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ROBUST ESTIMATION IN DYNAMYC SYSTEMS SIMULATION

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In the course of market relations' development the demand for new economic and mathematical tool formation has appeared for both analysis of economical dynamics and a strategy generation of economic process management. The robust algorithm of ellipsoidal state estimation of dynamic systems is applied to search for the solution to the current economical situation.

Let the evolution of discrete dynamic system and its measuring line are described by equations

$$x_{k+1} = A_k x_k + f_k(u_k), \tag{1}$$

$$y_k = h_k^T x_k + \xi_k. \tag{2}$$

Here k = 0, 1, …— discrete time; x_k — n-dimensional state vector inaccessible to immediate change, $x_k \in R_n$; R_n — n-dimensional substantial Euclidean space; u_k — known control vector, $u_k \in R_m$, $m \le n$; y_k — vector of observed signals, which are the results of measuring of outcome of object $h_k^T x_k$, deadened by action of additive disturbance ξ_k, $y_k^T = (y_{1k},\ldots,y_{mk})$, $\xi_k^T = (\xi_{1k},\ldots, \xi_{mk})$; T — transposition operation symbol;

A_k and $h_k^T = \begin{pmatrix} h_{1k}^T \\ \ldots \\ h_{mk}^T \end{pmatrix}$ – known n×n and m×n order matrixes accordingly, det $A_k \ne 0$ $\forall_k = 0, 1,\ldots$, $h_{ik} \in R_n$; $f_k(u_k)$ — given n-dimensional function. It is assumed that disturbance ξ_k satisfies component or standard constraint:

$$\xi_{ik}^2 \le c_{ip}^2,\ i = 1,\dots, m,\ k = 0, 1, \dots; \tag{3}$$

$$(\xi_{ik} - \xi_{i,k-1})^2 \le c_{ir}^2,\ i = 1,\dots, m,\ k = 1, 2, \dots; \tag{4}$$

$$\xi_k^T \xi_k \le c_p^2,\ k = 0, 1, \dots; \tag{5}$$

$$(\xi_k - \xi_{k-1})^T(\xi_k - \xi_{k-1}) \le c_r^2,\ k = 1, 2, \dots, \tag{6}$$

where c_{ip},c_i, c_{ir}, c_r — given constants.

Assumed that the results of current measuring y_k, $k = 0, 1, \dots$, are known, there is a problem of ellipsoid estimation of unknown state vector x_k

$$x_k \in E[\hat{x}_k , H_k] = E_k, \tag{7}$$

where $E[\hat{x}_k , H_k] = \{x: \sigma(x, \hat{x}_k ,H_k) \le 1\}$, $\sigma(x, \hat{x}_k ,H_k) = (x - \hat{x}_k)^T H_k^{-1}(x - \hat{x}_k)$, $\hat{x}_k$ — ellipsoid centre, $H_k = H_k^T > 0$. The characteristic of prior ellipsoid $\hat{x}_k$, H_k when $k = 0$ is considered to be given.

From equalizations (1), (2) we have

$$\xi_k = y_k - h_k^T x_k, \tag{8}$$

$$\xi_{k-1} = y_{k-1} - h_{k-1}^T x_{k-1}. \tag{9}$$

Having used equalization (1), from (8) and (9) we found

$$\xi_k - \xi_{k-1} = z_k - r_k^T x_k, \tag{10}$$

where

$z_k=(z_{1k},\dots,z_{mk})^T= y_k - y_{k-1} - h_{k-1}^T A_{k-1}^{-1} f_{k-1}(u_{k-1})$, $r_k^T=\begin{pmatrix} r_{1k}^T \\ \dots \\ r_{mk}^T \end{pmatrix}=h_k^T-h_{k-1}^T A_{k-1}^{-1}$.

Let $\zeta_k = \xi_k - \xi_{k-1}$, relation (10) can be represented as additional measuring line of system (1)

$$z_k = r_k^T x_k + \zeta_k, \qquad (11)$$

where disturbance $\zeta_k = (\zeta_{1k}, \ldots, \zeta_{mk})^T$ satisfies component or standard constraint

$$\zeta_{ik}^2 \le c_{ir}^2,\ i = 1,\ldots, m,\ k = 1, 2, \ldots, \qquad (12)$$

$$\zeta_k^T \zeta_k \le c_r^2,\ k = 0, 1, \ldots, \qquad (13)$$

Let us study component-wise constraint of disturbance. There are sets

$$S_{ip}(x_k) = \{x_k: (y_{ik} - h_{ip}^T x_k)^2 \le c_{ip}^2,\ k = 0, 1,\ldots,\ i = 1, \ldots m\}, \qquad (14)$$

$$S_{ir}(x_k) = \{x_k: (z_{ik} - r_{ik}^T x_k)^2 \le c_{ir}^2,\ k = 1, 2,\ldots,\ i = 1, \ldots m\}, \qquad (15)$$

Relations (14), (15) define in phase space of system (1), (2) sets $S_{ip}(x_k)$ and $S_{ir}(x_k)$ of systems states which are compatible with the results of measuring. These sets are 2m hyperbands each of that is restricted by two parallel hyperplanes $y_{ik} - h_{ik}^T x_k = \pm c_{ir}$, i =1, …m.

Apparently, inclusion $x_k \in D(x_k)$ is true for unknown state vector x_k of system (1), (2), where $D(x_k)$ is 2m hyperbands intersection of sets (14), (15). Also, according to assumption, inclusion (7) is satisfied. Therefore intersection $x_k \in E[\tilde{x}_k, \tilde{H}_k]$ is also satisfied, where $E[\tilde{x}_k, \tilde{H}_k]$ is ellipsoid like (7), when $D(x_k) \cap E[\hat{x}_k, H_k] \subset E[\tilde{x}_k, \tilde{H}_k]$ is true. For building ellipsoid $E[\tilde{x}_k, \tilde{H}_k]$ is reasonable to use robust algorithm of successive building of 2m ellipsoids, which contain intersection of ellipsoids and $S_{ip}(x_k)$, i = 1, …m и $S_{ir}(x_k)$, i = m+1, …, 2m. These ellipsoids can be built according to minimum criterion of determinant (many- ellipsoid dimensional volume) or track of congruent matrixes $\tilde{H}_{ik}$, $\tilde{H}_{2m,k} = \tilde{H}_k$. Estimative ellipsoid $E[\hat{x}_{k+1}, H_{k+1}]$, for which inclusion $x_{k+1} \in E[\hat{x}_{k+1}, H_{k+1}]$ is satisfied, where x_{k+1} is unknown state vector of system (1), (2), is direct image of ellipsoid $E[\tilde{x}_k, \tilde{H}_k]$ which is linear mapped under equation (1). Ellipsoid $E[\hat{x}_{k+1}, H_{k+1}]$ centre and matrix are defined by relations

$\hat{x}_{k+1} = A_k \widetilde{x}_k + f_k(u_k),\ H_{k+1} = A_k \widetilde{H}_k A_k^T.$

In case (4) disturbance ξ_k and the speed of its changing according to system (1), (2) states sets, which are compatible with the results of measuring, are restricting by relations

$$S_p(x_k)=\{x_k:(y_k - h_k^T x_k)^T(y_k - h_k^T x_k)\le c_p^2, \quad (16)$$

$$S_r(x_k)=\{x_k:(z_k - r_k^T x_k)^T(z_k - r_k^T x_k)\le c_r^2. \quad (17)$$

These sets are cylinders in phase space R^n of system (1), (2). For its unknown state vector the inclusion $x_k \in D(x_k)$ is satisfied, where $D(x_k)=S_p(x_k) \cap S_r(x_k)$. In accordance with the fact that condition (7) is met inclusion $x_k \in D(x_k) \cap E[\hat{x}_k, H_k]$ follows directly. Therefore estimating ellipsoid, which securely contains unknown vector x_k, can be built with the help of intersection ellipsoidal approximation of $E[\hat{x}_k, H_k]$ and set $S_p(x_k)$ and then intersection approximation of derived ellipsoid and set $S_r(x_k)$. Also, system (1), (2) state vector estimation boils down to building of parametric ellipsoid family $E[\widetilde{x}_k(\nu), \widetilde{H}_k(\nu)]$ and $E[\widetilde{x}_k(\nu,\tau), \widetilde{H}_k(\nu,\tau)]$ like (7), which contain intersections $E[\hat{x}_k, H_k] \cap S_p(x_k)$ and $E[\widetilde{x}_k(\nu), \widetilde{H}_k(\nu)] \cap S_r(x_k)$ accordingly, where $\nu \ge 0$, $\tau \ge 0$ – family parameters. Estimating ellipsoid center $\hat{x}_{k+1}$ and matrix H_{k+1}, for which inclusion $x_{k+1} \in E[\hat{x}_{k+1}, H_{k+1}]$ is satisfied surely, are defined by relations $\hat{x}_{k+1} = A_k \widetilde{x}_k(\nu,\tau) + f_k(u_k),\ H_{k+1} = A_k \widetilde{H}_k(\nu,\tau) A_k^T$. Robust algorithm of construction of ellipsoid family $E[\widetilde{x}_k(\nu), \widetilde{H}_k(\nu)]$ is of the form

$$\widetilde{x}_k(\nu) = \hat{x}_k + \nu H_k h_k (I_m + \nu e_k)^{-1} \widetilde{y}_k, \quad (18)$$

$$\widetilde{H}_k(\nu) = \chi_k(\nu)(H_k - (1-\beta_k^2)\nu H_k h_k (I_m + \nu e_k)^{-1} h_k^T H_K), \quad (19)$$

$$\chi_k(\nu) = 1 + \nu c_p^2 + (\alpha_k - 1)\mu(\nu, e_k, \widetilde{y}_k). \quad (20)$$

Here $e_k = h_k^T H_k h_k$, $\widetilde{y}_k = y_k - h_k^T \hat{x}_k$, $\mu(\nu, e_k, y_k) = \nu \widetilde{y}_k^T (I_m + \nu e_k)^{-1} \widetilde{y}_k$

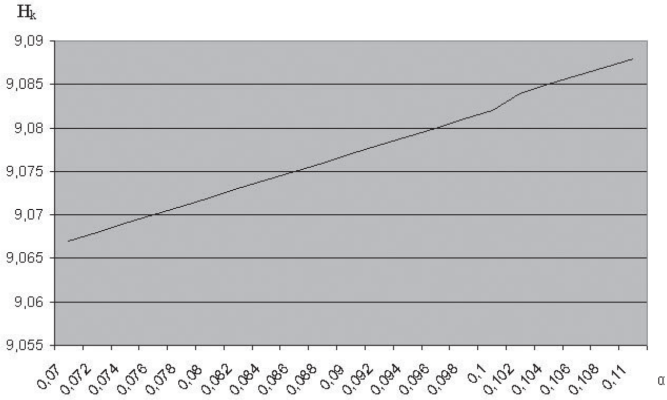

Figure 1 – Determinant det H_{10} α dependence diagram

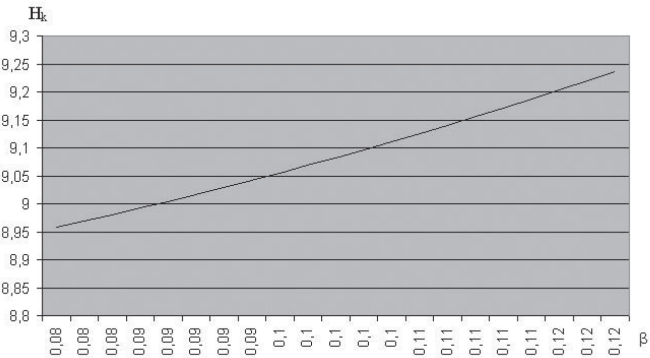

Figure 2 – Determinant det H_{10} β dependence diagram

I_m – unity m×m order matrix, $\beta_k \in (0,1)$ and $\alpha_k > 0$ – algorithm parameters. The centre $\widetilde{x}_k(\nu,\tau)$ and the matrix $\widetilde{H}_k(\nu,\tau)$ of ellipsoid $E[\widetilde{x}_k(\nu,\tau),\widetilde{H}_k(\nu,\tau)] \supset E[\widetilde{x}_k(\nu),\widetilde{H}_k(\nu)] \supset S_r(x_k)$ is calculating according to formulas (16) – (18) by replacement in their right parts variables ν, $\widetilde{x}_k(\nu)$, H_k, c_p and $\widetilde{y}_k = y_k - h_k^T \hat{x}_k$ by variables τ, $\widetilde{x}_k(\nu)$, $\widetilde{H}_k(\nu)$, c_r and $\widetilde{z}_k = z_k - r_k^T \widetilde{x}_k(\nu)$ accordingly.

The algorithm parameters verification is used for model

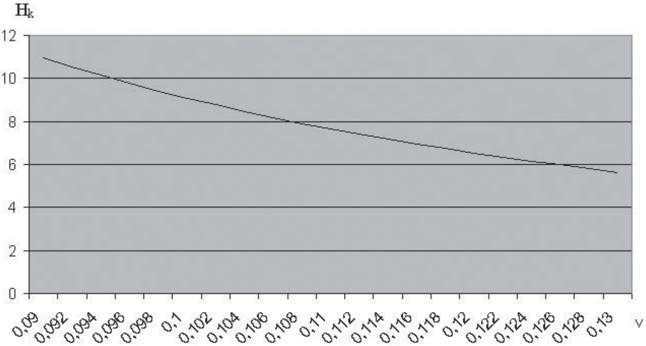

Figure 3 – Determinant det H_{10} ν dependence diagram

optimization. The determinant det H_k is fixed on step 10. Initial value α = 0.1 varies from 0.07 to 0.11 with fixed lead 0.002, β = 0.1 varies from 0.08 to 0.12 with fixed lead 0.002. There is a determinant det H_{10} variation caused by α and β diagram in the fig. 1-2.

The direct connection can be observed from fig.1-2. Therefore, α and β should be chosen as minimum.

Ellipsoid family parameter ν = 0.1 varies from 0.09 to 0.13 with fixed lead 0.002. There is a determinant det H_{10} variation caused by ν diagram in the fig. 3.

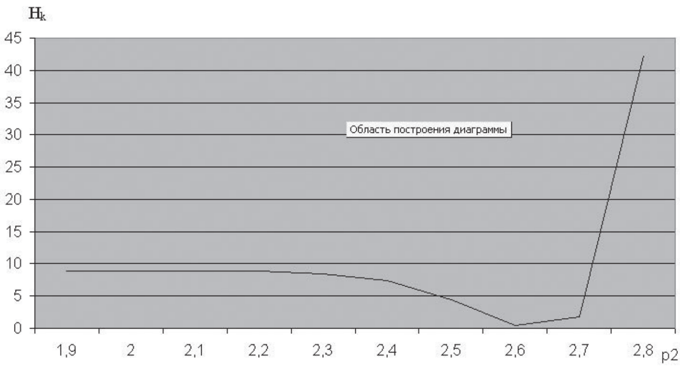

Figure 4 – Determinant det H_{10} p_2 dependence diagram

The indirect connection can be observed from fig. 3. Therefore, the bigger ν, the smaller determinant det H_{10}.

The control vector u_k parameter p_2 influence on determinant det H_k is also studied. Initial value $p_2 = 0.5$ varies from 1.9 to 2.9 with fixed lead 0.1. There is a determinant det H_{10} variation caused by p_2 diagram in the fig. 4.

Judging from pic. 4, it could be seen that when p_2 increases determinant det H_{10} decreases at first, then possesses the minimum value at the figure of 2.6, and then increases dramatically. However, when при $p_2 = 2.5$ determinant det H_k possesses the minimum value on the step 12, then starts growing. Therefore, the best value is $p_2 = 2.4$, where determinant det H_{10} possesses the value 7.364 at initial algorithm parameters. Current dependence can be observed starting from the step 9, as at previous steps the algorithm is inadequate and when p_2 decreases determinant det H_k keeps on decreasing.

References

[1] Volosov V.V. Robust algorithms of ellipsoidal state estimation of one type of nonlinear dynamic systems. // The problems of management and informatics, 2008, №1

[2] Volosov V.V. Shevchenko V.N. Robust methods of ellipsoidal estimation of dynamical systems state in the case of restrictions on the noise of measured output and the speed of its changing. // The problems of management and informatics, 2008, №5