ANALISYS OF DYNAMIC LOAD AND A DYNAMIC COEFFICIENT AT TIME-EFFECT STRENGTH

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The article presents the analysis which tends towards the load appointment caused by impacts. The mass falling from a particular height is the most frequent case causing the dynamic load. In order to illustrate the dynamic force (reaction) it is needed to appoint the character of the impact and find the restitution factor. Duration of the dynamic impulse allows us to discover the value of the dynamic force (reaction) as well as a dynamic coefficient. Calculated protection coefficients for an assumed model of a bending beam are higher than the ones used during designing.

1. Introduction

An impulse is the most frequent source of a dynamic load. During the impact there is a transformation of the energy from the one body to the other, as a consequence of a contact of two bodies. Time of the contact and its character are important. In this case, we do not deal with forces acting continuously on the substance but in a particular part of time [1]. In mining, the main reason of a dynamic load occurrence is an impact. Cracking rock layers, rocks falling off the mining ceiling and walls in an excavation, during dislocation hit a support. It causes a great hazard for its functionality. This article tries to define the influence of an impact on a reaction of a hitting body. It has an essential meaning for a designing process, e.g. of a drift support. A dynamic coefficient for a bending beam has been calculated.

2. Force impulse as a main reason of a dynamic load occurrence

Dynamic effect is an impulse acting of a load. It is generally a collision of two or more bodies having a particular mass or a hitting a body in state of rest by a force having a particular mass. A law of a momentum change is connected with that phenomenon.A momentum change as well as a velocity of a body is related to a force acting in a certain period of time. During dynamic loads we deal with bodies collision phenomena, when the time of the process is very short. A body moving with a particular velocity and having a particular mass when hitting another body it can lose its velocity or get it in a different direction. In that case it is very hard to calculate the impact forces. That is why, in such cases, I use impulse forces. Such impulse acting at a moment t_o and lasting Δt can be calculate from a relation:

$$d\overline{F} = \int_{t_o}^{t_o+\Delta t} \overline{F}\, dt$$

Treating Δt as an infinitesimal value, we get a force F as a value infinitely high. It is an impact force or a instantaneous force.

Assuming that the period of time presented by Δt is very small but finished we deal with a force average:

$$\overline{F}_r = \frac{\overline{F}}{\Delta t}$$

The velocity change suits the impulse of an impact :

$$m\overline{v}_k - m\overline{v}_p = \overline{F}$$

so, the velocity after a crash: $\overline{v_k} = \overline{v}_p + \frac{\overline{F}}{m}$

Dislocation of a body in the moment of the impact can be reckoned using the formula: ds = V_k dt, so

$$\Delta s = \int_{t_o}^{t_o+\Delta t} v_k dt = \left(v_p + \frac{F}{m} \right)\Delta t$$

According to above dependence it is seen that dislocation is very small for a small value Δt. In the case when we deal with an impact of a particular mass in a stopping or in a body of a big mass being in a state of rest, at an angle α and with a velocity V_p, the mass returns at an angle β with velocity V_k (Fig 1).

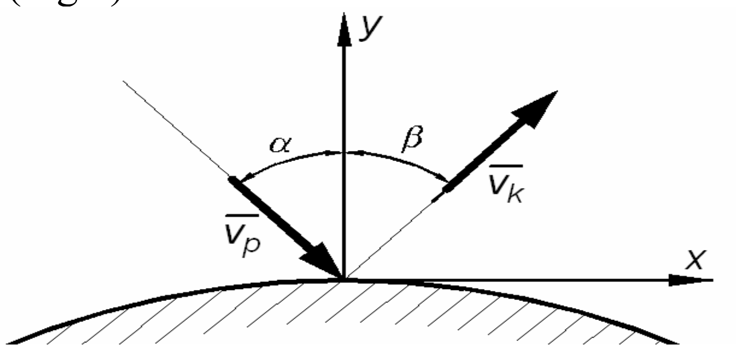

Fig. 1 Drawing of an impact in an unmoving stopping.

In order to find out a dependence between a falling angle and a glancing angle I will use a formula made by Newton (called the Newton's kinematics hypothesis) that allows us to calculate a restitution coefficient (it is also known as a Poisson Dynamic Hypothesis according to which the impulse of a relief phase is proportional to the impulse of a load phase, and is the same as a proportionality coefficient, called a restitution coefficient). It defina dependence between normal components of velocity and does not depend on size of the bodies but on the material the bodies are made of.

$$k = \frac{V_{kN}}{V_{pN}} = \frac{v_k \cos \beta}{v_p \cos \alpha}$$

from a Fig. 1appears that:

$$mv_k \cos \beta + mv_p \cos \alpha = \int_{t_o}^{t_o+\Delta t} F dt = F_{im}$$

These equations allows us to calculate an impulse value F_{im}.

$$F_{im} = mv_p (k + 1) \cos \alpha$$

and a final velocity:

$$v_k = v_p k \frac{\cos \alpha}{\cos \beta}$$

During the elastic impact (k=1) a falling angle is the same as a glancing angle, and a velocity after hitting is the same as before it. In the case of a deformation model (actual body) when the bodies touch they deform, during the period of time Δt and its velocity V_p lows to zero. A reaction R_1 of a hit body appears in that moment and the dependence can be calculated:

$$R_1 = \int_{t_o}^{t_o+\Delta t} F_1 dt = mv_p \cos \alpha$$

In the next phase the hit body responses. Thanks to elastic forces it tries to return to a starting state and causes a reaction R_2 which impulse can be calculated from:

$$R_2 = \int_{t_o+t_2}^{t_o+\Delta t+t_2} F_2 dt = mv_k \cos \beta$$

The ratio of impulses in both collision phases is:

$$\frac{R_2}{R_1} = \frac{mv_k \cos \beta}{mv_p \cos \alpha} = k$$

It is seen that a restitution coefficient depends on properties of the bodies that collide. In the case when the impact occurs vertically to a surface of a hit body, the angles α = β= 0. The restitution coefficient is:

$$k = \frac{v_k}{v_p}$$

Taking the energetic analysis into account, a difference between a kinematics energy before and after impact can be calculated :

$$E_2 - E_1 = \frac{mv_p^2}{2}(k^2 - 1)$$

At the moment of the impact a kinematics energy decrease as a restitution coefficient is lower. In the case of the plastic impact all kinematics energy is lost (k=0). In this case a velocity after deflection (V_k) is zero. In the case of the elastic impact (k=1) there is no loss of a kinematics energy. Experiments show that a restitution coefficient for actual bodies oscillate from zero to one. A Fig. 2 present some values of restitution coefficient for different materials (a case when both bodies are made of the same material for elastic impacts) [4].

3. Calculation of a dynamic force (reaction)

Fig. 2 shows the process of an impact force. We can distinguished a load phase from the beginning of the contact to reaching the maximum effect force as well as a relief phase from this moment to the end of the contact.

Taking a character of an impact into account a value of the impact force can be reckoned (surface reaction).

From the relation:

$$-\int_{t_o}^{t_o+\Delta t} R dt = -mv_k - mv_p$$

The reaction can be calculated:

$$R = \frac{m(v_k + v_p)}{\Delta t} = \frac{mv_p(k-1)}{\Delta t}$$

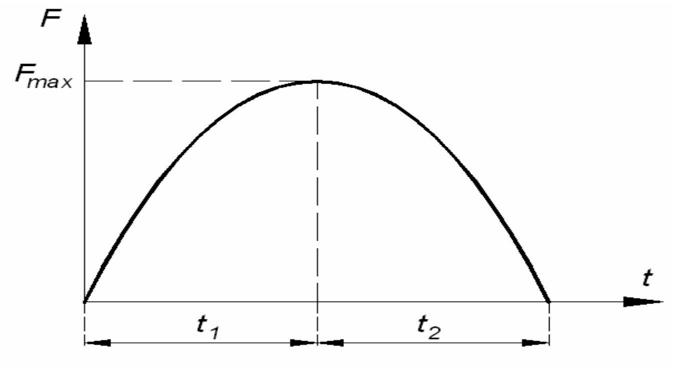

Fig. 2. Process of an effect force in the case of an elastic impact.

Fig. 3 shows a relation between a restitution coefficient and a reaction of a surface assuming that a mass of 1kg hits the surface from a height of 1m.

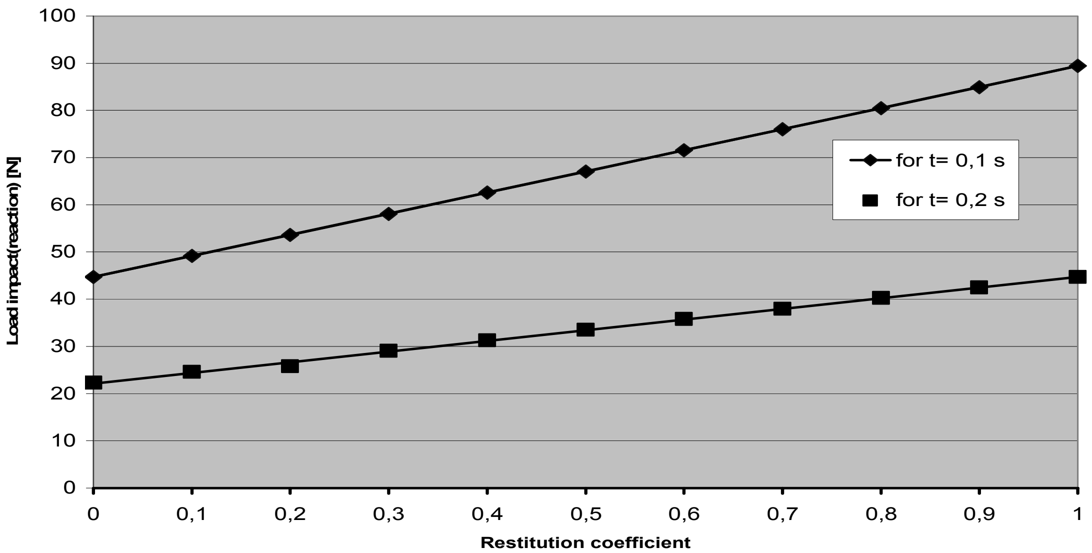

Fig. 3 Relation between a restitution coefficient and an impact force.

4. Calculating a dymanic force in a bending beam

As a model of a support subjected to a percussive load, a supported beam can be used (as in a Fig. 4). In the case when a beam is loaded with a static force Q=mg, acting in a middle of the beam, its bending is calculated from a dependence:

$$f_{st} = \frac{Q(2a)^3}{48EJ}$$

where: bending stiffness EJ=const.

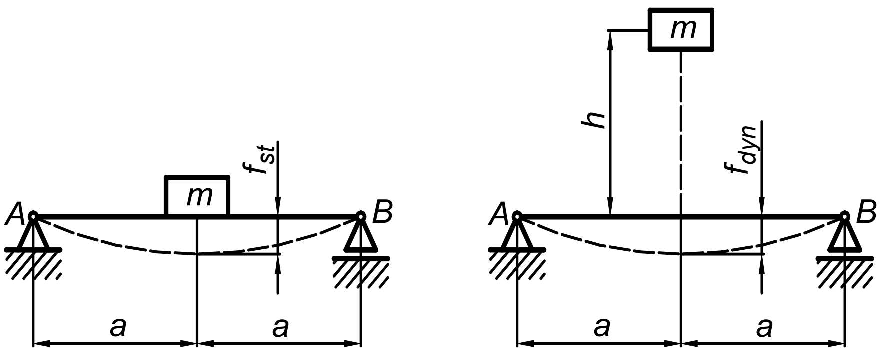

Fig. 4 Model of a beam loaded with a static and a dynamic force.

Assuming by analogy that a similar relation takes place between a bending resulted from the action of a dynamic load of a mass m, falling from a height h, a dynamic force P_d can be reckoned

$$f_{dyn} = \frac{P_d (2a)^3}{48EJ}$$

so:

$$P_{dyn} = \frac{f_{dyn} 48EJ}{(2a)^3} = \frac{Q}{f_{st}} f_{dyn}$$

A dynamic coefficient is the ratio of a dynamic deformation to a static deformation.

$$K_{dyn} = \frac{f_{dyn}}{f_{st}}$$

This coefficient can be reckoned from an energetic condition. The energetic condition: E_k=E_p. A kinematics energy of a weight Q is equal to a work done during a dislocation (h + f_{dyn}). It transforms into a potential energy of an elastic deformation of a beam.

$$E_k = \frac{mv^2}{2} = Q(h + f_{dyn}) \;;\; E_{spr} = W = 0{,}5 P_{dyn} f_{dyn}$$

$$Q(h + f_{dyn}) = \frac{1}{2} P_{dyn} f_{dyn} = \frac{1}{2} \frac{Q}{f_{st}} f_{dyn}^2$$

After transforming we have:

$$f_{dyn} = f_{st} + \sqrt{f_{st}^2 + 2hf_{st}}$$

From this formula we can reckon a dynamic coefficient:

$$K_{dyn} = 1 + \sqrt{1 + \frac{2h}{f_{st}}}$$

In the case when a value h is big to a f_{st} ratio, approximate dependence can be presented:

$f_{dyn} \approx \sqrt{2 f_{st} h}$ or in a velocity function : $f_{dyn} = \sqrt{\frac{v^2}{g} f_{st}}$;

5. Conclusions

On the basis of the undertaken analysis it can be claimed that a dynamic load is a very unfavourable kind of load. The same force (mass) depending on the time of acting and the height from it falls can cause very different loads. It has very essential influence on the effects that it produces. According to the above analysis the shorter contact and the bigger velocity at the moment of the beginning of the contact, the power of an impact is bigger. The character of the contact is also crucial.

The above analysis shows that in the case of a load resulting from an impact a load of a body (reaction) is a few times bigger than a static load loaded by the same mass. It is essential during a designing of e.g. a drift support where a static load is a base of its design. Dynamic coefficients are assumed basing on the forecast, in fact they are too low [2]. Loading a beam with a dynamic force it can be noticed that a minimum dynamic coefficient is bigger than two. Property of a material is also significant. Impulse loads radically change the characteristics of the material [3].

Material presented in this article should be assumed as the introduction to the further theoretical analysis as well as practical verification. The aim is to calculate real dynamic loads acting e.g. on a mining support.

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