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**THEORETICAL RESEARCHES OF PRESSURE LOSSES WHILE  
PNEUMATIC CONVEYING THROUGH HORIZONTAL PIPES**

*For listed particles flows by methods of a hydromechanics the equations intended for calculation of specific losses of pressure at pneumotransport are formed.*

The pipeline horizontal segment specific pressure losses due to friction are the important parameter, determining industrial pneumatic conveying systems power characteristics. The known techniques of the pneumatic conveying system hydraulic design are based on empirical dependences, the scope of which is limited as a rule by the experiment conditions and the design error comes up to 40 % and more.

One of the most common specific pressure losses formulas is the Geisterstadt formula, taking the form:

$$\frac{\Delta P}{L} = (1 + K\mu) \frac{\Delta P}{L}, \quad (1)$$

where  $\Delta P$  and  $\Delta P$  – pressure losses at the pipe segment with the length  $L$ ;

$\mu$  – mixture mass concentration;

$K$  – Geisterstadt empirical coefficient.

According to the author, the dependence (1) can be considered as general, and the coefficient  $K$  numerical values shall be estimated for every single case experimentally.

Provided the carefully carried out experiments the formula (1) can present the results that are accurate enough for engineering practice. Yet all the attempts for the coefficient  $K$  theoretical proving are unsuccessful so far.

The work purpose is in theoretical proving, accuracy improving of the Geisterstadt formula (1) and generally – in creating opportunities for a new suspension flows design technique.

Specific pressure losses are detected as a rule at the pipe length of 1 m. In this case the pressure difference at the segment ends is not big and the assumption of the gas medium incompressibility and isothermality can be accepted. The flow is considered one dimensional i. e. the velocity, pressure, temperature, density and the concentration are constant in the pipe cross section and vary while the sections change.

On the basis of the assumptions in the work [1] the equations of the suspension motion through the pipe segment with the length of  $L$  have been set up. One of the equations is the Bernoulli's equation. The Bernoulli's equation hasn't been solved in the work [1] just like there is no solution of the equation in the world practice. Later on in the work [2] the solution of the equation was found, resulting in the possibility of a new theoretically proven pneumatic conveying hydraulic design technique developing.

The Bernoulli's equation for suspension can be presented in the form [3]:

$$\rho_m \frac{u_m^2}{2} + P + \Delta P = const, \quad (2)$$

where  $\rho_m$  – gas mixture density;

$u_m$  – gas mixture motion mean velocity;

$P$  – pressure;

$\Delta P$  – friction pressure losses at the pipe segment with the length  $L$ .

Considering the respective dependences derived in the works [1] and [2] and presented in [3], the equation (2) is transformed to the form:

$$\left[ 1 + \frac{\rho_S}{\rho} \frac{C_v^3}{C^2 (1 - C_p)^2} \beta_S \right] \rho \frac{u^2}{2} + P + \Delta P = const, \quad (3)$$

where  $\rho_S$  and  $\rho$  – solids and gas density;

$u$  – gas mean velocity;

$\beta_S$  – Coriolis coefficient for solids.

(3) comprises the volume flow rate  $C_v$ , equal to the ratio of solids volume flow rate  $Q_S$  to the suspension volume flow rate  $Q = Q_S + Q$  and the mean volume concentration  $C$ . Yet in the pneumatic conveying practice the notions volume and actual flow rate concentrations are not used as  $C_v$  and  $C$  are hardly measured values. The notion of mass flow rate  $\mu$ , equal to the ratio of solids mass flow rate  $G_S$  to the gas mass flow rate  $G$  is used instead of them, so that

$$\mu = \frac{G_S}{G}, \quad (4)$$

at that, the value  $G_S$  is set as a rule and  $G$  is determined by the formula

$$G = \rho u \omega. \quad (5)$$

Let us express  $C_v$  and  $C$  through  $\mu$ . Considering

$$C_v = \frac{Q_S}{Q_S + Q}$$

and  $Q_S = G_S / \rho_S$ ,  $Q = G / \rho$ , we have

$$C_v = \frac{1}{1 + \frac{G \rho_S}{G_S \rho}}. \quad (6)$$

As  $\frac{G \rho_S}{G_S \rho} \gg 1$ , the one in the equation (6) right part can be neglected. As a result of this the equation, taking into account the change of notation from  $C_p$  to  $\alpha$ , takes the form

$$\alpha = \frac{G_S \rho}{G \rho_S} = \mu \frac{\rho}{\rho_S}. \quad (7)$$

While writing down the formula (7), changing of notation from  $C_v$  to  $\alpha$  is due to the fact that the symbols denote different by meanings volume flow rates:  $C_v$  – the ratio of solids volume flow rate to the suspension volume flow rate whereas  $\alpha$  – the ratio of solids volume flow rate to the gas volume flow rate.

Inserting the expression (7) into (3) instead of  $C_v$  we obtain

$$\left[ 1 + \frac{\mu^3 \left( \frac{\rho}{\rho_S} \right)^2}{C^2 \left( 1 - \mu \frac{\rho}{\rho_S} \right)^2} \beta_S \right] \rho \frac{u^2}{2} + P + \Delta P = const. \quad (8)$$

Regarding the functional connection of  $C$  with the mass flow rate  $\mu$ , the following equation obtained in the solids [4] hydraulic pipeline conveying research area is used for determining it.

$$C_v = C \left[ 1 - f(\text{Re}_S) \left( 1 - \frac{C}{C_m} \right)^{2,16} \left( \frac{u_{cr}}{u} \right)^{1,66} \right]; \quad (9)$$

$$f(\text{Re}_S) = 0,45 \left[ 1 + \text{Sign} X \cdot th(0,967|X|^{0,6}) \right], \quad (10)$$

$$X = \lg \text{Re}_S - 0,88, \quad (11)$$

In (9) – (11) the following notation are agreed:  $C_m$  – solids limit concentration;  $u_{cr}$  – critical velocity of pneumatic conveying through the horizontal pipeline, corresponding with the solids saltation as still sediment on the pipe lower wall;  $\text{Re}_S$  – Reynolds number expressed through the solids mean diameter  $d_S$  and the free fall velocity  $w_S$ , i. e.

$$\text{Re}_S = \frac{w_S d_S}{\nu},$$

where  $\nu$  – gas kinematic viscosity.

Inserting the expression (7) into (9) instead of  $C_v$  выражение (7), we obtain the constraint equation of  $C$  and  $\mu$ :

$$\mu \frac{\rho}{\rho_S} = C \left[ 1 - f(\text{Re} S) \left( 1 - \frac{C}{C_m} \right)^{2,16} \left( \frac{u_{cr}}{u} \right)^{1,66} \right]. \quad (12)$$

The solution of the equation (12) allows determining the volume concentration  $C$  for the given values  $\mu \frac{\rho}{\rho_S}$ ,  $\text{Re}_S$ ,  $C_m$  and  $\frac{u_{cr}}{u}$ . At that the equation (12) is solved by numerical or graphical method.

Let us note that in the particular case, when  $\mu = 0$ , the equation (8) is transformed into an ordinary Bernoulli's equation for a horizontal flow of an incompressible fluid with the density  $\rho$ :

$$\rho \frac{u^2}{2} + P + \Delta P = const. \quad (13)$$

As is known in hydraulics, the specific pressure loss  $\frac{\Delta P}{L}$ , conditioned by pipe walls and incompressible fluid friction, is proportional to the flow specific (per unit volume) kinetic energy  $\rho \frac{u^2}{2}$ , expressed by the first addend of the Bernoulli's equation (8) left member and is determined by the Darcy–Weisbach formula

$$\frac{\Delta P}{L} = \lambda \frac{1}{d} \rho \frac{u^2}{2}, \quad (14)$$

where  $\lambda$  – hydraulic friction coefficient;  
 $d$  – the circular pipe drift diameter.

Proceeding to the suspension flow and considering the expression of the first addend of the Bernoulli's equation (8) left member, we can write down by analogy with (14),

$$\frac{\Delta P}{L} = \left[ 1 + \frac{\mu^3 \left( \frac{\rho}{\rho_S} \right)^2}{C^2 \left( 1 - \mu \frac{\rho}{\rho_S} \right)^2} \beta_S \right] \lambda_m \frac{1}{d} \rho \frac{u^2}{2}, \quad (15)$$

where  $\lambda_m$  – suspension hydraulic friction coefficient. The notion is determined by the same formulas as incompressible fluids motion, but considering the Reynolds number  $Re_m$ , expressed through the gas mean velocity  $u$ , the pipe diameter  $d$  and the suspension kinematic viscosity  $\nu$ , i. e.

$$Re_m = \frac{ud}{\nu_m}. \quad (16)$$

The kinematic viscosity  $\nu_m$  is determined as  $\nu_m = \frac{\mu_m}{\rho_m}$ , where  $\mu_m$  and  $\rho_m$  – the gas suspension mean effective dynamic viscosity and density throughout the pipe section. For  $C$  weak volume concentrations the value  $\mu_m$  is determined by formula [5]:

$$\mu_m = \mu(1 + 3,5C). \quad (17)$$

The gas suspension mean velocity is:

$$\rho_m = \rho \left[ 1 + \left( \frac{\rho_S}{\rho} - 1 \right) C \right]. \quad (18)$$

Considering the formulas (17) and (18) the expression for  $\nu_m$  is as follows:

$$\nu_m = \nu \frac{1 + 3,5C}{1 + \left( \frac{\rho_S}{\rho} - 1 \right) C}. \quad (19)$$

If we multiply and divide the right member of the equation (15) by the hydraulic friction coefficient  $\lambda$  the expression in the square brackets denote with  $\varphi$  and consider formula (14), the equation can be written as:

$$\frac{\Delta P}{L} = \varphi \cdot \frac{\lambda_m}{\lambda} \frac{\Delta P}{L}, \quad (20)$$

where  $\frac{\Delta P}{L}$  – specific pressure losses while the pure gas motion.

It can be taken  $\frac{\lambda_m}{\lambda} \approx 1$ . for suspension flows. After that introducing the notation:

$$K = \frac{\mu^2 \left( \frac{\rho}{\rho_S} \right)^2}{C^2 \left( 1 - \mu \frac{\rho}{\rho_S} \right)^2} \beta_S, \quad (21)$$

we will obtain:

$$\frac{\Delta P}{L} = (1 + K\mu) \frac{\Delta P}{L} \quad (22)$$

and in this case formula (20) is converted into the Gastershtadt formula (1).

The linear dependence of type 1 was the development basis for the experimental data of specific pressure losses for friction measuring while pneumatic conveying. Thereafter it was established that the coefficient  $K$  is not constant for the flows of enhanced concentration. The coefficient value depends upon a variety of parameters, particularly the conveying gas mean velocity, solids size and density, solids mass and volume flow rates, the pipe diameter. The key parameters dependence of  $K$  coefficient solves the equation (21) following from (20). Therein lies the vital difference of the generalizing formula from the Gastershtadt formula (1).

Hence on the basis of the Bernoulli's equation for the steady gas suspension flow in the horizontal circular cylindrical pipe the scientifically proven formula of specific pressure losses for friction (20) has been obtained. Checking evaluation by the formula shows a good coincidence with the experimental data. The evaluation error is no more than 10 %.

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