STABILIZATION BY ROTATING RIGID BODIES FOR UNSTABLE ROTATION OF A RIGID BODY WITH CAVITIES CONTAINING A FLUID

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On XII International congress on theoretical and applied mechanics (ICTAM 1972) A. Y. Ishlinsky [1] has set the problem about definition of conditions for stability of uniform rotations of system in two heavy gyroscopes of Lagrange. The interesting effect of stabilization in unbalanced gyroscope of Lagrange by the second rotating [2, 3] has been found in the works of Donetsk school on mechanics under supervision of P.V.Kharlamov. In S. L. Sobolev's known work [4] it is shown that the Lagrange gyroscope if contains the perfect fluid is rather unstable. Therefore, the problem on the possibility of stabilization in unstable rotation of the gyroscope in question by one or two rotating rigid bodies has been put out.

The possibility for stabilization by rotating rigid bodies in unstable rotation of the Lagrange gyroscope containing a perfect fluid is shown in our study. The case of free motion and the case when the Lagrange gyroscope with fluid rotates about a fixed point are considered. The equations of the works [5, 6] are foundations of the conclusion on the motion equations for the considered mechanical system.

1. Free system of the connected rigid bodies with fluid (SCRBF).

Consider free motion of the rotating Lagrange gyroscope with a cavity containing a perfect fluid. Let the considered gyroscope (body S_2) have the common points O_2 and O_3 with rotating rigid bodies S_1^0 and S_3^0 accordingly (fig. 1). The body S_2 consists of a rigid body S_2^0 with a cavity entirely filled with a perfect fluid of the density ρ and is connected with rigid bodies S_1^0 and S_3^0 in points O_2 and O_3 by elastic restoring spherical hinges k_1 and k_2 accordingly ($k_1 > 0, k_2 > 0$). Rigid bodies are rotating with angular velocities ω_{0i} about the common axis of symmetry (i=1, 2, 3). We set the problem on the possibility for stabilization by rotating rigid bodies S_1^0 and S_3^0 for unstable free motion of rotating body S_2. The characteristic equation of motion of the system above is as follows [6]:

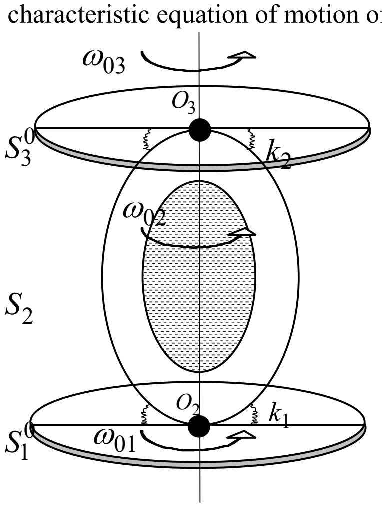

Fig. 1.

$$\begin{vmatrix} F_1 & \mu_1 + \dfrac{k_1}{\lambda^2} & \mu_2 \\ \mu_1 + \dfrac{k_1}{\lambda^2} & F_2 & \mu_3 + \dfrac{k_2}{\lambda^2} \\ \mu_2 & \mu_3 + \dfrac{k_2}{\lambda^2} & F_3 \end{vmatrix} = 0 \qquad (1)$$

Here

$$F_i = A_i' + \frac{C_i\omega_{0i}}{\lambda} - \frac{k_{i-1}+k_i}{\lambda^2}, \ (i = 1,3), \ k_0 = k_3 = 0,$$

$$F_2 = A_2' + \frac{C_2\omega_{02}}{\lambda} - \frac{k_1+k_2}{\lambda^2} - (\lambda + \omega_{02})\sum_{n=1}^{\infty}\frac{E_n}{\lambda + \omega_{02}\lambda_n^*},$$

$$A_1' = A_1 + \frac{m_1 m_{23}}{m}c_1^2, \ A_2' = A_2 + \frac{1}{m}(m_2 m_{31} c_2^2 - 2m_2 m_3 c_2 s_2 + m_3 m_{12} s_2^2), \ A_3' = A_3 + \frac{m_3 m_{12}}{m}c_3^2,$$

$$\mu_1 = \frac{m_1 c_1}{m}(m_2 c_2 + m_3 s_2), \ \mu_2 = \frac{m_1 m_3}{m}c_1 c_3, \ \mu_3 = \frac{m_3 c_3}{m}[m_1 s_2 + m_2(s_2 - c_2)],$$

$$m = m_1 + m_2 + m_3, \ m_{ij} = m_i + m_j, \ \lambda_n^* = 1 - \tilde{\lambda}_n, \ \tilde{\lambda}_n = \lambda_n/\omega_{02}, \ c_i = C_i O_i, \ s_2 = O_2 O_3,$$

A_i, C_i - equatorial and axial inertia moments of the bodies S_1^0, S_2 and S_3^0 as to their centers of weights. The definition of other variables is given in the work [6].

The necessary condition fort stability of uniform rotation in the considered system is the following: all roots of the characteristic equation (1) are real.

2. Not free system of the connected rigid bodies with fluid (SCRBF).

Let point O_2 be fixed (Fig. 1). In this case the characteristic equation is the following [5]:

$$\begin{vmatrix} F_1 & s_1 a_2 + \dfrac{k_1}{\lambda^2} & s_1 a_3 \\ s_1 a_2 + \dfrac{k_1}{\lambda^2} & F_2 & s_2 a_3 + \dfrac{k_2}{\lambda^2} \\ s_1 a_3 & s_2 a_3 + \dfrac{k_2}{\lambda^2} & F_3 \end{vmatrix} = 0. \qquad (2)$$

Here

$$F_i = A_i' + \frac{C_i \omega_{0i}}{\lambda} + \frac{a_i g - k_{i-1} - k_i}{\lambda^2}, \ (i = 1,3), \ k_0 = k_3 = 0,$$

$$F_2 = A_2' + \frac{C_2 \omega_{02}}{\lambda} + \frac{a_i g - k_1 - k_2}{\lambda^2} - (\lambda + \omega_{02}) \sum_{n=1}^{\infty} \frac{E_n}{\lambda + \omega_{02} \lambda_n^*},$$

$$A_1' = A_1 + m_{23} s_1^2, \ A_2' = A_2 + m_3 s_2^2, \ A_3' = A_3, \ m_{ij} = m_i + m_j,$$

$$a_1 = m_1 c_1 + s_1 m_{23}, \ a_2 = m_2 c_2 + s_2 m_3, \ a_3 = m_3 c_3,$$

$$\lambda_n^* = 1 - \tilde{\lambda}_n, \ \tilde{\lambda}_n = \lambda_n / \omega_{02}, \ c_i = C_i O_i, \ s_1 = O_1 O_2, \ s_2 = O_2 O_3.$$

From a kind of the characteristic equations (1) and (2) it follows that if one elastic restoring moment (ki=0) is absent and the center of weights of a rigid body S_i^0 is a common point, each and every of the equations (1) and (2) will be divided into two independent equations, and the rotation of a rigid body S_i^0 will not effect the conditions of stability (i=1, 2). If the both of elastic restoring moments (k_1=k_2=0) are absent and the centers of weights of bodies S_1^0 and S_3^0 are points O_2 and O_3 accordingly, each and every of the equations (1) and (2) will be divided into three independent equations, and, in this case, the stabilization effect for unstable rotation of a body S_2 by rotating rigid bodies S_1^0 and S_3^0 will be absent.

Numerical studies of the characteristic equations (1), (2) have been carried out for ellipsoidal and cylindrical cavities, as well as for rigid bodies S_1^0 and S_3^0 as flat, convex and concave circular disks for constant weight of a fluid.

Following analytical and numerical researches the conclusions are made:

1) To stabilize for unstable rotation of a rigid body with cavities containing a fluid is possible by virtue of one or two rotating rigid bodies.
2) If elastic restoring moments are absent and the centers of weights of rotating rigid bodies are common points SCRBF, stabilization will be impossible.
3) For the counterbalanced rotating rigid bodies (c_1<0, c_2<0) the effect of stabilization increases in comparison with unbalanced rigid bodies (c_1=0, c_2=0).

The literature:

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