Gas-transport Networks Modelling

on the Basis of Energetic Analogies Theory

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Abstract

Saukh S.Ye., Semagina E.P. Gas-transport Networks Modelling on the Basis of Energetic Analogies Theory. *The problem of computer models creation is considered for energy systems, which allows studying heterogeneous physical events. A common theory of energy concepts is proposed for such models building. This common theory allows selecting valid base variables from the mathematical description of heterogeneous events and applying circuit theory methods to build computer models of topologically complex objects. The efficiency of our approach is illustrated by the problem solution for a gas-transport system model building.*

Introduction

The creation of computational models of power supply systems includes many difficulties. A significant variety of the generating elements of the system, the heterogeneity of the observed physical phenomena and the complexity of their mathematical description are the main aspects of the problem .A general solution for this problem is possible with the help of the common theory which provides the harmonized development of mathematical models and computational algorithms.

Description of interrelated phenomena for components with lumped parameters

The basic motivation for the application of circuit theory analogies for constructing models of energetic circuit components with heterogeneous physical phenomena is the possibility to use the laws of energy conservation.

Introducing vector groups of generalized sequential and parallel variables allows describing the interrelations between variables by means of analogues of resistances, conductivities, capacities and inductances. As a result, the basic principles and analysis methods for electric circuits appear to be usable

for the energetic circuit analysis.

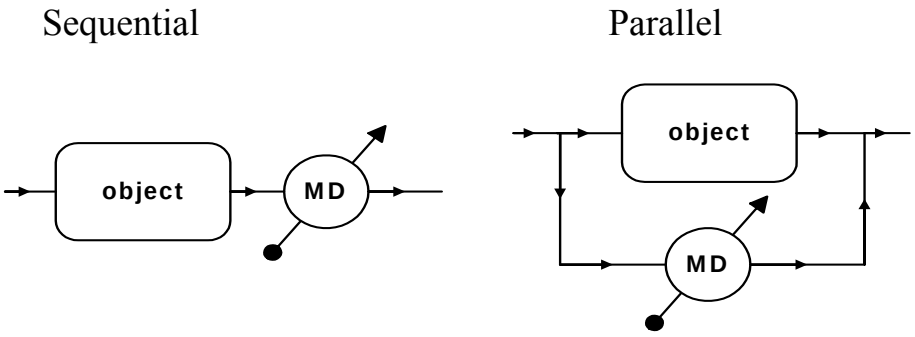

Figure.1. Method of measurement

All system variables are subdivided into two sets: sequential one, which may be measured by sequentially switched device, and parallel one, which may be measured by parallel switched device (Fig. 1).The main classification of variables was made on the basis of a generalized approach on a higher level of abstraction, i.e. the principle of energetic analogies, according to which: the concepts of generalized state variables (Table 1) and generalized action variables (Table 2) are introduced. The action variables determine the level of energy dissipation of the component and the state variables represent the integral characteristics of the action variables; the product of parallel and sequential action variables determines the level of energy dissipation in the system element; kinetic energy is defined as the integration of the sequential action variables with respect to the sequential state variables;potential energy is defined as the integration of the parallel action variables with respect to the parallel state variables.

Table 1. Parallel and sequential state variables

Type of interaction	Parallel variables	Sequential variables
Electricity	Charge q, K	Magnetic flow F, Wb
Mechanics	Displacement x, m	Impulse K, $\mathrm{kg \cdot m/sec}$
	Rotation angle θ, rad	Angular Moment M, $\mathrm{kg \cdot m^2/sec}$
Heat	Entropy S, J/K	Undefined
Hydraulics	Displacement x_H, m	Impulse of moving liquid $K_H = \int_V \rho \cdot \frac{dx_H}{dt} \cdot dV$, $\mathrm{kg \cdot m/sec}$ ρ – density, $\mathrm{kg/m^3}$; V – volume, $\mathrm{m^3}$.

Table 2. Parallel and sequential action variables

Type of interaction	Parallel variables	Sequential variables
Electricity	Voltage $u = \frac{d\Phi}{dt}$, V	Current $i = \frac{dq}{dt}$, A
Mechanics	Force $F = \frac{dK}{dt}$, N	Velocity $\upsilon = \frac{dx}{dt}$, m/sec
	Momentum $N = \frac{dM}{dt}$, $\mathrm{N \cdot m}$	Angular velocity $\omega = \frac{d\theta}{dt}$, rad/sec
Heat	Temperature T, K	Entropy flow $\sigma = \frac{dS}{dt}$, $\frac{\mathrm{Wt}}{\mathrm{K}}$
Hydraulics	Force $F_H = \int_S p \cdot d\Pi = \frac{dK_H}{dt}$, N S – surface of volume V, m^2 p – pressure, Pa.	Average velocity of liquid motion in cross-section $\upsilon_H = \frac{dx_H}{dt}$, $\frac{\mathrm{m}}{\mathrm{sec}}$

The law of energy conservation in passive components of energetic circuits with lumped parameters is represented as $dW^D + dW^K + dW^P = 0$, where the summands are the dissipation energy $dW^D = (\boldsymbol{u}, \boldsymbol{i}) \cdot dt = (\boldsymbol{R} \cdot \boldsymbol{i}, \boldsymbol{i}) \cdot dt = (\boldsymbol{u}, \boldsymbol{R}^{-1} \cdot \boldsymbol{i}) \cdot dt$,

$$\boldsymbol{R} = \begin{vmatrix} R_{E,E} & R_{E,FM} & R_{E,RM} & R_{E,T} & R_{E,H} \\ R_{FM,E} & R_{FM,FM} & R_{FM,RM} & R_{FM,T} & R_{FM,H} \\ R_{RM,E} & R_{RM,FM} & R_{RM,RM} & R_{RM,T} & R_{RM,H} \\ R_{T,E} & R_{T,FM} & R_{T,RM} & R_{T,T} & R_{T,H} \\ R_{H,E} & R_{H,FM} & R_{H,RM} & R_{H,T} & R_{H,H} \end{vmatrix},$$

the kinetic energy $dW^K = d(\boldsymbol{L} \cdot \boldsymbol{i}, \boldsymbol{i})/2 = (\boldsymbol{L} \cdot \boldsymbol{i}, d\boldsymbol{i})$,

$$\boldsymbol{L}=\begin{vmatrix} L_{E,E} & L_{E,FM} & L_{E,RM} & L_{E,T} & L_{E,H} \\ L_{FM,E} & L_{FM,FM} & L_{FM,RM} & L_{FM,T} & L_{FM,H} \\ L_{RM,E} & L_{RM,FM} & L_{RM,RM} & L_{RM,T} & L_{RM,H} \\ L_{T,E} & L_{T,FM} & L_{T,RM} & L_{T,T} & L_{T,H} \\ L_{H,E} & L_{H,FM} & L_{H,RM} & L_{H,T} & L_{H,H} \end{vmatrix},$$

the potential energy $dW^P = d(\boldsymbol{C}\cdot\boldsymbol{u},\boldsymbol{u})/2 = (\boldsymbol{C}\cdot\boldsymbol{u},d\boldsymbol{u})$.

$$\boldsymbol{C}=\begin{vmatrix} C_{E,E} & C_{E,FM} & C_{E,RM} & C_{E,T} & C_{E,H} \\ C_{FM,E} & C_{FM,FM} & C_{FM,RM} & C_{FM,T} & C_{FM,H} \\ C_{RM,E} & C_{RM,FM} & C_{RM,RM} & C_{RM,T} & C_{RM,H} \\ C_{T,E} & C_{T,FM} & C_{T,RM} & C_{T,T} & C_{T,H} \\ C_{H,E} & C_{H,FM} & C_{H,RM} & C_{H,T} & C_{H,H} \end{vmatrix}.$$

In these relations $(\boldsymbol{f}_1,\boldsymbol{f}_2)$ is the scalar product of the vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$; the parallel and sequential variables are organized in the vectors $\boldsymbol{u}=|u \quad F \quad N \quad T \quad F_H|^T$ and $\boldsymbol{i}=|i \quad \upsilon \quad \omega \quad \sigma \quad \upsilon_H|^T$; $\boldsymbol{R}$, $\boldsymbol{L}$ and $\boldsymbol{C}$ are the matrices of the parameters with the elements. Nondiagonal elements of matrix parameters are obviously defined by interrelations of the physical phenomena.

The formulas for determination of power, kinetic and potential energies for different physical areas are shown in Table 3.

Table 3. Energetic analogies

Type of interaction	Power (is defined by action variables)	Kinetic energy (is defined by sequential variables)	Potential energy (is defined by parallel variables)
Electricity	$R_E = u \cdot i$ $= R_E \cdot i^2$ $= G_E \cdot u^2$	$W_E^K = \int_0^{\Phi} i \cdot d\Phi$ $= L_E \frac{i^2}{2}$	$W_E^P = \int_0^{q} u \cdot dq$ $= C_E \frac{u^2}{2}$
Mechanics	$R_{FM} = F \cdot \upsilon$ $= R_{FM} \cdot \upsilon^2$ $= G_{FM} \cdot F^2$	$W_{FM}^K = \int_0^{K} \upsilon \cdot dK$ $= L_{FM} \frac{\upsilon^2}{2}$	$W_{FM}^P = \int_0^{x} F \cdot dx$ $= C_{FM} \frac{F^2}{2}$
	$R_{RM} = N \cdot \omega$ $= R_{RM} \cdot \omega^2$ $= G_{RM} \cdot N^2$	$W_{RM}^K = \int_0^{M} \omega \cdot dM$ $= L_{RM} \frac{\omega^2}{2}$	$W_{RM}^P = \int_0^{\theta} N \cdot d\theta$ $= C_{RM} \frac{N^2}{2}$
Heat	$R_T = T \cdot \sigma$ $= R_T \cdot \sigma^2$ $= G_T \cdot T^2$	Undefined	$W_T^P = \int_0^{S} T \cdot dS$ $= C_T \frac{T^2}{2}$
Hydraulics	$R_H = F_H \cdot \upsilon_H$ $= R_H \cdot \upsilon_H^2$ $= G_H \cdot F_H^2$	$W_H^K = \int_0^{K_\Gamma} \upsilon_H \cdot dK_H$ $= L_H \frac{\upsilon_H^2}{2}$	$W_H^P = \int_0^{x_H} F_H \cdot dx_H$ $= C_H \frac{F_H^2}{2}$

Description of homogeneous and heterogeneous phenomena in resistive type components

For the case of homogeneous phenomena the generalized law for components is defined as $J = \gamma \cdot E$, where γ is the specific conductance of the flow J induced by the intensity E. In particular, for components of homogeneous systems this law is formulated as following laws:

Ohm's law, i.e. the relation between density J_E of the electrical current and the potential gradient φ have the form:

$$J_E = -\gamma_E \cdot grad(\varphi) = \gamma_E \cdot E_E, \qquad E_E = -grad(\varphi), \ \gamma_E > 0;$$

Fourier's law, i.e. the relation between the density J_T of the heat flow

and the temperature gradient T, has the form:

$$J_T = -\gamma_T \cdot grad(T) = \gamma_T \cdot E_T, \qquad E_T = -grad(T), \ \gamma_T > 0;$$

Darcy's law, i.e. the relation between the density J_H of the mixture component flow and the pressure gradient p has the form:

$$J_H = -\gamma_H \cdot grad(p) = \gamma_H \cdot E_H, \qquad E_H = -grad(p), \ \gamma_H > 0.$$

For the case of non-homogeneous phenomena we have the generalization of Ohm's, Fourier's, Darcy's and other laws in the vector matrix form

$J = \acute{r}\ddot{Y} E$,

where $\boldsymbol{E} = |E_E \quad E_{FM} \quad E_{RM} \quad E_T \quad E_H|^T$ and $\boldsymbol{J} = |J_E \quad J_{FM} \quad J_{RM} \quad J_T \quad J_H|^T$. Here

$$\int_{\acute{r}\P} J \cdot d\acute{r}\P = i, \qquad \int_H E \cdot d H = u.$$

Kirchhoff's laws in energetic circuits and generalization of the principles of action

Following the accepted notation in a generalized form, the first and second Kirchhoff's laws can be formulated for energetic circuits:

differential form of the first Kirchhoff's law $\oint_{\acute{r}\P_n} J \cdot d\acute{r}\P_n = 0$,

integral form of the first Kirchhoff's law $\sum_n \boldsymbol{i} = 0$,

differential form of the second Kirchhoff's law $\oint_{\acute{r}\Gamma_k} E \cdot dH_k = 0$,

integral form of the second Kirchhoff's law $\sum_k \boldsymbol{u} = 0$.

The known principle of least action for electric circuits, the least entropy generation principle and the extreme principle of classical mechanics can be generalized with the extreme principle of action in energetic circuits, i.e. $min P_\Sigma = min \sum_z (\boldsymbol{u}_z, \ \boldsymbol{i}_z)$.

Energetic circuits as a model for gas transport systems

We extended the principles of model construction of generalized energetic circuits described above to gas-transport systems with arbitrary topologies. This extension allows the development of software tools for modeling the stationary and transitional operation modes of gas pipelines. The variables included in the mathematical models of the individual components of the system were analyzed according to the proposed classification. In particular, the following mathematical models of the components of the system were considered: the set of equations describing the nonstationary nonisothermal gas flow in the pipeline with a constant diameter:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = \frac{4C}{D} \cdot (p_{out} - p), \tag{1}$$

$$\frac{\partial (\rho v)}{\partial t} + \frac{\partial \left(p + \rho v^2\right)}{\partial x} = -\rho \cdot \left(g \frac{dh}{dx} + \lambda \frac{v|v|}{2D} \right), \tag{2}$$

$$\frac{\partial (\rho E)}{\partial t} + \frac{\partial \left[\rho v \cdot \left(E + \frac{p}{\rho} \right) \right]}{\partial x} = \frac{4\alpha}{D} \cdot (T_{out} - T) + \rho v g \frac{dh}{dx} + \frac{4C}{D} \cdot (p_{out} - p) \cdot \left(E + \frac{p}{\rho} \right), \tag{3}$$

$$\rho = \frac{p}{zRT}, \quad z = z(p,T), \quad E = \frac{v^2}{2} + u, \quad u = u(p,T), \tag{4}$$

where variables are
for gas flow: $p(x,t)$ – *pressure,* $T(x,t)$ – *temperature,* $\rho(x,t)$ – *density,* $v(x,t)$ – *velocity,* $E(x,t)$ – *energy;*
for environment parameters: g – *acceleration gravity,* p_{out} – *pressure on the tube surface,* T_{out} - *temperature on the tube surface;*
coefficients:
λ – *hydraulic resistance coefficient,* α – *heat transfer coefficient,* C – *loss factor;*
form factor: D – *pipe size,* L – *length of pipe,* h – *occurrence depth of tube;*
initial condition:

$$\boldsymbol{W}_{con}(x) = \left| \pi D^2 / 4 \cdot \rho(x,t_0) \cdot v(x,t_0) \quad p(x,t_0) \quad T(x,t_0) \right|^T, \tag{5}$$

one of the possible variants of boundary conditions:

$$\boldsymbol{W}_{lim}(t)=\begin{cases} \left|p(0,t)\;\; T(0,t)\;\; \pi D^2/4\cdot\rho(L,t)\cdot v(L,t)\right|^T, & v(0,t)>0,\; v(L,t)\geq 0, \\ \left|p(0,t)\;\; T(0,t)\;\; p(L,t)\;\; T(L,t)\right|^T, & v(0,t)>0,\; v(L,t)<0, \\ \left|\pi D^2/4\cdot\rho(0,t)\cdot v(0,t)\;\; p(L,t)\;\; T(L,t)\right|^T, & v(0,t)\leq 0,\; v(L,t)<0, \\ \left|\pi D^2/4\cdot\rho(0,t)\cdot v(0,t)\;\; \pi D^2/4\cdot\rho(L,t)\cdot v(L,t)\right|^T, & v(0,t)\leq 0,\; v(L,t)\geq 0, \end{cases} \tag{6}$$

which is determined by the direction of the gas flow or by the sign of the velocity v of the flow. In the equations (1)-(3) $t\geq t_0$ is an independent time variable, $x\in[0,l]$ is an independent space variable.

The following set of equations describes the gas-pumping process:

$$\left(\pi D^2/4\cdot\rho\cdot v\right)_{in}=\left(\pi D^2/4\cdot\rho\cdot v\right)_{out}, \tag{7}$$

$$p_{out}=p_{in}\cdot\left[1+\left(\frac{n}{n_0}\right)^2\cdot\frac{(zRT)_0}{(zRT)_{in}}\cdot\left(\varepsilon_0^{\alpha}-1\right)\right]^{1/\alpha}, \tag{8}$$

$$T_{out}=T_{in}\cdot\left[1+\left(\frac{n}{n_0}\right)^2\cdot\frac{(zRT)_0}{(zRT)_{in}}\cdot\left(\varepsilon_0^{\alpha}-1\right)\right]; \tag{9}$$

$$\alpha=\frac{k-1}{k\eta_0};\quad \varepsilon_0=\varepsilon_0(Q);\quad \eta_0=\eta_0(Q);\quad Q=\frac{n}{n_0}\cdot\left(\pi D^2/4\cdot v\right)_{in}. \tag{10}$$

The cooling process is described by the equation:

$$W_{out}=\acute{r}i_{cooling}(W_{in},T_a), \tag{11}$$

Where $$\boldsymbol{W}_{in}(x)=\left|\pi D_{in}^2/4\cdot\rho_{in}(t)\cdot v_{in}(t)\quad p_{in}(t)\quad T_{in}(t)\right|^T \tag{12}$$

and $$\boldsymbol{W}_{out}(x)=\left|\pi D_{out}^2/4\cdot\rho_{out}(t)\cdot v_{out}(t)\quad p_{out}(t)\quad T_{out}(t)\right|^T \tag{13}$$

are the vectors of gas flow parameters at the cooler's input and output. Nonlinear vector-function $ŕi_{cooling}(W_{in}, T_a)$ represents thermohydraulic phenomena at the temperature of air T_a.

Taking into account the continuity of heat and hydraulic phenomena observed in gas transport systems, the following variable vector functions are defined for model (7) – (13):

sequential variables (mass flow and energy flow)

$$\boldsymbol{i}(x,t) = \pi D^2 / 4 \cdot \begin{vmatrix} \rho(x,t) \cdot v(x,t) \\ \rho(x,t) \cdot v(x,t) \cdot T(x,t) \end{vmatrix}; \tag{14}$$

parallel variables (pressure and temperature)

$$\boldsymbol{u}(x,t) = \begin{vmatrix} p(x,t) \\ T(x,t) \end{vmatrix}. \tag{15}$$

The balance equations

$$\sum_n M(t) = 0 \quad \text{and} \quad \sum_n M(t) \cdot T(t) = 0 \tag{16}$$

for the mass flows $M(t)$ and the energy flows $M(t) \cdot T(t)$ on the nodes of the network of pipelines numbered with $n \in n_{in} \bigcup n_{out}$, where n_{in} and n_{out} are the sets of inner nodes and boundary nodes, respectively. The equations (1)-(16) are defined on the graph which represents the set of connections between the elements of the pipeline network. Therefore, the model of the network consists of the models of its elements of the forms (1)-(3), (7)-(9), (11) suitable to build the model of the whole system on the basis of equations (16). The equation of different network elements must be transformed to unified form for integrated system model obtaining. The required form can be obtained through algebraization of the system of equations of gas dynamics (1)-(3) and its further linearization under the conditions (5)-(6).To algebraize the equation set (1)-(3), the original numerical operator method is proposed, which based on the Newton polynomials [1]. This method provides an equally high accuracy of partial derivative approximation (1)-(3) not only inside of the domain of independent variables $t \geq t_0$ and $x \in [0, l]$, but also at their boundaries according to the conditions (5)-(6). After algebraization the model components are linearized and transformed in a common form:

$$\begin{vmatrix} \boldsymbol{i}_{in}(\tau) \\ \boldsymbol{i}_{out}(\tau) \end{vmatrix} = \boldsymbol{G} \cdot \begin{vmatrix} \boldsymbol{u}_{in}(\tau) \\ \boldsymbol{u}_{out}(\tau) \end{vmatrix} + \begin{vmatrix} \boldsymbol{i}^0_{in}(\tau) \\ \boldsymbol{i}^0_{out}(\tau) \end{vmatrix}, \qquad (17)$$

where τ is the set of discrete values of the independent variable t. The solution of system (1)-(16) is approximated with account of the parametrically given boundary conditions (5)-(6); $\boldsymbol{u}_{in}(\tau)$ and $\boldsymbol{u}_{out}(\tau)$ are the node vectors of unknown parallel action variables; $\boldsymbol{i}_{in}(\tau)$ and $\boldsymbol{i}_{out}(\tau)$ are the element vectors of unknown sequential action variables at the input and output correspondently; $\boldsymbol{G}$ is the matrix of element conductances; $\boldsymbol{i}^0_{in}(\tau)$ and $\boldsymbol{i}^0_{out}(\tau)$ are the linearization constants. Using the operator equations of network elements (17), the model of gas transport network of arbitrary topology can be easily constructed on the basis of balance equations (16). Now it is possible to perform the model construction using the method of nodal thermo hydraulic potentials, which is similar to that in the electric circuit theory.

Conclusion

The described approach allows to construct efficient computer models of different energy systems in spite of a variety of the elements of the system, the heterogeneity of the observed physical phenomena and the complexity of their mathematical description. On the basis of energetic analogies theory, a generalized energetic circuit principle is extended to gas-transport networks with arbitrary topologies.

References

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Дата надходження до редакції 25.10.2007 р.