alpha_1\in[-a_m,-a_{m-1}], |\alpha_2|\le b\}$, $D_{+}^{(m)}=\{(\alpha_1,\alpha_2): \alpha_1\in[a_{m-1},a_m], |\alpha_2|\le b\}$. The region $D^{(1)}$ is internal rectangular one-coherent area with the centre of gravity in the origin of coordinates: $D^{(1)}=\{(\alpha_1,\alpha_2): |\alpha_1|\le a_1, |\alpha_2|\le b\}$.

Each of the listed N regions $D^{(m)}$ is occupied by the homogeneous and isotropic elastic material with the shear modulus $\mu^{(m)}$, Poisson's ratio $\nu^{(m)}$ and density $\rho^{(m)}$. We assume that the wave field in the region of section D is determinate by action on borders $\alpha_1=\pm a, \alpha_2=\pm b$ by normal, harmoniously changing in time with the frequency ω, self-counterbalanced loads of intensity $q_1(\alpha_2)$ and $q_2(\alpha_1)$ accordingly. In the subsequent calculations the time multiplier $e^{i\omega t}$ which is general for all characteristics of wave fields falls. Taking into account symmetry of region, it is possible to consider a wave field of a part of region of section of the prism located in the first quarter.

Amplitude displacements and stresses of points of elastic media of regions $D^{(m)}$ we will designate through $u_{\beta}^{(m)}$ and $\tau_{\gamma\beta}^{(m)}$ accordingly ($\beta=1,2;\gamma=1,2$). The general boundary problem of own frequencies determination and of non-uniform region D fluctuations own forms is formulated in a following dimensionless form. In all regions $D^{(m)}$ it is introduced dimensionless local coordinates $x^{(m)}$ and we search for the functions $U_{\beta}^{(m)}(x^{(m)},y)$, satisfying the equations movement

$$\Delta U_{\beta}^{(m)}+(C_{12}^{(m)}+1)(U_{1,1}^{(m)}+U_{2,2}^{(m)})_{,\beta}+(\Omega^{(m)})^2 U_{\beta}^{(m)}=0, \tag{1}$$

where $U_{\beta}^{(m)}=u_{\beta}^{(m)}/a$, $C_{12}^{(m)}=C_{11}^{(m)}-2$, $C_{11}^{(m)}=2(1-\nu^{(m)})/(1-2\nu^{(m)})$, $f_{,1}^{(m)}=\dfrac{\partial f^{(m)}}{\partial x^{(m)}}$, $f_{,2}^{(m)}=\dfrac{\partial f^{(m)}}{\partial y}$, $\Omega^{(m)}=a\omega\sqrt{\dfrac{\rho^{(m)}}{\mu^{(m)}}}$, $x=\alpha_1/a, \delta_m=a_m/a, \delta_0=0$, $y=\alpha_2/a$, $\overline{h}^{(m)}=\delta_m-\delta_{m-1}$, $x^{(m)}\in[0,\overline{h}^{(m)}], y\in[0,\eta]$, $\eta=b/a$, $m=1,2,...,N$.

Dimensionless amplitude components of the stress tensor $\sigma_{\gamma\beta}^{(m)} = \tau_{\gamma\beta}^{(m)} / \mu^{(m)}$ are calculated according to the proportions of Hooke's law. On straight lines $S_{+}^{(m)}$ conditions of contact of areas with various elastic properties should be satisfied. Exactly on a joint of all regions $\bar{D}^{(m)} = \{(x^{(m)}, y): 0 \le x^{(m)} \le \bar{h}^{(m)}, 0 \le y \le \eta\}$ and $\bar{D}^{(m+1)}$ $(m = 1,2,...,N-1)$ a condition of rigid coupling is considered as satisfied

$$\sigma_{1\gamma}^{(m)}(\bar{h}^{(m)}, y) = r_m \sigma_{1\gamma}^{(m+1)}(0, y), \; U_{\gamma}^{(m)}(\bar{h}^{(m)}, y) = U_{\gamma}^{(m+1)}(0, y), \; r_m = \mu^{(m+1)} / \mu^{(m)} \tag{2}$$

On external border of section power boundary conditions are $(q_{\gamma}^{(m)} = q_{\gamma} / \mu^{(m)})$

$$\sigma_{11}^{(N)}(1-\delta_{N-1}, y) = q_1^{(N)}(y), \sigma_{22}^{(m)}(x^{(m)}, \eta) = q_2^{(m)}(x^{(m)}), \sigma_{12}^{(N)}(1-\delta_{N-1}, y) = \sigma_{12}^{(m)}(x^{(m)}, \eta) = 0, \tag{3}$$

1.2. Common solution construction. According to the algorithm of a method of superposition [7,8,11], the common decision $U_{\beta}^{(m)}$, satisfying the system of the equations (1) within region $\bar{D}^{(m)}$, we design in the form of the sum of two private solutions of this system, each of which describes symmetric fluctuations of infinite strips $G_1^{(m)} = \{(x^{(m)}, y): 0 \le x^{(m)} \le \bar{h}^{(m)}, |y| < \infty\}$ and $G_2^{(m)} = \{(x^{(m)}, y): |x^{(m)}| < \infty, |y| \le \eta\}$, forming at the crossing region $\bar{D}^{(m)}$. Thus it is necessary to consider that on function $U_{\beta}^{(m)}(x^{(m)}, y)$ of the coordinate $x^{(m)}$ are general view functions. As a result we have the problem common solution in regions $\bar{D}^{(m)} (m = 2,3,...,N)$ in the form

$$U_1^{(m)} = (\bar{H}^{(m)} sh(t^{(m)} x^{(m)}) + \bar{Q}^{(m)} ch(t^{(m)} x^{(m)})) \cos\alpha(y-\eta) + \bar{R}^{(m)} ch(r^{(m)} y) \sin(\chi^{(m)}(x^{(m)} - \bar{h}^{(m)})),$$
$$U_2^{(m)} = (H^{(m)} ch(t^{(m)} x^{(m)}) + Q^{(m)} sh(t^{(m)} x^{(m)})) \sin\alpha(y-\eta) + R^{(m)} sh(r^{(m)} y) \cos(\chi^{(m)}(x^{(m)} - \bar{h}^{(m)})) \tag{4}$$

For the region $D^{(1)}$ the solution is construction as in case with symmetric fluctuations of a homogeneous rectangle and [5,8] is given by

$$U_1^{(1)} = \bar{H}^{(1)} sh(t^{(1)} x^{(1)}) \cos\alpha(y-\eta) + \bar{R}^{(1)} ch(r^{(1)} y) \sin\chi^{(1)}(x^{(1)} - \delta_1),$$
$$U_2^{(1)} = H^{(1)} ch(t^{(1)} x^{(1)}) \sin\alpha(y-\eta) + R^{(1)} sh(r^{(1)} y) \cos\chi^{(1)}(x^{(1)} - \delta_1) \tag{5}$$

The set of constants $\bar{H}^{(m)}, H^{(m)}, \bar{Q}^{(m)}, ..., R^{(m)}$ in formulas (4) (5) provides a necessary degree of an arbitrariness for satisfaction of boundary conditions and interface conditions in considered compound area. As values $\alpha, \chi^{(m)}$ it is expedient to choose such sequences of numbers $\alpha_k, \chi_j^{(m)}$ so as systems of corresponding functions to be full and orthogonal on intervals $|y| \le \eta$ and $x^{(m)} \in [0, \bar{h}^{(m)}]$, i.e.,

$$\alpha_k = k\pi / \eta, \; \chi_j^{(m)} = j\pi / \bar{h}^{(m)}, \; k = 1,2,\ldots; j = 1,2,\ldots \tag{6}$$

After substitution of expressions (4), (5) in the equations (1), we receive for each value k and j systems of the linear homogeneous equations concerning coefficients $\bar{H}^{(m)}$ and $H^{(m)}$,…, $\bar{R}^{(m)}$ and $R^{(m)}$. From existence conditions of not trivial solution of these systems we find values of parameters $t^{(m)}$ and $r^{(m)}$

$$(t_{\beta k}^{(m)})^2 = \alpha_k^2 - (l_{\beta}^{(m)})^2, \; (r_{\beta j}^{(m)})^2 = (\chi_j^{(m)})^2 - (l_{\beta}^{(m)})^2, \; l_1^{(m)} = \Omega^{(m)} / \sqrt{C_{11}^{(m)}}, \; l_2^{(m)} = \Omega^{(m)}, \; \beta = 1,2 \tag{7}$$

and connection between the mentioned coefficients and completely defines the problem common decision in all regions $\bar{D}^{(m)}$ and allows to satisfy the conditions of interface (2) and to power boundary conditions (3).

1.3. The formulation and the solution of auxiliary problems. Similarly to [7,11], we introduce for consideration auxiliary boundary problems.

Regions $\bar{D}^{(m)}(m=2,...,N-1,N)$:

$$U_1^{(m)}(\bar{h}^{(m)},y)=f_1^{(m)}(y)\,,\sigma_{12}^{(m)}(\bar{h}^{(m)},y)=\phi_1^{(m)}(y)\,,\phi_1^{(N)}(y)=0\,,U_2^{(m)}(x^{(m)},\eta)=f_2^{(m)}(x^{(m)})\,,$$

$$\sigma_{12}^{(m)}(x^{(m)},\eta)=0\,,U_1^{(m)}(0,y)=f_1^{(m-1)}(y),\ \sigma_{12}^{(m)}(0,y)=r_m^{-1}\phi_1^{(m-1)}(y)\,. \tag{8}$$

Region $\bar{D}^{(1)}$:

$$U_1^{(1)}(\delta_1,y)=f_1^{(1)}(y)\,,\ \sigma_{12}^{(1)}(\delta_1,y)=\phi_1^{(1)}(y)\,,\ U_2^{(1)}(x^{(1)},\eta)=f_2^{(1)}(x^{(1)})\,,\ \sigma_{12}^{(1)}(x^{(1)},\eta)=0 \tag{9}$$

Here $f_1^{(m)}(y),\phi_1^{(m)}(y),f_2^{(m)}(x^{(m)})$ are unknown auxiliary functions. We display these (*3N-1*) functions in Fourier series on corresponding intervals and, using the problem common decision we make conditions (8) and (9). Then conditions on vertical sites of border $x=\delta_m$ will give at each value of an index *k* the $(4N-2)$ linear equations for definition of the same quantity of coefficients $H_{\gamma k}^{(1)},H_{\gamma k}^{(2)},Q_{\gamma k}^{(2)},...,H_{\gamma k}^{(N)},Q_{\gamma k}^{(N)}(\gamma=1,2)$ in the problem common decision. The remaining of $2N$ coefficients $R_{\gamma j}^{(m)}$ at each value j will be defined from $2N$ boundary conditions at $y=\eta$. Obtained sets of linear systems suppose the analytical decision and allow to express in an explicit form characteristics of a wave field in the whole compound region of section through Fur'e coefficients $f_{10}^{(m)},f_{1k}^{(m)},f_{20}^{(m)},f_{2j}^{(m)},\phi_{1k}^{(m)}$ of the introduced auxiliary functions.

1.4. An output of the system of the integrated equations and it asymptotical analysis. For determination of the introduced auxiliary functions we take into account (*3N-1*) non-used boundary conditions and interface conditions from (2), (3). Namely

$$\sigma_{11}^{(m)}(\bar{h}^{(m)},y)=r_m\sigma_{11}^{(m+1)}(0,y)\,,U_2^{(m)}(\bar{h}^{(m)},y)=U_2^{(m+1)}(0,y)\,,\sigma_{22}^{(m)}(x^{(m)},\eta)=q_2^{(m)}(x^{(m)})\ (m=1\div N-1)\,,$$

$$\sigma_{11}^{(N)}(1-\delta_{N-1},y)=q_1^{(N)}(y)\,,\sigma_{22}^{(N)}(x^{(N)},\eta)=q_2^{(N)}(x^{(N)}) \tag{10}$$

As all components of a wave field appearing in (10), are expressed through Fourier coefficients of auxiliary functions these conditions represent a system of the integral equations (SIE) for their definition. For the construction of effective algorithm for its decision and the subsequent successful selection of coordinate functions in the asymptotical method we investigate features of a wave field in a vicinity of irregular border points. In a considered problem such points are angular points of the areas joint $A_m(\bar{h}^{(m)},\eta)$ and an external angular point of section $B(1-\delta_{N-1},\eta)$. Taking into account the mechanical sense of auxiliary functions, we suppose, that their features in points A_m are defined by formulas (*m = 1,2, ..., N-1*)

$$\phi_1^{(m)}(\xi)=\Phi_1^{(m)}(\eta-\xi)^{p_m-1},\ f_1^{(m)/}(\xi)=F_1^{(m)}(\eta-\xi)^{p_m-1}\ \text{at}\ \xi\to\eta\,;$$

$$f_2^{(m)/}(\xi)=F_2^{(m)}(\bar{h}^{(m)}-\xi)^{p_m-1}\ \text{at}\ \xi\to\bar{h}^{(m)}\,,f_2^{(m+1)/}(\xi)=\bar{F}_2^{(m+1)}\xi^{p_m-1}\ \text{at}\ \xi\to 0\,.$$

In a vicinity of point *B* of the area $\bar{D}^{(N)}$:

$$f_1^{(N)/}(\xi)=F_1^{(N)}(\eta-\xi)^{p_N-1}\ \text{at}\ \xi\to\eta\,;f_2^{(N)/}(\xi)=F_2^{(N)}(1-\delta_{N-1}-\xi)^{p_{N-1}}\ \text{at}\ \xi\to 1-\delta_{N-1}$$

Here through p_m are designated LFP, defining required functions rupture character in the specified points, and through $\Phi_1^{(m)},...,F_2^{(N)}$ — any constants. Making integration in these formulas, we define asymptotic of auxiliary functions Fourier coefficients in a vicinity of all irregular points of border. It allows, to investigate the behaviour of received SIE at the approach to irregular points. The left part of the system limitation requirement allows to receive for each irregular point the system of the equations for determination of parameters p_m. Thus in points A_m asymptotically members, containing Fourier coefficients of functions $f_1^{(m)}, \phi_1^{(m)}, f_2^{(m)}, f_2^{(m+1)}$ will be significant. In a vicinity of an external angular point *B* the behavior of characteristics of a wave field will be defined by functions $f_1^{(N)}, f_2^{(N)}$. After a redefinitions of constants at features for their determination we come to following systems of the homogeneous equations.

In points $A_m (m = 1, 2, ..., N-1)$:

$$-s_m \sin\frac{\pi p_m}{2}\Phi_1^{(m)} + 2(n_m + r_m n_{m+1})\sin\frac{\pi p_m}{2}F_1^{(m)} + 2n_m p_m F_2^{(m)} + 2n_{m+1} p_m r_m \bar{F}_2^{(m+1)} = 0,$$

$$(t_m + r_m t_{m+1})\sin\frac{\pi p_m}{2}\Phi_1^{(m)} + 2s_m \sin\frac{\pi p_m}{2}F_1^{(m)} - -2(1 - n_m p_m)F_2^{(m)} - 2(1 - n_{m+1} p_m)\bar{F}_2^{(m+1)} = 0, \quad (11)$$

$$(n_m^{-1} + p_m)\Phi_1^{(m)} + 2p_m F_1^{(m)} + 2\sin\frac{\pi p_m}{2}F_2^{(m)} = 0, \ r_m^{-1}(n_{m+1}^{-1} + p_m)\Phi_1^{(m)} + 2p_m F_1^{(m)} + 2\sin\frac{\pi p_m}{2}\bar{F}_2^{(m+1)} = 0,$$

where $s_m = (C_{11}^{(m)})^{-1} + (C_{11}^{(m+1)})^{-1}$, $n_m = (C_{11}^{(m)} - 1) / C_{11}^{(m)}$, $t_m = (C_{11}^{(m)} + 1) / C_{11}^{(m)}$.

In a point *B*:

$$\sin\frac{\pi p_N}{2}F_1^{(N)} + p_N F_2^{(N)} = 0, \ p_N F_1^{(N)} + \sin\frac{\pi p_N}{2}F_2^{(N)} = 0 \quad (12)$$

It is obvious that constants $F_1^{(N)}, F_2^{(N)}$ in the system (12) will not be equal to zero if the parameter p_N satisfies the equation

$$\sin^2\frac{\pi p_N}{2} - p_N^2 = 0 \quad (13)$$

Equation (13) corresponds to the equation received in [12] for single wedge with loose sides and angle opening 90°. As we see, character of feature of a mechanical field in a point does not depend on elastic constant regions $\bar{D}^{(m)}$. Naturally, from mechanical sense of functions $f_1^{(N)}(\xi), f_2^{(N)}(\xi)$, we consider only a real root $p_N = 1$ of the equation (13) and countable set of complex roots [7] with a positive real part.

The parameters p_m, characterizing features of wave characteristics in internal angular points of compound region, do not depend on frequency and geometrical parameters η and δ_m and are defined only by values shear modulus and Poisson's ratio of joined areas. It is possible to define them from existence condition of not trivial decision of system (11)

$$\Delta(p_m, \mu^{(m)}, \nu^{(m)}, \mu^{(m+1)}, \nu^{(m+1)}) = 0 \quad (14)$$

The equation (14) is symmetric concerning elastic parameters of the neighboring regions $\bar{D}^{(m)}, \bar{D}^{(m+1)}$, and will not change at replacement of values $\mu^{(m)}, \nu^{(m)}$ on $\mu^{(m+1)}, \nu^{(m+1)}$ and vice versa. It is easy to prove it by means of elementary transformations of lines and columns of a system determinant (11). At certain proportions of joined areas materials mechanical properties the equation (14) has a real root $0 < p_m < 1$ and it characterizes occurrence local features in values of stresses in points A_m.

It is possible to construct a parameter p_m asymptotic at high values r_m. Searching for roots of the equation (14) in the form of expansion in series on small parameter: $\varepsilon = r_m^{-1}$: $p_m = p_{m0} + \varepsilon p_{m1} + ...$, we can receive simply enough consistency of the equations for determination $p_{m0}, p_{m1}, ...$. For example, the first member of expansion satisfies the equation

$$(\sin^2(\pi p_{m0}/2) - p_{m0}{}^2)(p_{m0}{}^2 - (3-4\nu^{(m)})\sin^2(\pi p_{m0}/2) - 4(1-\nu^{(m)})^2) = 0 \qquad (15)$$

The first factor in the equation (15) coincides with the left member of equation (13). As shows the numerical analysis the second factor has roots $0 < p_{m0} < 1$ practically at any value of Poisson's ratio $\nu^{(m)}$.

Further the Bubnov-Galerkin method for the solution of SIE is applied. This method takes into account the character of features of the solution at a choice of coordinate functions. As a result we come to infinite system of the algebraic equations with known asymptotic of unknowns which are defined by roots of the equations (13), (14).

2. Coupling of three different medias

Let the section of an infinite toward axis α_3 of a non-uniform elastic prism to occupy in the system of coordinates $\alpha_1 O \alpha_2$ region $D_3 = G^{(1)} \cup G^{(2)} \cup G^{(3)}$, where regions $G^{(m)}$ are stuck together with each other, and are defined by inequalities

$$G^{(1)} = \{(\alpha_1,\alpha_2): |\alpha_1| \le c; \alpha_2 \in [-b,-d] \cup [d,b]\},\ G^{(2)} = \{(\alpha_1,\alpha_2): \alpha_1 \in [-a,-c] \cup [c,a]; |\alpha_2| \le d\},$$
$$G^{(3)} = \{(\alpha_1,\alpha_2): \alpha_1 \in [-a,-c] \cup [c,a]; \alpha_2 \in [-b,-d] \cup [d,b]\}$$

On section outer sides $\alpha_1 = \pm a$, $\alpha_2 = \pm b$, is set harmoniously changing in time with frequency ω vibration loading of variable intensity $q_1(\alpha_2), q_2(\alpha_1)$ accordingly, and the internal border of section is free from loadings. Taking into account symmetry of region D, it is possible to consider a wave field of a part of the region located in the first quarter. This part of region is represented on Fig. 1 in dimensionless coordinates $x = \alpha_1/a, y = \alpha_2/a$.

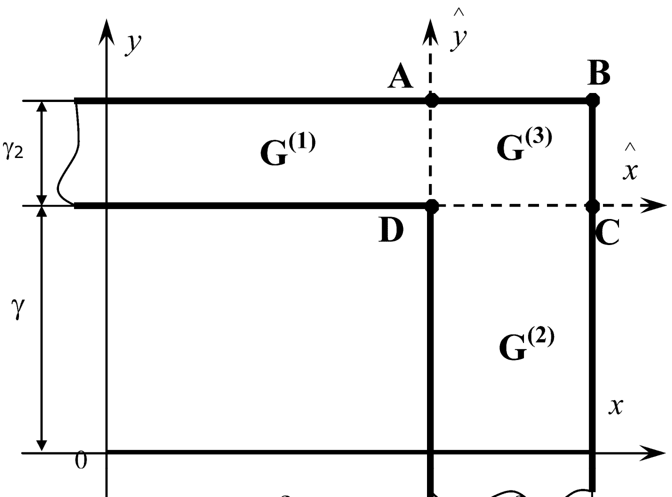

Fig. 1. Coupling of three different medias

For convenience in the field of section local dimensionless coordinates $\hat{x} = (\alpha_1 - c)/a, \hat{y} = (\alpha_2 - d)/a$ and dimensionless geometrical parameters $\eta = b/a$, $\delta = c/a$, $\gamma = d/a$, $\delta_2 = 1-\delta, \gamma_2 = \eta - \gamma$ are introduced. Boundary conditions of the problem include power conditions of loading on external and internal borders of section and a condition of rigid coupling of regions $G^{(m)}$.

We start to construct common solution. Similarly to previous, the common solution $U_\beta^{(m)}$, satisfying the system of the equations of movement within regions $G^{(m)}$, we construct on a superposition method. Thus it is necessary to consider that the functions $U_\beta^{(1)}(x,\hat{y}),U_\beta^{(3)}(\hat{x},\hat{y})$ on the coordinate $\hat{y}$, and the functions $U_\beta^{(2)}(\hat{x},y),U_\beta^{(3)}(\hat{x},\hat{y})$ on the coordinate $\hat{x}$ are general view functions. Thus, the problem common solution in regions $G^{(m)}$ will note in the following form

$$U_1^{(1)} = H_1^{(1)}sh(t^{(1)}x)\cos\alpha^{(1)}(\hat{y}-\gamma_2)+(R_1^{(1)}sh(r^{(1)}\hat{y})+S_1^{(1)}ch(r^{(1)}\hat{y}))\sin\chi^{(1)}(x-\delta)$$

$$U_2^{(1)} = H_2^{(1)}ch(t^{(1)}x)\sin\alpha^{(1)}(\hat{y}-\gamma_2)+(R_2^{(1)}sh(r^{(1)}\hat{y})+S_2^{(1)}ch(r^{(1)}\hat{y}))\cos\chi^{(1)}(x-\delta)$$

$$U_1^{(2)} = (H_1^{(2)}sh(t^{(2)}\hat{x})+Q_1^{(2)}ch(t^{(2)}\hat{x}))\cos\alpha^{(2)}(y-\gamma)+R_1^{(2)}ch(r^{(2)}y)\sin\chi^{(2)}(\hat{x}-\delta_2)$$

$$U_2^{(2)} = (H_2^{(2)}sh(t^{(2)}\hat{x})+Q_2^{(2)}ch(t^{(2)}\hat{x}))\sin\alpha^{(2)}(y-\gamma)+R_2^{(2)}sh(r^{(2)}y)\cos\chi^{(2)}(\hat{x}-\delta_2)$$

$$U_1^{(3)} = (H_1^{(3)}sh(t^{(3)}\hat{x})+Q_1^{(3)}ch(t^{(3)}\hat{x}))\cos\alpha^{(1)}(\hat{y}-\gamma_2)+(R_1^{(3)}sh(r^{(3)}\hat{y})+S_1^{(3)}ch(r^{(3)}\hat{y}))\sin\chi^{(2)}(\hat{x}-\delta_2)$$

$$U_2^{(3)} = (H_2^{(3)}sh(t^{(3)}\hat{x})+Q_2^{(3)}ch(t^{(3)}\hat{x}))\sin\alpha^{(1)}(\hat{y}-\gamma_2)+(R_2^{(3)}sh(r^{(3)}\hat{y})+S_2^{(3)}ch(r^{(3)}\hat{y}))\cos\chi^{(2)}(\hat{x}-\delta_2)$$

As values $\alpha^{(\beta)},\chi^{(\beta)}$ it is expedient to choose

$$\alpha_k^{(1)} = k\pi/\gamma_2,\ \alpha_k^{(2)} = k\pi/\gamma,\ \chi_j^{(1)} = j\pi/\delta,\ \chi_j^{(2)} = j\pi/\delta_2,\ k = 1,2,\ldots;\ j = 1,2,\ldots$$

According to the basic technique it is formulated auxiliary boundary problems which suppose the analytical solution. Boundary conditions of auxiliary problems in the case under consideration considerably will get complicated in comparison with considered earlier (see (8), (9)) in view of presence of two internal lines of section of areas $G^{(m)}$ and internal border of section. They will take the following form [13]

$$G^{(1)} = \{|x|\le\delta; 0\le\hat{y}\le\gamma_2\}:\ U_1^{(1)}(\delta,\hat{y}) = f_1(\hat{y}),\ \sigma_{12}^{(1)}(\delta,\hat{y}) = \phi_1(\hat{y})$$

$$U_2^{(1)}(x,\gamma_2) = f_2(x),\ \sigma_{12}^{(1)}(x,\gamma_2) = \sigma_{12}^{(1)}(x,0) = 0,\ U_2^{(1)}(x,0) = f_3(x)$$

$$G^{(2)} = \{0\le\hat{x}\le\delta_2; |y|\le\gamma\}:\ U_1^{(2)}(\delta_2,y) = f_4(y),\ \sigma_{12}^{(2)}(\delta_2,y) = \sigma_{12}^{(2)}(0,y) = 0,\ U_1^{(2)}(0,y) = f_5(y),$$

$$U_2^{(2)}(\hat{x},\gamma) = f_6(\hat{x}),\ \sigma_{12}^{(2)}(\hat{x},\gamma) = \phi_2(\hat{x}) \tag{16}$$

$$G^{(3)} = \{0\le\hat{x}\le\delta_2; 0\le\hat{y}\le\gamma_2\}:$$

$$U_1^{(3)}(\delta_2,\hat{y}) = f_7(\hat{y}),\ \sigma_{12}^{(3)}(\delta_2,\hat{y}) = 0,\ U_1^{(3)}(0,\hat{y}) = f_1(\hat{y});\ \sigma_{12}^{(3)}(0,\hat{y}) = r_{13}\phi_1(\hat{y})$$

$$U_2^{(3)}(\hat{x},\gamma_2) = f_8(\hat{x}),\ \sigma_{12}^{(3)}(\hat{x},\gamma_2) = 0,\ U_2^{(3)}(\hat{x},0) = f_6(\hat{x}),\ \sigma_{12}^{(3)}(\hat{x},0) = r_{23}\phi_2(\hat{x})$$

Here unknown auxiliary functions are designated through $f_1(\hat{y}),\phi_1(\hat{y}),\ldots f_8(\hat{x})$. It should be noted that the choice of boundary conditions of an auxiliary problem in the form of (16) allows satisfying automatically a part of boundary conditions of the initial boundary problem mentioning normal movement and transverse stresses on external and internal borders of region.

After replacement of an initial boundary problem auxiliary the part of boundary conditions and conditions of interface of medias remained unsatisfied. They represent SIE concerning unknown auxiliary functions. As far as the character of features in special points of section *A*, *B* and *C* (Fig. 1) has been investigated earlier, we set the problem of determination of a wave field feature in an internal point $D(\delta,\gamma)$ of a joint of three regions. For this purpose we assume that asymptotically significant in a vicinity of this point auxiliary functions have features of the following form

$$f_i^{'}(\xi) = F_i^D \xi^{\alpha-1},\ \phi_j(\xi) = \Phi_j^D \xi^{\alpha-1}\ \ (i = \overline{1,6};\ j = \overline{1,2})\ \text{ at }\ \xi \to 0;$$
$$f_3^{'}(\xi) = F_3^D (\delta-\xi)^{\alpha-1}\ \text{ at }\ \xi \to \delta;\ \ f_5^{'}(\xi) = F_5^D (\gamma-\xi)^{\alpha-1}\ \text{ at }\ \xi \to \gamma\,.$$

LFP in a point D is designated in these formulas through α, and through $F_i^D, \Phi_j^D (i = 1,3,5,6;\ j = 1,2)$ — an arbitrary constants. We define asymptotic of Fourier coefficients of considered functions and write down the boundary conditions non-used in auxiliary problems and conditions of interface of regions $G^{(m)}$ in a point D vicinity at limiting values of arguments. These conditions can be written down in the form

$$\sigma_{11}^{(1)}(\delta,\hat{y}) = r_{31}\sigma_{11}^{(3)}(0,\hat{y})\,,\ \hat{y} \to 0;\ \sigma_{22}^{(2)}(\hat{x},\gamma) = r_{32}\sigma_{22}^{(3)}(\hat{x},0)\,,\ \hat{x} \to 0\,;U_2^{(1)}(\delta,\hat{y}) = U_2^{(3)}(0,\hat{y})\,,\ \hat{y} \to 0;$$
$$U_1^{(2)}(\hat{x},\gamma) = U_1^{(3)}(\hat{x},0)\,,\ \hat{x} \to 0\,;\sigma_{11}^{(2)}(0,y) = 0\,;\ \sigma_{22}^{(1)}(x,0) = 0\,,\ x \to \delta \tag{17}$$

As a result of asymptotical analysis of SIE we come to homogeneous system of the equations defining specified constants

$$-m_{13}\sin\frac{\pi\alpha}{2}\Phi_1 + r_{21}(1+\alpha n^{(3)})\Phi_2 - 2(n^{(1)} + r_{31}n^{(3)})\sin\frac{\pi\alpha}{2}F_1 - 2n^{(1)}\alpha F_3 - 2r_{31}n^{(3)}\alpha F_6 = 0$$
$$r_{12}(1+\alpha n^{(3)})\Phi_1 - m_{23}\sin\frac{\pi\alpha}{2}\Phi_2 - 2r_{32}n^{(3)}\alpha F_1 - 2n^{(2)}\alpha F_5 + 2(n^{(2)} - r_{32}n^{(3)})\sin\frac{\pi\alpha}{2}F_6 = 0$$
$$-(s^{(1)} + r_{13}s^{(3)})\sin\frac{\pi\alpha}{2}\Phi_1 + r_{23}n^{(3)}\alpha\Phi_2 + 2m_{13}\sin\frac{\pi\alpha}{2}F_1 + 2(1-\alpha n^{(1)})F_3 + 2(1-\alpha n^{(3)})F_6 = 0$$
$$r_{13}n^{(3)}\alpha\Phi_1 - (s^{(2)} + r_{23}s^{(3)})\sin\frac{\pi\alpha}{2}\Phi_2 + 2(1-\alpha n^{(3)})F_1 + 2(1-\alpha n^{(2)})F_5 + 2m_{23}\sin\frac{\pi\alpha}{2}F_6 = 0$$
$$(\frac{1}{n^{(2)}} + \alpha)\Phi_2 - 2\sin\frac{\pi\alpha}{2}F_5 - 2\alpha F_6 = 0\,,\ (\frac{1}{n^{(1)}} + \alpha)\Phi_1 - 2\alpha F_1 - 2\sin\frac{\pi\alpha}{2}F_3 = 0\,, \tag{18}$$

where $n^{(m)} = \dfrac{1}{2(1-\nu^{(m)})}$, $s^{(m)} = \dfrac{3-4\nu^{(m)}}{2(1-\nu^{(m)})}$, $m_{ij} = \dfrac{2-3(\nu^{(i)}+\nu^{(j)})+4\nu^{(i)}\nu^{(j)}}{2(1-\nu^{(i)})(1-\nu^{(j)})}$

$\Phi_\beta = -2\Phi_\beta^D\Gamma(\alpha)\sin\dfrac{\pi\alpha}{2}$, $F_k = 2F_k^D\Gamma(\alpha)\sin\dfrac{\pi\alpha}{2}$, $\Gamma(\alpha)$ is gamma function, $k = 1,3,5,6$.

It is possible to define LFP α from existence condition of non-trivial solution of system (18)

$$\Delta_1(\alpha,\mu^{(m)},\nu^{(m)}) = 0 \tag{19}$$

Let's define asymptotic at high values of the shear modulus $\mu^{(3)}$ of angular area. For this purpose we introduce small dimensionless parameters $\varepsilon_j = \mu^{(j)}/\mu^{(3)} = r_{3j}^{-1} (j = 1,2)$ and search for the solution of the equation (19) in the form of a series on degrees of these parameters $\alpha = \alpha_0 + \varepsilon_1\alpha_{11} + \varepsilon_2\alpha_{12} + \ldots$. For example, the first member of expansion satisfies the equation

$$(\sin^2(\pi\alpha_0/2) - {\alpha_0}^2)\prod_{i=1}^{2}({\alpha_0}^2 - (3-4\nu^{(i)})\sin^2(\pi\alpha_0/2) - 4(1-\nu^{(i)})^2) = 0 \tag{20}$$

The first multiplier in the equation (20) coincides with the left member of equation (13) defining feature of the component stress tensor in the top of a homogeneous wedge with the angle opening of 90^0. The second and third multiplier have the form of the second multiplier of the asymptotic equation (15) answering to a case of coupling of two medias. Presence of an additional multiplier factor in the equation (20) allows varying values of Poisson's ratio of two areas $G^{(1)}$ and $G^{(2)}$ for control of LFP value.

3. Coupling of four different medias

Let the section of an infinite toward the axis α_3 of non-uniform elastic prism to occupy region $D_4 = G^{(1)} \cup G^{(2)} \cup G^{(3)} \cup G^{(4)}$ where regions $G^{(m)}$ are stuck together with each other, and are defined by inequalities

$$G^{(1)} = \{|\alpha_1| \le c; \alpha_2 \in [-b,-d] \cup [d,b]\};\ G^{(2)} = \{\alpha_1 \in [-a,-c] \cup [c,a]; |\alpha_2| \le d\};$$
$$G^{(3)} = \{\alpha_1 \in [-a,-c] \cup [c,a]; \alpha_2 \in [-b,-d] \cup [d,b]\}; G^{(4)} = \{|\alpha_1| \le c; |\alpha_2| \le d\}.$$

In each region $G^{(m)}$ we consider the Lame equations of movement which have been written in dimensionless displacements $U_\beta^{(m)}$ ($\beta = 1,2$) and dimensionless coordinates $x = \alpha_1 / a, y = \alpha_2 / a$. As above proceeding from reasons of symmetry of region D_4, it is possible to consider a wave field of a part of the region located in the first quarter (Fig. 2). Systems of coordinates and the sizes of section are similar to ones presented on Fig. 1. Boundary conditions of the problem include power conditions loading on external border of section and a condition of rigid coupling of regions. This problem was not investigated earlier in the presented form. Complexity of a task in view is caused by following factors. First, definition of features LSC in a vicinity of an irregular point on a joint of four diverse media demands the account of behavior of components of a wave field at the approach to a special point already in four directions. Second, the nature of a wave field for a considered difficult case of "chess" heterogeneity also undergoes essential changes. It is connected with presence in the detail section already four singular points. The research of features LSC in a vicinity of a point of interface of four media on the one hand has independent value, and on the other hand — it is necessary for representation of a full picture of a wave field. Thirdly, practically for certain it is necessary to expect mutual influence of LSC features in irregular points of border of the section, especially at the small sizes of angular region.

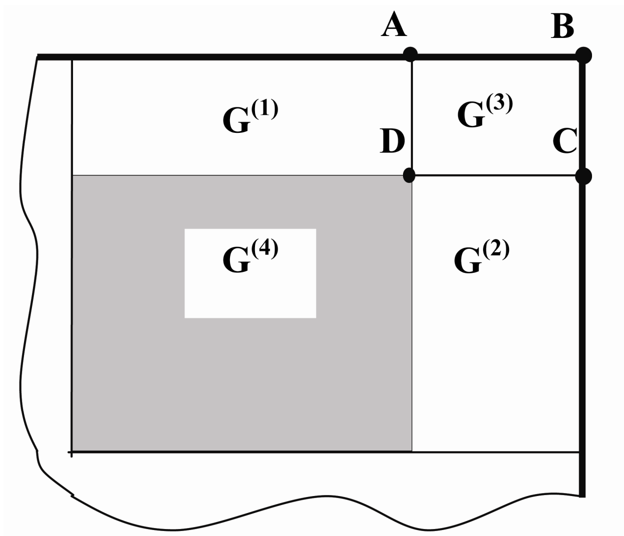

Fig. 2. Coupling of four different medias

At construction of the common solution of a problem we are guided again by a method of superposition and we write down the problem common solution in regions $G^{(m)}$ in the following form

$$U_1^{(1)} = H_1^{(1)} sh(t^{(1)} x) \cos\alpha^{(1)} (\hat{y} - \gamma_2) + (R_1^{(1)} sh(r^{(1)} \hat{y}) + S_1^{(1)} ch(r^{(1)} \hat{y})) \sin\chi^{(1)} (x - \delta)$$

$$U_2^{(1)} = H_2^{(1)} ch(t^{(1)}x)\sin\alpha^{(1)}(\hat{y}-\gamma_2) + (R_2^{(1)} sh(r^{(1)}\hat{y}) + S_2^{(1)} ch(r^{(1)}\hat{y}))\cos\chi^{(1)}(x-\delta)$$

$$U_1^{(2)} = (H_1^{(2)} sh(t^{(2)}\hat{x}) + Q_1^{(2)} ch(t^{(2)}\hat{x}))\cos\alpha^{(2)}(y-\gamma) + R_1^{(2)} ch(r^{(2)}y)\sin\chi^{(2)}(\hat{x}-\delta_2)$$

$$U_2^{(2)} = (H_2^{(2)} sh(t^{(2)}\hat{x}) + Q_2^{(2)} ch(t^{(2)}\hat{x}))\sin\alpha^{(2)}(y-\gamma) + R_2^{(2)} sh(r^{(2)}y)\cos\chi^{(2)}(\hat{x}-\delta_2)$$

$$U_1^{(3)} = (H_1^{(3)} sh(t^{(3)}\hat{x}) + Q_1^{(3)} ch(t^{(3)}\hat{x}))\cos\alpha^{(1)}(\hat{y}-\gamma_2) + (R_1^{(3)} sh(r^{(3)}\hat{y}) + S_1^{(3)} ch(r^{(3)}\hat{y}))\sin\chi^{(2)}(\hat{x}-\delta_2),$$

$$U_2^{(3)} = (H_2^{(3)} sh(t^{(3)}\hat{x}) + Q_2^{(3)} ch(t^{(3)}\hat{x}))\sin\alpha^{(1)}(\hat{y}-\gamma_2) + (R_2^{(3)} sh(r^{(3)}\hat{y}) + S_2^{(3)} ch(r^{(3)}\hat{y}))\cos\chi^{(2)}(\hat{x}-\delta_2);$$

$$U_1^{(4)} = H_1^{(4)} sh(t^{(4)}x)\cos\alpha^{(2)}(y-\gamma) + R_1^{(4)} ch(r^{(4)}y)\sin\chi^{(1)}(x-\delta),$$

$$U_2^{(4)} = H_2^{(4)} ch(t^{(4)}x)\sin\alpha^{(2)}(y-\gamma) + R_2^{(4)} sh(r^{(4)}y)\cos\chi^{(1)}(x-\delta).$$

Here $\alpha_k^{(1)} = k\pi/\gamma_2$, $\alpha_k^{(2)} = k\pi/\gamma$, $\chi_j^{(1)} = j\pi/\delta$, $\chi_j^{(2)} = j\pi/\delta_2$, $k = 1,2,\ldots; j = 1,2,\ldots$

At the formulation of auxiliary boundary problems we base on the principles used in the previous parts. Namely, we will bring into the formulation of auxiliary problems "cross" boundary conditions mentioning shearing stresses and normal displacements on corresponding parts of compound section border. In this case it is necessary to take into account that the section border consists of eight pieces (Fig. 2). Thus, boundary conditions of auxiliary boundary problems we formulate in the following form

$$G^{(1)}: U_1^{(1)}(\delta,\hat{y}) = f_1(\hat{y}),\ \sigma_{12}^{(1)}(\delta,\hat{y}) = \phi_1(\hat{y}),$$

$$U_2^{(1)}(x,\gamma_2) = f_2(x),\ \sigma_{12}^{(1)}(x,\gamma_2) = 0,\ U_2^{(1)}(x,0) = f_3(x),\ \sigma_{12}^{(1)}(x,0) = \phi_4(x);$$

$$U_1^{(2)}(\delta_2,y) = f_4(y),\ \sigma_{12}^{(2)}(\delta_2,y) = 0,\ U_1^{(2)}(0,y) = f_5(y),\ \sigma_{12}^{(2)}(0,y) = \phi_3(y),$$

$$U_2^{(2)}(\hat{x},\gamma) = f_6(\hat{x}),\ \sigma_{12}^{(2)}(\hat{x},\gamma) = \phi_2(\hat{x});$$

$$G^{(3)}: U_1^{(3)}(\delta_2,\hat{y}) = f_7(\hat{y}),\ \sigma_{12}^{(3)}(\delta_2,\hat{y}) = 0,\ U_1^{(3)}(0,\hat{y}) = f_1(\hat{y});$$

$$\sigma_{12}^{(3)}(0,\hat{y}) = r_{13}\phi_1(\hat{y}), U_2^{(3)}(\hat{x},\gamma_2) = f_8(\hat{x}),\ \sigma_{12}^{(3)}(\hat{x},\gamma_2) = 0,$$

$$G^{(4)}: U_1^{(4)}(\delta,y) = f_5(y),\ \sigma_{12}^{(4)}(\delta,y) = r_{24}\phi_3(y),\ U_2^{(4)}(x,\gamma) = f_3(x),\ \sigma_{12}^{(4)}(x,\gamma) = r_{14}\phi_4(x).$$

Here $f_1(\hat{y}), \phi_1(\hat{y}), \ldots f_8(\hat{x})$ are unknown auxiliary functions.

Let's take into consideration 12 non-used boundary conditions and conditions of coupling of an initial boundary problem which is SIE for determination of the introduced auxiliary functions, namely

$$\sigma_{11}^{(1)}(\delta,\hat{y}) = r_{31}\sigma_{11}^{(3)}(0,\hat{y}),\ \sigma_{22}^{(2)}(\hat{x},\gamma) = r_{32}\sigma_{22}^{(3)}(\hat{x},0),\ U_2^{(1)}(\delta,\hat{y}) = U_2^{(3)}(0,\hat{y}),\ U_1^{(2)}(\hat{x},\gamma) = U_1^{(3)}(\hat{x},0),$$

$$\sigma_{11}^{(2)}(0,y) = r_{42}\sigma_{11}^{(4)}(\delta,y),\ \sigma_{22}^{(1)}(x,0) = r_{41}\sigma_{22}^{(4)}(x,\gamma),\ U_2^{(2)}(0,y) = U_2^{(4)}(\delta,y),\ U_1^{(1)}(x,0) = U_1^{(4)}(x,\gamma), \quad (21)$$

$$\sigma_{22}^{(1)}(x,\gamma_2) = q_2^{(1)},\ \sigma_{11}^{(2)}(\delta_2,y) = q_1^{(2)},\ \sigma_{11}^{(3)}(\delta_2,\hat{y}) = q_1^{(3)},\ \sigma_{22}^{(3)}(\hat{x},\gamma_2) = q_2^{(3)},\ q^{(m)} = q/\mu^{(m)}.$$

We investigate features of a wave field in a vicinity of irregular points of border of the section. The algorithm of the modified method of superposition offered in this work allows to study local features of characteristics of a wave field in all singular points by a uniform technique. For this purpose it is necessary to make the asymptotic analysis of all equations of system (21) at the approach to each irregular point of border. Thus we consider that unknown functions of SIE will have various local features in a vicinity of points *A, B, C, D.* Taking into account the received in the previous parts results, we make the asymptotic analysis only for point *D* of a joint of four heterogeneous medias. For this purpose it is necessary to a make the asymptotic analysis of first eight equations SIE (21) at limiting values of arguments, namely

$$\begin{gathered}
\sigma_{11}^{(1)}(\delta,\hat{y}) = r_{31}\sigma_{11}^{(3)}(0,\hat{y}),\ U_2^{(1)}(\delta,\hat{y}) = U_2^{(3)}(0,\hat{y}),\ \hat{y} \to 0;\\
\sigma_{22}^{(1)}(x,0) = r_{41}\sigma_{22}^{(4)}(x,\gamma),\ U_1^{(1)}(x,0) = U_1^{(4)}(x,\gamma),\ x \to \delta;\\
\sigma_{11}^{(2)}(0,y) = r_{42}\sigma_{11}^{(4)}(\delta,y), U_2^{(2)}(0,y) = U_2^{(4)}(\delta,y),\\
\sigma_{22}^{(2)}(\hat{x},\gamma) = r_{32}\sigma_{22}^{(3)}(\hat{x},0),\ U_1^{(2)}(\hat{x},\gamma) = U_1^{(3)}(\hat{x},0),\ \hat{x} \to 0.
\end{gathered} \tag{22}$$

We assume that features of auxiliary functions in point D are defined by formulas

$$\begin{gathered}
\phi_\beta(\xi) = \bar{\Phi}_\beta^D(\xi)^{\alpha-1},\ f_i^/(\xi) = \bar{F}_i^D(\xi)^{\alpha-1},\ (\beta = 1,2; i = 1,6),\ \xi \to 0;\\
\phi_3(\xi) = \bar{\Phi}_3^D(\gamma-\xi)^{\alpha-1}, f_5^/(\xi) = \bar{F}_5^D(\gamma-\xi)^{\alpha-1}, \xi \to \gamma;\\
\phi_4(\xi) = \bar{\Phi}_4^D(\delta-\xi)^{\alpha-1}, f_3^/(\xi) = \bar{F}_3^D(\delta-\xi)^{\alpha-1}, \xi \to \delta.
\end{gathered}$$

Here through $\alpha = \alpha(D)$ the parameter characterizing features of required functions in point D, and through $\bar{\Phi}_\beta^D,...,\bar{F}_3^D$ — arbitrary constants are designated. The asymptotic analysis of expressions (22) leads to the following system of the homogeneous equations for determination of the specified constants

$$\begin{gathered}
-m_{13}\sin(0,5\pi\alpha)\Phi_1^D + r_{21}(1+n^{(3)}\alpha)\Phi_2^D - (1+n^{(1)}\alpha)\Phi_4^D - 2(n^{(1)}+r_{31}n^{(3)})\sin(0,5\pi\alpha)F_1^D - 2n^{(1)}\alpha F_3^D -\\
-2r_{31}n^{(3)}\alpha F_6^D = 0,\\
r_{12}(1+n^{(3)}\alpha)\Phi_1^D - m_{23}\sin(0,5\pi\alpha)\Phi_2^D - (1+n^{(2)}\alpha)\Phi_3^D - 2r_{32}n^{(3)}\alpha F_1^D - 2n^{(1)}\alpha F_5^D -\\
-2(n^{(2)}+r_{32}n^{(3)})\sin(0,5\pi\alpha)F_6^D = 0,\\
-(s^{(1)}+r_{13}s^{(3)})\sin(0,5\pi\alpha)\Phi_1^D + r_{23}n^{(3)}\alpha\Phi_2^D - n^{(1)}\alpha\Phi_4^D + 2m_{13}\sin(0,5\pi\alpha)F_1^D + 2(1-n^{(1)}\alpha)F_3^D +\\
+2(1-n^{(3)}\alpha)F_6^D = 0,\\
r_{13}n^{(3)}\alpha\Phi_1^D - (s^{(2)}+r_{23}s^{(3)})\sin(0,5\pi\alpha)\Phi_2^D - n^{(2)}\alpha\Phi_3^D + 2(1-n^{(3)}\alpha)F_1^D + 2(1-n^{(2)}\alpha)F_5^D +\\
+2m_{23}\sin(0,5\pi\alpha)F_6^D = 0,\\
(1+n^{(2)}\alpha)\Phi_2^D + m_{24}\sin(0,5\pi\alpha)\Phi_3^D - r_{12}(1+n^{(4)}\alpha)\Phi_4^D - 2r_{42}n^{(4)}\alpha F_3^D -\\
-2(n^{(2)}+r_{42}n^{(4)})\sin(0,5\pi\alpha)F_5^D - 2n^{(2)}\alpha F_6^D = 0,\\
(1+n^{(1)}\alpha)\Phi_1^D - r_{21}(1+n^{(4)}\alpha)\Phi_3^D + m_{14}\sin(0,5\pi\alpha)\Phi_4^D - 2n^{(1)}\alpha F_1^D -\\
-2(n^{(1)}+r_{41}n^{(4)})\sin(0,5\pi\alpha)F_3^D - 2r_{41}n^{(4)}\alpha F_5^D = 0,\\
-n^{(2)}\alpha\Phi_2^D - (s^{(2)}+r_{24}s^{(4)})\sin(0,5\pi\alpha)\Phi_3^D + r_{14}n^{(4)}\alpha\Phi_4 - 2(1-n^{(4)}\alpha)F_3^D - 2m_{24}\sin(0,5\pi\alpha)F_5^D -\\
-2(1-n^{(2)}\alpha)F_6^D = 0,\\
-n^{(1)}\alpha\Phi_2^D + r_{42}n^{(4)}\alpha\Phi_3^D - (s^{(1)}+r_{14}s^{(4)})\sin(0,5\pi\alpha)\Phi_4^D - 2(1-n^{(1)}\alpha)F_1^D - 2m_{14}\sin(0,5\pi\alpha)F_3^D -\\
-2(1-n^{(4)}\alpha)F_5^D = 0.
\end{gathered} \tag{23}$$

From a condition of equality to zero of a determinant of the system (23)

$$\Delta_D(r_{ij,}\nu^{(m)},\alpha) = 0 \tag{24}$$

it is received the characteristic equation for definition in point D. The feature order on tension, as well as earlier, is equal $|\mathrm{Re}\,\alpha - 1|$.

It should be noted that the received system (23) passes into the defining system (18) for determination of LFP in a point of a joint of three regions. For this purpose it is necessary to put $\phi_3(y)=\phi_4(x)=\sigma_{11}^{(4)}(\delta,y)=\sigma_{22}^{(4)}(x,\gamma)\equiv 0$ and to remove the seventh and eighth conditions of coupling.

4. Some results of numerical research

Numerical research of the received solutions we begin with the analysis of values of LFP in singular point of a joint of two media defined by the equation (14). In part expressions 1 we accept $N=3$ according to the internal region $D^{(1)}$ joined with two external surfacing $D_-^{(2)}\cup D_+^{(2)}$. On Fig. 3 diagrams of dependence of LFP from a proportion of rigidity of joined media are presented. As a material of internal area were taken steel, lead and tungsten, elastic characteristics surfacing varied by means of parameter $r_{2S}=\mu^{(2)}/\mu^{(St)}$ change, where $\mu^{(St)}$ is the shear modulus of a steel. The value of Poisson's ratio of a material наплавок is accepted as fixed and equal $\nu^{(2)}=0.29$.

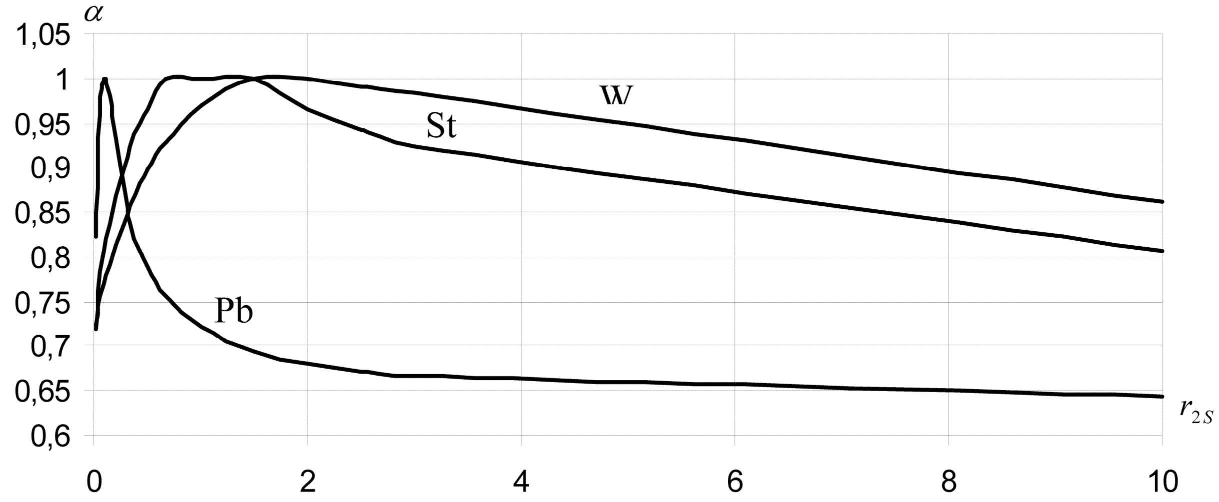

Fig. 3. Dependence of LFP $\alpha=\alpha(r_{2s})$ from a proportion of rigidity of joined media ($r_{2s}=\mu^{(2)}/\mu^{(st)}$, $\nu^{(2)}=0.29$)

When analyzing the results of the numerical solution of the equation (14) and the data Fig. 3 is to be noted: 1) LFP $\alpha=p_1$ essentially depends on elastic parameters of internal region. 2). For all considered combinations of elastic constant joined regions were not received complex roots of the equation (14) with a positive real part, smaller then unity. 3). Value $\alpha=1$ is a root of the equation (14) at any combinations of materials of joined areas. For the majority of combinations, however, this root turns out the second-large positive root of this equation. 4). At coupling of the regions made of identical materials, LSC is absent, LFP α is always equal to unity. 5). At some values of the parameter r_{2S} feature disappears and at coupling of different materials. 6). The variation of value of Poisson's ratio of a material surfacing makes the minimum impact on size LFP practically at any parities of shear modulus of joined regions. 7). At enough big and enough small values of shear modulus of surfacing value of LFP α is stabilized, aspiring to certain value.

The final conclusion can be concretized, investigating roots of the second multiplier of the asymptotic equations (15). For all real materials roots of this multiplier satisfy to an inequality $0<p_{m0}<1$. In table 1 are represented values of these roots for various materials of the first region.

Table 1

The asymptotic values of LFP corresponding to infinitely high value of shear modulus by one of regions of the section

Al, Mg	W	Au	Cu	Mo	Ni	Sn	Pt	Pb	Ag	Ti	Zn
0,680	0,718	0,638	0,692	0,781	0,705	0,692	0,656	0,633	0,662	0,711	0,857

Thus, there is a possibility to reach the greatest possible value of LFP through the change of Poisson's ratio of one of regions at very big distinctions in values of shear modulus of joined regions.

Let's begin the numerical analysis of values of LFP in an angular point D of a joint of three regions of the section represented on Fig. 1. It should be noted that in this case there is no possibility to introduce the compact parameters similar to Danders parameters [1,6], somewhat being analogue of LFP.

From the results of the made numerical analysis it is possible to make the following conclusions: 1) Values of LFP on stresses in internal angular point *D* of a joint of three media for any combinations of materials are essentially less values of LFP in points *A* and *C*, analyzed above. 2) For any combinations of materials there is a root of the resolving equation (19), smaller then unity, i.e. stresses will always have feature in an angular point of a joint of three media. 3) At coupling of any three identical materials LFP is always equal $\alpha = 0.544$. It testifies to essential intensity of LSC in point *D*. 4) For many combinations of materials unlike a case of coupling of two media there is not one, but two roots, smaller then unity. The account of these roots is important at construction asymptotic solutions SIE in point *D*. In table 2 the least valid roots of the equation (19) for a case of various materials of angular region are presented. At calculations it is accepted that joined regions, are made of a steel.

Table 2

The least values of LFP for various materials of angular region

Roots	Material of angular area														
	St	Al	W	Fe	Au	Mg	Cu	Mo	Ni	Sn	Pt	Pb	Ag	Ti	Zn
1	0.544	0.430	0.607	0.542	0.475	0.370	0.492	0.549	0.545	0.386	0.556	0.286	0.449	0.484	0.409
2	0.909	0.657	1.000	0.910	0.667	0.552	0.786	1.000	0.899	0.593	0.859	0.364	0.664	0.793	0.741
3	1.000	1.000		1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000	1.000	1.000	1.000

At the analysis of dependence of LFP from rigidities of joined regions through the control of reliability of received results was carried out check of following conditions: First, values of LFP should not change by interchanging the materials of regions $G^{(1)}$ and $G^{(2)}$; second, when one of shear modulus of joined regions aspires to zero, the parameter α should aspire to asymptotic value of LFP in a joint point of two regions and, third, at very high values of shear modulus $\mu^{(3)}$ of angular region LFP should aspire to the asymptotic values defined by the equation (20).

For example, on Fig. 4 are presented diagrams of of the specified dependence for a case when materials of regions $G^{(1)}$ and $G^{(2)}$ are identical and fixed ($\mu^{(1)} = \mu^{(2)}, \nu^{(1)} = \nu^{(2)}$), and elastic characteristics of angular region $G^{(3)}$ vary.

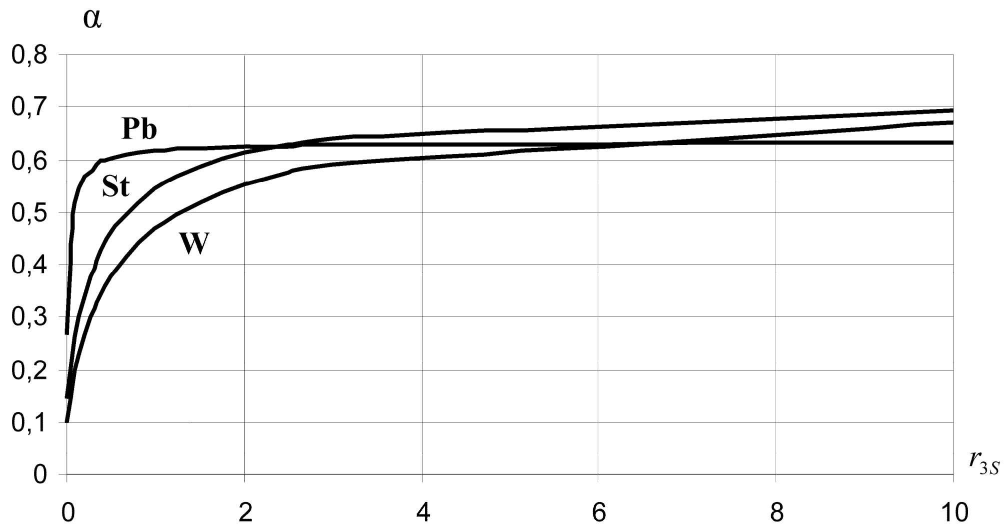

Fig. 4. Dependence of LFP from a parameter rigidity for a case when materials of regions $G^{(1)}$ and $G^{(2)}$ are identical

By the results of the numerical analysis it is possible to make following conclusions.

1) At aspiration $\mu^{(3)} \to 0$ the section loses bearing capacity that corresponds to mechanical sense of the problem.

2) At aspiration $\mu^{(3)}$ to infinity of LFP aspires to the asymptotic value defined by the equation (20) for α_0, and this value will be the greatest on all interval of change of the parameter r_{3S}.

3) The less rigid will be a material of regions $G^{(1)}$ and $G^{(2)}$, the at smaller values the parameter r_{3S} leaves on its asymptotic.

Possible recommendations in a considered variant of a combination of materials of regions $G^{(m)}$ are obvious enough: the material of angular region should be rigid as much as possible, it is desirable with enough high value $\nu^{(3)}$. The material of regions $G^{(1)}$ and $G^{(2)}$ needs to be chosen from materials with is minimum possible value of Poisson's ratio.

From the results of the numerical analysis of the equation (24) defining LFP on stresses in point *D* of a joint of four different media (Fig. 2) it is possible to make following conclusions.

First, for the majority of combinations of materials there is a positive root of this equation, smaller then unity. Second, practically for all considered combinations of materials of value of LFP are more than the values of LFP at interface of three environments. Third, at coupling of four regions to identical elastic characteristics of LFP $\alpha = 1$. Fourth, value of LFP $\alpha = 1$ is reached for some combinations of materials and at coupling of different regions. This property took place earlier at coupling only two different regions. Fifth, for many combinations of materials unlike a case of coupling of two media there are some roots smaller then unity. Sixth, value of LFP does not change at mutual replacement of elastic characteristics of crosswise lying regions, and also at their circular shift.

At the characteristic of considered combinations of materials we list them in a circular order, starting with the material of an region $G^{(1)}$. For example, combination **St-Pb-St-Pb** means that as a material for regions $G^{(1)}$ and $G^{(2)}$ serves steel, and for regions $G^{(3)}$ and $G^{(4)}$ — lead. If, for example, any elastic parameter of region $G^{(2)}$ varies, the combination will be written down in a form **St-Pb-G$^{(2)}$ — Pb**.

In table 3 are presented values of the least positive root of the equation (24) for a case when elastic parameters of regions $G^{(1)}$ and $G^{(2)}$ are fixed and there correspond steels, and elastic parameters of regions $G^{(3)}$ and $G^{(4)}$ vary. From the data of this table, in particular, follows that combination **St-Pb-St-Pb** will be the least preferable variant of a combination of materials.

Table 3
Value of LFP for various materials of two adjoined regions under condition when the material of two associate regions is steel

	Al	**W**	**Fe**	**Au**	**Mg**	**Cu**	**Pb**
Al	0.710	0.910	0.841	0.757	0.642	0.779	0.562
W	0.910	0.811	0.917	1	0.829	1	0.932
Fe	0.841	0.917	1	0.905	0.793	0.934	0.674
Au	0.757	1	0.905	0.821	0.681	0.913	0.586
Mg	0.642	0.829	0.793	0.681	0.585	0.710	0.504
Cu	0.779	1	0.934	0.913	0.710	0.857	0.623
Pb	0.562	0.932	0.674	0.586	0.504	0.623	0.428

At the analysis of a question of dependence of LFP from a proportion of rigidity of joined media we study a case of a combination of regions at which two adjoined regions made of identical material, the third area is made of another material, and value of the shear modulus of the fourth region varies. The results of numerical calculations for some combinations of materials are presented on Fig. 5. From them, in particular, follows: 1) The increase in rigidity of a material of region $G^{(3)}$ displaces area of extremely high values of LFP which are close to unity, in area of higher values of parameter r_{2S} and vice versa. 2) At aspiration of value of parameter r_{2S} to zero the increase in rigidity of a material of region $G^{(3)}$ reduces the minimum value of LFP. 3) At very high values of parameter r_{2S} the increase in rigidity of a material of region $G^{(3)}$ increases the limiting value of LFP.

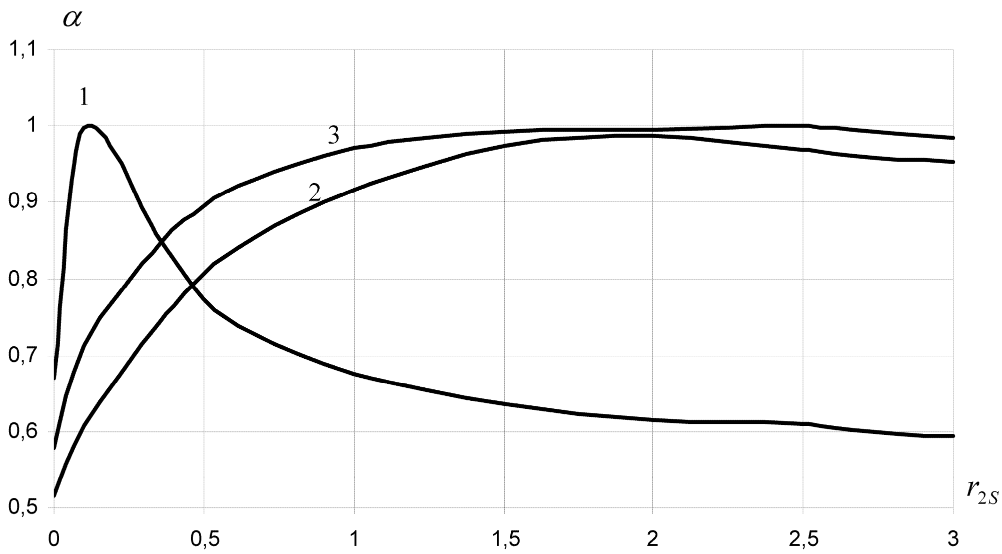

Fig. 5. Dependence of LFP in a point of joint of four different media from rigidity parameter (St-Pb-$G^{(2)}$-St – curve 1; St-W-$G^{(2)}$-St – curve 2; Pb-W-$G^{(2)}$-Pb – curve 3)

Conclusions

Taking into account the laws of change of LFP on the one hand allows to optimize the computational algorithm of wave characteristics construction in all non-uniform region, and on the other hand, gives the possibility to determine, both coefficient of concentration of stresses, and [14, 15] features of a spectrum of resonant frequencies and own forms of vibrations in a vicinity of an irregular point of a joint of two, three and four media.

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