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MODEL OF THE CATHODE AND ANODE SPOT PROCESSES

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As it was shown in analyzing the works dealing with the electrical discharge in water or other substances (such as petrol and oil), the erosion processes take place in the electrode spots. The review shown below describes the erosion processes in the electrode spots.

1. Erosion processes in the electrode spots

The theoretical investigation of the erosion processes will be carried out in the following way: the heat sources and heat sinks in the electrode spots will be examined, and then analyzes of calculated results will be carried out.

2. Heat sources and heat sinks in electrode spots. Statement of the problem

The heat transmission from the plasma discharge to the electrode body determines dynamics of the electrode processes, as well as mechanism of the electrode material erosion.

Let us examine the possible heat sources and heat sinks in the electrode spot, both in stationary one and moving with the certain velocity. The experimental investigations discovered these kinds of the electrode spots

2.1. Surface heat source generated by the energy transfer with the particles

The heat flow density due to the bombardment of the cathode surface with ions can be calculated in the form:

$$F_i = \frac{\beta\kappa}{1+\beta\kappa} j_\kappa (U_\kappa \mu_i + U_i), \quad (1)$$

where μ_1 - coefficient of ion accommodation in the cathode material; β - ion current fraction on the cathode; U_μ - cathode voltage drop; U_i - ionization potential of the bombarding ion; j_i – ion current density on the cathode surface.

The surface is cooled in the cathode spot due to the electron emission. The density of heat flow removed from with electrons is calculated:

$$F_e = \frac{\Phi - (e^3 E)^{1/2} + 2kT(o,y,z,t)}{e} j_e, \quad (2)$$

Where Φ –Electronic work function.

The heat provided by the electron flow:

$$F_{обр\,e} = \mu_e \frac{(\Phi + 2kTe)}{4\pi} n_e U_e \exp\left(-\frac{eU_k}{kTe}\right), \quad (3)$$

where μ - coefficient of electron accommodation; U_e - cathode voltage drop; n – electron concentration; U – electron velocity determined under the assumption of Maxwell velocity distribution; T_e – electron temperature.

2.2. Coefficient of ion and atom accommodation in the substrate material

In atom and ion bombardment of the crystalline solid surface, the energy interchange takes place as a result of collisions (both elastic and inelastic collisions) of the particles and atomic lattice. The particles are located on the surface according to the mean life time, and then leave the surface and removes some portion of energy.

The heat accommodation coefficient λ was proposed by Knoudsen. This coefficient can be determined in the form:

$$\lambda = \lim \frac{E_2 - E_o}{E_e - E_o},\ E_i \to E_o \tag{4}$$

where E_0 –particle energy; E_i – energy of the atom adsorbed on the surface; E_2 – energy of the particle leaving the surface.

It must be noted that the accommodation coefficient for molecule may be written on the form:

$$\alpha = \frac{\alpha_{\text{пос}_\text{р}} C_{\text{пост}} + \alpha_{\text{вр}} C_{\text{вр}} + \alpha_{\text{кол}} C_{\text{кол}}}{C_{\text{плась}} + C_{\text{вр}} + C_{\text{кол}}}, \tag{5}$$

where λ - accommodation coefficient for oscillatory and translational degrees of freedom; C – the corresponding addition to the molar heat capacity.

In the guess, that the separate atom of a surface participates only in one collision, it is ground of the prime laws of collision the relation for an accomodation coefficient is retrieved

$$\alpha = 1 - \frac{m^2 + M^2}{(m+M)^2}. \tag{11}$$

This formula provides rather low compliance with the experimental data.

2.3. Nottingen heat

With electrodes made of the fusible alloys, the Nottingem heat can provide a great addition to the surface heat source [10, 11]. This heat is released on the emitting surface and:

created the following heat flow density

$$F_н = 9{,}3.10^{-1}\ \mu_y E \sqrt{\Phi}\ j_k(o,y,z,t)\left(\frac{1}{\beta\kappa + 1}\right), \tag{12}$$

where E – electrical field intensity near the cathode, V/m; Φ – Electronic work function, eV; μ - electrical field amplification factor.

The Nottingem heat can be essential for the point (needle) cathodes, as well as for cathodes with the rough surface (with high electrical field amplification factor), i.e. with the roughness height – roughness radius being of 100 to 200, and electrical field amplification factor reaching the value of 50 to 100.

With no the electrical field available due to the surface roughness (this takes place for the usual cathodes), the Nottingem heat creates additional density of the heat flow being from several fractions of percents to 2 or 3 percents relative to the ion heat flow. At the same time, this value for the point cathode can be comparable or exceed the ion heat flow. So, the heat flow density in catode created by the taking into account the emission cooling, reverse electron current and the Nottingem heat, can be calculated:

$$F_к = F_i - F_e + F_{e\,обр} + F_н. \tag{13}$$

2.4 Volume source of heat

When studying the heat exchange processes in the electrode spot area, the various methods for specifying the volume heat source can be used.

The current spreading in the electrode spots area can be described with the Laplace's equation:

$$\frac{1}{\rho[T(x,y,z,t)]}\left(\frac{\partial^2\varphi}{\partial\chi^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}\right) = 0. \tag{14}$$

The boundary conditions for this equation can be found:

1. On the x=0 boundary:

$$\frac{1}{\rho[T(x,y,z,t)]}\left(\frac{\partial\varphi}{\partial\chi}\right) = \{\, j_k \text{ или } \textbf{or } j_a \text{ при } \textbf{at } \sqrt{}z^2 + y^2 < 0;\ 0 \text{ при } \textbf{at } \sqrt{}z^2 + y^2 > R. \tag{15}$$

2. On the $X=X_{k,}$ $Y=Y_н$, $Y=Y_k$, $Z=Z_н$, Z_k boundaries:

$$-\frac{\partial\varphi_0}{\partial\chi}=\rho[T_H]j_0;\quad j=j_o=\frac{j_{k,a}\quad\pi Rn^2}{2X_k(Y_k-Y_H)+2X_k(Y_k-Z_H)+(Y_k-Y_H)(Z_k-Z_H)},\quad(16)$$

where φ - electrode point potential, R – electrode spots radius (on the cathode and anode); $X_к$, $Y_н$, $Y_к$, $Z_н$, $Z_к$ - coordinates of the boundary under consideration; ρ[T(x,y,z,t)] - specific resistance as a function of a temperature.

Taking into consideration the above mentioned, the current density on the elementary volume will be found:

$$j(x,y,z,t)=\frac{j_k, a\pi R_n^2}{\rho[T(x,y,z,t)]}\left[\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial\varphi}{\partial z^2}\right]^{1/2}.\quad(17)$$

This solution is valid when using the numerical solution of current flow problem, i.e. when the temperature of each step n is slightly different from this of n+1 step. Otherwise, the system of equations shall be studied. For the volume heat source the relation is used:

$$Q_{об}=\rho[T(x,y,z,t)]j^2(x,y,z,t).\quad(18)$$

2.5. Heat dissipation due to evaporation border movement

Generally it is possible to write to heat rejection, owing to bias of front of transpiration, following view:

$$Q_{исп}=C_v[T(x,y,z,t)]V_ф\,\frac{\partial T(x,y,z,t)}{\partial x},\quad(19)$$

Where C_v[T(x,y,z,t)] - heat capacity of a material of an electrode in the given point at relevant temperature T(y,x,z,t), which one is set by an array with usage of experimental values of this quantity or with the help of a linear extrapolation; $V_ф$ - the quantity of velocity of bias of front of transpiration for a case, when on a surface of an electrode does not occur a stratum of metal, superheated above than melting point, i.e. T(o,y,z,t)<$T_{пл}$ ($T_{пл}$ - effective melting point of a material of an electrode taking into account heat input& on a fusion). This velocity of bias can be spotted:

$$V_ф=\frac{1}{\gamma}\sqrt{\frac{M}{2\pi RT(o,y,z,t)}}\left[\exp\left(A-\frac{B}{T}\right)-P_{OCT}\right],\quad(20)$$

where A and B – tabulated constants [14], R - gas constant, γ - electrode material density, M – atom weight of the heated electrode, and P – residual gas pressure.

With the superheated and melted metal on the surface, the Frenkel mechanism comes into effect. This being the case, the evaporation border movement velocity is

$$V_ф=V_o\exp\left(-\frac{T^*}{T(o,y,z,t)}\right),\quad(21)$$

where V and T - evaporation border movement velocity and spot surface temperature under the steady-state conditions, to be calculated from the system (15).

2.6. Heat removal from the electrode spot due to heat conductivity

The heat flow removed from the elementary volume can be found from the formula: (22) where a is the thermal diffusivity.

$$Q_T=C_V[T(x,y,z,t)]\lambda[T(x,y,z,t)]\Delta T(x,y,z,t),\quad(22)$$

2.7. Heat exchange due to the radiation

The heat supply to the electrode spot surface due to the plasma radiation increases the heat flow density by the value:

$$F_{ип}=\varepsilon_п\,QT_п^{\,4},\quad(23)$$

where Q – Stephan-Boltzmann's constant, T – plasma temperature, E – plasma emissivitity factor.

The heat extraction from electrode spot due to the radiation will be found:

$$F_и=\varepsilon\,Q[T(o,y,z,t)]^4\,.\quad(24)$$

here E – electrode material emissivitity factor, Q – Stephan-Boltzmann's constant ($Q = 5.67\cdot10^{-8}$ W/m$^2\cdot$grad^4).

2.8. Decreasing the surface heat source intensity due to the evaporation

The surface heat source intensity is decreased due to the evaporative gear removal according to the formula:

$$F_{исп} = L_{исп}\, \gamma V_{ф},, \quad (25)$$

where L - Evaporation heat. The rate of evaporation border movement (with the temperature below the melting point) is determined according to the equation (27).

2.9. Heat exchange due to the electrode spot movement

The heat exchange in the electrode spot area can be stipulated by the spot movement. The heat flow being rejected from the spot due to the spot movement can be calculated:

$$Q_П = C_v\,[T(x,y,z,t)]V_n \frac{\partial T(x,y,z,t)}{\partial y}, \quad (26)$$

where V – the heat source velocity (electrodynamic displacement speed for some electrode spot modes).

2.11. Statement of elementary volume heat exchange problem

Taking into account the heat sources/sinks described, the heat balance in the spot elementary volume (Ref. Fig. 1) can be described by the following system:

$$C_v[T(x,y,z,t)] \frac{\partial T(x,y,z,t)}{\partial t} - C_V[T(x,y,z,t)]V_n \frac{\partial T(x,y,z,t)}{\partial y} =$$

$$= C_V\,[T(x,y,z,t)]\nabla\lambda[T(x,y,z,t)]\ \nabla T(x,y,z,t)+C_V\,[T(x,y,z,t)]x$$

$$x\ V_Ф \frac{\partial T(x,y,z,t)}{\partial x} + \rho[T(x,y,z,t)]j^2(x,y,z,t) + \frac{Bj_{i,e}\ \mu_{i,e}\ \partial E}{C_V[T(x,y,z,t)]\partial x}. \quad (27)$$

The boundary conditions are composed taking into consideration the surface heat source and the surface source intensity change due to mutual radiation exchange in the plasma – electrode system, as well as due to the heat removal with the evaporated metal:

1. With $t < \tau_Ж$:

$$\frac{\partial T(o,y,z,t)}{\partial x} = \frac{F_1}{\lambda} - \frac{F_{ÈÑ\tilde{I}}}{\lambda_1} + \frac{Q}{\lambda}\left[E_{\tilde{I}}T_{\tilde{I}}^4 - ET^4(o,y,z,t)\right] - \frac{F_{Êîí}}{\lambda}, \quad (28)$$

With $\sqrt{z^2+(y^1)^2}$ {<R, then F=F_K, or F = F_a. where R – spot radius.

2. With $t > \tau_Ж$ F = 0

The boundary conditions for X_K, Y_H, Y_K, Z_H и Z_K planes can be put down in the form:

$$X_K = \sqrt{6at} \qquad T(x_K,y,z,t) =$$
$$Y_H = -R-\sqrt{6at} \qquad = T(x,y_H,z,t)=$$
$$Y_K = R+\sqrt{6at}+V_П\,\tau_Ж \qquad = T(x,y_K,z,t)=$$
$$Z_H = -R-\sqrt{6at} \qquad = T(x,y,z_H,t)=$$
$$Z_K = R+\sqrt{6at} \qquad = T(x,y,z_K,t) = T_H. \quad (29)$$

The electrode area with the coordinates X_H = 0, X_K, Y_H, Y_K, Z_H and Z_K Is shown in the Fig. 1. These coordinates bound the space area involved in this heat problem.

The initial conditions: t=0, T(x,y,z,o)=T_H. (30)

The volume heat source mode is determined by the solution of Laplace's equation (20) followed by calculation of the current density in the electrode point specified (23) and volume heat source intensity (18).

2.12. Calculation of erosion factor

The problem stated can be calculated by the numerical methods. The mass of evaporated metal is determined by integrating the evaporation rate over the electrode surface during an electrode spot effective time (the doubled time of electrode spot average life):

$$M_{ИСП} = \int_{Z_H}^{Z_Ê}\int_{Y_{\tilde{I}}}^{Y_Ê}\int_0^{2\tau_æ} W[T(o,y,z,t)]dzdydt, \quad (40)$$

where $W\,[T(o,y,z,t)] = V_{\hat{O}}\gamma$.

To determine the metal mass evacuated in the fluid phase, the main mechanisms of liquid metal ejection must be examined.

2.13. Liquid metal ejection

The liquid metal can be ejected from the electrode surface due to the following phenomena:

1. Electrostatic mechanism of the material ejection. This mechanism takes place in case of high electrical field intensity near the electrode surface (15 – 16).

2. Electromagnetic and electrodynamic ejection of a liquid metal stipulated by the magnetic pressure (17 – 18). This mechanism takes place in case of high current value in the electrode spot.

3. Gas-dynamic ejection at a high temperature in the electrode spot, with high evaporated metal pressure (19 – 21).

4. Thermoelastic and thermo-elasticoplastic ejection of a liquid metal stipulated by the non-stationary temperature stresses, causing the thermoelastic and thermoelasticoplastic stress wave in the liquid metal resulting in metal ejection (22, 23, 24).

The electromagnetic pressure in the dicharge channel can be found:

$$P_{\text{ÝÌ}} = \frac{H^2}{8\pi}\frac{S}{S_B}, \tag{41}$$

Where H - magnetic field intensity near the electrode spot; this value is determined by the spot current. Then:

$$P_{\text{ÝÌ}} = \frac{j_K^2 R_0^2}{128\pi}\frac{S_0}{S_B}, \tag{42}$$

Where j – electrode spot current density.

The current spreading is characterized by the complex pattern. So, electromagnetic ejection physics will be examined only. It is evident that the self-magnetic field will create the Ampere force detaching the molten metal from the surface. The surface tension forces prevents metal from detaching.

The magnetic field and electrodynamic force distribution in the spot area. Depending upon the current density distribution, the magnetic pressure can be distributed according to the parabolic or Gaussian (normal) distribution.

The gas kinetic (gas-dynamic) mechanism of the metal ejection consists in the fact that the metal vapors create the reactive pressure upon the liquid metal surface in the electrode spot area. The pressure value can be estimated:

$$P = nkT(o,y,z,t)\frac{S_0}{S_B}, \tag{43}$$

Where K – Boltzmann's constant, n – evaporated metal density over the electrode (depends on a temperature and evaporation mechanism). The pressure distribution in the spot area stipulated the above mechanism can be seen that the metal can be evacuated along the spot perimeter.

Thermoelastic and thermoelasticoplastic ejection of a liquid metal consists in the following. The considerable non-stationary thermal stress can be created on the liquid-solid metal border. The above stress causes the stress wave in the solid metal body and rises the P pressure; this provides ejection of a liquid metal. The stress value can be determined by joint solution of the thermal conductivity equation and thermoelastic equation. The maximum probability of liquid metal ejection corresponds to the spot perimeter.

The electrodynamic ejection of a liquid metal is connected with the electrodynamic (Lorenz - F_n) force applied to the molten metal in the area of current spreading (Ref. Fig. 6.2 г). So, the electrodynamic pressure applied to the molten metal will be found:

$$P_{\text{ÝÄ}} = \frac{F_n}{S_B} = \frac{1}{S_B}\left|g\left(V_P \cdot B\right)\right| \approx j^2 R_0 b \cdot 5 \cdot 10^{-6}\left[\frac{H}{M^2}\right], \tag{44}$$

Where S – area of metal ejection; q – the charge value accumulated in the ejected metal, B - magnetic induction of the own current; V – charge velocity.

The metal ejection due to the charged particle flux occurs in pressure transmission to the molten metal surface and metal ejection in the low-pressure spits. The pressure value

$$P_{i,e} = n_{i,e}\, kT_{i,e}\frac{S}{S_0}, \tag{45}$$

Where S_0- spot surface area, n and T – ion/electron concentration and temperature.

The surface tension, viscosity, and external pressure P_c prevents metal from ejection.

The surface tension pressure can be calculated:

$$P_{\text{ÏÍ}} = \frac{2Q}{R_{\text{Ì}}}, \tag{46}$$

Where Q – surface tension coefficient; R – meniscus radius.

The viscosity force preventing a metal from ejection:

$$F_B = -\int \eta(T)\frac{\partial V}{\partial x} \approx \eta(T)\frac{\Delta P}{\gamma_B V}S_0, \qquad (47)$$

Where η(T) - viscosity coefficient (as function of the liquid metal temperature); $\frac{\partial V}{\partial x}$ - the velocity change along the molten pool depth; δP – excess of pressure created by the ejecting forces over the forces preventing the ejecting; V – liquid metal velocity; b – liquid metal pool depth.

The inertial forces preventing a metal from ejection:

$$P_И = - m\,W \approx \Delta P S_0, \qquad (48)$$

i.e. the inertial forces pressure is comparable with the excess of pressure created by the ejecting forces over the forces preventing the ejecting.

The ejecting condition consists in exceeding the containment forces by the ejecting forces (taking into consideration the forces directions, i.e. signs):

$$P_\Sigma = P_{ЭС} + P_М + P_Г + P_{ТУ} + P_{ЭД} + P_{i,e} > P_{ПН} + P_В + P_И + P_С. \qquad (49)$$

In general case, all components of this inequation must be determined; then the pressure distribution allows finding the areas with a molten metal to be ejected. The summary pressure distribution over the molten metal along the radius determines profile of the frozen metal, i.e. bulge, needle point, or bulge with a needle point can be created.

The molten metal mass, as well as ejected mass are determined by the volume with a temperature exceeding the molting point (provided that the ejecting forces exceed the containment forces). This allows tracing the metal ejection time dependence [29, 52, 55]. The erosion coefficient for electrode spot under investigation is calculated in the form relation of the evaporated mass (K_1 coefficient) or summary evaporated mass (K_2 coefficient) – value of the charge transferred in the electrode spot (to be determined from the equation):

$$g = \int_{\text{Ó}_{\text{Í}}}^{\text{Ó}_{\text{Ê}}}\int_{Z_{\text{Í}}}^{Z_{\text{Ê}}}\int_{0}^{\tau\text{æ}} j(o,y,z,t)\,\partial y\,\partial z\,\partial t, \qquad (50)$$

where $j(o,y,z,t) = j_{K(a)}$ with $\sqrt{(y^1)^2 + z^2 \le R_n}$;$j(o,y,z,t) = 0$ with $\sqrt{(y^1)^2 + z^2 > R_n}$.

The erosion coefficient is determined as:

$$K_1 = \frac{m_{\text{ÈÑÏ}}}{g}, \quad K_2 = \frac{m_{\text{ÈÑÏ}} + m_{\text{ÏËB}}}{g}. \qquad (51)$$

When calculating the steps quantity accounts for 10 along the axis; the time stem was divided to 20 increments.

3. Calculated erosion coefficient and geometrical characteristics of the blasted area

The numerical calculations allow determination of the erosion coefficients in the electrode spots (in the cathode spot – Ref. Fig. 2, and anode spot – Ref. Fig. 3) depending on the spot life time for various current densities (1 – 10^{11}, 2 – 10^{10}, 3 – 10^9, 4 – 10^8 A/m^2), ferrous electrode. The jump discontinuities can be found in both 2 and 3 figures for high current densities. This discontinuities are connected with the liquid metal ejection (on the cathode at $j=10^{11}$ and 10^{10} A/m^2, and on the anode at $j=10^{10}$ and 10^9 A/m^2). These dependencies ensure calculations of the erosion coefficients using the experimental data on current density and spot life time. Then, the ejected mass can be determined by use of the above erosion coefficients and spots quantity N_n:

$$m_{PK} = K_{ПК}[j_K, \tau_K]\cdot J_{ПК}\cdot \tau_{ПК}\cdot N_K,$$
$$m_{PA} = K_{ПА}[j_A, \tau_A]\cdot J_{ПА}\cdot \tau_П\cdot N_A.\,. \qquad (52)$$

The geometrical characteristics of the electrode blasted area can be found with help of experimental data on electrode spot areas and blasted area depth on the anode and cathode (Figs. 4 and 5, 1 – 10^{11}, 2 – 10^{10}, 3 – 10^9, 4 – 10^8 A/m^2) depending on the spot life time. These relations are also characterized by the jump discontinuities with the parameters corresponding to the metal ejection. If the complete ejection took place in this case, the blasted area depth reached the maximum possible value. With the partial ejection, blasted area volume can be calculated:

$$V_{ПОРК} = N_{ПК}\cdot S_{ПК}\cdot \Delta l_{ПК},$$
$$V_{ПОРА} = N_{ПА}\cdot S_{ПА}\cdot \Delta l_{ПА}. \qquad (53)$$

4. CONCLUSION

The lead analytical investigations allow to make following deductions:

1. The model allows calculating the mass transfer in electrode spots.
2. The maximum possible productivity for electric erosion machining can be calculated.

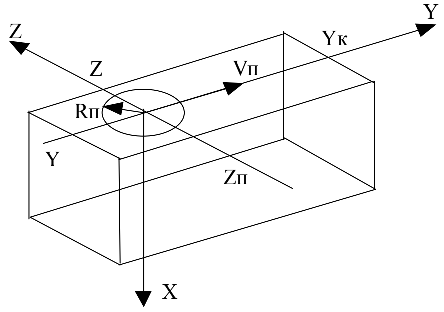

fig.6.1. The electrode area with the coordinates $X_H = 0$, X_K, Y_H, Y_K, Z_H

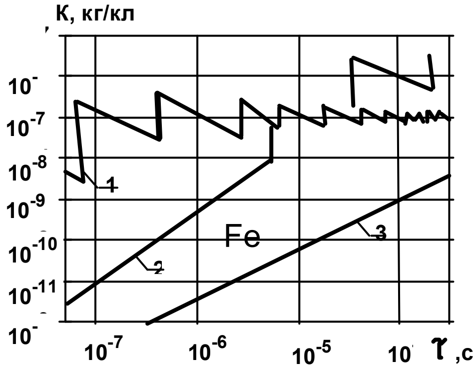

fig.3. the erosion coefficients in the anode spots

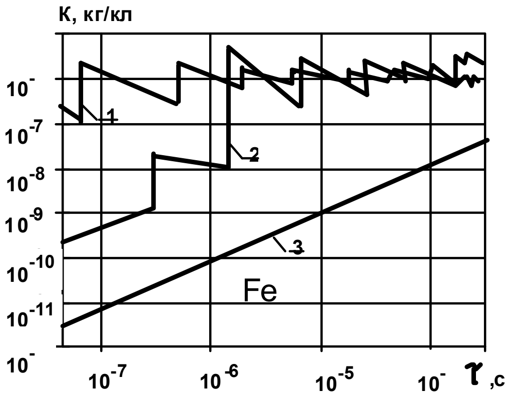

fig.2. the erosion coefficients in the cathode spots

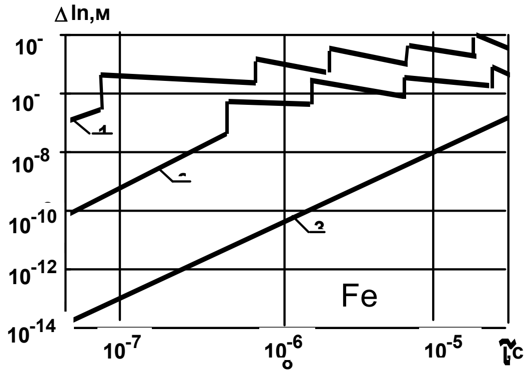

fig.4. the geometrical characteristics depth on the cathode

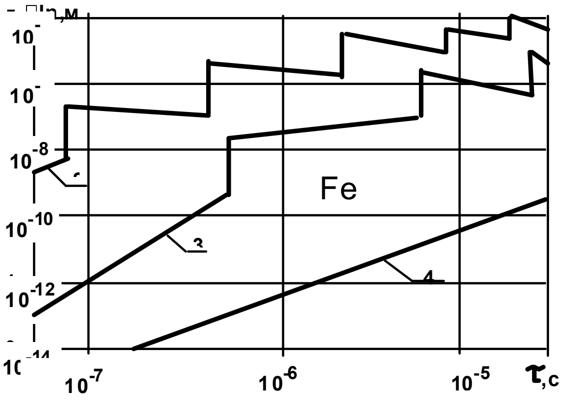

fig.5. the geometrical characteristics depth on the anode

Reffereses

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