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GENERATION OF SPECIALY COORDINATED SURFACES APPLICABLE TO THE STRENGTH ANALYSIS OF SHELLS

The article is dedicated to geometric research in the area of thin-shell structural design. Thing shells represent a widely-spread type of structural forms used in modern machines and civil engineering works. A method for the analytical transformation of a parametric line on higher-order surfaces from normal to special coordinates is proposed.

Keywords: *curvilinear coordinate, congruence, curvature line, special parameterization, thin construction*

1. Introduction

Any methods of the strength analysis of thin constructions named shells require the representation of the surfaces using special coordinates. Here, two cases are possible [1].

First case. The coordinate lines $\alpha = const,\ \beta = const$ on middle surfaces are orthogonal.

Second case. The coordinate lines on middle surfaces are curvature lines.

Hence the problem of special parameterization of a surface is relevant.

2. Terminology and definitions on the theory of curvilinear coordinates and congruences

Let the curvilinear coordinates $u,\ v,\ w$ be introduced by the functions:

$$x = f(u,\ v),\ y = g(u,\ v),\ z = h(u,\ v), \tag{1}$$

where $x,\ y,\ z$ are Cartesian coordinates.

As we assume that we do not have $\dfrac{D(f,\ g,\ h)}{D(u,\ v,\ w)} = 0.$

According to Lane [2] and Eisenhart [3], the functions (1) may be applied for the representation of curve congruence. For this, one of three coordinates $u,\ v,\ w$ will be a point on a congruence line.

Hence functions (1) represent not one but three congruences of lines which are lines of the system (1). In the domain where the coordination by functions (1) is regular, it is possible to solve (1) as parametric equations relative to $u,\ v,\ w.$

Let

$$u = u(x,\ y,\ z),\ v = v(x,\ y,\ z),\ w = w(x,\ y,\ z), \tag{2}$$

be obtained solutions.

In the case when one of three couples out of the coordinates $u,\ v,\ w$ is represented by fixed values and the two remaining couples are variable, the functions (1) are transformed into parametric equations of one of three types of coordinate lines of the system (1). The type of the line is determined by fixed values of the couple and by the remaining variable coordinate.

In the case when three coordinates $u,\ v,\ w$ are variable, the functions (1) may be consi-dered as parametric equations of three coordinate lines of the system (1). Let`s introduce $u-,\ v-,\ w-$ as a congruence and $u-,\ v-,\ w-$ as a surface. Hence the theory of curvilinear coordinates and the theory of curvilinear congruences are linked by factors resulting from the initial representation.

3. Principles of the surface generation in special coordinates

In general, a form of any surface may be determined analytically by the forms of its parametric lines and by the structure of its parametric families.

It is possible to choose the structure of both parametric families and for one of them – the form of its parametric lines by choice of the coordinate system (1), in which one of three congruences will be containing the parametric lines of the surface S.

The second parametric family of required surface and the form of its elements are determined by the line m immersed into the above mentioned congruence of coordinate lines.

If none of known coordinate systems is applicable to the surface generation of required form, it is necessary to generate a special coordinate system based on the structure of the parametric families and the forms of their elements of the required surface S.

Let (1) be the functions introducing the coordinate system u, v, w and m be the line, represented by the parametric equations

$$x = p(\alpha),\ y = q(\alpha),\ z = s(\alpha) \tag{3}$$

immersing of which in $u-$congruence, the $u-$surface will be generated.

Three functions (1), the second and third functions (2) and three functions (3) complete the base for the generation of parametric equations of $u-$surface. The substitution of the functions (3) for the second and third functions (2) gives the substituents for the functions (1). So parametric equations of the required $u-$surface, passing through the line m will be represented as superposition of the functions

$$\begin{aligned} x &= f\left\{u, v\left[p(\alpha), q(\alpha), s(\alpha)\right], w\left[p(\alpha), q(\alpha), s(\alpha)\right]\right\}, \\ y &= g\left\{u, v\left[p(\alpha), q(\alpha), s(\alpha)\right], w\left[p(\alpha), q(\alpha), s(\alpha)\right]\right\}, \\ z &= h\left\{u, v\left[p(\alpha), q(\alpha), s(\alpha)\right], w\left[p(\alpha), q(\alpha), s(\alpha)\right]\right\}. \end{aligned} \tag{4}$$

Curvilinear coordinates for $v-$surface are:

$$u\left[p(\alpha), q(\alpha), s(\alpha)\right], v, w\left[p(\alpha), q(\alpha), s(\alpha)\right] \equiv v,\ \alpha$$

and for $w-$surface are:

$$u\left[p(\alpha), q(\alpha), s(\alpha)\right], v\left[p(\alpha), q(\alpha), s(\alpha)\right], w \equiv w,\ \alpha.$$

Let us consider the cases when the surface (4) is specially coordinated according to demands of the strength analysis of shells.

Theorem. For parametric lines $u = const,\ \alpha = const$ of $u-$surface (4), let`s assume that:

1) the system of coordinates (1) will be orthogonal;

2) curvature lines of the coordinates of surfaces $u = const$ of the orthogonal coordinate system (1) should be the spheres or the planes.

Proving. In order to prove the theorem, it is sufficient to make the following consequence of arguments provided that the coordinate system (1) is orthogonal:

– the coordinate surface $u = const$ and lines of $u-$congruence are orthogonal;

– $u-$surface containing the family of lines belonging to $u-$ congruence makes the right angles with each surface of the family $u = const$;

– for $u-$surface (4) the lines $\alpha = const$ as lines of $u-$ congruence and the lines $u = const$ as intersections of $u-$surface with surfaces of the family $u = const$ are orthogonal;

– any line on the sphere is its curvature line;

– providing that coordinate surfaces $u = const$ of the system (1) are the spheres, intersections $u = const$ with $u-$surface (4) form the curvature lines in accordance with Ioachimsthal's theorem [4] both for spheres and $u-$surface. But the lines of different families $u = const$, $\alpha = const$ are orthogonal and lines $u = const$ are the curvature lines.

Hence lines of the family $\alpha = const$ on $u-$surface are also curvature lines. The theorem is proven.

By following corollary we discriminate the case which is very important for the strength analysis of shells.

Corollary. For the immersing line m belongs to the parametric family of $u-$lines on $u-$surface (4), it will be situated on one of coordinates $u = const$ of the system (1).

Remark. Our considerations were applicable to the generation of $u-$surfaces. They are also applicable to the generation of $w-$surfaces.

4. Introducing the cyclic system of coordinates

Let coordinates u, v be the radii of conjugated pencils of circles and w be the angle of pencils rotation about the circle centers of one of them. Make coinciding the axis of rotation with axis oz. The functions (1) in traducing the coordinate system are:

$$\begin{aligned} x &= u\frac{u\sqrt{v^2+c^2}+v\sqrt{u^2-c^2}}{u^2+v^2}\cos w, \\ y &= u\frac{u\sqrt{v^2+c^2}+v\sqrt{u^2-c^2}}{u^2+v^2}\sin w, \\ z &= v\frac{u\sqrt{v^2+c^2}+v\sqrt{u^2-c^2}}{u^2+v^2}, \end{aligned} \tag{5}$$

where $c \geq 0$ is constant, $k = -1, 0, 1-$ coefficient.

<u>First case.</u> In this case $k = 1$, $u \geq c$ and axis oz are the centers of the line of circles belonging to hyperbolic pencil. Introduced system is known as toroidal with other content of u, v, w [5, 6].

<u>Second case.</u> In this case $k = -1$, $v \geq c$ and axis oz are the centers of the line of circles belonging to elliptic pencil. Introduced system is known as bipolar [5, 6] with other content of u, v, w.

<u>Third limiting case.</u> In this case $k = 0$, $c = 0$. Both conjugated pencils of circles are parabolic.

Domain of the functions (5):

$$\left.\begin{array}{l} k = 1, \quad c > 0,\ c \leq u < \infty,\ 0 < v < \infty \\ k = -1,\ c > 0,\ 0 < u < \infty,\ c \leq v < \infty \\ k = 0, \quad c = 0,\ 0 < u < \infty,\ 0 < v < \infty \end{array}\right\} 0 \leq w \leq 2\pi .$$

Coordinate surfaces of the system (5):

– $u = const-$ the spheres, the centers of which are situated on the axis oz;

– $v = const-$ toroidal surfaces of revolution, the meridians of which are the arcs with centers on the plane xoy;

– $w = const-$ demi-planes of the pencil with axis oz.

Coordinate lines of the system (5):

– $u-$line – an arc of a circle with the center on the plane xoy situated on a demi-plane of the pencil with axis oz;

– $v-$line – an arc of a circle with the center on the axis oz situated on a demi-plane of the pencil with axis oz;

– $w-$line – a circle with the center on the axis oz situated on the plane parallel to the plane xoy.

5. Applications

Let us demonstrate, by two examples, the proposed method of the surface generation with special coordinates.

Example 1. To generate the $u-$, $v-$ and $w-$surfaces with special coordinates based on the cyclic coordinate system (5) by immersing the helices:

$$x = p(\alpha) = r\cos\alpha,\ y = q(\alpha) = r\sin\alpha,\ z = s(\alpha) = b\alpha, \tag{6}$$

in $u-$, $v-$ and $w-$congruencies in turn.

Solutions. a) as the coordinate system (5) is orthogonal and its coordinate surfaces $u = const$ are the spheres, we obtain the parametric equations of $u-$surface as the superposition of functions received according to the scheme: $(9)\dfrac{\nearrow(7)\searrow}{\searrow(8)\nearrow}(5)$.

The parametric lines of $u-$surface represented by the form (4) are: $u = const-$ spherical lines of the curvature, $\alpha = const - u-$circles which are the lines of the curvature;

b) the scheme for representation of the parametric equations of $v-$surface is the following: $(9)\dfrac{\nearrow(6)\searrow}{\searrow(8)\nearrow}(5)$.

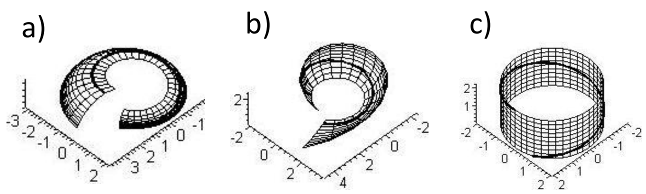
As the coordinate surfaces $v = const$ of the system (4) are the tori (not spheres or planes), the $v-$surface will be represented by parametric lines $v = const$ on the surface of the tori and $\alpha = const - v-$circles of the coordinate system (5). The parameterization of $v-$surface is orthogonal;

c) as the coordinate surfaces $w = const$ are the planes of $w-$surface with be the curvature lines, the scheme for obtaining parametric equations of $w-$surface is the following: $(9)\dfrac{\nearrow(6)\searrow}{\searrow(7)\nearrow}(5)$.

In general case, $w-$surface is the surface of revolution, and in this particular case it is the cylinder of revolution.

It should be noted that in all three cases, the immersing line m does not belong to the parametric families of the required surface.

The computer aided representations of $u-$, $v-$ and $w-$surfaces are given in the Figure 1.



a) $k=1, \quad 0{,}4 \le u \le 8$; b) $k=-1,\ 0{,}5 \le v \le 2{,}5$; c) $k=0, \quad 0 \le w \le 2\pi$

Figure 1 – $U-$, $v-$and $w-$surfaces passing through helices $a = 2,\ b = 0{,}3,\ c = 1$

Example 2. To generate $u-$surface by immersing in $u-$congruence of the system (5) of the curve in given by the equation:

$$t = t_1 + b\cos(n\alpha), \tag{7}$$

where $t-$ variable, t_1- fixed angle measured in horizontal plane, $\alpha-$ variable angle in vertical plane, $b-$ amplitude, $n-$ number of periods.

The equation (7) is internal to the parametric equations:

$$\begin{gathered} x = p(t,\,\alpha) = u\cos t\cos\alpha,\; y = q(t,\alpha) = u\cos t\sin\alpha, \\ z = s(t,\,\alpha) = \sqrt{u^2 - k\,c^2} + u\sin t, \end{gathered} \tag{8}$$

which represent one of coordinate spheres $u = const$ of the coordinate system (5).

Solutions. The parametric equations of the required surface designated as the curvature lines, are obtained according to the following scheme: $(10) \to (11)\dfrac{\nearrow(7)\searrow}{\searrow(8)\nearrow}(5)$.

As the immersing line m given by equations (7), (8) belongs to one of the coordinate surface $u = const$ which is the sphere, it is an element of the parametric family $u = const$ of $u-$surface.

The Figure 2 shows the required $u-$surface.

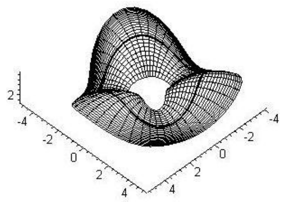


Figure 2 – $U-$surface passing through the line, belonging to the coordinate sphere

6. Conclusion

The proposed method of the surface generation gives possibilities to foresee its form and correct it using computer added visualization, as well as use them in generation of thin constructions having a form of a middle surface.

References

1. Krivoshapko, S. N. Static Analysis of Shells With Developable Middle Surfaces / S. N. Krivoshapko // Applied Mechanics Reviews. – 1998. – Vol. 51, Issue 12. – Part 1. – P. 731–746.
2. Lane, E. P. Surfaces and Curvilinear Congruences / E. P. Lane // Transactions of the American Mathematical Society. – 1932. – Vol. 34, № 3. – С. 676–688.
3. Eisenhart, L. P. Congruencies of Curves / L. P. Eisenhart // Transactions of the American Mathematical Society. – 1903. – Vol. 4, № 4. – P. 470–488.
4. Joachimsthal, F. Anwendung der Differential- und Integralrechnung auf die Allgemeine Theorie der Flächen und der Linien doppelter Krümmung / F. Joachimsthal. – 1872. – Leipzig. – P. 208.
5. Moon, P. Field Theory Handbook: Including Coordinate Systems Differential Equations and Their Solutions / P. Moon, D. E. Spencer. – 2nd Ed. – Berlin [et al.] : Springer – Verlag, 1971. – 637 p. – ISBN 97-80-387-027-326.
6. Spiegel, R. Mathematical Hand book of Formulas and Tables / R. Spiegel, R. Murray. – New York, McGraw Hill Book Company, 1968. – 265 p.

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Формообразование поверхностей в специальных координатах, применяющихся в анализе прочности оболочек

Статья посвящена геометрическим исследованиям в области проектирования тонкостенных конструкций. Тонкостенные оболочки являются распространенным типом конструктивных форм современных машин и разнообразных инженерных сооружений. Предложен метод аналитического преобразования параметрической линии на поверхностях высшего порядка из общих в специальные координаты.

Любой метод анализа прочности тонкостенных конструкций, называемых оболочками, заключается в представлении поверхности с помощью специальных координат.

Формообразование поверхностей при условии их отнесения к линиям кривизны представляет значительный интерес для расчета оболочек и для составления программ обработки на оборудовании с ЧПУ, поэтому способы получения таких поверхностей являются актуальными, следовательно, проблематика специальной параметризации поверхностей является актуальной.

В теории оболочек и практике их расчета на прочность важным условием является такое ее аналитическое представление, при котором серединная поверхность отнесена к линиям кривизны, а края совпадают с координатными линиями. Для первичной оценки прочности конструкции, а также ее эстетической выразительности желательно иметь многовариантную аналитическую модель, способы варьирования ее формой, а также возможность использования программ компьютерной графики для визуализации вариантов с целью окончательного выбора.

КРИВОЛИНЕЙНАЯ СИСТЕМА КООРДИНАТ, КОНГРУЭНТНОСТЬ, ЛИНИЯ КРИВИЗНЫ, СПЕЦАЛЬНАЯ ПАРАМЕТРИЗАЦИЯ, ТОНКОСТЕННАЯ КОНСТРУКЦИЯ

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Generation of Specially Coordinated Surfaces Applicable to the Strength Analysis of Shells

The article is dedicated to geometric studies in the area of a thin-shell structural design. Thing shells represent a widely-spread type of structural forms used in modern machines and various engineering structures. A method for the analytical transformation of a parametric line on higher-order surfaces from general to special coordinates is proposed.

Any method for analyzing the strength of thin-walled structures called shells is to represent the surface using special coordinates.

The generation of surfaces, provided they are assigned to the lines of curvature, is of significant interest for calculating shells and for processing programming on the CNC equipment, therefore, methods for generating such surfaces are relevant, consequently, the problems of the surface special parameterization is actual.

In the theory of shells and practice of calculating their strength, an important condition is its analytical representation in which the middle surface is assigned to the lines of the curvature, and the edges coincide with coordinate lines. For initial assessment of the structural strength, as well as its aesthetic expressiveness, it is desirable to have a multivariate analytical model, ways to vary its shape, as well as the ability to use computer graphics programs to visualize options for the final choice.

CURVILINEAR COORDINATE SYSTEM, CONGRUENCE, CURVATURE LINE, SPECIAL PARAMETERIZATION, THIN CONSTRUCTION

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