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SUPERHARMONIC RESONANCES IN TWO-MASSES VIBRATING MACHINES

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Abstract. *Two masses vibrating machine with inertial exciter and polynomial characteristic of elastic ties is considered. Its mathematical model is constructed and the stationary motions in frequency zone located between two natural ones are studied. With use of complex form of harmonic balance method the analysis of its dynamics is reduced to the solving of complex system of algebraic equations. Its subsequent numerical solution under changing parameters of the system gives an opportunity to build the bifurcation curves and discover pure combination resonances of different orders. One of the most suitable among them from the practical point of view is the resonance of the order 2:1 which, by this reason, is the focus of the article. With the help of original software the oscillations of vibrating machine are analyzed, bifurcation curves and spectral properties are studied. There are established certain correlations between parameters, which are necessary for designing of such machines, one approach for projecting of nonlinear elastic ties is discussed.*

1 INTRODUCTION

To date a great number of investigations concerning the use of combination resonances was fulfilled for one masses vibrating machines [1, 2]. These investigations are being continued till now [3, 4]. Here we consider two-masses vibrating machine and study its oscillations in frequency range located between the natural ones that is in, so-called, “antiresonance” zone. Our previous investigations show [5] that one of the most suitable for practical realization is the resonance of the order 2:1. Our purpose here is to get some instructions for choosing parameters of the machine and discuss approaches to designing of its elastic ties. By our opinion this is one of the main problems in creation of such machines.

2 MODEL, METHOD OF INVESTIGATION

The principal scheme of vibromachine is represented in Figure 1. There m_1 is mass of a frame, m_2 – of a box, m_0 – of unbalance masses, characteristics of the main and supported elastic ties are $f_m(x) = k_1\,x + k_2\,x^2 + k_3\,x^3$ and $f_s(x) = k_{01}\,x + k_{02}\,x^2 + k_{03}\,x^3$ correspondently, resistance forces are $f_{r.m.}(x) = \mu(k_1' + k_2'\,x + k_3'\,x^2)\dot{x}$ and $f_{s.m.}(x) = \mu(k_{01}' + k_{02}'\,x + k_{03}'\,x^2)\dot{x}$, ω – rotation speed of the exciter, r is its eccentricity.

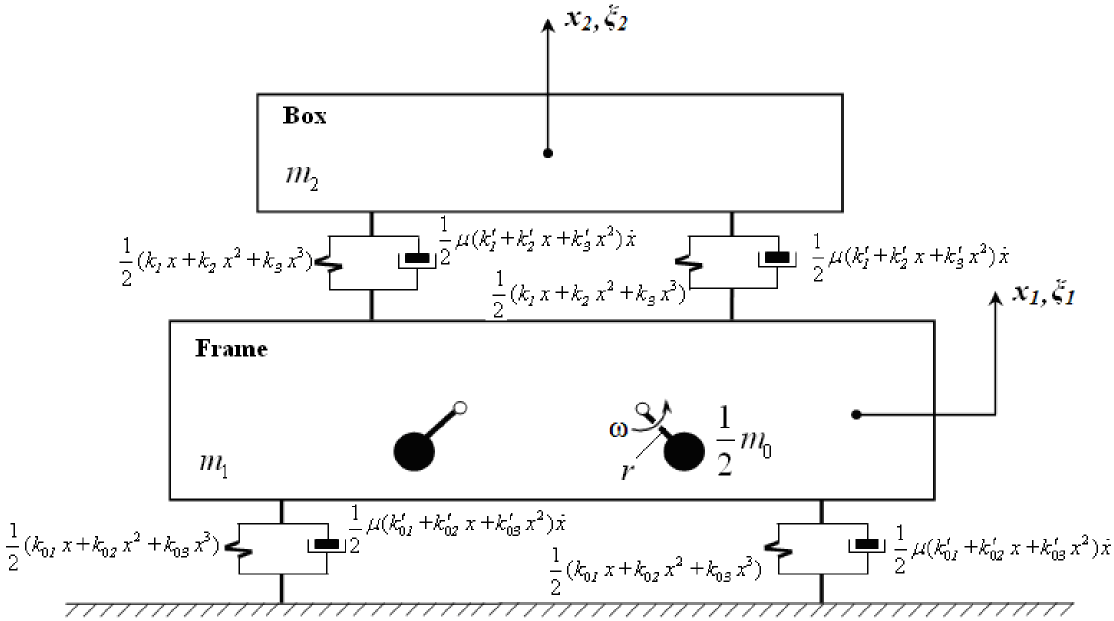

Figure 1: Principal scheme of the vibromachine.

In dimensionless form the motion of the machine is described by the following equations

$$\begin{cases} \dfrac{d^2\xi_1}{d\tau^2} + b_{101}\dfrac{d\xi_1}{d\tau} + b_{102}\,\xi_1\dfrac{d\xi_1}{d\tau} + b_{103}\,\xi_1^2\dfrac{d\xi_1}{d\tau} + b_{11}\dfrac{d\xi}{d\tau} + b_{12}\,\xi\dfrac{d\xi}{d\tau} + b_{13}\,\xi^2\dfrac{d\xi}{d\tau} + \\ \quad + k_{101}\,\xi_1 + k_{102}\,\xi_1^2 + k_{103}\,\xi_1^3 + k_{11}\,\xi + k_{12}\,\xi^2 + k_{13}\,\xi^3 = P_1\eta^2\cos\eta\,\tau, \\ \dfrac{d^2\xi}{d\tau^2} + b_{201}\dfrac{d\xi_1}{d\tau} + b_{202}\,\xi_1\dfrac{d\xi_1}{d\tau} + b_{203}\,\xi_1^2\dfrac{d\xi_1}{d\tau} + b_{21}\dfrac{d\xi}{d\tau} + b_{22}\,\xi\dfrac{d\xi}{d\tau} + b_{23}\,\xi^2\dfrac{d\xi}{d\tau} + \\ \quad + k_{201}\,\xi_1 + k_{202}\,\xi_1^2 + k_{203}\,\xi_1^3 + k_{21}\,\xi + k_{22}\,\xi^2 + k_{23}\,\xi^3 = P_2\eta^2\cos\eta\,\tau, \end{cases} \tag{1}$$

where $b_{101}=\frac{\mu k'_{01}}{m'_1\omega_1}$, $b_{102}=\frac{\mu k'_{02}\Delta}{m'_1\omega_1}$, $b_{103}=\frac{\mu k'_{03}\Delta^2}{m'_1\omega_1}$, $b_{11}=-\frac{\mu k'_1}{m'_1\omega_1}$, $b_{12}=-\frac{\mu k'_2\Delta}{m'_1\omega_1}$, $b_{13}=-\frac{\mu k'_3\Delta^2}{m'_1\omega_1}$, $b_{201}=-\frac{\mu k'_{01}}{m'_1\omega_1}$, $b_{202}=-\frac{\mu k'_{02}\Delta}{m'_1\omega_1}$, $b_{203}=-\frac{\mu k'_{03}\Delta^2}{m'_1\omega_1}$, $b_{21}=\frac{\mu(m'_1+m_2)k'_1}{m'_1 m_2\omega_1}$, $b_{22}=\frac{\mu(m'_1+m_2)k'_2\Delta}{m'_1 m_2\omega_1}$, $b_{23}=\frac{\mu(m'_1+m_2)k'_3\Delta^2}{m'_1 m_2\omega_1}$, $k_{101}=\frac{k_{01}}{m'_1\omega_1^2}$, $k_{102}=\frac{k_{02}\Delta}{m'_1\omega_1^2}$, $k_{103}=\frac{k_{03}\Delta^2}{m'_1\omega_1^2}$, $k_{11}=-\frac{k_1}{m'_1\omega_1^2}$, $k_{12}=-\frac{k_2\Delta}{m'_1\omega_1^2}$, $k_{13}=-\frac{k_3\Delta^2}{m'_1\omega_1^2}$, $k_{201}=-\frac{k_{01}}{m'_1\omega_1^2}$, $k_{202}=-\frac{k_{02}\Delta}{m'_1\omega_1^2}$, $k_{203}=-\frac{k_{03}\Delta^2}{m'_1\omega_1^2}$, $k_{21}=\frac{k_1(m'_1+m_2)}{m'_1 m_2\omega_1^2}$, $k_{22}=\frac{k_2(m'_1+m_2)\Delta}{m'_1 m_2\omega_1^2}$, $k_{23}=\frac{k_3(m'_1+m_2)\Delta^2}{m'_1 m_2\omega_1^2}$, $P_1=\frac{m_0 r}{m'_1\Delta}$, $P_2=-P_1$, $m'_1=m_0+m_1$, $\eta=\omega/\omega_1$.

Using harmonic balance method we find the periodic solutions in the form

$$\xi_1(\tau)=\sum_{n=-N}^{N}c_n^{(1)}e^{in\eta\tau},\ \xi(\tau)=\sum_{n=-N}^{N}c_n e^{in\eta\tau}, \tag{2}$$

where N is the number of considered harmonics. After substitution (2) into (1) and equating coefficients we get the system of polynomial equations with respect to c_n

$$\begin{cases}
(k_{101}+b_{101}i\eta n-\eta^2n^2)c_n^{(1)}+\sum\limits_{j=-N}^{N}c_j^{(1)}c_{n-j}^{(1)}(k_{102}+b_{102}i\eta(n-j))+\\
\quad+\sum\limits_{j=-N}^{N}\sum\limits_{m=-N}^{N}c_j^{(1)}c_m^{(1)}c_{n-j-m}^{(1)}(k_{103}+b_{103}i\eta(n-j-m))+\\
\quad+(k_{11}+b_{11}i\eta n)c_n+\sum\limits_{j=-N}^{N}c_jc_{n-j}(k_{12}+b_{12}i\eta(n-j))+\\
\quad+\sum\limits_{j=-N}^{N}\sum\limits_{m=-N}^{N}c_jc_mc_{n-j-m}(k_{13}+b_{13}i\eta(n-j-m))=\begin{cases}P_1\eta^2/2, & n=\pm1\\ 0, & n\neq\pm1\end{cases},\\
(k_{201}+b_{201}i\eta n)c_n^{(1)}+\sum\limits_{j=-N}^{N}c_j^{(1)}c_{n-j}^{(1)}(k_{202}+b_{202}i\eta(n-j))+\\
\quad+\sum\limits_{j=-N}^{N}\sum\limits_{m=-N}^{N}c_j^{(1)}c_m^{(1)}c_{n-j-m}^{(1)}(k_{203}+b_{203}i\eta(n-j-m))+\\
\quad+(k_{21}+b_{21}i\eta n-\eta^2n^2)c_n+\sum\limits_{j=-N}^{N}c_jc_{n-j}(k_{22}+b_{22}i\eta(n-j))+\\
\quad+\sum\limits_{j=-N}^{N}\sum\limits_{m=-N}^{N}c_jc_mc_{n-j-m}(k_{23}+b_{23}i\eta(n-j-m))=\begin{cases}P_2\eta^2/2, & n=\pm1\\ 0, & n\neq\pm1\end{cases},
\end{cases} \tag{3}$$

where $n, n-j, n-j-m\in[-N,N]$.

Supposing the trigonometric view of the solutions in the form $\sum_{j=0}^{N} A_j \cos(j\eta\tau - \varphi_j)$, where $\varphi_j \in [-\pi, \pi)$, you can get the amplitude of the harmonic $A_j = 2\sqrt{c_j\, c_{-j}}$ and its initial phase $\varphi_j = \arccos\dfrac{c_j + c_{-j}}{2\sqrt{c_j\, c_{-j}}}$ or $\varphi_j = -\arccos\dfrac{c_j + c_{-j}}{2\sqrt{c_j\, c_{-j}}}$, if $\left(\Im c_{-j} = 0 \ \wedge\ \Re c_{-j} < 0\right) \vee\ \Im c_{-j} < 0$.

Analysis of oscillations below is realized with the help of original software [6] in which construction of the bifurcation curves is based on solving of system (3). The points of bifurcation in it are found by the control of change of sign of the Jacobian of the system (3), stability of the solutions in the first approximation is being analyzed with use of Floquet theory.

The effectiveness of vibrational processes, such as, moving and packing of granular material, depends upon asymmetry of laws of motion. For quasi harmonic regimes sometimes it is described as the ratio of peak values of acceleration. But for more complex types of the motion it is reasonable to use and more integrated approaches. By analogy with probability theory [7] and considering the law of motion or acceleration as a continuous random variable you can also enter for it an asymmetry coefficient $A_s = \mu_3 / \sigma^3$ as the ratio of the third central moment to the third power of standard deviation where

$$\mu_3 = \left(\int_0^T (x(t) - m)^3 dt\right) / T\,,\ \sigma^2 = \left(\int_0^T (x(t) - m)^2 dt\right) / T\,,\ m = \int_0^T x(t) dt / T\,, \tag{4}$$

here $x(t)$ is the law of motion or acceleration of period T. Impact of this factor for the effectiveness of the vibration process can be discovered in the subsequent experiments.

3 RESULTS, ITS ANALYSIS

Taking in mind here experimental machine values of some parameters were taken as follows $m_1 = 98\ kg$, $m_0 = 2\ kg$, $m_2 = 45\ kg$, $k_{01} = 2.4 \cdot 10^4\ \ N/m$, $k_1 = 0.6 \cdot 10^6\ \ N/m$, $m_0 \cdot r = (0.05 - 0.5)\ kg \cdot m$ then $\omega_1 = 12.8\ rad \cdot \sec^{-1}$. Presumable ranges of the others are $k_2, \in [-2, 0] \cdot 10^3 \cdot k_1$; $k_3, \in [0, 3] \cdot 10^6 \cdot k_1$, where k_2 and k_3 characterize the level of nonlinearity of the main elastic ties. The similar correlations were taken for parameters k_{02}, k_{03} of the supported elastic ties. Angular velocity of the exciter ω is supposed to be variable, parameters k'_i and k'_{0i} of resistance forces are dependent on the design of elastic ties, coefficient of dissipation in material of elastic ties is equal to $\mu = 8 \cdot 10^{-4}$ sec. Keeping below in mind the elastic ties of magnetic type we suppose $k'_i = k'_{0i} = 0$. Then in accordance to (1) the dimensionless values of the parameters are $b_{101} = 0.015$, $b_{102} = b_{103} = 0$, $b_{11} = -0.375$, $b_{12} = b_{13} = 0$, $k_{101} = 1.465$, $k_{102} = k_{103} = 0$, $k_{11} = -36.621$, $k_{12} / k_{11} \in [-2, 0]$, $k_{13} / k_{11} \in [0, 3]$, $P_1 \in [0.5, 5.0]$, $b_{201} = -b_{101}$, $b_{202} = b_{203} = 0$, $b_{21} = 1.208$, $b_{22} = b_{23} = 0$, $k_{201} = -k_{101}$, $k_{202} = k_{203} = 0$, $k_{21} = 118.001$, $k_{22} / k_{21} = k_{12} / k_{11}$, $k_{23} / k_{21} = k_{13} / k_{11}$, $P_2 = -P_1$.

Firstly we'll demonstrate the influence of each of free parameters upon behavior of the machine in the zone of the 2:1 resonance. Calculations and the solving of the system (3) have been done for five harmonics in the solutions (2), that is $N = 5$, but for simplification only three of them are shown in the presented figures.

3.1 The influence of the external force

It is demonstrated in Figure 2, where amplitude and phase-frequency characteristics (AFC and PFC) of the frame and the box and diagrams of acceleration of the box in certain points are given. Thin lines correspond there to unstable regimes.

It may be noted that under small values of exciting forces there exists only one superharmonic regime and it is globally stable (Figure 2a). But after increasing of it the number of superharmonic regimes may also increase (Figures 2b, 2c). In particular, for $P_1 = 5$ the number of such regimes is equal already to three (Figure 2c), two of them are stable and opposite (this fact is characterized by the shift of initial phases of even harmonics which is equal to pi), but the third one is unstable. Superharmonic oscillations become more intensive and take place for the higher frequencies of the exciting force. The ratio of the peak values of acceleration may be equal to 2 and even more, its asymmetry coefficient in indicated points $A_s = 0.95 \div 1.56$. From the practical point of view it is important to note that the spectral and phase composition of oscillations on the upward curve section is close to the recommended [7].

3.2 The influence of the nonlinearity of elastic ties

It is demonstrated in Figure 3. In this case the value of k_{13} / k_{11} equals consequently 0, 1.5, 3.0 the values of other parameters are $k_{12} / k_{11} = -1.0$ and $P_1 = 2.5$. The marked peculiarities, in main, remain valid here too. The only difference is that the intensity of oscillations does not practically change.

3.3 The influence of the asymmetry of elastic ties

It is shown in Figure 4. Here the value of k_{12} / k_{11} equals sequentially 0.0, -1.0, -2.0 and $k_{13} / k_{11} = 1.5$, $P_1 = 2.5$. Intensity of oscillations remains practically the same, but the frequency range of superharmonic oscillations becomes broader.

3.4 Recommendations

The desire to decrease energy consumption and ensure “natural” conditions of the starting causes the intention to reduce exciting forces and choose parameters of the machine which guarantee the global stability of the operating regime. The analysis which has been done above shows the expediency of choice of the parameters of the elastic ties close to their extreme values. Such opportunities are demonstrated in Figure 5. For values $P_1 > 1.5$ the opposite regimes may already appear. So, for $k_1 >> k_{01}$, at least for $k_1 = (10 \div 50) k_{01}$, the ranges

$$0.5 < \frac{m_0 r}{(m_0 + m_1)\Delta} < 1.5, \quad \frac{k_2 \Delta}{k_1} \cdot \frac{m_1}{m_0 + m_1} \approx -2, \quad \frac{k_3 \Delta^2}{k_1} \approx 3,$$

may be considered as the preliminary intervals for choosing parameters of the machine.

4 DESIGN OF ELASTIC TIES

Requirements for elastic elements of vibromachines often have quite opposite character. Thus, vibration isolators should be of less stiffness in order to minimize dynamic loads on the foundation. On the contrary the main elastic elements need to have relatively big stiffness in order to ensure operation of the machine in the resonant or antiresonant mode or close to it. And usually it is quite difficult to provide these requirements simultaneously. Partially this

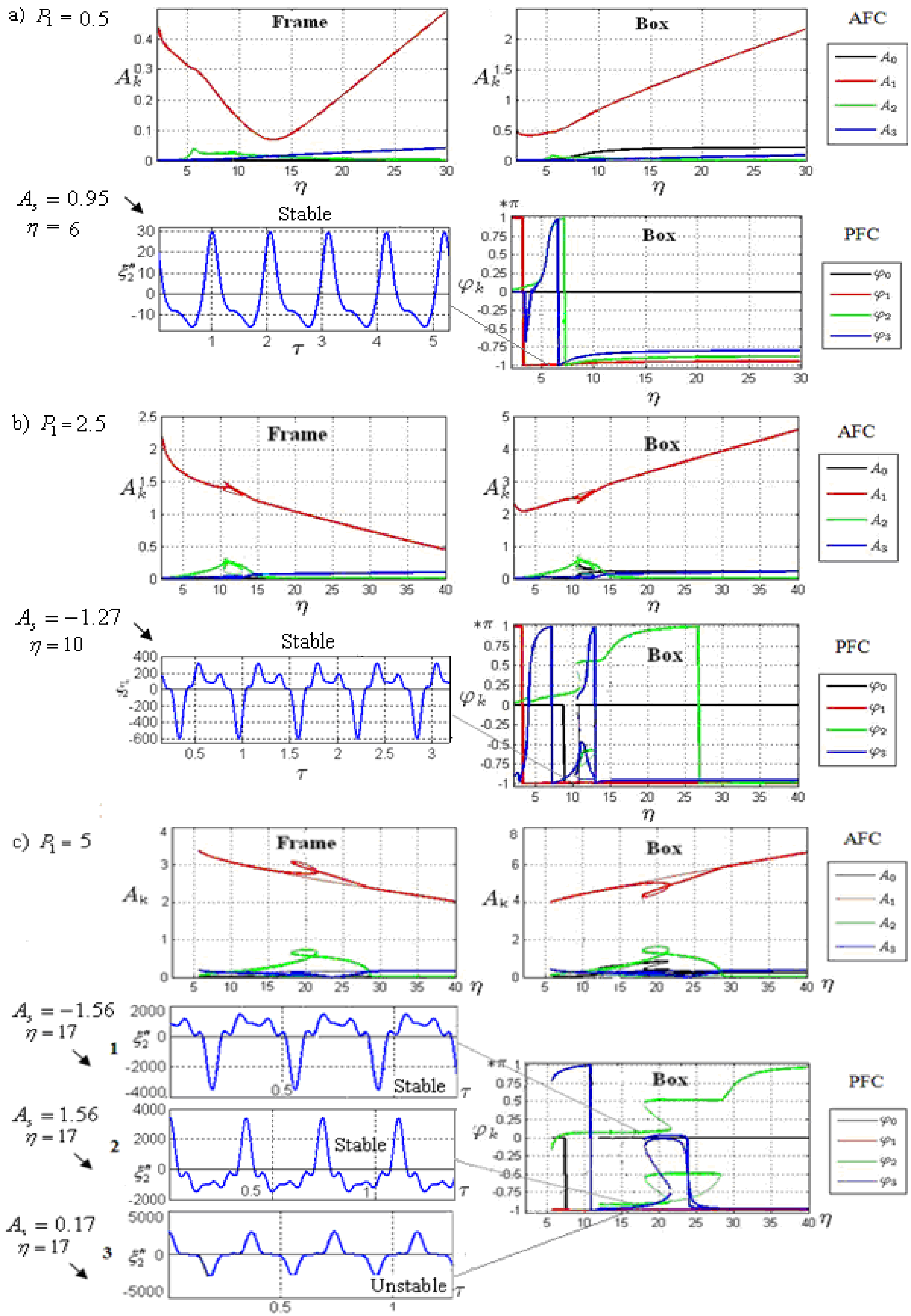

Figure 2: The effect of the external force for $k_2 / k_1 = -1.0$, $k_3 / k_1 = 1.5$.

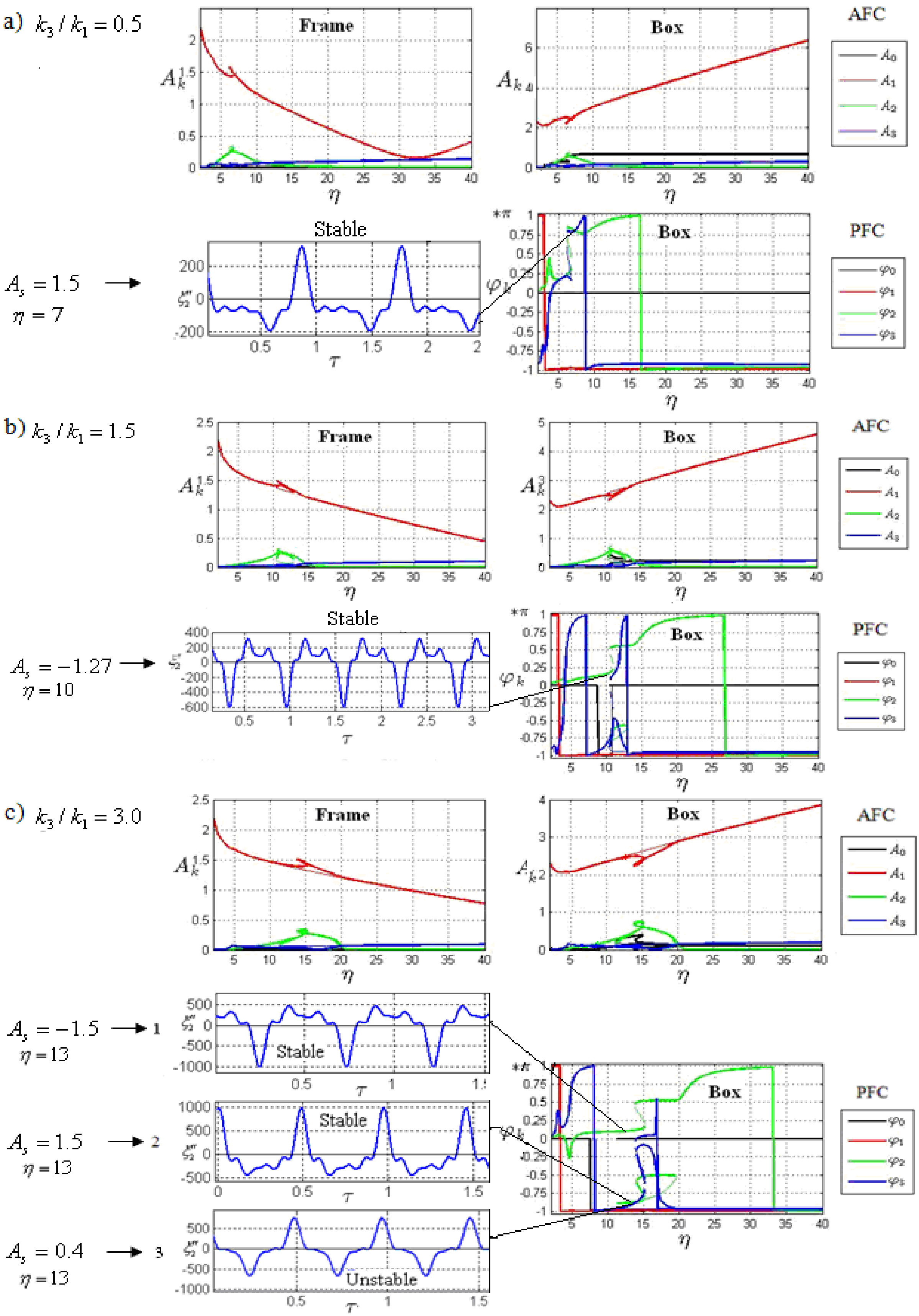

Figure 3: The effect of the nonlinearity of elastic ties for $P_1 = 2.5$, $k_2 / k_1 = -1$.

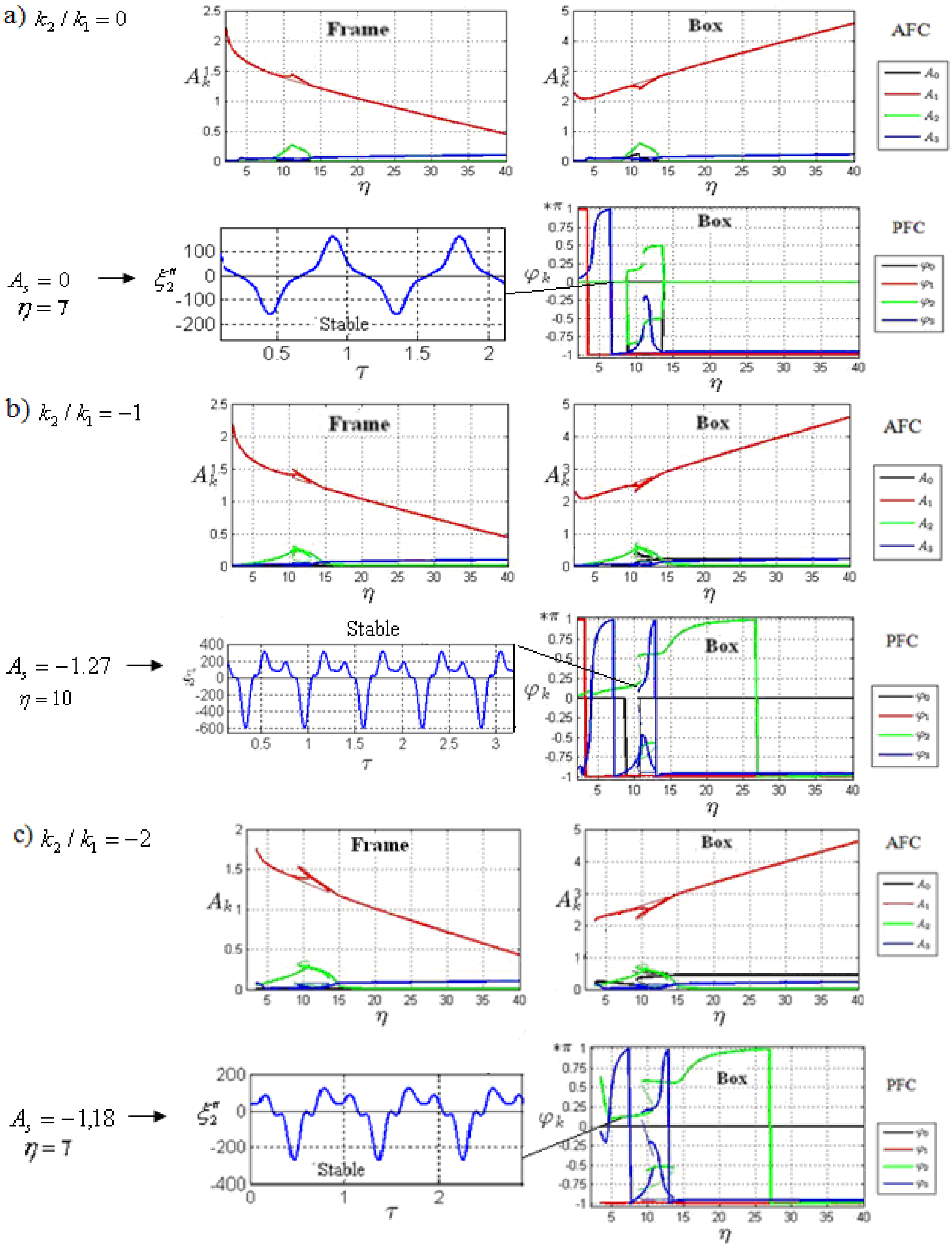

Figure 4: The effect of the asymmetry of elastic ties for $P_1 = 2.5$, $k_3 / k_1 = 1.5$.

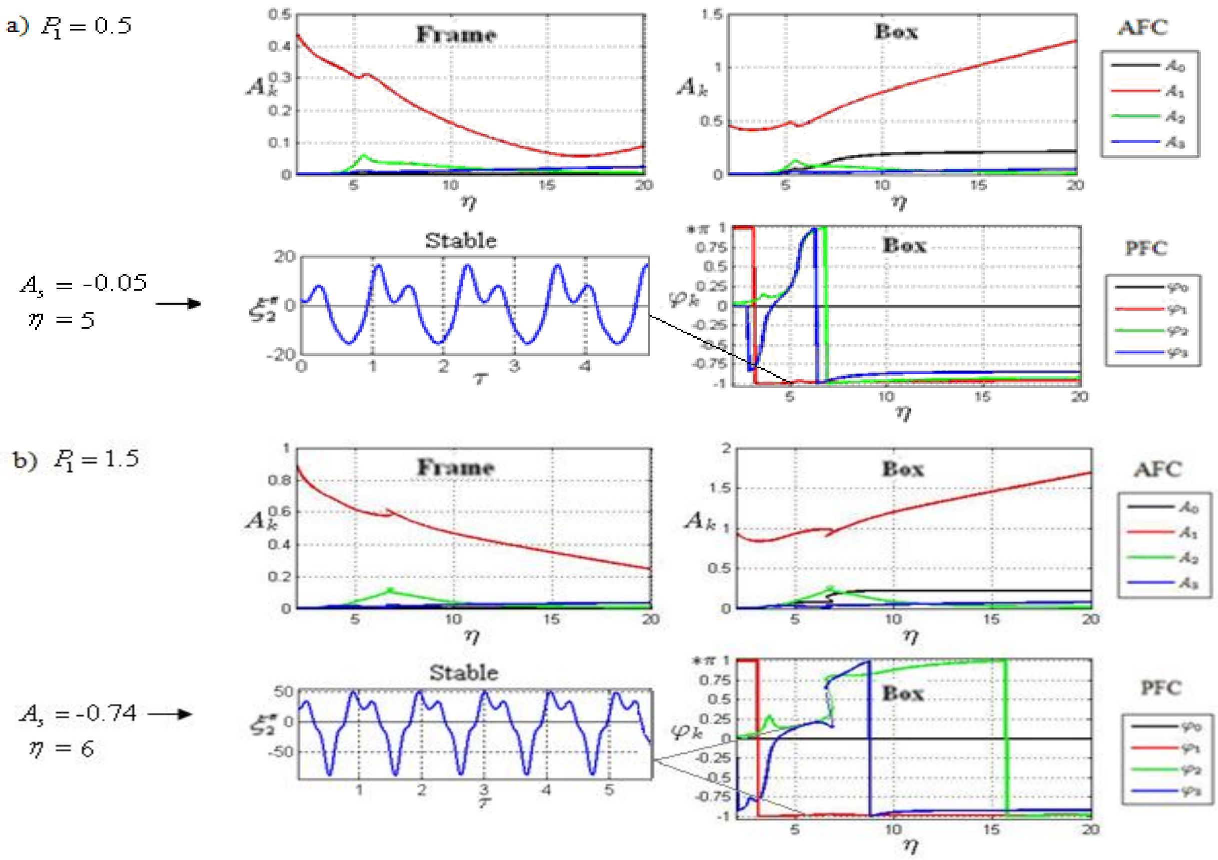

Figure 5: Superresonance for extreme values of parameters of elastic ties $k_2/k_1 = -2$, $k_3/k_1 = 3$.

problem is the restraining factor in development of effective many-masses vibrating machines having resonant or antiresonant regimes and polyharmonic oscillations.

At present for main elastic ties there are usually used metal springs, rubber elements of different forms, buffer elements, torsions, leaf springs made of steel or other special material and so on. But the analysis of literature sources gives the opportunity to pay attention to the use of permanent magnets [9]. In the last years different types of magnetic supports, shock absorbers and muffs are getting more and more applications in the connection with the development of technology of high-energy permanent magnets.

Figure 6 shows the design of the proposed nonlinear elastic element of magnetic type, currently it passes the state examination of the invention. The element consists of a body 1, rigidly connected with the frame of vibromachine by means of the supports 2. Working organ of the machine is connected with the elastic element by means of trunnion 3. Rubber elements 4 with linear elastic characteristic work on a shift. Nonlinearity of elastic element is formed by means of permanent magnets 5 and 6 that are set the same poles to each other. The size of gap δ between the magnets is regulated with the help of bolts 7. Accordingly to this the form of elastic characteristic of the magnetic element changes (Figure 6b) and we get a vibrating machine with adjusting nonlinearity.

5 CONCLUSION

The fulfilled investigations gave an opportunity to get certain information for designing of experimental sample of nonlinear super harmonic vibrating machines. At present some elements of its construction have been already produced, experimental studies are ahead.

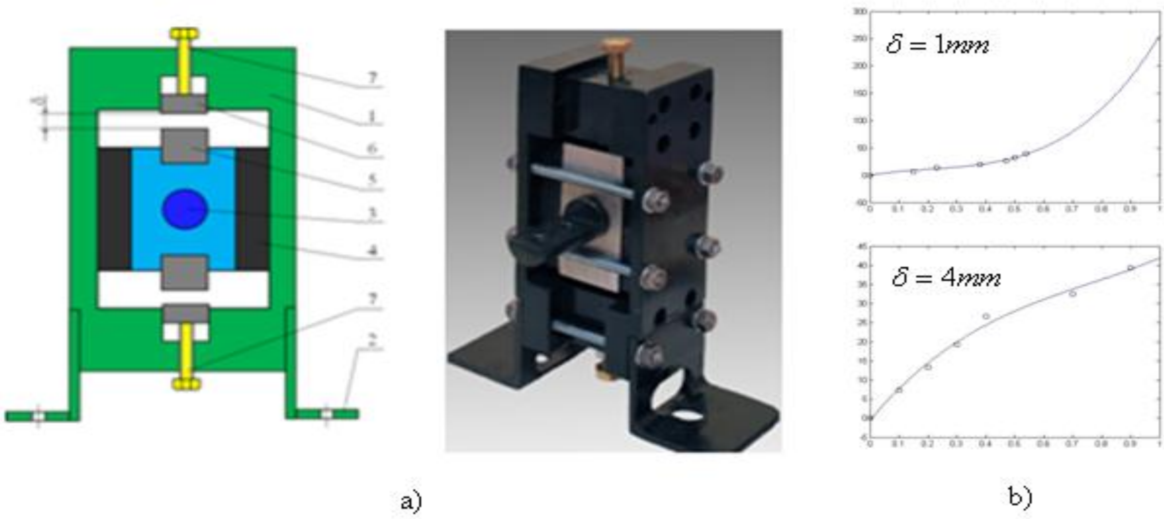

Figure 6: Magnetic elastic element a) A scheme and sample: 1 – body; 2 – support; 3 – trunnion; 4 – rubber element; 5, 6 – magnets; 7 – adjusting bolt; b) View of elastic characteristics for different gaps δ.

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