MATHEMATICAL MODELS AND METHODS FOR DECISION MAKING IN CAE OF TRANSMISSION SYSTEMS

N. N. Guschinsky, G. M. Levin

Institute of Engineering Cybernetics, Belarus
Surganova Str. 6, 220012 Minsk, Belarus,
gyshin@newman.bas-net.by

ABSTRACT

This paper presents mathematical models and optimization methods for decision making in CAE of mechanical transmission systems. The design problem is considered as a multi-criteria graph optimization problem. To solve it, a multilevel decomposition scheme is used in combination with methods of quadratic, dynamic and of nonlinear programming.

INTRODUCTION

Transmission systems are very important components of machines and mechanisms. The transmission design is a highly complex problem. In the literature, the main results concern the design of specific elements such as spring, gears, and shaft, see for example [1, 2, 3]. Some results were obtained in structural and parametric synthesis for certain type of transmission systems [4, 5, 6].

The expert systems are widely used in order to take into account engineering experience and to integrate the partial optimization models in design process [7, 8, 9].

In this paper, we focus on the preliminary design stage of transmission systems of complex structure. Such systems consist of tens shafts and gears, and the number of kinematic chains (speeds) can be greater than 20. The transmission is considered as a multiunit system taking into account main interconnections between elements and main constraints. The design problem is stated as non-linear multi-criteria optimization problem with discrete, combinatorial and continuous variables. We propose a general scheme of optimization and iterative procedure for decision making. This scheme is based on multilevel parametric decomposition approach [10, 11]. We develop also models and methods for the main sub-problems.

FORMULATION OF DESIGN PROBLEM

We investigate complex power transmissions, which can include an engine, speed gear box, chains of various gears, passing a motion from the engine to input shaft of speed gear box as well as from output shaft of speed gear box to actuator.

We suppose that the structural scheme of the transmission has been selected at previous stages of design and that load conditions of actuator are known. We consider the problem to determine the following design parameters of the transmission system:

- set of nominal speeds of output shaft of the transmission;
- assigning the above set of speeds to kinematic chains;
- transmission ratios (tooth numbers) of gears;
- basic design parameters of transmission elements (types, diameters, widths of gears, diameters of shafts).

We take into account the following basic functional, kinematic, strength and constructional factors to define the required design parameters:

- the desired set of actuator speeds;
- load conditions and total working times of actuator for each kinematic chain;
- ranges of transmission ratios of gears;

- ranges of absolute speeds of shafts;
- total transmission life taking into account contact and bending endurance of gears and static rolling strength of shafts;
- characteristics of used materials.

We consider a deviation of the obtained set of output shaft speeds to the desired set, total transmission life and total mass of the transmission elements as criteria for design decision making.

MATHEMATICAL MODEL

We use a finite acyclic directed multigraph $G=(V,E)$ for representing the structural scheme of the transmission. This graph has one initial node s and one terminal node t. The nodes s, t and $v\in V' =V\backslash\{s,t\}$ correspond to the input shaft of the transmission (shaft of the engine), output shaft and intermediate shafts of the transmission, respectively, arcs from E correspond to the gears of the transmission. Each kinematic chain from the input shaft s to the shaft $v \in V\backslash\{s\}$ defines in one-to-one manner a path in multigraph G.

Let us denote:

- a nominal speed of the engine (input shaft s) by n_0;
- an unknown transmission ratio of the gear $e\in E$ and its range by $x(e)$, and $[\underline{x}(e), \bar{x}(e)] \in R$;
- unknown tooth numbers of the gear $e\in E$ and a set of their feasible values by $z_i(e), i=1,2,$ and $Z_i(e)$;
- a range of absolute speeds of the shaft $v\in V$ by $[\underline{n}(v), \bar{n}(v)]$;
- a collection of unknown design parameters (including teeth numbers) of the gear $e\in E$ and a set of its feasible values by $u(e)$ and $U(e)$;
- a collection of unknown design parameters of the shaft $v\in V$ and a set of its feasible values by $w(v)$ and $W(v)$;
- input and output shafts of the gear $e\in E$ by $v_1(e)$ and $v_2(e)$;
- the set of paths in G from node s to node v by $L(v)=\{L_k(v) \mid k=1,\ldots,r(v)\}$.

The permutation $\pi= (\pi_1,..., \pi_{r(t)})$ of elements of set $\{1,...,r(t)\}$ determines the one-to-one correspondence between the kinematic chains into the output shaft and the desired set $C(t)=(C_1, \ldots, C_{r(t)})$ of speed of this shaft. Here π_k is a speed number from C, which is assigned to the kinematic chain k from $L(t)$. Let Π be a set of all feasible permutations of the set $\{1, ...,r(t)\}$. This set is mainly defined by the selected scheme of control of speed gearbox.

Later on:

$x = (x(e) \mid e\in E)$;

$X = \{ x \mid x(e)\in[\underline{x}(e), \bar{x}(e)] \}$;

$u = (u(e) \mid e\in E)$;

$w = (w(v) \mid v\in V)$;

$n_k(v,x) = n_0 / \prod_{e\in L_k(v)} x(e),\ L_k(v) \in L(v),\ x \in X,\ v \in V$;

$N(v,x) = (n_1(v,x),\ldots, n_{r(v)}(v,x))$.

For the given engine power, load conditions of the actuator and each value from C, we assume that load conditions of the shaft $v\in V$ and of the gear $e\in E$ can be defined by collections $(\pi,N(v,x))$ and $(\pi,x(e),N(v_1(e),x))$. The following functions then can be constructed:

(i) functions $T_v(\pi,N(v,x),w(v))$ and $T_e(\pi,x(e),N(v_1(e),u(e)))$ which determine a longevity of the shaft $v \in V$ and a longevity of the gear $e\in E$ for fixed values of unknown parameters x,w and u;

(ii) functions $M_v(w(v))$ and $M_e(u(e))$ which determine a mass of the shaft $v \in V$ and a mass of the gear $e\in E$ for fixed values of unknown parameters w and u.

Under such assumptions, the considered design problem can be reduced to the following optimization problem:

$$g_1(\pi,x)=\sum_{k=1}^{r(t)}\alpha_{\pi_k}(\ln(C_{\pi_k})-\ln(n_k(t,x)))^2\to\min, \quad (1)$$

$$g_2(\pi,x,u,w)=\min(\min\{T_v(\pi,N(v,x),w(v)) \mid v\in V\},\ \min\{T_e(\pi,x(e),\ N(v_1(e),\ u(e)) \mid e\in E\})\to\max, \quad (2)$$

$$g_3(u,w)=\sum_{e\in E}M_e(u(e))+\sum_{v\in V}M_v(w(v))\to\min, \quad (3)$$

subject to:

$$x\in X, \quad (4)$$

$$n_k(v,x)\in[(\underline{n}(v),\bar{n}(v)],\ v\in V,\ k=1,\dots,r(v), \quad (5)$$

$$\pi\in\Pi, \quad (6)$$

$$x(e)=z_2(e)/z_1(e),\ e\in E'\subseteq E, \quad (7)$$

$$z_i(e)\in Z_i(e),\ i=1,2,\ e\in E', \quad (8)$$

$$w(v)\in W(v),\ v\in V, \quad (9)$$

$$u(e)\in U(e),\ e\in E. \quad (10)$$

Here the set E' corresponds to the set of gears for which its transmission ratio is determined by ratio of tooth numbers, the coefficients $\alpha_k(t)$, $k=1,\dots,r(t)$ characterize the importance of closeness of the obtained values $n_k(t,x)$ to desired values C_k.

The first criterion is to minimize a deviation of an obtained set of output shaft speeds from the desired set. The second criterion is to maximize the transmission life, and the third criterion is to minimize the total mass of the transmission.

DECOMPOSITION SCHEME FOR SOLVING PROBLEM

The problem (1-10) is a highly complicated multi-criteria optimization problem where the functions $T_v(\cdot)$, $T_e(\cdot)$, $M_v(\cdot)$ and $M_e(\cdot)$ are defined algorithmically. We propose a multilevel man-machine decomposition scheme to obtain an approximate solution of the problem taking into account the character of connections between the unknown design parameters, specified factors and criteria. This scheme is based on simultaneous use of several decomposition techniques in combination with various methods of nonlinear programming. The scheme includes consecutive execution of the following procedures.

At the first stage, as a result of minimizing the function $g_1(\pi,x)$ on a set of all feasible (π,x) we determine a vector $C^*=(\ C^*_k=n_k(t,x^*)\mid k=1,\dots,r(t)\ )$ (set of output shaft speeds) which is the nearest vector to the vector C as well as the optimal permutation π^* (Problem **A**).

The designer performs an informal analysis of the obtained design decision. He can either repeat the first stage with modified data or go to next stage.

At the second stage, for each value τ_i of the total transmission life from sequence $\{\tau_1,\tau_2,\dots\}$ specified by the designer, we determine optimal (according to the total mass of the transmission) values of unknown parameters by minimizing the function $g_3(u,w)$. Simultaneously we provide that the set of the obtained speeds of output shaft is equal to C^* (Problem **B**). The designer selects himself the most suitable combination of values τ and $g_3^*(\tau)$ where $g_3^*(\tau)$ is the optimal value of function $g_3(u,w)$ for fixed τ.

SOLUTION OF THE PROBLEM A

The problem **A** is stated as follows:

$$g_1(\pi,x)\to\min, \quad (11)$$

$$\pi\in\Pi, \quad (12)$$

$$x\in X, \quad (13)$$

$$n_k(v,x)\in[(\underline{n}(v),\bar{n}(v)],\ v\in V,\ k=1,\dots,r(v), \quad (14)$$

$$x(e)=z_2(e)/z_1(e),\ e\in E'\subseteq E, \quad (15)$$

$$z_i(e)\in Z_i(e),\ i=1,2,\ e\in E'. \quad (16)$$

The problem **A** is also solved in two stages. At the first stage, we solve it without constraints (15-16) (Problem $\mathbf{A}_1$) and obtain preliminary values of (π^*,x^*). At the second stage (Problem $\mathbf{A}_2$), we determine $z_i(e)\in Z_i(e)$, $i=1,2$, $e\in E'$, which minimize the function $g_1(\pi^*,x)$ for $C_k = C_{\pi_k^*}$, $k=1,...,r(t)$, under constraints (13)-(16).
For solving the problem $\mathbf{A}_1$ we consider it in logarithmic coordinates but for convenience we keep the same notations. In this case, it is a mixed optimization problem with linear constraints (14) including continuous variables x as well as combinatorial variables π. An approximate solution of the problem $\mathbf{A}_1$ can be obtained by using the following decomposition scheme. If we fix variables x, then assignment problem arises. We denote this problem by $\mathbf{A}_{11}(x)$ and its solution by $\pi^*(x)$. If we fix variables π then a quadratic program arises. We denote this problem by $\mathbf{A}_{12}(\pi)$ and its solution by $x^*(\pi)$.
The number of constraints (14) in the problem $\mathbf{A}_{12}(\pi)$ is very large and in general case is equal to the number of all paths in graph G from the vertex s to others. However, as a rule, the number of essential constraints is not more than $|V|$. For solving the problem $\mathbf{A}_{12}(\pi)$, we use a general scheme of relaxation of constraints. First of all, the problem $\mathbf{A}_{12}(\pi)$ is solved without constraints (14) and then we add step by step the constraints that are not valid for the obtained solution. At each step for each vertex v we add only those invalid constraints (14) which have the greatest discrepancies. The paths which correspond to above constraints can be also found by shortest path algorithms.
We obtain the approximate solution of the problem $\mathbf{A}_1$ as a result of "coordinate-wise" (x and π) descent by solving in turn problems $\mathbf{A}_{11}(x)$ and $\mathbf{A}_{12}(\pi)$.
One of the modifications of above scheme was proposed in [12]. It uses parametric properties of the problems $\mathbf{A}_1(x)$ and $\mathbf{A}_2(\pi)$ and takes into account that it is not obligatory to obtain the exact solution of the specified sub-problems during the iterative process.
For solving the problem $\mathbf{A}_2$, we use one of the modifications of "branch and bound" method. To the current step of searching process, a feasible set of the problem $\mathbf{A}_2$ is partitioned into several subsets. The subset P_k is defined by segment $[\underline{x}_k(e),\overline{x}_k(e)]$ for each $e\in E'$, by a set $E_k = \{e \in E' | \underline{x}_k(e) \neq \overline{x}_k(e)\}$ and by a bound $q_k = g_1(\pi^*, x_k^*)$, where x_k^* is the solution of the problem which is obtained from the problem $\mathbf{A}_{12}(\pi^*)$ by replacing constraints (13) on constraints $x(e) \in [\underline{x}_k(e),\overline{x}_k(e)]$, $e\in E'$.
For branching at the current step, we choose the set P_k with minimal in lexicographic sense bound (E_k,q_k). Among arcs $e\in E_k$ we find the arc e^* with minimal value $|x^0(e)-x_k^*(e)|$, where $x^0(e)$ is the nearest value to $x_k^*(e)$ in the set $X_k^0(e)$ of values $x(e)\in[\underline{x}_k(e),\overline{x}_k(e)]$ and satisfying constraints (15)-(16). For the arc e^* we also determine values $x^1(e^*)$, $x^2(e^*)$ which are the nearest lesser and the nearest greater values to $x^0(e^*)$ in $X_k^0(e)$. Then the set P_k is partitioned into three (in general case) subsets with segments $[\underline{x}_k(e^*),x^1(e^*)]$, $[x^0(e^*),x^0(e^*)]$, and $[x^2(e^*),\overline{x}_k(e^*)]$.

SOLUTION OF THE PROBLEM B

The problem **B** is to minimize the function $g_3(u,w)$ for fixed $\pi=\pi^*$, $\tau \in \{\tau_1,\tau_2,\dots,\}$ and $C_k = C_{\pi_k^*}$, $k=1,\dots, r(t)$, under constraints (4), (5), (7)-(10) and the additional constraints:
$g_2(\pi,x,u,w) \geq \tau$,
$N(t,x)=C$.
To solve it, we propose the method, which is based on a system of invariants.
Let D be a family of subsets $d=\{e_j(d)|j=1,\dots, r(d)\}$ of arcs $e\in E$ which connect the same vertices, and $\mu(k,v)$ the minimal number in $\{1,\dots,r(t)\}$ such that $L_k(v)$ is the subpath of path $L_{\mu(k,v)}(t)$. Assume that sets $L_k(v)$ are reordered in ascending code of $\mu(k,v)$. Under these assumptions, it is easy to prove

Proposition. For any $d \in D$, $v \in V$ and $x \in X$, such that $N(t,x)=C$ values $\gamma(e_j(d))\, x(e_j(d))=\gamma(e_1(d))\, x(e_1(d))$, $j=1,\dots,r(d)$, $n_k(v,x)=n_1(v,x)\lambda_k(v)$, $n_k(v, x \times \gamma)= n_1(v,x)$, and $\prod_{e \in L_k(v)} \gamma(e) = \lambda_k(v)$, $k=1,\dots, r(v)$, where $\lambda_k(v)$ and $\gamma(e)$ are constants.

Here $x \times \gamma$ is component-wise multiplication of vectors x and γ. It should be noted that $\lambda_1(v)=1$ and $\gamma(e)=1$ for all $v \in V$ and $e \in L_1(v)$.

Values $\lambda_k(v)$, $\gamma(e)$ for all $v \in V$ and $e \in E$ can be determined before solving the problem **B**. Using these values we can determine more precise ranges $[a(v),b(v)]$ of $n_1(v,x)$. If we introduce change of variables $q(e)=\gamma(e)\, x(e)$ then functions $T_v(\pi,N(v,x),w(v))$ and $T_e(\pi,x(e),N(v_1(e),x), u(e))$ can be redefined as follows:

$T_v(\pi,N(v,x),w(v))=T_v^*(\pi,n_1(v,q),w(v))$

$T_e(\pi,x(e),N(v_1(e),x),u(e))=T_e^*(\pi,q(e),n_1(v_1(e),q),u(e))$.

On the base of well-known techniques (for instance, [2]) we can construct the procedures for determining

$M_v^*(n_1(v,q))=\min\{M_v(w(v)) | T_v^*(\pi^*, n_1(v,q), w(v)) \geq \tau, w(v) \in W(v)\}$

and

$M_e^*(q(e),n_1(v,q))= \min\{M_e(u(e)) |\ T_e^*(\pi^*,q(e),n_1(v_1(e),q), u(e)) \geq \tau, u(e) \in U(e)\}$.

Then the problem **B** can be stated as follows:

$$g(q)= \sum_{e \in E} M_e^* (q(e), n_1(v_1(e),q))) + \sum_{v \in V} M_v^* (n_1(v,q)) \rightarrow \min \tag{17}$$

$$q(e) \in [\underline{q}(e), \overline{q}(e)],\ e \in E, \tag{18}$$

$$q(e)/ \gamma(e) = z_2(e) / z_1(e),\ e \in E' \subseteq E, \tag{19}$$

$$z_i(e) \in Z_i(e),\ i=1,2,\ e \in E', \tag{20}$$

$$n_1(v,q) \in [a(v),b(v)],\ v \in V, \tag{21}$$

$$n_k(t,q)=C_1,\ k=1,\dots,r(t), \tag{22}$$

$$w(v) \in W(v),\ v \in V, \tag{23}$$

$$u(e) \in U(e),\ e \in E, \tag{24}$$

where $[\underline{q}(e), \overline{q}(e)]= [\underline{x}(e)\ \gamma(e), \overline{x}(e)\ \gamma(e)]$.

To solve problem **B**, we use the method of parametric decomposition [10, 11], introducing parameters $y(v)=n_1(v,q)$, $v \in V$ and set $Y=\{y=(y(v)|\ v \in V, y(v) \in [a(v),b(v)], y(v_1(e))/y(v_2(e)) \in [\underline{q}(e), \overline{q}(e)], e \in E\}$ of their possible values. In this case, the lower level problem is very simple, i.e. to find for fixed $y \in Y$ values $q^*(e,y)$, $e \in E$ such that

$$q^*(e,y) = y(v_1(e))/ y(v_2(e)),\ e \in E, \tag{25}$$

$$q^*(e,y)/\gamma(e)=z_2(e)/z_1(e),\ z_i(e) \in Z_i(e),\ i=1,2,\ e \in E', \tag{26}$$

Let $f_e^*(y(v_1(e)), y(v_2(e))= M_e^*(q^*(e,y), y(v_1(e)))$ if there exists $q^*(e,y)$, satisfying (25)-(26), and $f_e^*(\cdot)=\infty$ otherwise. Then the upper level problem is to minimize function

$F(y)= \sum_{e \in E} f_e^*(y(v_1(e)), y(v_2(e)) + \sum_{v \in V} M_v^*(y(v))$ on set Y (Problem **C**).

Problem **C** is a multiextremal problem in which functions $f_e^*(\cdot)$ and $M_{v^*}(\cdot)$ are defined algorithmically. It should be noted that effectiveness of applied methods depends on a topology of graph G. When G is a path then the problem can be solved using method of dynamic programming if we replace segments $[a(v),b(v)]$ by their discrete representations. In general case we propose the decomposition scheme, which is based on recursive procedures of consecutive-parallel decomposition of graph and its sub-graphs.

CONCLUSIONS

Multilevel parametric decomposition approach for multiunit transmission systems design is proposed. The models and methods for main sub-problems are developed.

The proposed approach is oriented to active participation of experienced designer in decision making. It was used for creating a computer aided decision support system of transmission design in the Institute of Engineering Cybernetics of Belarus National Academy of Sciences [13, 14, 15].
The developed system was tested at Minsk tractor plant and Minsk wheel tractor plant for solving real design problems. Use of the system provides improving the design decisions, including decreasing total metal consumption of the transmission by up 5-10% and considerable cutting down of labor effort at the considered stages of design.

ACKNOWLEDGMENT

This work is financially supported by ISTC project B-104-98.

REFERENCES

1. Litvin F.L. Gear geometry and applied theory. Englewood Cliffs, N.J.: Prentice Hall, 1994.
2. Кудрявцев В.Н. Элементы машин. М: Машгиз, 1980.
3. Shigley J.E., Mischke C.R. Mechanical engineering design. 5th Ed., New York: McGraw Hill, 1989.
4. Hsieh H.-I., Tsai L.-W. ASME Journal of Mechanical Design. 1996, 118. P. 295-299.
5. Кудрявцев В.Н., Державец Ю.А., Глухарев Е.Г. Конструирование и расчет зубчатых редукторов. Л.:Машиностроение,1971. 328 с.
6. Тарасик В.П., Станиловский С.В. Известия высших учебных заведений, Машиностроение. 1992, №1-3. С. 41-44.
7. Su D., Proceedings of the 2nd International Conference on Adaptive Computing in Engineering Design and Control, Plymouth University, 1996. P. 316-318.
8. Su D. Journal of Materials Processing Technology. 1998, 76. P. 23-28.
9. Dixon J.R. ASME Journal of Mechanical Design. 1995, 117. P. 11-16.
10. Левин Г.М., Танаев В.С. Декомпозиционные методы оптимизации проектных решений.- Мн.: Наука и техника, 1978. 240 с..
11. Левин Г.М., Танаев В.С.Весці АН БССР. Сер. фіз.-мат. навук, 1998, № 4. С. 121–131.
12. Верина Л.Ф., Левин Г.М. Весці АН БССР. Сер. фіз.-мат. навук, 1991, № 4. С. 106-111.
13. Гущинский Н.Н., Левин Г.М. Модели и алгоритмы автоматизации проектирования конструкций и технологий. Мн.: ИТК АНБ, 1994. С. 4-21.
14. Гущинский Н.Н., Левин Г.М. Моделирование и информационные технологии проектирования. Мн.: ИТК АНБ, С. 50-57.
15.Dolgui A., Guschinsky N., Levin G. Proceedings of the Fifth International Conference on Advanced Computer Systems (ACS'98). Szczecin University, 1998, P. 450–456.