SHORT COMMUNICATIONS

Dynamic Drag of Dislocations in Crystal with Structural Imperfections

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Abstract—A new mechanism of dynamic drag of dislocations is proposed and analyzed. A pair of dislocations is treated as a linear harmonic oscillator. The dissipation mechanism under investigation involves an irreversible conversion of the kinetic energy of moving dislocations into the vibrational energy of the dislocation oscillator. The proposed mechanism is used for calculating the drag force exerted by stationary trapped dislocations on a moving pair of dislocations and the drag of a solitary dislocation by dislocation dipoles. Radiative drag force acting on a moving pair of dislocations is also calculated.

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INTRODUCTION

Real crystals usually contain a certain number of dislocations rigidly trapped by impurity complexes, dislocation lattice sites, etc., as well as dislocation dipoles formed by a pair of dislocations with opposite Burgers vectors. The interaction of moving dislocations with stationary ones plays a very important role in strain hardening and plastic deformation processes; for this reason, a large number of theoretical and experimental research works are devoted to analysis of this problem [1–3]. In most theoretical publications, the motion of a solitary dislocation through a forest of flexible or rigid parallel dislocations intersecting the slip plane of a test dislocation was studied using computer simulation; the problem was solved in the quasi-static approximation (for low velocities of dislocations). In [4], the motion of a single screw dislocation through a system of parallel screw dislocations was studied for a high velocity (i.e., for external stresses $\sigma > \sigma_i =$ $(\mu b/2\pi)n^{1/2}$, where μ is the shear modulus and n is the number density of trapped dislocations). At such velocities, the motion of the dislocation is controlled by the dynamic drag mechanisms. Swinging of segments of forest dislocations by the moving dislocation led to irreversible loss of its kinetic energy; the drag mechanism studied in [4] was associated precisely with this process. The results of this publication were used in review [5]. In [6, 7], the results obtained in [4] were verified experimentally.

It is well known that edge dislocations located in parallel slip planes may form stable configurations by aligning one above another [8]. This process forms the basis of polygonization, as a result of which dislocation walls are formed in crystals. The presence of small groups and dislocation walls is quite typical of the structure formed during easy glide especially under large deformations or during local action of bending moments, when a high density of dislocations predominantly of the same sign appears in a crystal [9]. Under the action of external stresses, such formations can move over the crystal. The motion of a pair of edge dislocations in parallel slip planes of a crystal containing randomly distributed point defects was studied in [10, 11]. Energy dissipation occurring in this case due to the conversion of the kinetic energy of dislocations into the vibrational energy of dislocation elements about the dislocation center of mass.

Here, we analyze the motion of a pair of edge dislocations gliding at high velocities over parallel planes through a system of edge dislocations parallel to this pair, as well as the slip of a single dislocation interacting with stationary dislocation dipoles parallel to it. A pair of dislocations is a linear harmonic oscillator the vibrations of which can be excited due to the interaction with stationary dislocations. The dissipation mechanism involves irreversible conversion of the kinetic energy of moving dislocations into the energy of their vibrations relative to the center of mass of the dislocation pair. Such a mechanism has not been proposed or analyzed earlier.

THEORETICAL ANALYSIS

Let us suppose that two infinitely long edge dislocations move under the action of a constant external stress σ_0 in parallel planes: one dislocation moves in the x, z

plane (i.e., y = 0), and the other moves in the plane y = a, where a is the distance between two slip planes. Dislocation lines are parallel to the z axis, their Burgers vectors have coordinates (b, 0, 0) (i.e., these lines are parallel to the x axis), and the center of mass of the given dislocation pair moves at a constant velocity v in the positive direction of this axis. The lines of stationary edge dislocations are assumed to be rigid; these lines are also parallel to the z axis. For simplicity, we assume that their Burgers vectors are the same as the vectors of glide dislocations. The interaction of moving dislocations with stationary ones gives rise to oscillations of moving dislocations in their slip planes relative to plane x = vt perpendicular to these planes. The position of dislocations is described by the functions

$$X_1(y = 0; t) = vt + w_1(y = 0; t),$$

 $X_2(y = a; t) = vt + w_2(y = a; t),$
(1)

where $w_1(y = 0, z, t)$ and $w_2(y = a, z, t)$ are random quantities the value of which averaged over the dislocation ensemble is zero. The motion of each dislocation is defined by the equation

$$m\frac{\partial X_k^2}{\partial t^2} = b[\sigma_0 + \sigma_{xy}^k(vt + w_k; z)] + F_{dis} - B\frac{\partial X_k}{\partial t}. \quad (2)$$

Here, k = 1, 2 is the number of a moving dislocation; m is its mass per unit length (for simplicity, we assume that the masses of dislocations are identical); B is the damping constant controlled by the phonon, magnon, electron, or other dissipation mechanisms characterized by a linear dependence of the drag force acting on a dislocation on its sliding velocity; c is the velocity of propagation of transverse acoustic waves in the crystal; σ_{xy}^k is the tensor component of stresses produced by stationary dislocations on the line of the kth moving dislocation,

$$\sigma_{xy}^k = \sum_{i=1}^N \sigma_{xy,i}^k,$$

N is the number of stationary dislocations in the crystal; and $F_{\rm dis}$ is the force of interaction of dislocations with one another, which is defined, according to [8], as

$$F_{\text{dis}} = b^{2} M \frac{x(x^{2} - y^{2})}{r^{4}} \approx -\frac{b^{2} M w}{a^{2}},$$

$$M = \frac{\mu}{2\pi(1 - \gamma)},$$
(3)

where γ is the Poisson ratio. Here, we assume that $w \le a$ (approximation of small oscillations) and $r \approx a$. Two edge dislocations lying in parallel slip planes located one above another form a linear harmonic oscillator. To verify this, let us consider these dislocations in a coordinate system associated with their center of mass and write the equation of their motion,

$$m\ddot{w}_{k} = -\frac{b^{2}M}{a^{2}}w_{k}; \quad \ddot{w}_{k} + \omega_{0}^{2}w_{k} = 0;$$

$$\omega_{0}^{2} = \frac{b^{2}M}{a^{2}m} = \frac{2c^{2}}{a^{2}\ln(D/L)} \approx \frac{c^{2}}{a^{2}},$$
(4)

where L is the dislocation length and D is a quantity on the order of the crystal size. Let us numerically estimate the vibrational frequency of the dislocation oscillator. For $v \approx 10^{-2}c \approx 30$ m/s, $b \approx 3 \times 10^{-10}$ m, and $a \approx 10b \approx 3 \times 10^{-9}$ m, we obtain $\omega_0 \approx 10^{12} \, \mathrm{s}^{-1}$. The influence of viscous drag produced by the phonon subsystem on damping of dislocation oscillations can be neglected provided that $\omega_0 > B/m$; this condition can approximately be written in the form

$$\frac{mc}{a} \gg B.$$
 (5)

For $m \approx 10^{-15}$ kg m⁻¹, $a \approx 10b \approx 3 \times 10^{-9}$ m, and $c \approx 3 \approx 10^3$ m s⁻¹, we find that this condition is satisfied for $B \le 10^{-4}$ Pa s (i.e., practically for any value of the damping constant).

Using the methods developed earlier in [10–13], we can derive the following expression for the drag force exerted on each dislocation:

$$F = b \left\langle \frac{\partial \sigma_{xy}}{\partial X} w \right\rangle$$

$$= \frac{nb^2}{4\pi m} \int dp_x dp_y |p_x| |\sigma_{xy}(p)|^2 \delta(p_x^2 v^2 - \omega_0^2),$$
(6)

where n is the number density of stationary dislocations and $\delta(p_x^2 v^2 - \omega_0^2)$ is the Dirac delta function reflecting the dissipation mechanism under investigation, viz., conversion of the kinetic energy of translational motion of a dislocation into the energy of its vibrations at frequency ω_0 . Furthermore, $\sigma_{xy}(p) = \sigma_{xy}(p_x, p_y, 0)$ is the Fourier transform of the tensor of stresses produced by a stationary dislocation, which in our case has the form

$$\sigma_{xy}(\mathbf{p}) = \frac{2\mu bi p_x p_y^2}{1 - \gamma p^4}.$$
 (7)

 $(p_z = 0$, since not a single quantity depends on coordinate z). Symbol $\langle ... \rangle$ indicates averaging over a random configuration of stationary dislocations in the crystal,

$$\langle f(r_i) \rangle = \int \prod_{s,i=1}^{N} f(\mathbf{r}_i) \frac{dr_i}{S^N}.$$
 (8)

Here, *S* is the cross-sectional area of the crystal perpendicular to dislocation lines. In averaging in accordance with the standard procedure, number of dislocations *N* and cross-sectional area *S* tend to infinity, and their ratio remains constant and equal to the average dislocation density. After transformations, we obtain the fol-

lowing expression for the dynamic drag force exerted by a system of stationary dislocations on a moving dislocation:

$$F = \frac{nb^{4}\mu^{2}}{16m\omega_{0}(1-\gamma)^{2}v} \approx nb^{2}\mu a \frac{c}{v} = n_{0}\mu a \frac{c}{v}.$$
 (9)

Here, $n_0 = nb^2$ is the dimensionless density of trapped dislocations.

Thus, the dislocation drag force controlled by this mechanism is inversely proportional to the dislocation slip velocity; in other words, such a force cannot ensure dynamic stability of motion of dislocations since it can be stable only in the presence of quasi-viscous forces (e.g., of phonon or magnon origin). The presence of force (9) leads to the emergence of a critical velocity, below which stationary motion of dislocations is ruled out. This velocity can be determined from the condition F = Bv and is given by

$$V_c = \frac{\mu b^2}{4(1-\gamma)} \sqrt{\frac{n}{m\omega_0 B}}.$$
 (10)

RESULTS AND DISCUSSION

Let us estimate numerically the drag force exerted by stationary dislocations on a glide dislocation for typical values of parameters for metals: $\mu = 3 \times 10^{10}$ N m⁻² and $b \approx 3 \times 10^{-10}$ m. Then drag force $F \approx 10^{-4}$ N/m for $n \approx 10^{12}$ m⁻², $v \approx 10^{-2}c \approx 30$ m/s, and $a \approx 10b \approx 3 \times 10^{-9}$ m. This drag force is comparable in order of magnitude to the quasi-viscous force of the phonon origin for a damping constant of $B \approx 10^{-5}$ Pa s. For $a \approx 100b \approx 3 \times 10^{-8}$ m, we obtain $F \approx 10^{-3}$ N/m and $B \approx 10^{-4}$ Pa s.

It should be noted that our results differ from those obtained in [10, 11], where the slip of a pair of edge dislocations was also studied. In these publications, the string model was used, the equation of motion contained the second derivative with respect to the coordinate, dislocations were decelerated by point defects (local obstacles of spherical symmetry), the drag mechanisms involved excitation of flexural oscillations of dislocations, and the role of the dislocation interaction was reduced to a rearrangement of the spectrum of these oscillations. Here, we are not using the string model, the derivative with respect of the coordinate does not appear in the equation of motion, and the drag is due to stationary dislocations, viz., extended linear objects with cylindrical symmetry. It is this symmetry of the problem that has made it possible to study the new mechanism involving the excitation of oscillations of a dislocation oscillator.

This energy dissipation mechanism is also operative when a single dislocation moves through a system of stationary dislocation dipoles parallel to this dislocation. In this case, the kinetic energy of the moving dislocation is transformed into the vibrational energy of a dipole, which is also a linear harmonic oscillator. Passing to a system of coordinates associated with the dipole, we can easily verify that the drag force exerted on the solitary dislocation by dislocation dipoles parallel to it can also be described by formula (9), the only difference being that in this case a is the distance between the dislocations forming the dipole and n is the number density of dipoles [12].

Vibrations of a dislocation relative to the center of mass of a dislocation pair must lead to the emission of elastic waves (i.e., radiative friction) by the dislocation. Radiative friction was studied by many authors (see review [5]), but they analyzed nonuniformity of the motion of dislocations associated with slip over the Peierls relief. According to [5], to solve the problem of radiative friction correctly, it is necessary to determine the law of motion of a dislocation self-consistently taking into account the response of radiation. In our case, it is impossible to solve this problem analytically; however, for rough estimates, we can use the result obtained in [14] under the assumption that the entire radiation is emitted at a single mode. It was demonstrated in [15] that such an assumption is equivalent to the hypothesis of the smallness of perturbation; i.e., radiative friction in this case is a small correction to the drag of a dislocation pair, which is produced by its interaction with stationary dislocations.

According to [14], the energy emitted per unit time by a unit length of dislocation oscillating at frequency ω is defined as

$$R = \frac{1}{32}\mu b^2 L^2 k^2 \omega; \quad k = \frac{\omega}{c}.$$
 (11)

Here, L is the amplitude of dislocation oscillations. The radiative friction force is calculated by the formula $F_R = R/bv$. In the case considered here, we can obtain a rough estimate by calculating the mean square deviation $\langle w^2 \rangle$ of a dislocation from its stable equilibrium position:

$$L^2 \approx \langle w^2 \rangle \approx n_0 a^2 \frac{c^2}{V^2}.$$
 (12)

Using formula (11), we obtain the radiative friction force acting of a dislocation in the form

$$F_R \approx n_0 \mu b \left(\frac{b}{a}\right) \left(\frac{c}{v}\right)^3$$
. (13)

Let us compare drag force (9) emerging due to the interaction of a moving dislocation with trapped dislocations to the radiative friction force:

$$\frac{F_R}{F} \approx \left(\frac{b c}{a v}\right)^2. \tag{14}$$

Formula (13) is valid for $F_R \ll F$, i.e., for

$$\frac{V}{c} \geqslant \frac{b}{a}$$
. (15)

Since the model used here is valid for velocities $c \gg v$, the maximal admissible velocity is on the order of $v \approx 10^{-1}c$; consequently, condition (15) is valid only for $a \ge 10^2b$. Let us estimate the order of magnitude of the radiative friction force for $v \approx 10^{-1}c$, $a \approx 10b$, $\mu = 3 \times 10^{10}$ N m², and $b \approx 3 \times 10^{-10}$ m. This gives $F_R \approx 10^{-5}$ N/m for stationary dislocation density $n \approx 10^{12}$ m⁻² m (i.e., $n_0 \approx 10^{-7}$).

The proposed drag mechanism may considerably affect the motion of dislocations (especially in metals).

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