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FINITE-DIFFERENCE APPROXIMATION AT DECISION OF WAVE EQUATIONS OF WORKS OF DRILL STRING

The mathematical models of many physical situations are described by means of differential equations in partials derivatives [1]. Many of these processes are related to the origin of vibrations in some environment and described by the wave equation which related to equations of hyperbolic type.

Independent variables in physical problems it is usually been time *t* and spatial coordinates, for example *x*. The decision of problem requires to be found in some area of change of independent variables *A(t, x, ...)*. The mathematical raising of problem contains differential equation and additional conditions allowing to distinguish an only decision among family of decisions of differential equation. Additional conditions are usually set on a border to the area of *A*.

Unfortunately, the analytical decision of equations of mathematical physics is possible only for a very limited number of problems. In most cases the decision of differential equations in partials derivatives is possible only with the using of numeral iteration methods.

The most widespread numeral method of decision of equations with partials(equations of mathematical physics) is a method of eventual differences or method of nets. A method supposes the discretisation of differential equations on the so-called rectangular coordinate scales and allows to take the approximate decision of equations in partials derivatives to the decision of the systems of algebraic equations.

Idea of method of nodes [2,3] consists of that instead of any continuous function *u(x, t)* we will examine a discrete function, that is certain in the nodes of grid Ω_h^τ, instead of derivatives of function we will examine their simplest differential approximations in the nodes of grid. What less size of *h* and τ, the more precisely a approximate decision will be the method of nodes. Most widespread variant of grid is a uniform grid.

We will enter the two-dimensional system of coordinates, setting aside the independent variable *x* on abscise axis, and for *y*-axes – independent variable of time *t*. In this case area *Ω*, in that it is necessary to find a decision, broken up on rectangular areas by direct, parallel to the axes of *x* and *t* (fig. 1). Nodes of grid $x_i = x_0 + i \cdot h$, *i=0, 1,...n;* $t_j = t_0 + j \cdot \tau$, *j=0, 1,...m,* where $h = \dfrac{x_n - x_0}{n}$, $\tau = \dfrac{t_n - t_0}{m}$ are steps of grid.

Then differential operator it is necessary to replace some differential operator, and also to execute the differential approximation for regional terms and for initial data.

After it a decision is taken to the decision of the algebraic system of equations. Thus, a problem about the numeral decision of linear differential equation is taken to the decision of the got algebraic system.

If the system of grid equations got thus is solvable, at least, on a shallow enough grid, that is to the grid with the thick location of nodes, and her decision at the unlimited growing of grid shallow approaches (meets) the decision of initial equation (problems), that decision got on any fixed grid and sets to the close decision of initial equation (problems).

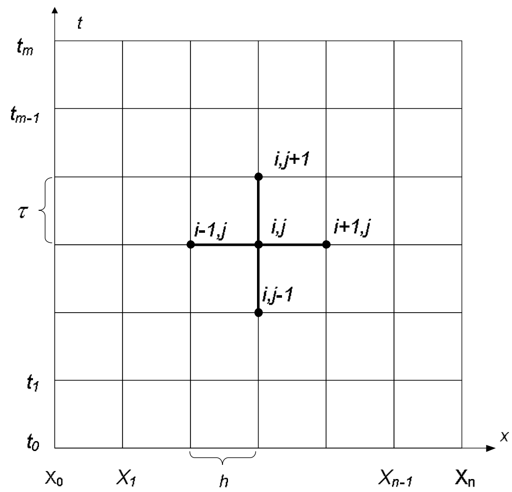


Figure 1. Finite-difference mesh

The schematic image of the nodes of differential grid which bound by equation of differential chart, names a differential template. A differential template can serve as a good reference-point at the choice of method of decision of differential chart and drafting of algorithm of decision.

If in a differential chart in every equation there is only one value of function on a next layer; than value easy obviously to express through the known values functions on this layer. Such charts are named obvious.

If in a chart in every equation there are a few unknown values of function on a new layer, that such charts are named non-obvious.

One of important descriptions of differential charts is her stability. For some of the differential equations small errors made on some stage of calculation in future strongly increase and do impossible the receipt of suitable result.

For steady charts at $h \to 0$ an error aspires to the eventual size that is linked with the error of initial data. If the error of initial data disappears; that steady charts allow to get high exactness of calculation, if the errors of rounding off are absent.

If a differential chart is unsteady, then at small steps error initial or any other data strongly increases during a calculation and at $h \to 0$ an error aspires to endlessness.

We will consider as an example the longitudinal vibrations of drilling string during the first stage of duty cycle of percussion mechanism. A percussion mechanism is used for liquidation of clips of drilling projectile. The first stage of work of percussion mechanism is an acceleration at that the energy accumulated in pipes passes to kinetic energy of drilling string. Efficiency of application of percussion mechanisms is determined by not only the size of crushing power but also wave processes excited in a string.

Wave equation of longitudinal vibrations of drilling string looks described by expression

$$U_{tt} - c^2 \cdot U_{xx} = 0 \qquad (1)$$

where c – speed of distribution of wave of resilient deformation in material of pipes.

$x = 0...L$, where L is length of drilling string .

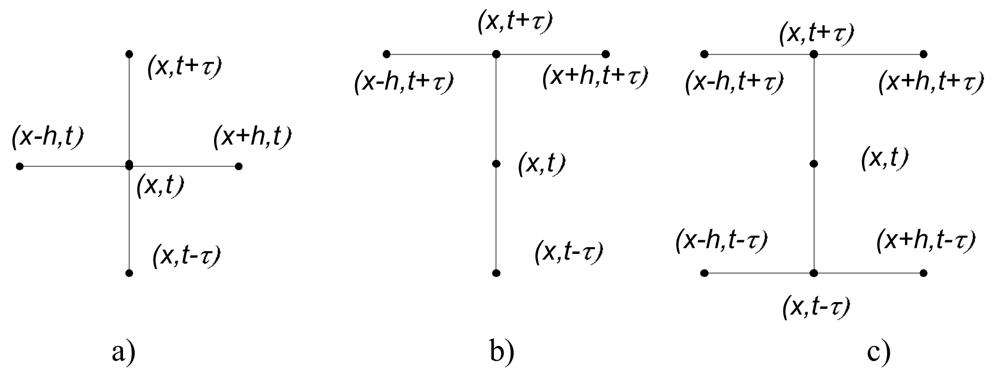
Time of completion of the first stage is from a condition $u(L, T_1) < U^I$, $U^I = U_{tensile} - U_{movement}$, where $U_{tensile}$ is initial tensile of lower section of string under act of the attached force, $U_{movement}$ - the movement of the lower section of the string, defined by the design of the mechanism.

The initial and boundary conditions for the first stage have the form

$$u(x,0) = \frac{Px}{EF}, \qquad u_t(x,0) = 0, \quad u_x(0,t_1) = \frac{z}{EF} u\,(0,t_1) + \frac{M_1}{EF} u_{tt}(0,t_1) + \frac{P}{EF},$$

$$u_x(L,t_1) = -\frac{M_2}{EF} u_{tt}^I(L,t_1) \qquad (2)$$

For equation (1) for the record of differential operator it is necessary to use the values of net function in three moments of time $t - \tau$, t, $t + \tau$. In this case a minimum template is a five-point template(fig. 2).



Picture 2. Variants of templates of differential charts for decision of equation

We will consider a differential scheme of the cross (fig. 2,a). Partials derivatives we replace divide differences $U_{tt} = \frac{U_i^{j-1} - 2 \cdot U_i^j + U_i^{j+1}}{\tau^2}$, $U_{xx} = \frac{U_{i-1}^j - 2 \cdot U_i^j + U_{i+1}^j}{h^2}$

Differential equations for the internal nodes of grid will have form

$$\frac{U_i^{j-1} - 2 \cdot U_i^j + U_i^{j+1}}{\tau^2} - c^2 \cdot \frac{U_{i-1}^j - 2 \cdot U_i^j + U_{i+1}^j}{h^2} = 0 \qquad (2)$$

Approximation of initial and regional conditions :

$$U_i^0 = \frac{Px_i}{(EF)}, \quad \frac{U_i^1 - U_i^0}{\tau} = 0, \quad \frac{U_1^j - U_0^j}{h} = \frac{z}{EF} U_0^j + \frac{M_1}{EF} \cdot \frac{U_o^{j+1} - 2 \cdot U_0^j + U_0^{j-1}}{\tau^2} + \frac{P}{EF}$$

$$\frac{U_n^j - U_{n-1}^j}{h} = -\frac{M_2}{EF} \cdot \frac{U_n^{j+1} - 2 \cdot U_n^j + U_n^{j-1}}{\tau^2} \qquad (3)$$

We will get an obvious finite-differential schema for this problem, where for each equations with number j all values of grid function are known, after an exception, that can be certain obviously from correlations (2).

In the beginning all values settle accounts and on zero and first layers at $j = 0$ and $j = 1$. Settle accounts of value of the following then. At the calculation of next layer on x values are determined U_0^j and U_n^j.

A condition of stability of numeral decision is a number Kuranta $\frac{c \cdot \tau}{h} \leq 1$ [2].

If differential operator on a spatial variable to approximate the relation of eventual differences on a temporal layer $U_{xx} = \frac{U_{i-1}^{j+1} - 2 \cdot U_i^{j+1} + U_{i+1}^{j+1}}{h^2}$, that we will get non-obvious finite-differential schema for this problem (fig. 2, b)

$$\frac{U_i^{j-1} - 2 \cdot U_i^j + U_i^{j+1}}{\tau^2} - c^2 \cdot \frac{U_{i-1}^{j+1} - 2 \cdot U_i^{j+1} + U_{i+1}^{j+1}}{h^2} = 0 \qquad (4)$$

Initial and regional conditions are approximated also as in a schema "cross". This schema consists of equation(4) and initial and regional conditions (3) from a schema "cross". Value of net function on a temporal layer it is possible to get from a decision the systems of linear algebraic equations with a three-diagonal matrix. Deciding this system is possible some general numeral method. However a schema has a specific type and it can be effectively decided by the special method named the method of tuning-up.

For the template represented on a picture 2,c differential equations for the internal nodes of grid have the form

$$\frac{U_i^{j-1} - 2 \cdot U_i^j + U_i^{j+1}}{\tau^2} - c^2 \cdot \frac{\left(U_{i-1}^{j+1} - 2 \cdot U_i^{j+1} + U_{i+1}^{j+1}\right) + \left(U_{i-1}^{j-1} - 2 \cdot U_i^{j-1} + U_{i+1}^{j-1}\right)}{2 \cdot h^2} = 0 \qquad (4)$$

The amount of equations is equal to the amount of nodes of grid area Ω. The more nodes, that is, the more shallow grid, the less error of calculations. However it is necessary to remember that with reduction of step h the dimension of the system of equations increases and consequently, time of decision. Therefore at first it is recommended to execute trial calculations with the large enough step h, to estimate the got error of calculations, and only after to pass to more shallow net in all area or in some her part.

As is generally known, non-obvious schema practically are always steady. However their use is related to certain difficulties. The calculation of data on a next layer at times requires the decision of the linear system of equations. From the point of view of realization obvious differential schema are far simpler, but during their realization it is necessary to take into account their stability and rightness of choice of step of net.

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