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Excitation of Polyharmonic Vibrations in Single-Body Vibration Machine with Inertia Drive and Elastic Clutch

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Abstract—The authors analyze excitability of polyharmonic vibrations in a single-body vibration machine. The developed mathematical model of the vibration system accounts for an elastic component element included in the design of the unbalance vibration exciter drive. The operating limits, frequency content and effect of the main design factors on the flow data of the vibration machine are examined. It is found that superharmonic vibration largely contributes to the polyharmonic spectrum at certain frequencies, depending on the stiffness of the elastic component element of the clutch connecting the vibration exciter and the rotary drive.

Keywords: Vibration machine, unbalance vibration exciter, transmission, elastic clutch, spectrum, superharmonic vibrations.

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INTRODUCTION

Vibration machines with centrifugal vibration exciters have found wide application in different industries, e.g. in coal and ore mining, chemical industry. The most widely used are the single-body vibration machines with the far-above-resonance operating mode.

Currently researchers focus on polyharmonic vibrations to be used to intensify technological properties [1–3], for instance, in mineral dressing. Polyharmonic vibration spectrum of an operating element of a machine will improve processing conditions in terms of both qualitative and quantitative data of the vibration machine performance.

Polyharmonic vibrations are excited in linear and nonlinear dynamic systems. Although limited, the application area of nonlinear vibrations is vividly studied in many countries.

Higher attention is attracted to the vibration machines with excited sub- and superharmonic stable resonance vibrations, using nonlinear perturbances and nonlinear-parametric properties of elastic elements [4].

It is known that the harmonic disturbing force effect on the system with nonlinear restoring force generates harmonic vibrations with the excitation frequency ω and vibrations with the frequency $m\omega$, divisible by the excitation frequency (superharmonic vibrations), or with the frequency ω/n (subharmonic vibrations), where m and n are integers [5].

Operational behavior of a superharmonic centrifugal vibration drive may be based on the properly enhanced nonuniformity of rotation of unbalances. In real-life conditions, angular velocity of rotation of centrifugal vibration exciters is not constant within a vibration cycle, e.g. [6]. Nonuniform rotation of unbalances can be caused by: nonuniform gravitation moment relative to rotation axis if the axis is not vertical; acceleration of axis of rotation that moves along with the housing; change of rotating resistance. Angular velocity of an unbalance is the sum of a constant (averaged) and infinite series of even harmonics; acceleration of the housing is the sum of an infinite series of odd harmonics, and amplitude of the harmonics decreases with their number. Ordinary vibration exciters have odd harmonics of small amplitude. For instance, amplitude of the third harmonic of acceleration (the largest amplitude of the ultraharmonics amplitudes) is hundred or dozen times smaller than the first harmonic amplitude [5].

Nearly all Russian and foreign researchers assume infinite-stiff connection between the electric motor rotor and unbalance vibration exciter rotor, and, based on that, their angular velocities are assumed equal. However, this assumption fits with only centrifugal vibration exciters of the type of motor–vibrator. If the system has, for example, an elastic clutch, the overall motion of the vibration machine parts to be described requires addition of differential equation of the relevant type motor and equation of the relevant type clutch motion [7].

In the present authors' opinion, inertia and properly selected elastic parts in transmission of a centrifugal vibration drive enables enhancing superharmonic vibrations of an unbalance vibration exciter and gaining efficient contribution of supreme frequencies in the spectrum of polyharmonic vibration of an operating element.

1. GOAL AND OBJECTIVES

The goal of this study is to determine conditions for polyharmonic vibrations in inertia vibration machines with an unbalance vibration exciter and an elastic clutch in the drive. To reach the goal, it is required to:

—develop the mathematical model of the vibration system, considering the elastic element in the drive of the unbalance vibration exciter;

— analyze the domain of the working modes, frequency content and conditions of polyharmonic vibrations;

—study the effect of the main design parameters of the vibration system on the performance of the vibration machine.

2. SCOPE AND RESULTS OF WORK

The single-body vibration machine (Fig. 1) has operating element *1* (for instance, inertia screen box) mounted (or suspended) on vibration absorbers *2*, with attached centrifugal unbalance vibration exciter *5* rotated on bearings *6* by electric motor *3* via elastic clutch *4*.

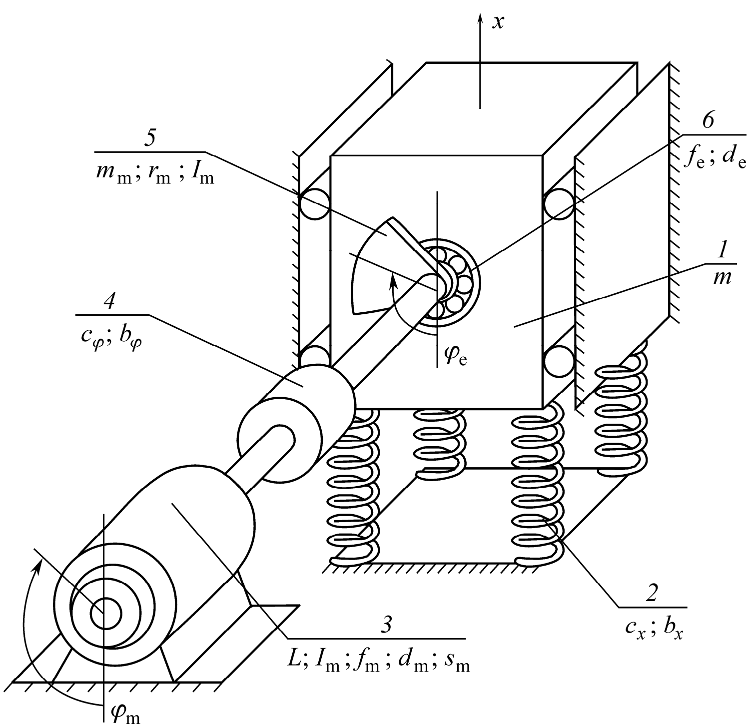

Fig. 1. Calculation scheme for vibration machine: *1*—operating element; *2*—vibration absorbers; *3*—electric motor; *4*—elastic clutch; *5*—unbalance; *6*—bearing.

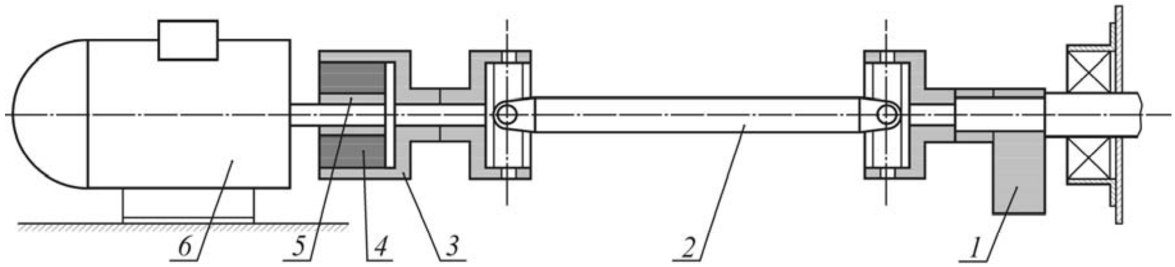

Fig. 2. Schematic circuit of transmission in the drive of vibration exciter: *1*—unbalance vibration exciter; *2*—cardan shaft; *3*—becket; *4* —elastic element of clutch; *5*—sleeve; *6*—electric motor.

A schematic circuit of the transmission in the drive of the inertia vibration machine is shown in Fig. 2. Electric motor *6* rotates centrifugal unbalance vibration exciter *1* with the help of cardan shaft and elastic clutch composed of becket *3*, elastic element *4* and sleeve *5*, mounted on the electric motor shaft.

The calculation scheme and motion equations of movable masses of vibration machine include the conventional assumptions that:

—operating element is a perfectly rigid body;

—axis of rotation of unbalances is horizontal;

—deformation of elastic elements is linear;

—accounting for internal resistances in the elastic elements is based on the viscous friction hypothesis;

—lateral rigidity of the clutch is negligible due to smallness.

Introduce the following: m—mass of the operating element; m_{e}, I_{e}—mass and inertia moment of unbalance parts of the vibrator exciter, respectively; I_{m}—central moment of inertia of rotor in the electric motor of the drive of the vibration exciter; r_{e}—excentricity of the unbalance vibration exciter (distance from the axis of rotation to the center of the unbalance masses); c_x—overall coefficient of rigidity of the vibration absorbers in the direction of the axis x; c_φ—coefficient of rigidity of the elastic clutch in the direction of the axis φ (torsion); b_x, b_φ—coefficients of viscous resistance of the vibration absorber and elastic clutch, respectively.

Current position of the operating element is defined by the absolute shift x of the center of masses off the static equilibrium position; position of the unbalance of centrifugal vibration exciter is defined by the angle of deflection, φ_{m}, of the excentricity vector $\vec{r}_{\mathrm{m}}$ of the unbalance mass center off the stable equilibrium; the angle of the elastic clutch torsion, $\varphi_{\mathrm{m}} - \varphi_{\mathrm{e}}$. The generalized coordinates q_i are assumed to be the absolute coordinates: $q_1 = x$, $q_2 = \varphi_{\mathrm{e}}$, $q_2 = \varphi_{\mathrm{m}}$.

The motion equations are constructed in the form of the second order Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} + \frac{\partial \Pi}{\partial q_i} = Q_{q_i},$$

where T, Π, Φ—respectively, the kinetic and potential energies and the dissipative function; q_i, $\dot{q}_i$ and Q_i— generalized coordinates, velocities and forces ($i = 1, 2, 3$).

The kinetic energy is:

$$T = \frac{1}{2}(M\dot{x}^2 + I\dot{\varphi}_{\mathrm{e}}^2 + I_{\mathrm{m}}\dot{\varphi}_{\mathrm{m}}^2) + m_{\mathrm{e}} r_{\mathrm{e}} \dot{\varphi}_{\mathrm{e}} \dot{x} \sin\varphi_{\mathrm{e}},$$

where $M = m + m_{\mathrm{e}}$; $I = I_{\mathrm{e}} + m_{\mathrm{e}} r_{\mathrm{e}}^2$.

The potential energy consists of the energy of deformation of the vibration absorbers, energy of torsion of the elastic clutch and the energy of position of the vibration exciter unbalances, and can be expressed by:

$$\Pi = \frac{1}{2}[c_x x^2 + c_\varphi(\varphi_m - \varphi_e)^2] + g m_e r_e (1 - \cos\varphi_e),$$

where g—acceleration of gravity.

The dissipative function is:

$$\Phi = \frac{1}{2}[b_x \dot{x}^2 + b_\varphi(\dot{\varphi}_m - \dot{\varphi}_e)^2],$$

where $b_x = \mu_x c_x$, $b_\varphi = \mu_\varphi c_\varphi$; μ_x, μ_φ—coefficients of resistance of material the elastic elements of the vibration absorbers and clutch are made of.

The generalized forces in the lines of x, φ_e, φ_m are, respectively:

$$Q_x = 0, \quad Q_{\varphi_e} = -R_e(\dot{\varphi}_e), \quad Q_{\varphi_m} = -L(\dot{\varphi}_m) - R_m(\dot{\varphi}_m),$$

where $L(\dot{\varphi}_m)$—moment developed by the electric motor of the drive of the vibration exciter.

Assuming the excentricity of the unbalance vibration exciter is much higher than the vibration amplitude of the operating element and the angular velocity sign is constant, the moment of friction in the bearings of the vibration exciter can be found from the known expression:

$$R_e(\dot{\varphi}_e) = 0.5 f_e m_e r_e d_e \dot{\varphi}_e^2,$$

where f_e—friction factor reduced to the inner diameter of the bearing; d_e—diameter of the inner race of the bearing.

The moment of friction in the bearings of the electric motor of the vibration exciter drive is:

$$R_m(\dot{\varphi}_m) = 0.5 f_m s_m d_m \dot{\varphi}_m^2,$$

where f_m—friction factor reduced to the inner diameter of the electric motor bearing; s_m—residual disbalance (mass static moment) of the electric motor rotor; d_m—diameter of the inner race of the electric motor bearing.

Asynchronous electric motors with short-circuited rotors are mostly used as drives in centrifugal vibration exciters of inertia vibration machines. The method of setting the drive torque of the electric motor governs the motor idealization degree and the domain of the vibration machine modes. It is known that static characteristics of the asynchronous electric motor of the centrifugal vibration exciter drive can be used to study motion of vibrating masses of vibration machine near the steady-state regime.

The present study is unconcerned with starting regimes, therefore, we use the classical model of the asynchronous electric motor based on Kloss's formula [8]:

$$L(\dot{\varphi}_m) = \frac{2L_c}{s/s_c + s_c/s},$$

where $s = (\omega_s - \dot{\varphi}_m)/\omega_s$, $s_c = (\omega_s - \omega_c)/\omega_s$—respectively, the current and critical slip of the electric motor rotor; ω_s, ω_c—synchronous and critical angular velocities of rotation of the motor; L_c—critical moment of the motor; $\dot{\varphi}_m$—rotation velocity of the motor, close to the nominal rotation velocity ω_n.

The values of L_c and s_c at ω_c are found using the motor specification data:

$$L_c = b_r L_c, \quad s_c = s_n(b_r + \sqrt{b_r^2 + 1}),$$

where b_r—multiplicity of the maximum moment of the motor; s_n—nominal slip; L_n—nominal moment of the motor.

Then, the analyzed vibration system can be described using the differential equations below:

$$M\ddot{x} + c_x(\mu_x \dot{x} + x) = -m_e r_e(\ddot{\varphi}_e \sin\varphi_e + \dot{\varphi}_e^2 \cos\varphi_e),$$

$$I\ddot{\varphi}_e + c_\varphi[\mu_\varphi(\dot{\varphi}_e - \dot{\varphi}_m) + (\varphi_e - \varphi_m)] = -m_e r_e[(\ddot{x} + g)\sin\varphi_e + 0.5 f_e d_e \dot{\varphi}_e^2)],$$

$$I_m\ddot{\varphi}_m + c_\varphi[\mu_\varphi(\dot{\varphi}_m - \dot{\varphi}_e) + (\varphi_m - \varphi_e)] = L(\dot{\varphi}_m) - 0.5 f_m s_m d_m \dot{\varphi}_m^2.$$

The mathematical calculations used the program package Matlab (Fig. 3) to analyze the case where movement of the operating element (m = 2000 kg; $c_x = 0.2$ MN/m; $\mu_x = 8 \cdot 10^{-4}$ s) was started by the unbalance exciter ($I_e = 0.14$ kg·m^2; $m_e = 50$ kg; $r_e = 0.24$ m; $\mu_\varphi = 2 \cdot 10^{-4}$ s; $f_e = 0.003$; $d_e = 0.1$ m) rigidly or elastically (elastic clutch) connected with the shaft of the electric motor ($f_m = 0.0013$; $s_m = 0.0001$ kg·m; $d_m = 0.04$ m; $I_m = 0.065$ kg·m^2; $P_n = 5$ kW; $\omega_n = 93.2$ rad/s; $L_n = 53.6$ N·m; $L_c = 126.2$ N·m; $s_n = 0.11$; $s_c = 0.853$).

Figure 3a shows the vibration records of displacements, velocities and accelerations of the operating element, angular velocities of rotation of the vibration exciter unbalances and the electric motor rotor, as well as the moment of the asynchronous electric motor of the vibration exciter drive in case of the rigid connection of the shafts of the exciter and motor.

The amplitude spectrum of vibro-displacements in Fig. 3b allows picking low-frequency subharmonic components in the vibration spectrum of the operating element. The vibro-velocity spectrum highlights vibrations in the area of the first harmonic; the vibro-acceleration spectrum—the high-frequency superharmonic components. So, the vibro-acceleration spectrum is to be used in the analysis of the high-frequency vibrations.

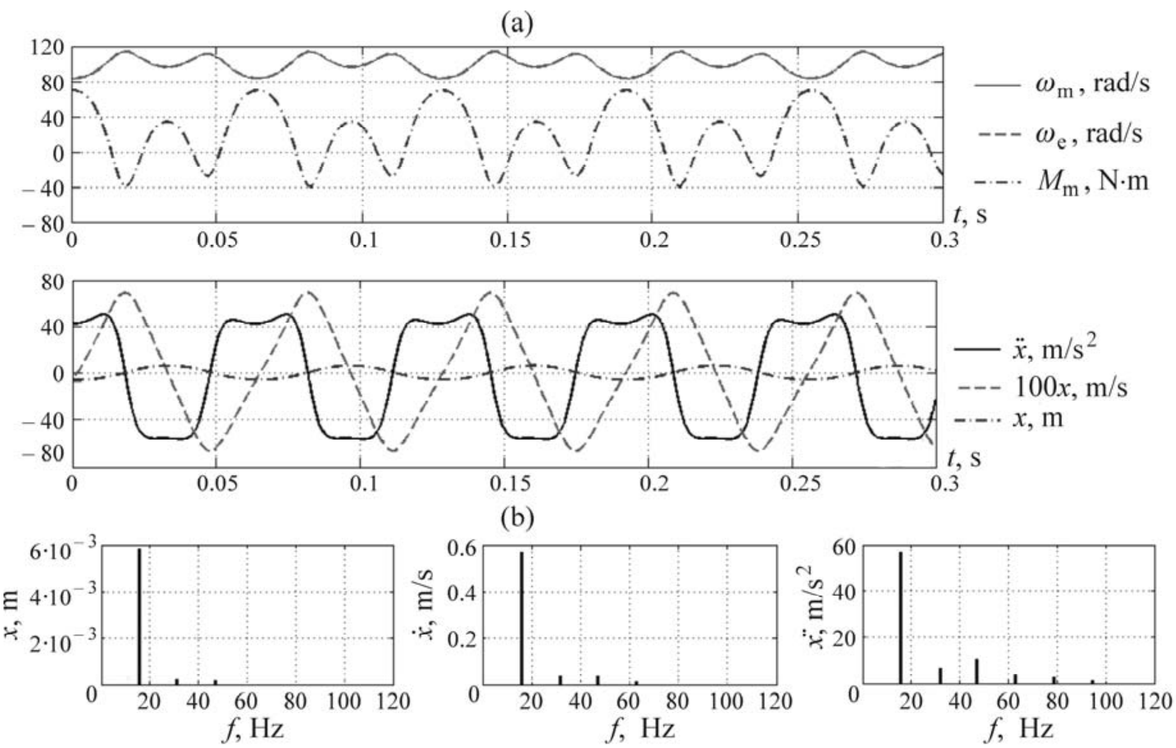

Fig. 3. (a) The vibration records of the steady-state operation in case of rigid connection of the motor and exciter shafts ($c_\varphi \to \infty$) and (b) the amplitude spectra of the vibro-displacement, vibro-velocity and vibro-acceleration of the operating element.

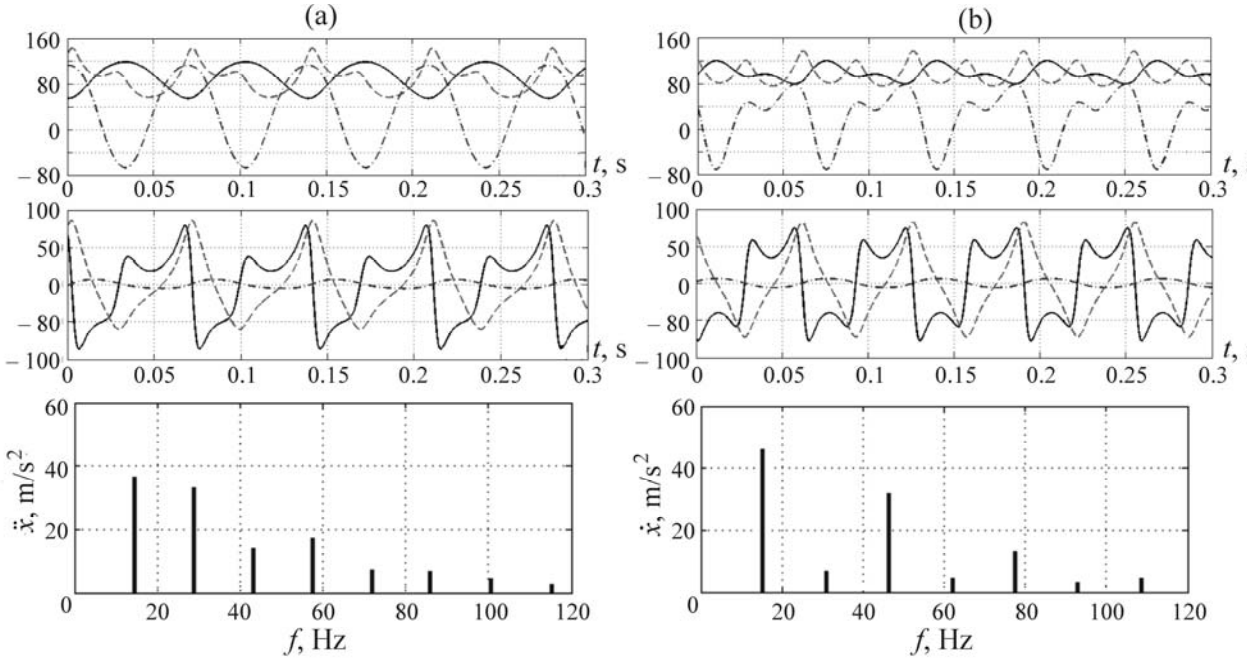

Fig. 4. The vibration records and the amplitude spectra of the operating element vibro-acceleration at various rigidity coefficients of the clutch: (a) $c_{\varphi} = 350$ N·m/rad; (b) $c_{\varphi} = 787.5$ N·m/rad (for identification of the curves refer to Fig. 3).

By the frequency analysis, the frequency spectrum, apart from the basic excitation frequency of 15.8 Hz (Fig. 3b), contains the superharmonic components (31.6; 47.4; 63.2 Hz, etc.). The vibration amplitudes are however not more than 12% (second harmonic with 31.6 Hz) and 19% (third harmonic with 47.4 Hz) of the first harmonic amplitude. The contribution of the higher harmonics in the vibration spectrum of the operating element is even less.

Figure 4 presents the results of the spectrum analysis performed for various rigidity elastic clutches of the vibration exciter drive transmission, c_{φ}. The contribution of the superharmonic vibrations in the polyharmonic spectrum at definite frequencies versus rigidity markedly grows. For instance, at $c_{\varphi} = 350$ N·m/rad (Fig. 4a) the second harmonic vibration amplitude was 92% and the third harmonic vibration amplitude was 39% of the first harmonic amplitude. At $c_{\varphi} = 787.5$ N·m/rad (Fig. 4b) the second harmonic amplitude was 15% and the third harmonic amplitude was 69% of the first harmonic amplitude.

Thus, it is possible to set mechanically the vibration spectrum of the operating element of a vibration machine by varying torsional rigidity of the elastic element in the clutch of the centrifugal vibration exciter drive transmission. It is advisable to make the elastic element of rubber. In this case, static rigidity of the elastic element in coaxial torsion can be determined as for a rubber–metal hinge [5]:

$$c_{\varphi}^{st} = \frac{4\pi\sigma l r_1^2 r_2^2}{r_2^2 - r_1^2},$$

where r_1 and r_2—maximum and minimum radii of the rubber element, respectively; l—rubber element length; σ—shear modulus of rubber.

The dynamic rigidity to be used in the modeling source data for the frequency range 150–180 rad/s is recommended to be found as [5]:

$$c_{\varphi} = 1.1 c_{\varphi}^{st}.$$

The desired rigidity of the elastic clutch in the body of the transmission of the vibration exciter drive within an inertia vibration machine can be obtained by changing geometry of elastic elements made of the same rubber, or from using different rubber elements of the same geometry.

CONCLUSIONS

The carried out theoretical investigations have allowed for:

—the mathematical model of the single-body vibration system with the inertia unbalance vibration exciter with the elastic clutch in the vibration drive transmission;

—finding that at definite rigidity and inertia characteristics of the transmission parts, it is possible to enhance contribution of the superharmonic vibrations in the overall vibration spectrum of the operating element of the vibration machine;

—using the obtained relationships in designing and upgrading vibration machines for coal and ore mining industries (e.g. screens, crushers, mills, separators, conveyors, feeders, etc.).

It is thought actual to continue the research in the laboratory and mine conditions.

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