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## MACHINING OF CYLINDRICAL TOOTH-WHEELS BY HYPERBOLA CUTTING TOOLS

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**Abstract.** The article investigates cutting of cylindrical tooth wheels using multi-passing hyperbola instrument. The instrument is produced as an enveloping a tooth wheel being machined one. Hyperbola cutters are obtained in a space machine-tool engagement. A principally new scheme of the instrument form shaping has been developed for tooth cutting instrument production. At this, a conventional gear machining equipment has been used. Major geometric and kinematic parameters of tooth cutting using the developed instrument have been defined in an analytical form. Main instrument surface as well as contact lines on the surfaces being machined have been defined. Main elements of a machine-tool contact of a hyperbola instrument with a work-piece being cut have been defined. The developed instrument may be used in all spheres of machine-building.

**Key words:** hyperbola, tooth-wheel, relative speed, specific slip, specific curvature, rolling, engagement.

### 1.Introduction

While designing cutting instrument working on principle of rolling, the main problem is to determine profiles of instrument cutting edges [1].

Cutting edge is usually determined in two ways. The first way consists in selecting a definite line as a cutting edge on the surface which is a line enveloping surface of the treated tooth, by sectioning it with the other surface. The second one consists in choosing a definite line on the item surfaces of engagement and its envelope by sectioning it with the other surface; determining the corresponding contact line on the surface enveloping item surface, and consider this contact line as a cutting edge; e.g., in the case of tooth grinding; or this line is given a helical motion and pass a section through helical surface received in this way by plane or helical surface. As a result of this, surfaces intersection the cutting edge of the instrument is received, for example in tooth milling.

### 2. Research object

The given paper is devoted to development of new processes in order to obtain tooth-working instrument based on the work-pieces of one-cavity hyperbola type. Such instrument is obtained in space machine-tool engagement according to the third class form-formation scheme. The aim of the presented research has been to

obtain the profile of such instrument and its basic geometric and kinematic parameters.

### 3. Results of experimental research

Let’s view the problem of industrial cutting instrument profiling. For this, let’s take two coordinate systems.

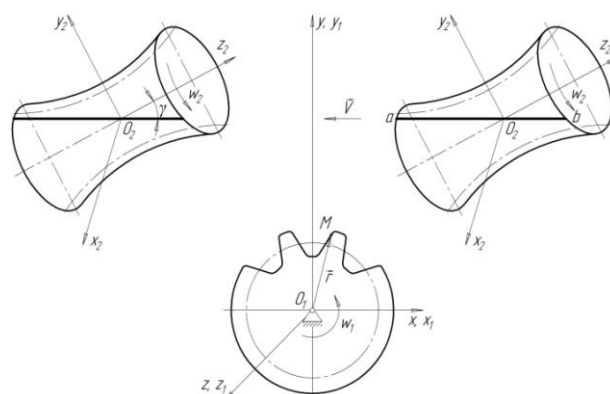


Fig.1. Coordinate system

The first system  $x, y, z$  – is a stationary system where axis  $z$  coincides with rotation axis of treated wheel. We shall determine engagement surfaces equation in coordinate system  $x, y, z$ . The second system  $x_2, y_2, z_2$  – is a stationary system, connected with instrument.

Let's write a system of equations by means of transition matrix giving a connection between system rotation coordinates, which is rigidly related to treated cylindrical tooth-gear and coordinates of rotating system.

Partial derivatives necessary for obtaining equations of the engagement surfaces are found from the system of equations (1):

$$\begin{aligned} \partial x' / \partial \varphi &= (1 + u \cos \nu) y - u \sin \nu \sin \varphi z' + \\ &+ u a_w \cos \nu \sin \varphi; \\ \partial y' / \partial \varphi &= -(1 + u \cos \nu) x + u \sin \nu \sin \varphi z' + \\ &+ u a_w \cos \nu \cos \varphi; \end{aligned} \quad (1)$$

$$\partial z' / \partial \varphi = u \sin \nu (x' \cos \varphi - y' \sin \varphi - a_w).$$

Dating:

$$1 + u \cos \nu = B; u \sin \nu = C; u a_w \cos \nu = D, \quad (2)$$

We get:

$$\begin{aligned} \partial x' / \partial \varphi &= B y' - C \cos \varphi z' + D \sin \varphi; \\ \partial y' / \partial \varphi &= -B x' + C \sin \varphi z' + D \cos \varphi; \end{aligned} \quad (3)$$

$$\partial z' / \partial \varphi = C(x' \cos \varphi - y' \sin \varphi - a_w).$$

Now, if we pass a section through helical surface with the plane, perpendicular to its axis, we get a curve, which can be written in polar coordinates as:

$$\theta = f(\rho) \quad (4)$$

where:  $\theta$  is a polar angle and  $\rho$  is a polar radius vector.

Let a curve given in equation (4) take a position in the primary moment in coordinate system  $x', y', z'$ , where angel between radius vector of some point  $\rho_0$  and optional radius vector equals to  $\theta - \theta_0$ . Let's give a helical movement with parameter P to the curve given by equation (4). This curve will describe a helical surface. Let this curve in its movement rotate by some angle  $\delta$ , then it will move along axis  $z'$  by  $P\delta$ . Consequently, a coordinate of any point on the helical surface, described by a given curve, will be as follows:

$$\begin{aligned} x' &= \rho \cos(\sigma_0 + \theta + \delta); \\ y' &= \rho \sin(\sigma_0 + \theta + \delta); \\ z' &= P\delta. \end{aligned} \quad (5)$$

Given equations (5) are item surface equations in parametric form, where  $\rho$  and  $\delta$  are variable arguments.

Items surface is engaged with the surface which describes it and is rigidly related to revolving coordinate system  $x_u, y_u, z_u$ . The given surfaces have contact by characteristics.

If item surface is involved in complex movement relative to instrument axis, we shall get a set of such surfaces which depend on turning angle  $\varphi$  (see system1).

As we known from differential geometry [2, 3, and 4] item surface characteristic equations in coordinate system  $x', y', z'$ , if it is given in parametric form, is as follows:

$$x' = \rho \cos(\sigma_0 + \theta - \delta);$$

$$y' = \rho \sin(\sigma_0 + \theta + \delta);$$

$$z' = P\delta.$$

$$\begin{vmatrix} \partial x' / \partial \varphi & \partial y' / \partial \varphi & \partial z' / \partial \varphi \\ \partial x' / \partial \rho & \partial y' / \partial \rho & \partial z' / \partial \rho \\ \partial x' / \partial \delta & \partial y' / \partial \delta & \partial z' / \partial \delta \end{vmatrix} = 0. \quad (6)$$

The first three equations are item surface in coordinate system  $x', y', z'$ , without parameter  $\varphi$ .

To find determinant in equations (6) we use descriptions  $\sigma_0 + \theta + \delta = \tau$ . In this case, partial derivatives will be as follows:

$$\begin{aligned} \partial x' / \partial \rho &= \cos \tau - \rho \partial \theta / \partial \rho \sin \tau; \\ \partial y' / \partial \rho &= \sin \tau + \rho \partial \theta / \partial \rho \cos \tau; \\ \partial z' / \partial \rho &= 0; \quad \partial x' / \partial \delta = -\rho \sin \tau; \\ \partial y' / \partial \delta &= \rho \cos \tau; \quad \partial z' / \partial \delta = P. \end{aligned}$$

Substituting these derivatives and values  $\rho \partial \theta / \partial \rho = tg \alpha_x$  [2] in equation determinant (6) we get:

$$\begin{vmatrix} \partial x' / \partial \varphi & \partial y' / \partial \varphi & \partial z' / \partial \varphi \\ \cos \tau - tg \alpha_x \sin \tau & \sin \tau + tg \alpha_x \cos \tau & 0 \\ -\rho \sin \tau & \rho \cos \tau & P \end{vmatrix} = 0;$$

$$\begin{aligned} \partial x' / \partial \varphi P (\sin \tau + tg \alpha_x \cos \tau) - \\ - \partial y' / \partial \varphi P (\cos \tau - tg \alpha_x \sin \tau) + \\ \partial z' / \partial \varphi \rho = 0. \end{aligned} \quad (7)$$

Setting designation  $\tau + \alpha_x = \eta$ , we get:

$$\begin{aligned} \sin \tau + tg \alpha_x \cos \tau &= \sin \eta / \cos \alpha_x; \\ \cos \tau - tg \alpha_x \sin \tau &= \cos \eta / \cos \alpha_x. \end{aligned}$$

Setting designation  $\tau + \varphi = \psi$  and having in mind that  $x' = \rho \cos \tau; y' = \rho \sin \tau$ , we get:

$$\begin{aligned}
\partial x' / \partial \varphi &= B y' + C \cos \varphi z' + D \sin \varphi = \\
&= B \rho \sin \tau - C \cos \varphi z' + D \sin \varphi; \\
\partial y' / \partial \varphi &= -B x' + C \sin \varphi z' + D \cos \varphi = \\
&= -B \rho \cos \tau + C \sin \varphi z' + D \cos \varphi; \\
\partial z' / \partial \varphi &= C[\rho \cos(\tau + \varphi) - a_w] = \\
&= C(\rho \cos \psi - a_w).
\end{aligned} \tag{8}$$

Substituting equations (8) into determinant (7) we get:

$$\begin{aligned}
&(B \rho \sin \tau - C \cos \varphi z' + D \sin \varphi) P \sin \eta / \cos \alpha_x - \\
&- (C \sin \varphi z' - B \rho \cos \tau + D \cos \varphi) P \cos \eta / \cos \alpha_x + \\
&+ \rho C(\rho \cos \psi - a_w) = B \rho P - C P z' \sin(\eta + \varphi) \cos \alpha_x - \\
&- D P \cos(\eta + \varphi) \cos \alpha_x + \rho C(\rho \cos \psi - a_w).
\end{aligned}$$

Setting designation  $\eta + \varphi = \omega$  we get:

$$\begin{aligned}
&B P \cos \alpha_x - C P z' \sin \omega - D P \cos \omega + \\
&+ \rho \cos \alpha_x C(\rho \cos \psi - a_w) = 0.
\end{aligned} \tag{9}$$

As in equation (9) dependence  $z'$  depends on parameters  $\rho$  and  $\varphi$  then equation of characteristics in coordinate system will be written as follows:

$$\begin{aligned}
&x' = \rho \cos \tau; \quad y' = \rho \sin \tau; \\
&B P \rho \cos \alpha_x - C P z' \sin \omega - \\
&- D P \cos \omega - C \rho \cos \alpha_x (\rho \cos \psi - a_w) = 0.
\end{aligned} \tag{10}$$

As engagement surface represents geometrical place of contact line in stationary space we have to rewrite equations (10) into stationary coordinate system  $xyz$  in order to get equations of this surface:

$$\begin{aligned}
&x = \rho \cos \tau \cos \varphi - \rho \sin \tau \sin \varphi = \rho \cos \psi; \\
&y = \rho \cos \tau \sin \varphi + \rho \sin \tau \cos \varphi = \rho \sin \psi; \\
&z = z'.
\end{aligned} \tag{11}$$

Cutting edges equations for all particular cases of treatment by means of knurling can be defined using these equations. From all equations above we can see, that it is not so easy to realize production of instrument cutting edges treating tooth of cylindrical wheels both with straight and helical tooth in practice.

The author of this article has theoretically grounded and practically realized tooth cutting instruments on quasi-globoids [5]. It turned out that such instruments are not relieved or sharply grounded. In the result, received instruments can be equaled to instruments with cutting surfaces. Suggested instruments are got by the second method of Olivie. It can be explained by the fact that definite cylindrical wheel can be cut by definite quasi-globoid instrument. So, if we have kinematic pair with linear contact of conjugated surfaces and if we try to get technological pair "work-piece - instrument", by means of which we can cut surface on a work-piece by means of knurling,

then we can take any line on the surface of this element, if this line has points all over tooth length involved into engagement, as an instrument cutting edge, got from one element of this kinematic pair.

#### 4. Conclusions

From the mentioned above, it is clear that if the instrument is formed out of one wheel of kinematic pair having a linear contact, one should find the common describing item surface in its motion relative to instrument axis and place the front cutting face at the angle depending on strength and hardness of treated material, strength and hardness of instrument to define cutting edge of this instrument. In this case, no distortion of the tooth being cut takes place because the instrument is not relieved and is not grounded sharply.

If there is a kinematic pair of cylindrical wheels having point-wise contact of teeth conjugated surfaces and if one tries to obtain a technological pair "work-piece - instrument" using which it is possible to cut tooth on the work-piece by means of knurling, then instrument cutting edge obtained of one element of this kinematic pair should be one line – a line of contact on this element tooth surface with conjugated surface of other kinematic pair element teeth surface.

Cutting edge in the form of line of contact on the tooth surface of one of element of the pair having a point-wise contact is very difficult to obtain in practice. If it is necessary to produce instrument under investigation, front face cutting edges of formed instrument may be produced on one side only, that is to produce tooth spaces it is necessary to cut preliminary space and then to produce space finishing at one and then at the other side. Thus, in this case it is necessary to design two instruments for finishing treatment of one preliminary treated space and then the other one.

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