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ANALYSIS OF OBSERVED-BASED CONTROL SYSTEMS WITH UNMEASURED DISTURBANCE

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Abstract

Transfer functions and equivalent transformed structures of observed-based linear time-invariant control systems with the deterministic unmeasured disturbance have been found. They allow simplifying the analysis of these systems.

Keywords: control system, estimation, observer, analysis

Introduction

Control quality in dynamical systems depends on the amount and accuracy of information about the plant, in particular on the possibility of measuring the state variables. If necessary for system design feedbacks are difficult or impossible to measure, then for their estimation the observers can be used (state estimators) [1-4 and many others]. Observed-based control systems have complicated structures for analysis, especially when we investigate a system response to unmeasured disturbance. Common examples of deterministic unmeasured disturbances include load torque in electric drive systems.

The goal of this paper is an analysis of the mutual influence of the separately designed control system and observer on static and dynamic properties of the resulting control systems with an unmeasured deterministic disturbance.

The research results

We consider a linear time-invariant SISO-system with state feedback controller (mode controller), in which some of the state variables can be measured, and some – cannot [5]. In this case, the non-measurable state variables can be estimated by the full-order Luenberger-observer, open-loop part of which is a model of an appropriated plant part. Thus then some of feedbacks of state controller are partly measured and some of feedbacks are partly estimated.

In general, the plant in this system can also include the internal closed-loops in its structure. For example, often it is necessary to insert the current control loop in the mode control system of DC electric drive; it provides a limitation of current. This part of system usually does not include in observer model.

Structure of described system in state space is given in Fig. 1, in which such signals and parameters have been specified:

u – control input of plant;

v – reference input of system;

d – unmeasured disturbance;

$\mathbf{X}_{\text{p1}}$, $\hat{\mathbf{X}}_{\text{p1}}$ – m×1-measured (sensed) and estimated parts of plant state vector, respectively;

$\mathbf{X}_{\text{p2}}$ – r×1 plant state vector, which is not estimated by the observer;

$y_2 = u_1$ – control input of the observer;

$\tilde{y}_1 = y_1 - \hat{y}_1$ – estimation error of the plant output y_1;

$\mathbf{L} = [l_1 \quad l_2 \quad \ldots \quad l_m]^T$ – observer gain vector;

$\hat{y}_f = \mathbf{K}_{\text{o1}}\hat{\mathbf{X}}_{\text{p1}}$ – resulting feedback signal by the observer;

$\mathbf{K}_{\text{p1}}$, $\mathbf{K}_2$, $\mathbf{K}_{\text{o1}}$ – feedback gains of the state controller by vectors $\mathbf{X}_{\text{p1}}$, $\mathbf{X}_{\text{p2}}$, $\hat{\mathbf{X}}_{\text{p1}}$ respectively:

$$\mathbf{K}_{\text{p1}} = [k_{\text{p11}} \quad k_{\text{p12}} \quad \ldots \quad k_{\text{p1}m}], \qquad \mathbf{K}_2 = [k_{21} \quad k_{22} \quad \ldots \quad k_{2r}],$$

$\mathbf{K}_{o1} = [k_{o11} \quad k_{o12} \quad \dots \quad k_{o1m}]$.

Vectors $\mathbf{K}_{p1}$, $\mathbf{K}_2$ and $\mathbf{K}_{o1}$ are associated with the mode controller gains without observer through such expressions:

$$\mathbf{K} = [k_1 \quad k_2 \quad \dots \quad k_m \mid k_{m+1} \quad k_{m+2} \quad \dots \quad k_n] = [\mathbf{K}_1 \mid \mathbf{K}_2], \tag{1}$$

where $n = m + r$ – the order of mode control system,

$$\mathbf{K}_1 = \mathbf{K}_{p1} + \mathbf{K}_{o1}. \tag{2}$$

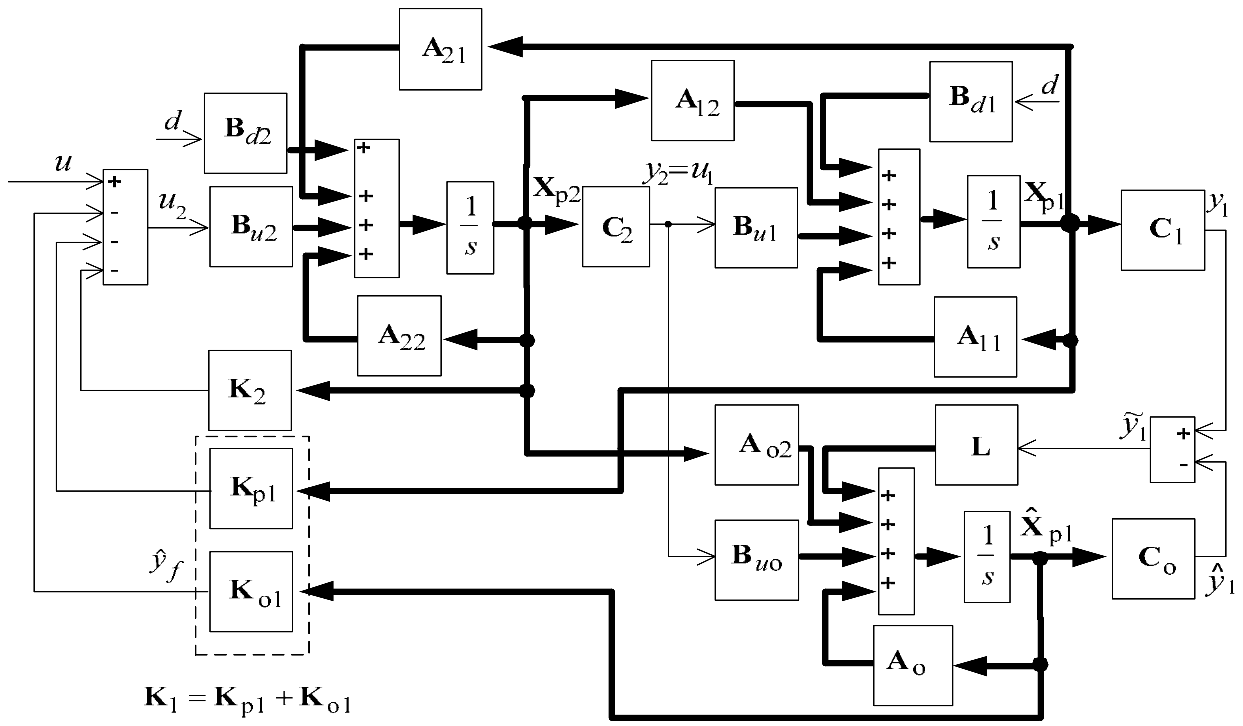

Figure 1 – Structure of mode control system with the observer, which estimates the part of plant state

Obviously, the vector $\mathbf{K}_{o1}$ has zero-elements in those positions at which the vector $\mathbf{K}_{p1}$ contains nonzero-elements, and vice versa.

Assume that the parameters of the direct part of observer-structure coincide with the corresponding parameters of the first part of the plant:

$$\mathbf{A}_o = \mathbf{A}_{11}, \mathbf{B}_{uo} = \mathbf{B}_{u1}, \ \mathbf{C}_o = \mathbf{C}_1, \ \mathbf{A}_{o2} = \mathbf{A}_{12}. \tag{3}$$

The dynamic of this system is described by equation

$$s\mathbf{X}_{p1} = \mathbf{A}_{11}\mathbf{X}_{p1} + (\mathbf{A}_{12} + \mathbf{B}_{u1}\mathbf{C}_2)\mathbf{X}_{p2} + \mathbf{B}_{d1}d, \tag{4}$$

$$s\mathbf{X}_{p2} = (\mathbf{A}_{21} - \mathbf{B}_{u2}\mathbf{K}_{p1})\mathbf{X}_{p1} + (\mathbf{A}_{22} - \mathbf{B}_{u2}\mathbf{K}_2)\mathbf{X}_{p2} - \mathbf{B}_{u2}\mathbf{K}_{o1}\widehat{\mathbf{X}}_{p1} + \mathbf{B}_{u2}v + \mathbf{B}_{d2}d, \tag{5}$$

$$s\widehat{\mathbf{X}}_{p1} = \mathbf{L}\mathbf{C}_1\mathbf{X}_{p1} + (\mathbf{A}_{12} + \mathbf{B}_{u1}\mathbf{C}_2)\mathbf{X}_{p2} + (\mathbf{A}_{11} - \mathbf{L}\mathbf{C}_1)\widehat{\mathbf{X}}_{p1}. \tag{6}$$

Subtracting from equation (4) equation (6) and denoting the difference between the plant state and the observer state, let us call the estimation state error as

$$\widetilde{\mathbf{X}}_{p1} = \mathbf{X}_{p1} - \widehat{\mathbf{X}}_{p1}, \tag{7}$$

then we obtain

$$s\tilde{\mathbf{X}}_{\text{p}1} = (\mathbf{A}_{11} - \mathbf{L}\mathbf{C}_1)\tilde{\mathbf{X}}_{\text{p}1} + \mathbf{B}_{d1}d \quad . \tag{8}$$

In view of (7) and (2) the expression (5) will be as follows

$$s\mathbf{X}_{\text{p}2} = (\mathbf{A}_{21} - \mathbf{B}_{u2}\mathbf{K}_{p1})\mathbf{X}_{\text{p}1} + (\mathbf{A}_{22} - \mathbf{B}_{u2}\mathbf{K}_2)\mathbf{X}_{\text{p}2} + \mathbf{B}_{u2}\mathbf{K}_{\text{o}1}\tilde{\mathbf{X}}_{\text{p}1} + \mathbf{B}_{u2}v + \mathbf{B}_{d2}d \quad . \tag{9}$$

After combining equations (4), (9) and (8) into one system, we obtain

$$s\begin{bmatrix}\mathbf{X}_{\text{p}1}\\ \mathbf{X}_{\text{p}2}\\ \tilde{\mathbf{X}}_{\text{p}1}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}+\mathbf{B}_{u1}\mathbf{C}_2 & \mathbf{0}\\ \mathbf{A}_{21}-\mathbf{B}_{u2}\mathbf{K}_{p1} & \mathbf{A}_{22}-\mathbf{B}_{u2}\mathbf{K}_2 & \mathbf{B}_{u2}\mathbf{K}_{\text{o}1}\\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{11}-\mathbf{L}\mathbf{C}_1\end{bmatrix}\begin{bmatrix}\mathbf{X}_{\text{p}1}\\ \mathbf{X}_{\text{p}2}\\ \tilde{\mathbf{X}}_{\text{p}1}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{B}_{u2}\\ \mathbf{0}\end{bmatrix}v + \begin{bmatrix}\mathbf{B}_{d1}\\ \mathbf{B}_{d2}\\ \mathbf{B}_{d1}\end{bmatrix}d \quad . \tag{10}$$

In order to simplify the expression (10) we perform a mathematical description of mode control system without the observer on the basis of the expanded structure shown in Fig. 2:

$$\left.\begin{aligned} s\mathbf{X}_{\text{p}1} &= \mathbf{A}_{11}\mathbf{X}_{\text{p}1} + (\mathbf{A}_{12} + \mathbf{B}_{u1}\mathbf{C}_2)\mathbf{X}_{\text{p}2} + \mathbf{B}_{d1}d,\\ s\mathbf{X}_{\text{p}2} &= (\mathbf{A}_{21} - \mathbf{B}_{u2}\mathbf{K}_1)\mathbf{X}_{\text{p}1} + (\mathbf{A}_{22} - \mathbf{B}_{u2}\mathbf{K}_2)\mathbf{X}_{\text{p}2} + \mathbf{B}_{u2}v + \mathbf{B}_{d2}d.\end{aligned}\right\} \tag{11}$$

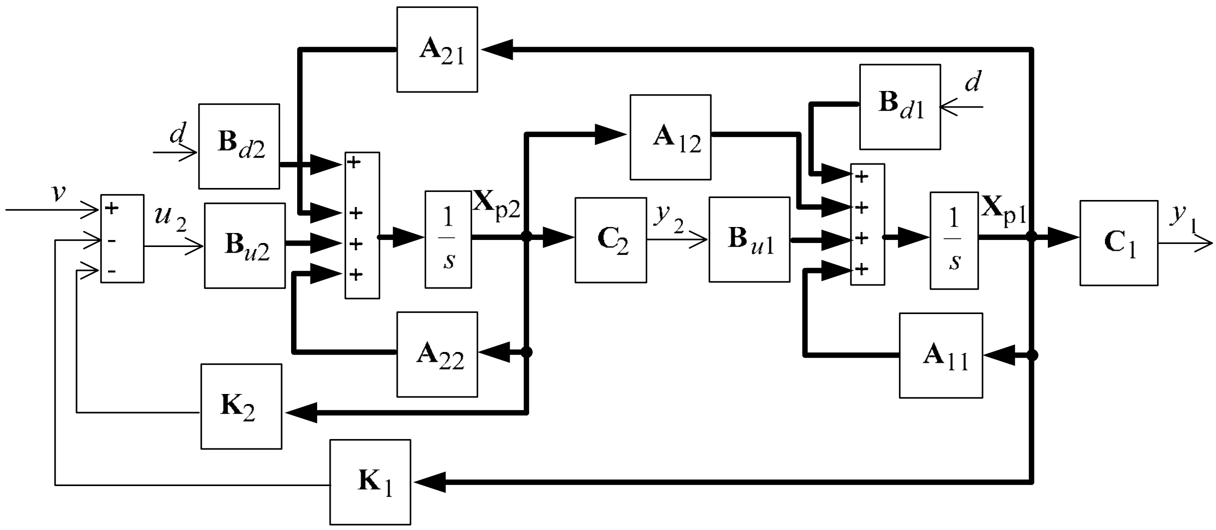

Figure 2 – Structure of the mode control system without observer

Uniting equation (5) and (11) into a single system, we obtain:

$$s\begin{bmatrix}\mathbf{X}_{\text{p}1}\\ \mathbf{X}_{\text{p}2}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}+\mathbf{B}_{u1}\mathbf{C}_2\\ \mathbf{A}_{21}-\mathbf{B}_{u2}\mathbf{K}_1 & \mathbf{A}_{22}-\mathbf{B}_{u2}\mathbf{K}_2\end{bmatrix}\begin{bmatrix}\mathbf{X}_{\text{p}1}\\ \mathbf{X}_{\text{p}2}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \mathbf{B}_{u2}\end{bmatrix}v + \begin{bmatrix}\mathbf{B}_{d1}\\ \mathbf{B}_{d2}\end{bmatrix}d \,. \tag{12}$$

The same equation without the division of the plant in two parts is:

$$s\mathbf{X}_{\text{p}} = \mathbf{A}_{\text{cp}}\mathbf{X}_{\text{p}} + \mathbf{B}_u v + \mathbf{B}_d d \quad , \tag{13}$$

where

$$\mathbf{A}_{\text{cp}} = \mathbf{A} - \mathbf{B}_u\mathbf{K} \tag{14}$$

– state matrix of the mode control system, closed by its own (measured) states,

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}+\mathbf{B}_{u1}\mathbf{C}_2\\ \mathbf{A}_{21} & \mathbf{A}_{22}\end{bmatrix} \tag{15}$$

– state matrix of the plant (open-loop system).

Comparison of equations (12), (13) and (14) gives:

$$\mathbf{X}_{\text{p}} = \begin{bmatrix}\mathbf{X}_{\text{p}1}\\ \mathbf{X}_{\text{p}2}\end{bmatrix}; \qquad \mathbf{B}_u = \begin{bmatrix}\mathbf{0}\\ \mathbf{B}_{u2}\end{bmatrix}; \qquad \mathbf{B}_d = \begin{bmatrix}\mathbf{B}_{d1}\\ \mathbf{B}_{d2}\end{bmatrix}; \tag{16}$$

$$\mathbf{A}_{\text{cp}}=\left[\begin{array}{c:c}\mathbf{A}_{11} & \mathbf{A}_{12}+\mathbf{B}_{u1}\mathbf{C}_2\\ \hdashline \mathbf{A}_{21}-\mathbf{B}_{u2}\mathbf{K}_1 & \mathbf{A}_{22}-\mathbf{B}_{u2}\mathbf{K}_2\end{array}\right]. \quad (17)$$

Let us combine two upper cells of the third column of the cell matrix (10):

$$\left[\begin{array}{c}\mathbf{0}\\ \hdashline \mathbf{B}_{u2}\mathbf{K}_{\text{o}1}\end{array}\right]=\left[\begin{array}{c}\mathbf{0}\\ \hdashline \mathbf{B}_{u2}\end{array}\right]\mathbf{K}_{\text{o}1}=\mathbf{B}_u\mathbf{K}_{\text{o}1}. \quad (18)$$

Inserting (16)-(18) in equation (10), we obtain the simpler form:

$$s\left[\begin{array}{c}\mathbf{X}_{\text{p}}\\ \hdashline \tilde{\mathbf{X}}_{\text{p}1}\end{array}\right]=\left[\begin{array}{c:c}\mathbf{A}_{\text{cp}} & \mathbf{B}_u\mathbf{K}_{\text{o}1}\\ \hdashline \mathbf{0} & \mathbf{A}_{\text{co}}\end{array}\right]\left[\begin{array}{c}\mathbf{X}_{\text{p}}\\ \hdashline \tilde{\mathbf{X}}_{\text{p}1}\end{array}\right]+\left[\begin{array}{c}\mathbf{B}_u\\ \hdashline \mathbf{0}\end{array}\right]v+\left[\begin{array}{c}\mathbf{B}_d\\ \hdashline \mathbf{B}_{d1}\end{array}\right]d, \quad (19)$$

where $\mathbf{A}_{\text{co}}=\mathbf{A}_{11}-\mathbf{L}\mathbf{C}_1$ – state matrix of the closed-loop observer.

Thus, the state matrix of described system is reduced to a standard triangular form:

$$\mathbf{A}_{\text{c}s}=\left[\begin{array}{c:c}\mathbf{A}_{\text{cp}} & \mathbf{B}_u\mathbf{K}_{\text{p}1}\\ \hdashline \mathbf{0} & \mathbf{A}_{\text{co}}\end{array}\right]. \quad (20)$$

On basis of matrix (20) we can write:

$$G_{\text{cs}}(p)=\det(s\mathbf{I}-\mathbf{A}_{cs})=\det(s\mathbf{I}-\mathbf{A}_{\text{cp}})\cdot\det(s\mathbf{I}-\mathbf{A}_{\text{co}})=G_{\text{cp}}(s)\cdot G_{\text{co}}(s). \quad (21)$$

From this equation follows the well-known theorem of separation, which allows designing independently the state controller and the observer with the desired poles location. Expression (21) confirms the fact that the separation theorem holds for mode control system with a full-order observer. This observer can be built on basis of the model either of whole plant, or a part of it.

Let us define the matrix transfer functions of investigated mode control system from the control (v) and perturbing (d) inputs to the state vector:

$$\mathbf{W}_x(p)=(p\mathbf{I}-\mathbf{A}_{\text{cs}})^{-1}\cdot\mathbf{B}=\left[\begin{array}{c:c}p\mathbf{I}-\mathbf{A}_{\text{cp}} & -\mathbf{B}_u\mathbf{K}_{\text{o}1}\\ \hdashline \mathbf{0} & p\mathbf{I}-\mathbf{A}_{\text{co}}\end{array}\right]^{-1}\cdot\left[\begin{array}{c:c}\mathbf{B}_u & \mathbf{B}_d\\ \hdashline \mathbf{0} & \mathbf{B}_{d1}\end{array}\right]. \quad (22)$$

From linear algebra [6] we know that the elements of matrix **b**, inverse to a four-cellular matrix **a**

$$\mathbf{b}=\left[\begin{array}{c:c}\mathbf{b}_{11} & \mathbf{b}_{12}\\ \hdashline \mathbf{b}_{21} & \mathbf{b}_{22}\end{array}\right]=\left[\begin{array}{c:c}\mathbf{a}_{11} & \mathbf{a}_{12}\\ \hdashline \mathbf{a}_{21} & \mathbf{a}_{22}\end{array}\right]^{-1},$$

can be defined by formulas:

$$\left.\begin{array}{l}\mathbf{b}_{12}=-\mathbf{a}_{11}^{-1}\mathbf{a}_{12}(\mathbf{a}_{22}-\mathbf{a}_{21}\mathbf{a}_{11}^{-1}\mathbf{a}_{12})^{-1},\\ \mathbf{b}_{11}=\mathbf{a}_{11}^{-1}-\mathbf{b}_{12}\mathbf{a}_{21}\mathbf{a}_{11}^{-1},\\ \mathbf{b}_{22}=(\mathbf{a}_{22}-\mathbf{a}_{21}\mathbf{a}_{11}^{-1}\mathbf{a}_{12})^{-1},\\ \mathbf{b}_{21}=-\mathbf{b}_{22}\mathbf{a}_{21}\mathbf{a}_{11}^{-1}.\end{array}\right\} \quad (23)$$

In the particular case ($\mathbf{a}_{21}=\mathbf{0}$) expression (23) can be simplified:

$$\mathbf{b}_{12}=-\mathbf{a}_{11}^{-1}\mathbf{a}_{12}\mathbf{a}_{22},\quad \mathbf{b}_{11}=\mathbf{a}_{11}^{-1},\quad \mathbf{b}_{22}=\mathbf{a}_{22}^{-1},\quad \mathbf{b}_{21}=\mathbf{0}. \quad (24)$$

Using (24) for the cell matrix inversion (see (22)) gives:

$$(s\mathbf{I}-\mathbf{A}_{cs})^{-1}=\left[\begin{array}{c:c}(s\mathbf{I}-\mathbf{A}_{\text{cp}})^{-1} & (s\mathbf{I}-\mathbf{A}_{\text{cp}})^{-1}\mathbf{B}_u\mathbf{K}_{\text{o}1}(s\mathbf{I}-\mathbf{A}_{\text{co}})^{-1}\\ \hdashline \mathbf{0} & (s\mathbf{I}-\mathbf{A}_{\text{co}})^{-1}\end{array}\right]. \quad (25)$$

After substituting (25)in (22) and matrix multiplication, we obtain:

$$\mathbf{W}_x(s)=\left[\begin{array}{c|c}\dfrac{\mathbf{X}_\text{p}(s)}{v(s)} & \dfrac{\mathbf{X}_\text{p}(s)}{d(s)} \\ \hline \dfrac{\tilde{\mathbf{X}}_\text{p1}(s)}{v(s)} & \dfrac{\tilde{\mathbf{X}}_\text{p1}(s)}{d(s)}\end{array}\right]=\left[\begin{array}{c|c}\mathbf{W}_{xv}(s) & \mathbf{W}_{xd}(s) \\ \hline \mathbf{W}_{\tilde{x}v}(s) & \mathbf{W}_{\tilde{x}d}(s)\end{array}\right]=$$

$$=\left[\begin{array}{c|c}(s\mathbf{I}-\mathbf{A}_\text{cs})^{-1}\mathbf{B}_u & (s\mathbf{I}-\mathbf{A}_\text{cs})^{-1}\mathbf{B}_d+(s\mathbf{I}-\mathbf{A}_\text{cs})^{-1}\mathbf{B}_u\cdot\mathbf{K}_\text{o1}(s\mathbf{I}-\mathbf{A}_\text{co})^{-1}\mathbf{B}_{d1} \\ \hline \mathbf{0} & (s\mathbf{I}-\mathbf{A}_\text{co})^{-1}\mathbf{B}_{d1}\end{array}\right]. \tag{26}$$

Expression (26) shows that the transfer function of the observed-base mode control system from the reference input to the plant state vector $\mathbf{W}_{xv}(s)$ coincides with the corresponding transfer function mode control system without observer $\mathbf{W}_{xvp}(s)$ (see diagram in Fig. 3):

$$\mathbf{W}_{xv}(s)=\frac{\mathbf{X}_\text{p}(s)}{v(s)}=\mathbf{W}_{xv\text{p}}(s)=(s\mathbf{I}-\mathbf{A}_\text{cp})^{-1}\mathbf{B}_u=\frac{Adj(s\mathbf{I}-\mathbf{A}_\text{cp})\mathbf{B}_u}{det(s\mathbf{I}-\mathbf{A}_\text{cp})}, \tag{27}$$

and the observer estimation error is identically equal to zero:

$$\mathbf{W}_{\tilde{x}v}(p)=\frac{\tilde{\mathbf{X}}_\text{o1}(p)}{v(p)}=\mathbf{0}. \tag{28}$$

By comparing the denominator of transfer functions (27) and characteristic polynomial (21) we conclude that transfer functions (27) contain the characteristic polynomial of the observer in numerator, which is reduced with the same polynomial in denominator.

These conclusions were obtained mathematically, they correspond to the analysis of the block diagram (Fig. 1). If structure and parameters of plant and open-loop observer correspond exactly to each other, the estimation error is equal to zero, and the observer works as an ideal model of plant (observer feedbacks in this case are not applied).

The estimate error vector $\tilde{\mathbf{X}}_\text{p1}$ in observer-based closed-loop control system can be found, if we know the transfer function of the closed-loop observer from disturbance to its output (we assume here, that the disturbance applies to the observer directly, see diagram in Fig. 4):

$$\mathbf{W}_{\tilde{x}d}(s)=\frac{\tilde{\mathbf{X}}_\text{p1}(s)}{d(s)}=(s\mathbf{I}-\mathbf{A}_\text{co})^{-1}\mathbf{B}_{d1}=\frac{Adj(s\mathbf{I}-\mathbf{A}_\text{co})\mathbf{B}_{d1}}{det(s\mathbf{I}-\mathbf{A}_\text{co})}, \tag{29}$$

and the behavior of the coordinates of the plant depends on the parameters both of the plant and observer.

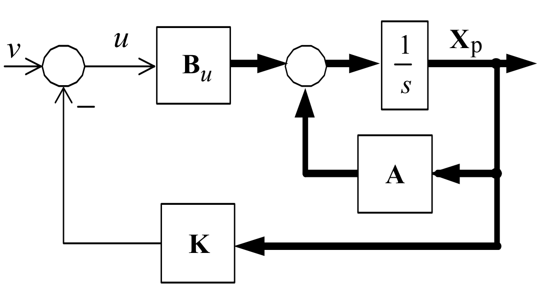

Figure 3 - Structure allows determining the state transfer function of observer-based mode control system by the reference input

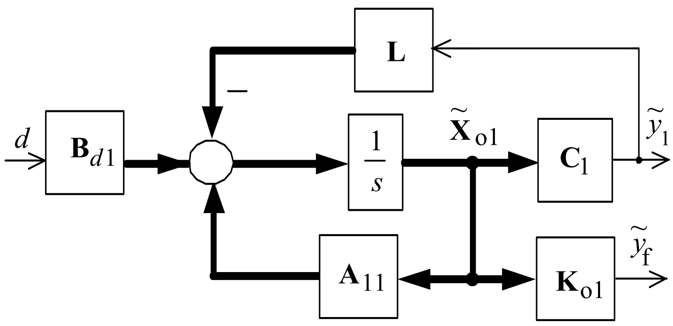

Figure 4 - Structure allows determining the transfer function of observer-based mode control from the unmeasured disturbance to the estimation errors

Analysis of the upper right cell of the matrix (26) gives:

$$\mathbf{W}_{xd}(p)=\frac{\mathbf{X}_{\mathrm{p}}(s)}{d(s)}=(s\mathbf{I}-\mathbf{A}_{\mathrm{cp}})^{-1}\mathbf{B}_d+(s\mathbf{I}-\mathbf{A}_{\mathrm{cp}})^{-1}\mathbf{B}_u\cdot\mathbf{K}_{\mathrm{o}1}(s\mathbf{I}-\mathbf{A}_{\mathrm{co}})^{-1}\mathbf{B}_{d1}=$$

$$=(s\mathbf{I}-\mathbf{A}_{\mathrm{cp}})^{-1}\left(\mathbf{B}_d+\mathbf{B}_u\cdot\mathbf{K}_{\mathrm{o}1}(s\mathbf{I}-\mathbf{A}_{\mathrm{co}})^{-1}\mathbf{B}_{d1}\right)=\mathbf{W}_{xd\mathrm{p}}(s)+\mathbf{W}_{xv\mathrm{p}}(s)\cdot W_{\tilde{y}_{\mathrm{f}}d\mathrm{o}}(s), \quad (30)$$

where

$$W_{\tilde{y}_{\mathrm{f}}d\mathrm{o}}(s)=\tilde{y}_{\mathrm{f}}(s)/d(s)=\mathbf{K}_{\mathrm{o}1}(s\mathbf{I}-\mathbf{A}_{\mathrm{co}})^{-1}\mathbf{B}_{d1}=\mathbf{K}_{\mathrm{o}1}\mathbf{W}_{\tilde{x}d}(s). \quad (31)$$

Structures, which are equivalent to the structure of Fig. 1, are shown in Fig. 5. They have been drawn on the basis of the matrix transfer function (26) and using (27)-(31).

Comparison of structures shows that the primary structure with observer in the feedback path can be transformed into a structure with a series-parallel connection of the plant and observer; this fact greatly simplifies the analysis of this dynamic system.

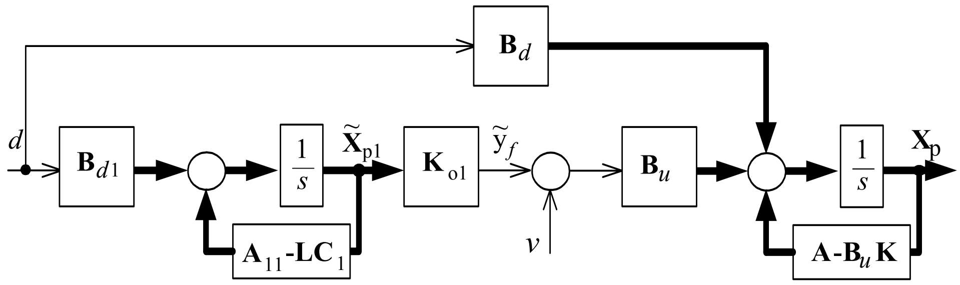

Figure 5 – Transformed structure for the analysis of the mode control system with the observer, which is built on the basis of the model of plant part.

Let's consider the special case of system in Fig. 1, when open-loop observer presents a model of the full plant, and all feedbacks are estimated states [7].

The block diagram of such system is shown in Fig. 6.

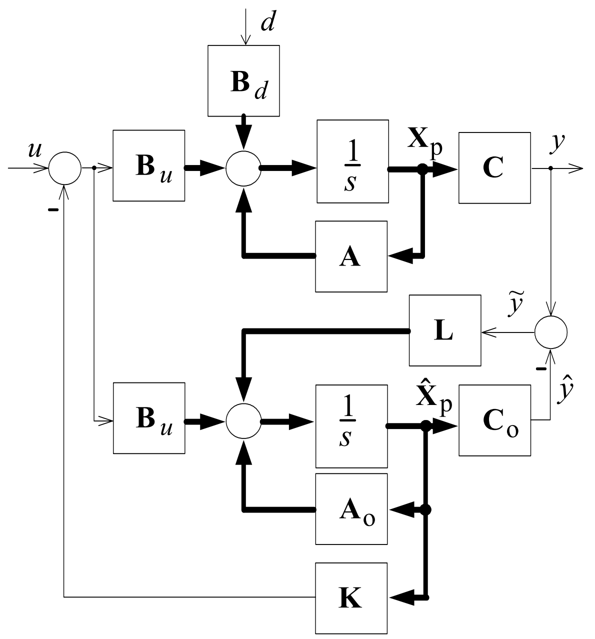

Figure 6 – The structure of mode control system with the observer, which estimates all the plant state

For the transformation from the structure in Fig. 1 to structure in Fig. 6 should be taken

$$\mathbf{K}_{\mathrm{p}1}=\mathbf{0},\ \mathbf{K}_2=\mathbf{0},\ \mathbf{K}_{\mathrm{o}1}=\mathbf{K},$$

$$\mathbf{B}_{d1}=\mathbf{B}_d,\ \mathbf{B}_{u1}=\mathbf{B}_u. \quad (32)$$

With the full identity of plant and open-loop observer ($\mathbf{A}_{\mathrm{p}}=\mathbf{A}_{\mathrm{o}}=\mathbf{A}$; $\mathbf{B}_{\mathrm{p}}=\mathbf{B}_{\mathrm{o}}=\mathbf{B}_u$; $\mathbf{C}_{\mathrm{o}}=\mathbf{C}_{\mathrm{н}}=\mathbf{C}$) a mathematical description of the system in Fig. 6 can be transformed to the form:

$$s\begin{bmatrix}\mathbf{X}\\ \tilde{\mathbf{X}}\end{bmatrix}=\mathbf{A}_{\mathrm{cs}}\begin{bmatrix}\mathbf{X}\\ \tilde{\mathbf{X}}\end{bmatrix}+\begin{bmatrix}\mathbf{B}_u\\ \mathbf{0}\end{bmatrix}u+\begin{bmatrix}\mathbf{B}_d\\ \mathbf{B}_d\end{bmatrix}d,$$

where

$$\mathbf{A}_{\text{cs}} = \left[\begin{array}{c|c} \mathbf{A}_{\text{cp}} & \mathbf{B}_u\mathbf{K} \\ \hline \mathbf{0} & \mathbf{A}_{\text{co}} \end{array}\right]; \quad \mathbf{A}_{\text{cp}} = \mathbf{A} - \mathbf{B}_u\mathbf{K}; \; \mathbf{A}_{\text{co}} = \mathbf{A} - \mathbf{LC}. \tag{34}$$

Matrix transfer function of this system can be obtained from (26), taking into account expressions (32):

$$\mathbf{W}_x(p) = \left[\begin{array}{c|c} \mathbf{W}_{xu}(s) & \mathbf{W}_{xd}(s) \\ \hline \mathbf{W}_{\tilde{x}u}(s) & \mathbf{W}_{\tilde{x}d}(s) \end{array}\right] =$$

$$= \left[\begin{array}{c|c} (s\mathbf{I} - \mathbf{A}_{\text{cp}})^{-1}\mathbf{B}_u & (s\mathbf{I} - \mathbf{A}_{\text{cp}})^{-1}\mathbf{B}_d + (s\mathbf{I} - \mathbf{A}_{\text{cp}})^{-1}\mathbf{B}_u\mathbf{K}(s\mathbf{I} - \mathbf{A}_{\text{co}})^{-1}\mathbf{B}_d \\ \hline \mathbf{0} & (s\mathbf{I} - \mathbf{A}_{\text{co}})^{-1}\mathbf{B}_d \end{array}\right]. \tag{35}$$

Block diagram of the equivalent structure in Fig. 6, is shown in Fig. 7. It is composed on the base of the matrix transfer function (35).

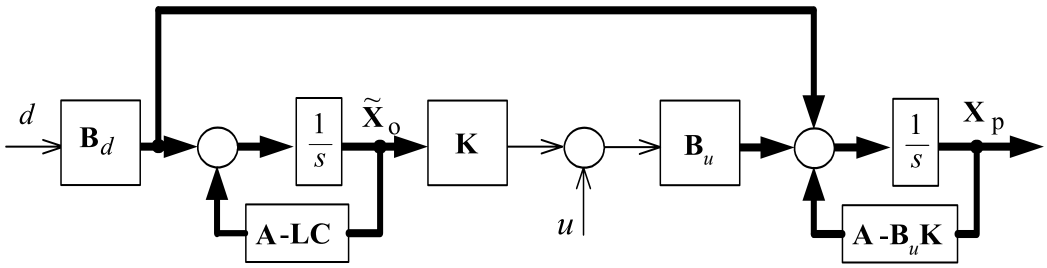

Figure 7 – Equivalent transformed structure of the mode control system with an observer, which estimates all the plant state

Since the application of observer is not limited to mode control systems, let us consider any given linear time-invariant with the full-order Luenberger-observer, which is built on the model of a certain part of the plant [8]. System is divided into two interrelated parts – with measured and estimated state variables. Structure of this system in state space is shown in Fig. 8.

We assume that the parameters of an open-loop observer coincide with the corresponding parameters of the first plant part

$$\mathbf{A}_{\text{o}} = \mathbf{A}_{11}, \; \mathbf{C}_{\text{o}} = \mathbf{C}_1. \tag{36}$$

Using the methodology of studies that is described above for the mode control system, we obtain the following results:

$$s\left[\begin{array}{c} \mathbf{X}_{\text{p}} \\ \hline \tilde{\mathbf{X}}_{\text{p1}} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}_{\text{c}} & -\mathbf{A}_{\text{po}} \\ \hline \mathbf{0} & \mathbf{A}_{\text{co}} \end{array}\right]\left[\begin{array}{c} \mathbf{X}_{\text{p}} \\ \hline \tilde{\mathbf{X}}_{\text{p1}} \end{array}\right] + \left[\begin{array}{c} \mathbf{B}_u \\ \hline \mathbf{0} \end{array}\right]u + \left[\begin{array}{c} \mathbf{B}_d \\ \hline \mathbf{B}_{d1} \end{array}\right]d, \tag{37}$$

$$\mathbf{W}_x(p) = \left[\begin{array}{c|c} (p\mathbf{I} - \mathbf{A}_{\text{c}})^{-1}\mathbf{B}_u & (p\mathbf{I} - \mathbf{A}_{\text{c}})^{-1}\mathbf{B}_f - (p\mathbf{I} - \mathbf{A}_{\text{c}})^{-1}\mathbf{A}_{\text{po}}(p\mathbf{I} - \mathbf{A}_{\text{co}})^{-1}\mathbf{B}_{d1} \\ \hline \mathbf{0} & (p\mathbf{I} - \mathbf{A}_{\text{co}})^{-1}\mathbf{B}_{d1} \end{array}\right]. \tag{38}$$

where

$$\mathbf{A}_{\text{c}} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right], \quad \mathbf{B}_u = \left[\begin{array}{c} \mathbf{B}_{u1} \\ \hline \mathbf{B}_{u2} \end{array}\right], \quad \mathbf{B}_d = \left[\begin{array}{c} \mathbf{B}_{d1} \\ \hline \mathbf{B}_{d2} \end{array}\right]; \quad \mathbf{A}_{\text{po}} = \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{A}_{21\text{o}} \end{array}\right]; \tag{39}$$

$$\mathbf{A}_{21} = \mathbf{A}_{21\text{p}} + \mathbf{A}_{21\text{o}}, \quad \mathbf{A}_{\text{co}} = \mathbf{A}_{11} - \mathbf{LC}_1. \tag{40}$$

From (38)it follows that the matrix transfer functions of the considered dynamic system have the same form as in the observed-based mode control system in Fig. 1 (see formulas (28) –

(31)). The only difference is the replacement in (26), (27), (30) matrix $\mathbf{A}_{cp}$ by matrix $\mathbf{A}_c$ and matrix expression $\mathbf{B}_u\mathbf{K}_{o1}$ – by matrix $\mathbf{A}_{op}$, in particular

$$\mathbf{W}_{xd}(p)=\frac{\mathbf{X}_p(s)}{d(s)}=(s\mathbf{I}-\mathbf{A}_c)^{-1}\mathbf{B}_d-(s\mathbf{I}-\mathbf{A}_c)^{-1}\mathbf{A}_{po}(s\mathbf{I}-\mathbf{A}_{co})^{-1}\mathbf{B}_{d1}=$$

$$=\frac{Adj(s\mathbf{I}-\mathbf{A}_c)\left(\mathbf{B}_d Adj(s\mathbf{I}-\mathbf{A}_c)-\mathbf{A}_{po}Adj(s\mathbf{I}-\mathbf{A}_{co})\mathbf{B}_{d1}\right)}{det(s\mathbf{I}-\mathbf{A}_c)det(s\mathbf{I}-\mathbf{A}_{co})}. \quad (43)$$

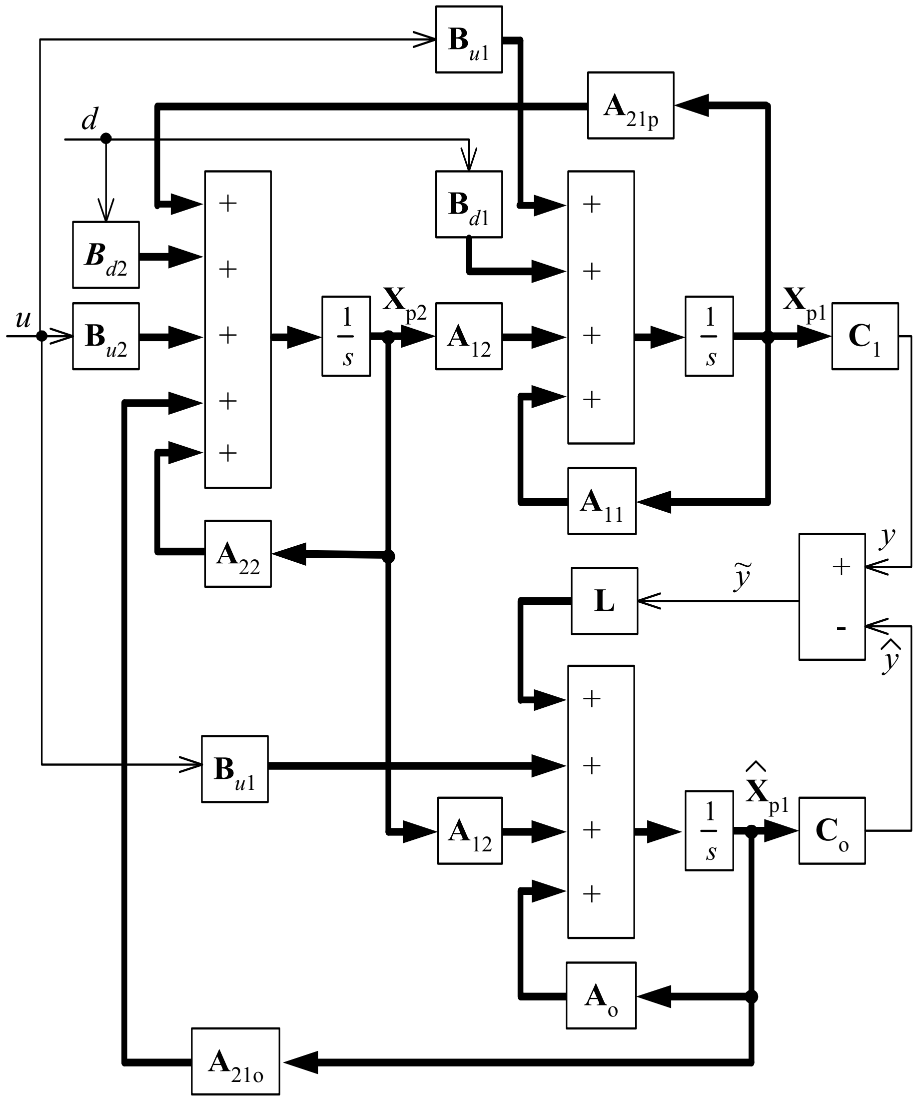

Figure 8 – Structure of control system of the general form
with the full-order observer, built on the model of the part plant

We can make equivalent transformed structure of the analyzed system (in Fig. 9), which is based on the matrix transfer function (41).

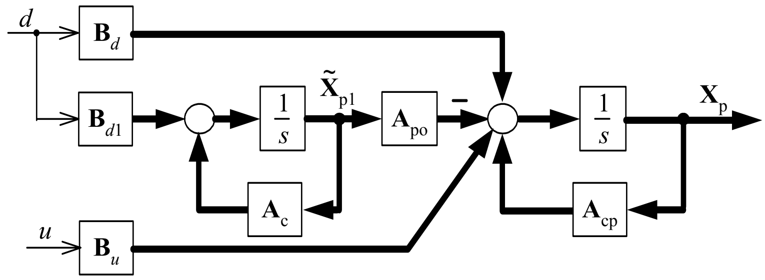

Figure 9 –Transformed structure, equivalent to Fig. 8

It is much simpler than primary one (Fig. 8), and thus makes it possible to simplify the analysis of the primary system.

Thus, arbitrary configurations observed-based control system as well as the observed-based mode control systems can be represented as a series-parallel connection of the observer and the closed-loop system with sensed feedbacks.

Conclusion

Structures of control systems with the partly measured and partly estimated feedbacks can be greatly simplified by presenting them in a series-parallel connection of the observer and system with measured feedbacks (compare primary structures in Fig. 1, 6, 8, with the equivalent transformed structures in Fig. 5, 7, 9). Transformations have been done on the basis of matrix transfer functions (26), (35), (38).

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