ON UNSTABLE GYROSCOPE CONTAINING FLUID ROTATION STABILIZATION BY ROTATING RIGID BODY

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The interesting effect of stabilization in unbalanced gyroscope of Lagrange by the second rotating has been found in the works of Donetsk school of mechanics under supervision of P. V. Kharlamov [1-5]. In S.L. Sobolev's known work [6] it is shown that the Lagrange gyroscope if contains the ideal fluid is rather unstable. In Y.N.Kononov's work [7] there is shown a possibility of rotation stabilization of the gyroscope by introduction in a cavity transversal and coaxial partitions. However, in practice it cannot always be carried out.

The possibility of stabilization by rotating rigid body in unstable rotation of the Lagrange gyroscope containing an ideal fluid is shown in our study. The equations of the works [8, 9] are foundations of the motion equations for the considered mechanical system. Some results of the work have been informed on the ICTAM04 [10] .

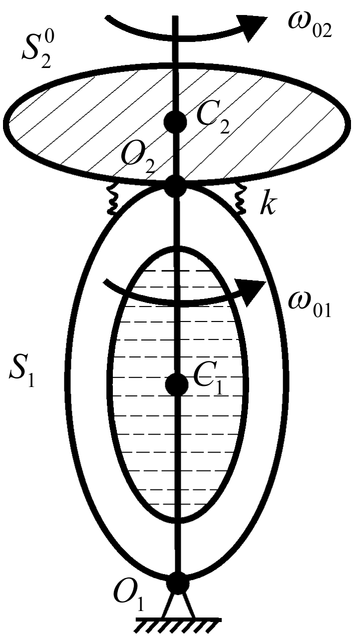

Fig. 1

Consider rotation of the Lagrange gyroscope with cavity containing an ideal incompressible fluid around a fixed point O_1. The considered gyroscope (body S_1) has the common point O_2 with the second rotating rigid body S_2^0. The body S_1 consists of a rigid body S_1^0 and the ideal fluid containing in the rigid body cavity (fig. 1).

Rigid bodies S_1^0 and S_2^0 are connected in a point O_2 by the elastic restoring spherical hinge with coefficient of elasticity k ($k > 0$). Let us consider the possibility of stabilization for unstable rotation of a body S_1 by rotating rigid body S_2^0.

The first rigid body S_1^0 and a fluid are rotating completely with angular velocity ω_{01} around an axis of geometrical and dynamic symmetry O_1O_2, and the second rigid body S_2^0 - with angular velocity ω_{02} around an axis O_2C_2. The common point O_2 lies on a straight line O_1C_2, where C_1 and C_2 - accordingly the centers of mass of bodies S_1 and S_2^0.

The considered system is a partial case of system of the connected rigid bodies with the cavities containing a fluid, investigated in works [8, 9] and therefore the characteristic equation of motion is as follows:

$$\begin{vmatrix} F_1 & \mu + \dfrac{k}{\lambda^2} \\ \mu + \dfrac{k}{\lambda^2} & F_2 \end{vmatrix} = 0 \qquad (1)$$

Here

$$F_1 = A_1^{'} + \frac{C_1^{'}}{\lambda} + \frac{a_1^* g - k}{\lambda^2} - (\lambda + \omega_{01}) \sum_{n=1}^{\infty} \frac{E_n}{\lambda + \lambda_n^{'}}, \qquad F_2 = A_2 + \frac{C_2^{'}}{\lambda} + \frac{a_2^* g - k}{\lambda^2},$$

$$A_1^{'} = A_1 + m_2 s_1^2, \ \ \mu = s_1 a_2^*, \ \ a_1^* = m_1 c_1 + s_1 m_2, \ \ a_2^* = m_2 c_2,$$

$$s_1 = O_1 O_2, \ \ c_i = O_i C_i, \ \ C_i^{'} = C_i \omega_{oi} \ \ \ i = (1, 2),$$

m_1 and m_2 - accordingly mass of a body S_1 and a rigid body S_2^0; A_i and C_i - accordingly the equatorial and axial inertia moments of the bodies S_1 and S_2^0 concerning a point O_i $(i = 1, 2)$; $\lambda_n^{'} = \widetilde{\lambda}_n \omega_{0i}$, $\widetilde{\lambda}_n = 1 - \lambda_n / \omega_{01}$.

Coefficient of inertial connection E_n and eigen numbers λ_n are found from the solution of a corresponding boundary value problem and are defined only by geometry of a cavity. Values of the sizes for ellipsoidal, cylindrical and conical cavities are given in [11].

The necessary condition for stability of permanent rotation in the considered system is the following: all roots of the characteristic equation (1) are real.

The equation (1) in case absence of relative motion of a fluid ($E_n \equiv 0$, a "frozen" fluid) coincides with the equation obtained and investigated in works [4, 5].

At $k = \infty$ (the cylindrical hinge) the equation (1) is reduced to the equation

$$\widetilde{F}_1 + \widetilde{F}_2 + 2\mu = 0, \qquad (2)$$

where

$$\widetilde{F}_1 = A_1^{'} + \frac{C_1^{'}}{\lambda} + \frac{a_1^* g}{\lambda^2} - (\lambda + \omega_{01}) \sum_{n=1}^{\infty} \frac{E_n}{\lambda + \lambda_n^{'}}, \qquad \widetilde{F}_2 = A_2 + \frac{C_2^{'}}{\lambda} + \frac{a_2^* g}{\lambda^2}.$$

If elastic restoring moment is absent ($k = 0$) and the center of mass of the second rigid body S_2^0 coincides with the common point O_2 ($c_2 = 0$, $\mu = 0$) the characteristic equation (1) is divided into two independent equations and in this case the possibility of stabilization for unstable rotation of a rigid body with a fluid by rotating rigid body is absent.

As is known [11] in the majority of practically important cases in the equation (1) it is enough to take into account only the basic tone of fluid oscillation ($n = 1$). It is always true for ellipsoidal cavities because from an infinite spectrum of eigen frequencies λ_n the harmonic corresponding to a unique value λ_1 is raised [11].

If take into account only the first harmonic ($n = 1$) in the equation (1) this equation can be written as a polynomial of the fifth degree

$$a_0 \lambda^5 + a_1 \lambda^4 + a_2 \lambda^3 + a_3 \lambda^2 + a_4 \lambda + a_5 = 0, \qquad (3)$$

where

$$a_0 = A_1^* A_2 - \mu^2 > 0, \quad a_1 = (A_1^{'} A_2 - \mu^2)\lambda_1^{'} + A_2 C_1^* + A_1^* C_2 \omega_{02},$$

$$a_2 = A_2 C_1^{'} \lambda_1^{'} + g(A_1^* a_2^* + A_2 a_1^*) - (A_1^* + A_2 + 2\mu)k + (A_1^{'} \lambda_1^{'} + C_1^*) C_2 \omega_{02},$$

$$a_3 = [g(A_1^{'} a_2^* + A_2 a_1^*) - (A_1^{'} + A_2 + 2\mu)k]\lambda_1^{'} - C_1^* k + g(a_2^* C_1^* + a_1^* C_2 \omega_{02}) + (C_1^{'} \lambda_1^{'} - k) C_2 \omega_{02},$$

$$a_4 = (a_2^* g - k) C_1^{'} \lambda_1^{'} + [a_1^* a_2^* g - k(a_1^* + a_2^*)] g + (a_1^* g - k) \lambda_1^{'} C_2 \omega_{02},$$

$$a_5 = g[a_1^* a_2^* g - k(a_1^* + a_2^*)]\lambda_1^{'},$$

$$A_1^* = A_1^{'} - E_1, \ \ C_1^* = C_1^{'} - E_1^{'}, \ \ C_1^{'} = C_1 \omega_{01}, \ \ E_1^{'} = E_1 \omega_{01}.$$

Conditions of the reality of roots of the equation of the fifth degree are as follows :

$$\begin{aligned} &d_1 = M_1^2 - M_1 M_3 > 0, \\ &d_2 = 4d_1 d_{10} - 9d_{11} > 0, \\ &d_3 = d_2 h_2 - 2h_1^2 > 0, \\ &d_4 = d_3(4h_1h_3 - h_2h_4) - 2(2d_2h_3 - h_1h_4)^2 > 0. \end{aligned} \tag{4}$$

Here

$$a_0 = M_1 > 0,\quad a_1 = 5M_2,\quad a_2 = 10M_3,\quad a_3 = 10M_4,\quad a_4 = 5M_5,\quad a_5 = M_6;$$
$$d_{10} = 6M_3^2 - 5M_2M_4 - M_1M_5,\quad d_{11} = M_2M_3 - M_1M_4,$$
$$h_1 = d_1(16\tilde{h}_1 - 15h_{25}) - 6h_{23}h_{24},\quad h_2 = 8d_1h_{35} + 48h_{23}\tilde{h}_2 - 8h_{24}d_{10},$$
$$h_3 = 6h_{35}h_{23} - h_{25}d_{10},\quad h_4 = 8d_1h_{35} - 3h_{23}h_{25},$$
$$\tilde{h}_1 = M_3M_4 - M_1M_6,\quad \tilde{h}_2 = 6M_3^2 - 5M_2M_4 - M_1M_5,$$
$$h_{23} = M_2M_3 - M_1M_4,\quad h_{24} = M_2M_4 - M_1M_5,$$
$$h_{25} = M_2M_5 - M_1M_6,\quad h_{35} = M_3M_5 - M_2M_6.$$

After simple transformations it is possible to show that the system of inequalities (4) is equivalent to inequalities

$$\begin{cases} d_1 > 0, \\ d_2 > 0, \\ \tilde{d}_3 > 0, \\ \tilde{d}_4 > 0, \end{cases} \tag{5}$$

where $d_3 = 2d_1\tilde{d}_3,\ d_4 = 2d_1^2\tilde{d}_4;\ \tilde{d}_3$ and $\tilde{d}_4$ - accordingly a polynomials 6 and 8 degree on $a_i\ (i = \overline{0, 5})$.

Stabilization of rotation of a rigid body with a fluid can be carried out by the following parameters of the second rigid body: ω_{02}, k, C_2, A_2, m_2, c_2. As parameters C_2 and ω_{02} enter into coefficients a_i by product we shall appoint this product through ω_0.

We research influence of parameter ω_0 on a possibility of stabilization. For this purpose we designate

$$a_1 = 5(\tilde{a}_1\omega_0 + b_1),\quad a_2 = 10(\tilde{a}_2\omega_0 + b_2),\quad a_3 = 10(\tilde{a}_3\omega_0 + b_3),\quad a_4 = 5(\tilde{a}_4\omega_0 + b_4). \tag{6}$$

We receive after substitution ratio (6) in inequalities (5)

$$\begin{cases} d_{12}\omega_0^2 + d_{11}\omega_0 + d_{10} > 0, \\ d_{24}\omega_0^4 + d_{23}\omega_0^3 + \ldots + d_{21}\omega_0 + d_{20} > 0, \\ d_{36}\omega_0^6 + d_{35}\omega_0^5 + \ldots + d_{31}\omega_0 + d_{30} > 0, \\ d_{48}\omega_0^8 + d_{47}\omega_0^7 + \ldots + d_{41}\omega_0 + d_{40} > 0, \end{cases} \tag{7}$$

where

$$d_{12}=\tilde{a}_1^2>0,\quad d_{24}=5\tilde{a}_1^2(3\tilde{a}_2^2-4\tilde{a}_1 a_3),$$

$$d_{36}=28\tilde{a}_1\tilde{a}_2\tilde{a}_3\tilde{a}_4-9\tilde{a}_1^2\tilde{a}_4^2-16\tilde{a}_1\tilde{a}_3^3-12\tilde{a}_2^3\tilde{a}_4+8\tilde{a}_3^2\tilde{a}_2^2=d_{33k}k^3+d_{32k}k^2+d_{31k}k+d_{30k},$$

$$d_{36}=72\tilde{a}_1\tilde{a}_2\tilde{a}_3\tilde{a}_4-27\tilde{a}_1^2\tilde{a}_4-32\tilde{a}_1\tilde{a}_3^3-32\tilde{a}_2^3\tilde{a}_4+16\tilde{a}_3^2\tilde{a}_2^2=d_{44k}k^3+d_{42k}k^2+d_{41k}k+d_{40k},$$

$$\tilde{a}_1=A_1^*,\quad 10\tilde{a}_2=A_1^{'}\lambda_1^{'}+C_1^*>0,\quad 10\tilde{a}_3=a_1^*g-k,\quad 5\tilde{a}_4=(a_1^*g-k)\lambda_1^{'},$$

$$d_{33k}=2A_1^*/625>0,\quad d_{43k}=2d_{33k}>0.$$

At $k>ga_1^*$ $\tilde{a}_3<0$, $\tilde{a}_4<0$ and $d_{24}>0$. Coefficients d_{36} and d_{48} are cubic polynomials respecting parameter k with positive coefficients at the higher degrees. Thus, at big enough elastic restoring moment $d_{24}>0$, $d_{36}>0$ and $d_{48}>0$ also there is such value ω_0 at which inequalities (7) are valid. Hence, at big enough ω_0 and k stabilization for unstable rotation of a rigid body with a fluid is possible.

In work [5] it is marked that influence of rigidity in the spherical hinge on effect of stabilization for unbalanced rigid body has the complicated character. Therefore we consider influence of the elastic restoring moment on a possibility of stabilization for unstable rotation of a rigid body with a fluid. For this purpose we designate

$$a_2=10(\tilde{a}_2k+b_2),\quad a_3=10(\tilde{a}_3k+b_3),\quad a_4=5(\tilde{a}_4k+b_4),\quad a_5=\tilde{a}_5k+b_5. \tag{8}$$

After substitution (8) in inequalities (5) we receive

$$\begin{cases} d_{11}k+d_{10}>0,\\ d_{23}k^3+d_{22}k^2+d_{21}k+d_{20}>0,\\ d_{35}k^5+d_{34}k^4+\ldots+d_{31}k+d_{30}>0,\\ d_{47}k^7+d_{46}k^6+\ldots+d_{41}k+d_{40}>0. \end{cases} \tag{9}$$

Here

$$d_{11}=-a_0\tilde{a}_2,\quad d_{23}=-24a_0\tilde{a}_2^3,\quad d_{35}=160a_0\tilde{a}_2^3(3\tilde{a}_2\tilde{a}_4-2\tilde{a}_3^2),$$

$$d_{47}=128a_0\tilde{a}_2^3(40\tilde{a}_3^3\tilde{a}_5-25\tilde{a}_3^2\tilde{a}_4^2+27\tilde{a}_2^2\tilde{a}_5^2+50\tilde{a}_2\tilde{a}_4^3-90\tilde{a}_2\tilde{a}_3\tilde{a}_4\tilde{a}_5),$$

$$10\tilde{a}_2=-(A_1^*+A_2+2\mu)<0,\quad 10\tilde{a}_3=-[(A_1^{'}+A_2+2\mu)\lambda_1^{'}+C_1^*+\omega_0]<0, \tag{10}$$

$$5\tilde{a}_4=-[(C_1^{'}+\omega_0)\lambda_1^{'}+(a_1^*+a_2^*)g]<0,\quad \tilde{a}_5=-(a_1^*+a_2^*)g\lambda_1^{'}<0.$$

From (10) follows that $d_{11}>0$, $d_{23}>0$ and coefficients d_{35} and d_{47} are polynomials accordingly 2-nd and 4-th degree relative to ω_0 with positive coefficients at the higher degrees. At big enough ω_0 and k inequalities (9) are valid and as it was earlier marked stabilization for unstable rotation of a rigid body with a fluid can be possible.

Let's consider a case of the cylindrical hinge ($k=\infty$). In the case instead of inequalities (9) it is more handy to use the equation (2) and at $n=1$ to write down conditions of the reality of roots of the cubic equation

$$g_0\lambda^3+3g_1\lambda^2+3g_2\lambda+g_3=0$$

as

$$d=4(g_1^2-g_0g_2)(g_2^2-g_1g_3)-(g_1g_2-g_0g_3)^2>0$$

or

$$d_4\omega_0^4 + d_3\omega_0^3 + d_2\omega_0^2 + d_1\omega_0 + d_0 = 0, \qquad (11)$$

where

$$d_4 = 3\tilde{a}_2^2 > 0, \quad d_3 = 6\tilde{a}_2(\tilde{a}_2 b_1 + b_2) - 4(\tilde{a}_2^3 a_0 + a_3),$$

$$d_2 = 3[(b_1^2 - 4a_0 b_2)\tilde{a}_2^2 + 2(a_0 a_3 + 2b_1 b_2)\tilde{a}_2 + b_2^2 - 4b_1 a_3],$$

$$d_1 = 6[(a_0 a_3 b_1 + b_1^2 b_2 - 2a_0 b_2^2)\tilde{a}_2 + a_0 a_3 b_2 + b_1 b_2^2 - 2a_3 b_1^2],$$

$$d_0 = 4(b_1^2 - a_0 b_2)(b_2^2 - b_1 a_3) - (b_1 b_2 - a_0 a_3)^2,$$

$$g_0 = A_1^* + A_2 + 2\mu, \quad 3g_1 = (A_1' + A_2 + 2\mu)\lambda_1' + \tilde{C}_1^* + \omega_0,$$

$$\tilde{a}_1 = 1/3, \quad \tilde{a}_2 = 1/3\lambda_1', \quad b_1 = (A_1' + A_2 + 2\mu)\lambda_1', \quad b_2 = (a_1^* + a_2^*)g + C_1'\lambda_1'.$$

So as $d_4 > 0$ if to assume that corresponding equation to an inequality (11) has three positive roots and the inequality has the solution

$$\{\omega_1 < \omega_0 < \omega_2\} \bigcup \{\omega_0 < \omega_3\},$$

and if one positive root ω_0^* then $\omega_0 > \omega_0^*$.

Thus at big enough elastic restoring moment and the big angular velocity rotations of the second rigid body stabilization for unstable rotation of rigid body with a fluid is possible.

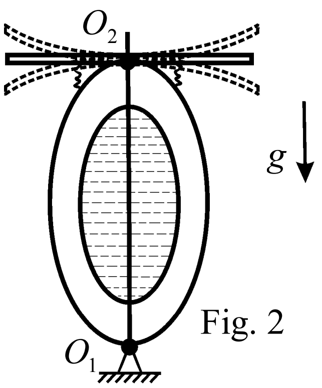

Fig. 2

For confirmation of results of analytical researches numerical calculations have been carried out for ellipsoidal cavity on formulas (7), (9) at the following values of parameters: $\omega_{02} = 0, 10, 10^2, 10^3$; $k = 0, 1, 10, 10^2, 10^3$; $\omega_{01} = 1 \div 500$; $m_1 = const$; $\beta_1 = 0{,}02 \div 4$ $\beta = c/a$; $A_{01} = C_{01} = 0$. The second rotating rigid body was slightly concave, convex and flat thin circular disk (fig. 2).

The results of numerical calculations for not free system are presented on fig. 3-6 ($c_2 = 0,\ m_1 = const,\ E_1 \neq 0$). The areas of stability are dark.

Following analytical and numerical researches the conclusions are made:

1. To stabilize for unstable rotation of a rigid body with cavities containing a fluid is possible by rotating rigid body.
2. If elastic restoring moments are absent and the center of mass of rotating rigid body coincides with the common point of two rigid body, stabilization will be impossible.
3. The effect similar to action of restoring moment on considered system is observed at the big angular velocity of rotation of a rigid body ($\omega_0 > 100$) and at the big elastic restoring moment ($k > 100$).
4. For the counterbalanced rotating rigid body ($c_2 < 0$) the effect of stabilization increases in comparison with unbalanced rigid body ($c_2 \geq 0$).

$k = 100,\quad \omega_{02} = 0$

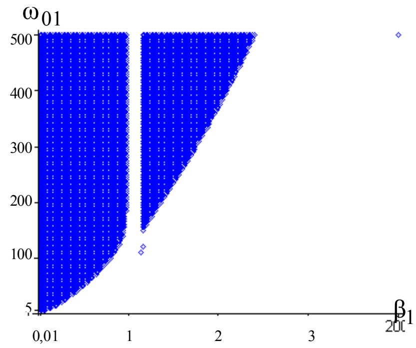

Fig. 3

$k = 100,\quad \omega_{02} = 0$

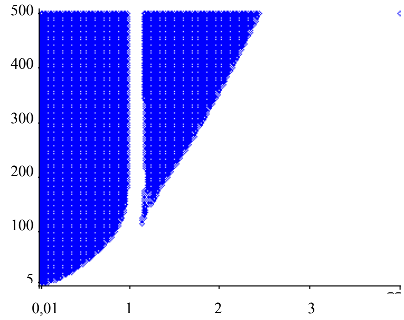

Fig. 4

$k = 100,\quad \omega_{02} = 1000$

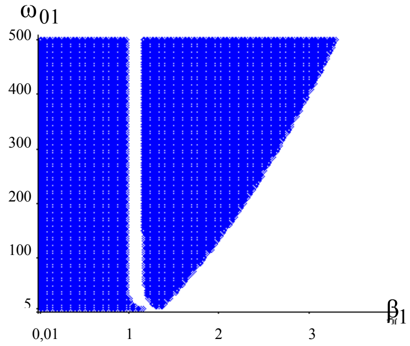

Fig. 5

$k = 1000,\quad \omega_{0} = 1000$

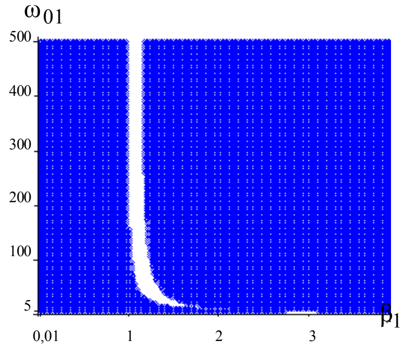

Fig. 6

References

1. Savchenko A. J. Stability for stationary motion of mechanical systems. Kiev: Naukova dumka, 1977, 160 p. (in Russian).
2. Kovalev A. M. Stability for uniform rotations of heavy gyrostat around of the principal axis. *Appl. Math. and Mech*. (AMM), 1980, 44, № 6, (in Russian).
3. Lesina M. E. About stabilization of based counterbalanced Lagrange gyroscope. *Mech. rigid body* , 1979, 11, p. 88 – 92, (in Russian).
4. Varkhalev Y. N., Savchenko A. Y., Svetlichnaja N. V. To a question for stabilization of based unbalanced Lagrange gyroscope. *Mech. rigid body,* 1982, 14, p. 105 – 109, (in Russian).
5. Svetlichnaja N. V. About effect of stabilization of a based unbalanced gyroscope by the second rotating. *Mech. rigid body,* 1989, 21, p. 74 – 76, (in Russian).
6. Sobolev S. L. On the motion of a symmetric top with a cavity filled a fluid. *Appl. Math. Tech. Phiz.* (AMTP), 1960, №3, p. 20-55, (in Russian).
7. Kononov Y. N. About influence of partitions in a cylindrical cavity on stability of uniform rotation of Lagrange gyroscope. *Math. physics and mechanics*, 1992, 7 (51), p. 33 – 37, (in Russian).
8. Kononov Y. N. On the motion of the system of two rigid bodies with cavities containing a fluid. *Mech. rigid body*, 1997, 29, p. 76-85, (in Russian).
9. Kononov Y. N. On the motion of the system of connected rigid bodies with cavities containing a fluid *Mech. rigid body,* 2000, 30, p. 207-216, (in Russian).
10. Kononov Yu. N., Khomyak T. V. Stabilization by rotating rigid bodies for unstable rotation of a rigid body with cavities containing a fluid. *ICTAM04 Abstracts and CD-ROM Proceedings*. Warsaw, Poland: IPPT PAN, Warszawa, 2004, 320 p.
11. Dokuchayev L. V., Rvalov R. V. About stability for stationary rotation of a rigid body with a cavity containing a fluid. Akad. Nauk SSSR, *Mech. Rigid body*, 1973, № 2, p. 6 – 14, (in Russian).