Tooth wear modeling and prognostication parameters of engagement of spur gear power transmissions

Valentin Onishchenko*

Donetsk National Technical University, 58 Artjom Str., 83000 Donetsk, Ukraine

Abstract

In this paper the wear model of tooth surface contact has been developed. This model takes into consideration the conditions of machine operation, corresponding tribological theories, the eccentricity of pitch circle and the instant temperature in the contact. The prognostication model of gear teeth characteristics takes into account the continuous influence of profile form on the contact parameters and the influence of parameters of contact on the profile form. The full model is done in the form of a package of computer programs. This model includes the kinematic model of tooth engagement with any form of profiles, the elastic dynamic model with four degrees of freedom, the tooth wear model for the boundary lubrication regime of friction and the model of synthesis of tooth wear profile.

Key words: Gear transmissions; Specific friction power; Specific wear intensity; Wear model; Synthesis of tooth wear profile; Prognostication parameters of engagement

1. Introduction

In the modern world machines are obviously the major means of mechanization used for manufacturing and transportation of huge quantities of all sorts of production. Efficiency of these highly mechanized processes appreciably depends on the reliability of work of tooth gearings of machine drives. Operating conditions for machines used in mining, mining-concentrating, metallurgy, transport etc. are extremely difficult because of high loadings, high speeds, rough environments which are significantly polluted by abrasive dust. The basic work performance criteria for tooth gearings are based on the computations of bending durability as well as contact endurance, scoring and wear of working surfaces. The wear calculation is the least reliable of all kinds of calculations in spite of a huge number of investigations devoted to this problem. There are two possible reasons for such a phenomenon.

The first reason of insufficient reliability of wear computation has to do with a wide range of lubrication regime being the major factor determining process of wear. Many researchers divide the whole range of lubrication regime into three subdivisions in which processes determining the intensity of wear process of contacting surfaces are qualitatively different. They are:

- the boundary lubrication regime;
- the elastohydrodynamic lubrication (EHL) regime;
- the partial- or mixed-elastohydrodynamic lubrication regime.

The boundary lubrication regime is actually realized at a high load per unit width W_n, low sliding velocities υ_s and high temperatures (causing a decrease of the dynamic viscosity η). It is characterized not only by a high value of the factor of friction f, but also by continual wearing of contacting surfaces.

A prerequisite to realization of the elastohydrodynamic lubrication regime is the existence of a lubricant film with the thickness which at the applied loadings exceeds the total height of roughness of contacting surfaces. In the EHL regime the direct contact of rubbed

* E-mail address: onival@r6.dgtu.donetsk.ua (V. Onishchenko). Tel. +380-62-3010-835.

bodies is completely excluded which results in a significant decreasing of the coefficient of friction, a practical cessation of wear (as the process of wear occurs only at the time of starting up and stopping of the machine).

In the partial-EHL regime some parts of contacting surfaces are divided up by a hydrodynamic film, while on other parts the boundary lubrication regime takes place. In this regime of lubrication both viscosity of the lubricant η and its capacity to create strong boundary films on surfaces of friction are of basic importance. As the EHL regime part increases, the coefficient of friction decreases and thus raises the wear resistance of surfaces.

The regime of elastohydrodynamic lubrication is most fully realized in sliding bearings. In tooth gearings the boundary regime of lubrication or the partial-EHL regime is realized in the overwhelming majority of cases

Wu S., Cheng H. S. [1] and Chichinadze A.V. [2] have suggested that the probability of realization of one or other lubrication regime should be estimated by the factor of thickness of a lubricant film between the teeth λ. So, in paper [2] the coefficient λ represents the relation of the thickness of a lubricant film in a zone of minimal backlash between the rubbed surfaces $h_{\min}$ to the characteristic of height of roughness of these surfaces

$$\lambda = \frac{h_{\min}}{\sqrt{R_{ap}^2 + R_{aw}^2}}, \tag{1}$$

where R_{ap} and R_{aw} are the arithmetical mean of the profile deviation of roughness of tooth surfaces of pinion and wheel[1].

At $\lambda > 3$ the elastohydrodynamic lubrication regime takes place, at $\lambda < 1$ the boundary lubrication regime occurs, and at $1 \le \lambda \le 3$ the partial-EHL regime is in action.

With the boundary lubrication regime the calculation of key parameter determining the wear resistance of a tooth gearing (the coefficient of friction f) is made by empirical formulas which have proven their efficiency by a long-term practice of their application [3,4,5]. With the use of these formulas the basic requirement is the conformity of conditions for which the calculation of wear resistance of teeth is carried out to a range of conditions at which they have been obtained.

The theory of the partial-EHL has appeared quite recently. A great contribution into the development of this theory was made by Wu S. and Cheng H. S. [1,6]. The scientific approach of these authors as to the formation of superficial films in tooth contact and the influence of these films on the tribological effect of lubrication deserves a particular attention. Of special interest are papers by H. Xu et al. who researched the coefficient of friction in the elastohydrodynamic lubrication regime [7].

However, this theory requires further improvements. So A. Kahraman et al. [7,8], pointing out the important implications of this theory as the first approximation to the determination of coefficient of friction, also reveal its shortcomings, such as: the wear models developed on the basis of the EHL theory "…were not practical since they required significant CPU time to run. In addition, they might be more accurate in EHL aspects of the problem, their modeling of gears was limited to simple spur gears with ideal load distributions and no tooth deformations". "Almost all of them assumed rigid gear teeth (no bending deflections, base rotations or gear blank deformations) and relied on theoretical idealized load distributions along the contact line. These models were not capable of including modified profiles and any type of geometric deviations resulting from the manufacturing processes, heat treatment distortions, assembly errors and deflections of support structures" [8].

---

[1] In a number of countries R_a instead of the root mean square the deviation heights of roughness S_{rms} is used ($S_{rms} \approx 1{,}3 R_a$)

It is possible to agree with the assertion of Wu S. and Cheng H. S. [6], who suggest that in tooth gearings the partial-EHL regime takes place, because the thickness of a hydrodynamic film between the mating teeth is in the same order of magnitude as their surfaces roughness. It is clearly indicative of the impossibility of existence of a normal hydrodynamic film between mating teeth. Hence it follows that the wear at sliding teeth will be inevitable. However, it seems to be necessary to add the further remark to the above. At a pitch point of engagement at the zero value of the sliding velocity - $\upsilon_s = 0$ the contact of roughness does not disappear, even if in those parts of the contact where there is an EHL regime, the coefficient of friction tends to zero. In the general meaning of friction coefficient the part of boundary friction is much more than that of the liquid friction even in view of generated strong superficial films. And in the boundary friction regime at $\upsilon_s = 0$ the coefficient of friction can decrease, but can never assume the zero value [9,10,11]. In this connection the statement made by the authors of works [1,6,7] casts the doubts upon the conclusion that at the zero value of sliding velosity with the partial-EHL regime the coefficient of friction is equal to zero.

Also it is difficult to agree with H. Xu [7] who argues that empirical formulas are inapplicable for determination of the coefficient of friction in tooth gearings. As mentioned above, these formulas are intended for calculations in the boundary lubrication regime ($\lambda < 1$) and are completely inapplicable for the partial-EHL regime and the more so for the regime of elastohydrodynamic lubrication (in the calculations proposed by H. Xu the parameter of height of roughness is taken as equal to $s_{rms} = 0{,}07$ μm, whereas the Drozdov and Misharin formulas [3,4,5] are applicable in the cases when the roughness is 20 to 40 times bigger). So, for instance, in the calculation of the coefficient of friction, presented in work [5], the value $s_{rms} = 4{,}16$ μm ($R_a = 3{,}2$ μm) is used.

The second reason for low reliability of wear calculations of tooth gearings is the determination of parameters of wear by the initial form of tooth profile. The most used shape of teeth of gearing of machine drives is the involute shape. On the base of this shape all calculations for obtaining a satisfactory strength, fatigue, and wear properties are carried out. However, during the operation of machines a significant wear of working surfaces occurs. Nevertheless, the influence of wear on the engagement parameters is either neglected or not given sufficient consideration.

In order to solve this problem the models of process of tooth engagement which take into account the influence of changes of a tooth shape upon the parameters of contact were developed [12-19]. So in the papers of Wang C.C. [12], Kasuba R. and Evans J. W. [14] the influence of profile errors of teeth manufacturing on a dynamic loading in engagement is investigated. In the papers of Wilk A. [15], Kuang J.H. and Lin A.D. [17], the influence of teeth wear on the parameters of contact is investigated, but their simplified artificial models of wear, which do not take into account the physical nature of wear process, have been developed. In the papers of Bajpai et al. [19,20] and Kahraman A. et al. [21] the process of gearing is modeled with the use of the method of finite elements, and the wear is determined with the use of the equation of Archard J. F. [24]. It is necessary to note that the coefficient of friction is not included directly into the Archard equation and it is taken into account indirectly in the wear coefficient k. This coefficient is taken as constant in the process of engagement, which does not represent the process of tooth wear accurately. Yuksel C. [22] has applied the technique of Kahraman A. to the planetary gearing. Flodin, A., and Andersson, S. [23] have suggested that in the model of gearing the change of teeth rigidity depending on the number of simultaneously mating teeth should be taken into account. On the whole, in papers [19-23] the obtained values of wear are commensurable with the manufacturing errors of the involute shape, which complicates their comparison with the results of tooth wear while in operation. Besides, the research of dynamic processes is limited to the determination of frequencies of teeth' own vibrations without taking into account the dynamic characteristics of the machine drive.

The physically proved model of wear process in the boundary friction regime and the model of prediction of the characteristics of gearing on its basis is presented in the papers by Wojnarowski J. and Onischenko V [13,16,18].

The analysis of toothed wheels with worn teeth shows that because of changeability of contact parameters on the line of action the teeth have been wearing along their height non-uniformly. As a result, tooth outlines cease to be involute. According to the fundamental law of toothed gearing [25] the gear ratio becomes variable, the line of action ceases to be a direct line, the radiuses of curvature of tooth outlines are changed so sharply that it may result in the change of sign of curvature. All the other parameters of engagement become different from the parameters of involute gearing.

In this study the influence of parameters of tooth contact on the process of tooth wearing (distortion) alongside with the influence of a distorted shape on the parameters of tooth contact is being investigated. The results of this research will allow to prognosticate the tooth shapes and parameters of engagement after a certain period of operation of toothed gearing. The opportunity of comparing the expected characteristics of tooth gearing with the demanded characteristics will allow to project long-lived transmissions of high reliability.

Nomenclature

$m =$	Module; (mm)
z_p and z_w =	Numbers of teeth of pinion and wheel;
$u = \frac{z_w}{z_p} =$	Gear ratio for unworn teeth or average gear ratio for worn teeth;
x_p and x_w =	Shift profile coefficients of pinion and wheel;
d_{ap} and d_{aw} =	Diameters of addendum circle of pinion and wheel; (mm)
$\rho_p =$	Reduced radius of curvature in the contact point of pinion; (m)
$\rho_w =$	Reduced radius of curvature in the contact point of wheel; (m)
$\rho_{red} = \frac{\rho_p \rho_w}{\rho_p + \rho_w} =$	Reduced radius of curvature in the contact point of pinion and wheel;(m)
$a =$	Center distance; (mm)
$b_{\min} =$	Minimal face width from two contacting toothed wheels; (mm)
$T_p =$	Input torque; (Nm)
$T_w =$	Output torque; (Nm)
$W_n =$	Load Per Unit Width; (N/m)
$\omega_p =$	Angular velocity of pinion; (rd/s)
BHN_p and BHN_w =	Brinell Hardness Numbers of working surfaces of the teeth of pinion and wheel; (N/m^2)
RHN_p and RHN_w =	Rockwell Hardness Numbers of working surfaces of the teeth of pinion and wheel;
σ_{ssp} and σ_{ssw} =	Limits of stretching strain of materials of toothed wheels; (N/mm^2)
$s_c =$	Contact stress (of Hertz); (N/mm^2)

E_p and E_w =	Modulus of elasticity for pinion and wheel; (N/mm^2)
$E_{red} = \dfrac{2E_p E_w}{E_p + E_w}$ =	Reduced module of elasticity of pinion and wheel materials (N/mm^2),
ν_p and ν_w =	Poisson's ratio for materials of pinion and gear;
R_{ap} and R_{aw} =	Average arithmetic deviation of roughness of tooth shapes of contacting materials; (mm)
f_{pb1} and f_{pb2} =	Errors of the basic tooth step; (mm)
j_n =	Normal lateral backlash; (mm).
J_p and J_w =	Polar mass of inertia of pinion and wheel of the researched tooth gearing; (kgm^2)
$J_{d,p-1}$ =	Reduced polar mass of inertia from motor to tooth wheel previous to pinion; (kgm^2)
$J_{r,w+1}$ =	Reduced polar mass of inertia from working mechanism to the tooth wheel nearest to the researched one; (kgm^2)
$C_{d,p}$ =	Reduced torsion rigidity factor of shafts of drive from motor to pinion of the researched transmission; (Nm/rd)
$C_{r,w}$ =	Reduced torsion rigidity factor of shafts of drive from working mechanism of machine to wheel of the researched transmission. (Nm/rd)
υ_{typ} and υ_{tyw} =	Rolling velocities of moving of zone of contact on tooth shapes of pinion and wheel; (m/s)
$\upsilon_{sz} = \upsilon_{typ} - \upsilon_{tyw}$ =	Sliding velocity between teeth; (m/s)
b_H	Half Hertzian width; (mm)
P_z =	Specific power of friction forces in teeth contact with relative rolling and sliding; (W/mm^2)
f_z =	Coefficient of friction between the teeth;
K_d =	Dynamic factor;
K_β =	Load distribution factor on the length of face width;
ν_{t^o} and η_{t^o} =	Kinematic and Dynamic viscosity of oil at a working temperature of teeth t^o; (m^2/s)

2. General scheme of prognostication of tooth shapes of toothed gearing

The engagement of toothed gears with worn teeth in the process of operation may be named the quasi-conjugate toothed gearing. This transmission as a result of wear has arbitrary (though close to involute) tooth shapes. As against the conjugate tooth shapes, having a constant value of the gear ratio, the quasi-conjugate tooth shapes keep only an average value of the gear ratio (equal to the relation of numbers of teeth of the mating gear). The instant value of the gear ratio of the quasi-conjugate tooth shapes is changeable and so is the function of rotation angle of the drive gear (pinion). This gear ratio varies in due course as the tooth wear out.

Since this work examines only the spur cylindrical tooth gearing, the flat problem of the theory of engagement with any form of tooth shapes has been put to the basis of the research. It is assumed that all the teeth in a wheel wear out equally and the error of a basic tooth step after the period of operation is rather insignificant, nearly zero. The non-uniformity of loading on the face width of a tooth was taken into account by the introduction of the load distribution factor.

The general flowchart of the model of prediction of tooth shapes is shown in Fig. 1 [16]. The scheme consists of separate models, which form one complex.

By means of kinematic and dynamic models the parameters of contact in function of time for the period of engagement of one tooth of pinion (with a step equal to 0.01 from the angular step of the pinion tooth) are determined. Depending on these parameters the wear of tooth in the contact points is determined and new coordinates are calculated and then the approximation of tooth shape is made. After that the approximated coordinates are entered again in the kinematic and dynamic models of gearing.

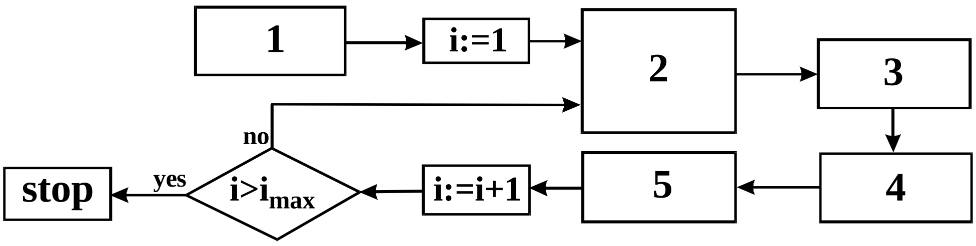


Fig. 1. Flowchart of the model of prediction of tooth shapes
1 - block of data preparation, 2 - kinematic model of engagement of worn teeth, 3 - elastic dynamic model of toothed gearing and drive, 4 - model of tooth wearing, 5 - model of synthesis of worn teeth outlines.

This iterative process lasts up to the moment when the maximum number of cycles of the wear process ($i_{\max}$) proportional to the time of machine operation is set. The value $i_{\max}$ is accepted so that the average wear for one iterative cycle could be at least ten times higher than the computation error (according to the tests for theoretical tooth shapes this error is equal to 0.001 mm). On the other hand, the average wear should not exceed 1-2 % of the maximum wear for the period of operation, which allows the wear process to be close to the one that takes place in operation.

3. Functional purpose of data preparation blocks as well as kinematic and dynamic models of engagement of worn teeth

The contents of data preparation blocks, kinematic and dynamic models of engagement of worn teeth are described in detail in many publications [13,16,18,26,27], therefore it would be enough to give their brief description only.

Data block preparation. Irrespective of the teeth's initial condition (worn or unworn) the following parameters of toothed gearing are entered in the block of preparation of initial data: m, z_p, z_w, x_p, x_w, d_{ap}, d_{aw}, a, $b_{\min}$, T_p, ω_p, RHN_p, RHN_w, σ_{ssp}, σ_{ssw}, E_p, E_w, ν_p, ν_w, R_{ap}, R_{aw}.

The orthogonal systems of co-ordinates are more suitable for the mathematical description of tooth shapes of worn teeth (Fig.2). The axes of abscissa of these systems coincides with the axes of tooth symmetry (of theoretical tooth), and the axes of ordinates goes through the starting point of an involute on the base circle. Mobile systems are attached to the center of rotation of mating gears by the distance x_0.

The co-ordinates of unworn tooth shapes are determined by the analytical method with the help of the involute equation. These co-ordinates are represented as the sets $C_{xp,yp} = \{x_p, y_p\}$ and $C_{xw,yw} = \{x_w, y_w\}$ with the capacity of 50 elements.

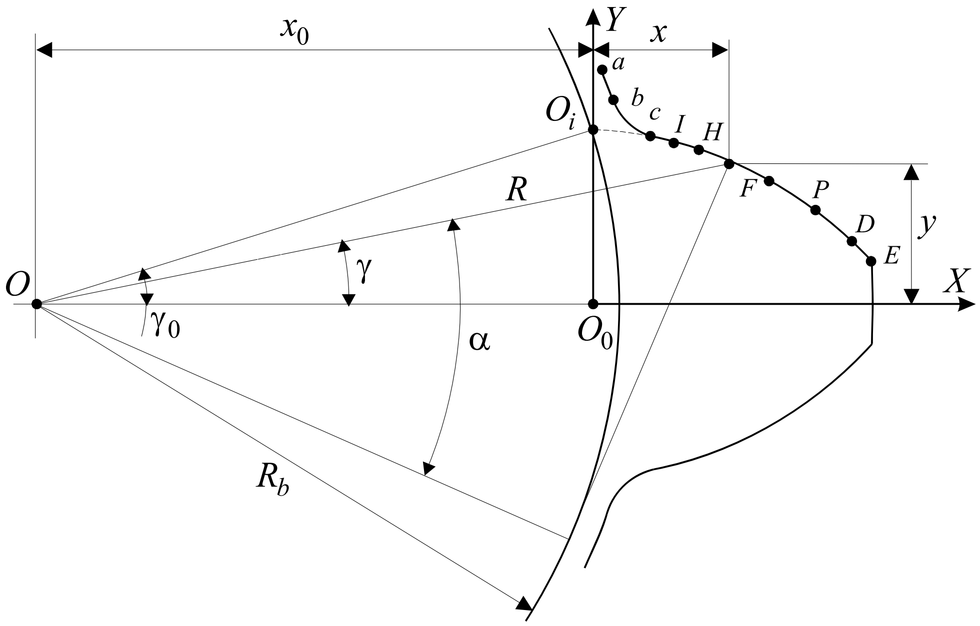


Fig. 2. System of coordinates of the tooth shape

ab - an arch of dedendum circle, *bc* - an arch of fillet, *I* - the bottom border of wear, *HF* and *DE* - sites of theoretical two pair gearings, *FD* - accordingly a site of one-pair gearing, P – pitch point, R_b - radius of the base circle, O_i - starting point of involute, *x* and *y* - coordinates of any point of the shape of a tooth.

The co-ordinates of outlines of worn teeth are entered in the data preparation block as the following sets: $C^w_{x_p, y_p} = \{x_{pw}, y_{pw}\}$ and $C^w_{x_w y_w} = \{x_{ww}, y_{ww}\}$.

In the data preparation block there is a possibility to choose the type of polynomial approximation (a power polynomial, Tczebyshev or Legandr polynomials), and also to choose the degree of polynomial (no more than 10) depending on the following conditions:

- the average quadratic deviation of the approximation error should be less than the error of wear measurement,
- the number of points with absolute error smaller than the error of wear measurement should not be less than 80 % of the set capacity.

Since the determination of the radius of curvature and the position of a normal to the tooth outlines by means of the differentiation of approximating function does not provide the required accuracy, these parameters were determined with the help of a circle that has been carried out through the three adjacent points of a set.

Kinematic model of engagement of worn teeth. This block is meant for the determination of geometrical and kinematic parameters of teeth contact. The integrated algorithm of solution of the problem looks as follows:

- before the model's work the range and the angular step of pinion and also the number of steps of iterative cycle are determined (in the data preparation block),
- for each value of the angular step the numerical method is used to determine the coordinates of contact point in the fixed and mobile systems of coordinates,
- for each tooth depending on the coordinates of contact point and normal force in this contact the displacement of this force with the consideration for bending and contact deformations of teeth is determined,
- by comparison of the displacement of a point of application of the normal force affecting a tooth with the displacement of corresponding contact points of neighboring teeth pairs the numerical value of the normal force working on this tooth in view of two-pair gearing is determined,

- the rigidity of teeth pair and contact stress are calculated according to the obtained value of normal force,
- the instant values of the gear ratio and pressure angle are determined[2] according to the coordinates of contact point in absolute movement,
- the values of the rolling velocities[3] and of sliding velocity are calculated according to the coordinates of contact point in relative movement.

Elastic dynamic model of toothed gearing and its drive. This block is meant for the determination of a dynamic component in contact loading, estimated by the value of dynamic factor. To maintain the model's functioning the inertial and elastic characteristics of toothed gearing and machine drive are entered: T_w, J_p, J_w, $J_{d,p-1}$, $J_{r,w+1}$, $C_{d,p}$, $C_{r,w}$.

The estimated influence of the varying gear ratio and of the rigidity of engagement at dynamic loading was carried out in two stages. At the first stage the rigid model with one degree of freedom was considered. At the second stage the elastic model with four degrees of freedom [27] was considered, in which the kinematic parameters of portable movement found with the help of the rigid model were used. The model allows (after finding the solution to the system of four differential equations) to determine the angular acceleration of the pinion ε_p [rd/s^2]. This acceleration is the result of the distortions of the tooth shape of gearing owing to the wear of teeth. Accordingly, the dynamic factor K_d will be equal to

$$K_d = 1 - \frac{\varepsilon_p \cdot J_{r,p}}{T_{r,p}}, \qquad (2)$$

where $J_{r,p}$ - Reduced polar mass of inertia from the working mechanism to the pinion; (kgm^2),

$T_{r,p}$ - Torque reduced to a shaft of pinion; [Nm].

The rigidity of teeth was determined in accordance with the deformation of bending and tangential displacement. For this purpose the Castigliano theorem was used. This theorem was built directly in the dynamic model. Besides, the displacements of teeth as a result of their contact deformation were taken into account [28].

In the process of creation of these dynamic models certain assumptions have been used. The rotating masses were considered as concentrated. The gear ratio of all gear transmissions of drive, except for the examined one is accepted as constant. It has enabled to reduce the moments of inertia and rotating moments from the motor and working mechanism to the wheels next to the examined transmission.

4. Model of teeth wearing

The model of wear process is developed on the basis of the results of analysis of operation conditions of the power machines, in particular, the coal-mining combines, and also the tribological theories corresponding to the work conditions of the researched toothed gearing

---

[2] The gear ratio of toothed gearing was determined in accordance with the fundamental law of toothed gearing. In a two-pair engagement the position of pitch point P is determined by the cross point of an action line of the force resultant (the vector sum of normal forces in the two neighboring teeth pairs) with the center distance. It means that normal forces in these pairs of worn teeth do not always lie on the same straight line.

[3] The rolling velocity was accepted as equal to the ratio of the path ΔS_j, going by the contact point along the outlines of mating teeth during the time interval Δt. Therefore, the rolling velocity is calculated as follows: $\upsilon_{tyj} = \Delta S_j / \Delta t, \quad j = p, w$.

[18] was taken into account. The next stage will be the determination of the regime of lubrication in order to select the corresponding model of wear process.

The establishment of the lubrication regime was made according to the value of the coefficient of thickness of a lubricant film between the teeth of gearing λ (1). The parameters of this gearing correspond to a range of parameters of tooth gearings of coal combines drivers. The thickness of a lubricant film in a zone of minimal backlash between the rubbed details h was determined by two formulas: by the Kodnir-Ratner formula[29] and by the Reshetov formula [30].

The Kodnir-Ratner formula after its reduction to the SI[4] looks as follows

$$h_1 = \frac{1{,}12 \cdot 10^{-6} \left[\eta_{t^o} \cdot (\upsilon_{typ} + \upsilon_{tyw})\right]^{0{,}75} \cdot n^{0{,}6} \cdot \rho_{red}^{0{,}4}}{W_n^{0{,}15}.} , [\mu m] \tag{3}$$

where η_{t^o} - Dynamic viscosity of a lubricant film at the working temperature of oil in a reducing gear, [Pa·s].

$(\upsilon_{typ} + \upsilon_{tyw})$ - Total rolling velocities of pinion and wheel; [m/s],

n - Pressure-viscosity coefficient; [m^2/N],

ρ_{red} - Reduced radius of curvature, [m];

W_n - Load per unit width of a tooth; [N/mm]

Accordingly, the Reshetov formula looks like

$$h = 2{,}68 \cdot 10^{-6} \sqrt{\frac{\eta_{t^o} \cdot (\upsilon_{typ} + \upsilon_{tyw}) \cdot \rho_{red}}{E_{red}}} , [\mu m] \tag{4}$$

Where η_{t^o} - Dynamic viscosity of a lubricant film at the working temperature of oil in a reducing gear, [MPa·s].

E_{red} - Reduced module of elasticity of materials of contacting surfaces; [MPa];

The parameters of a tooth gearing needed for the calculation of thickness of a lubricant film as well as the results of calculations by the formulas (3) and (4) are presented in Table 1.

Table 1 The initial data and results of calculation of thickness of a lubricant film

Parameters	Units	Means
η_{80^o}	Pa·s MPa·s	0,0144 $0{,}0144 \cdot 10^{-6}$
$(\upsilon_{typ} + \upsilon_{tyw})$	m/s	1,57+1,57=3,14
n	m^2/N	$0{,}015 \cdot 10^{-6}$
ρ_{red}	m	0,023
W_n	N/mm	408
E_{red}	MPa	210000
R_{ap} and R_{aw}	μm	6,3 and 6,3
h_1 and h_2	μm	0,20 and 0,19
λ_1 and λ_2		0,022 and 0,021

[4] The International System of Units (Standard ISO 31).

As seen from this table, $\lambda_1 << 1$ and $\lambda_2 << 1$. Therefore, in the zone of tooth contact of the gearing under consideration the boundary lubrication regime takes place, while the partial-EHL regime and the regime of hydrodynamic lubrication are not possible.

In this paper the wear model is based on the Chudakov criterion [31] which states that the wear in the direction of a normal to a surface of contact is simply proportional to specific power P_z [W/мм^2], spent on the overcoming of friction forces between contact surfaces

$$I_z = i_z P_z = i_z f_z s_c \upsilon_s , \tag{5}$$

where f_z - Coefficient of friction,

i_z - Specific intensity of teeth wear process (mm^3/W).

A similar criterion was offered in 1937 by Ketov and Kolchin [32]. Much later, in 1968, Kragelsky [33] suggested to use the power intensity of wear process i [mm^3/J], determined by the volume of the worn material per unit of work of friction forces, as one of the basic characteristics of wear

$$i = \frac{V}{F_{fr} L} = \frac{i_z}{t} , \qquad [6]$$

where $V = AI_z$ - Volume of the worn material; [mm^3]

A - Area of contact; [mm^2]

$L = \upsilon_s t$ - Way of friction; [m]

$F_{fr} = fN$ - Force of friction; [N]

$N = qA$ - Force compressing contacting surfaces; [N]

q - Specific pressure in the contact of the compressed surfaces; [N/mm^2]

t - Time of wear process; [s]

From the formula (6) the volume of the worn material is equal to

$$V = ifNL . \tag{7}$$

When determining the values included in the formula (6), the following assumption is made: during the time of moving on a distance equal to the Hertzian width the velocity of relative moving of contact surfaces υ_s [m/s] is constant and the area of contact A [mm^2] does not vary in the wear process.

With the substitution of these values in the formula (7) and all the corresponding transformations the formula (8) takes the form

$$I_z = i_z fq\upsilon_s = i_z P_z , \tag{8}$$

which fully complies with the formula (5).

The obtained formula has been put in the basis of the techniques of conventional calculation wear of the flat surfaces and bearings of sliding. For the calculation of the wear of tooth gearing it is necessary to substitute the value q with another term, such as the contact stress s_c. The volumetric wear dependence on the work of friction forces has also been used by Flesher [34] in his development of the power theory of wear. The thesis about the proportionality of wear to the work of forces of friction for the calculation of wear of machines was used by Glagolev [35], Iosilevich [36] and other researchers.

On the basis of formula (5) the method [37] of calculation of tooth wear for the boundary friction regime has been developed. This method has been tested experimentally for the operating conditions of hardly loaded drives of mining machines.

Except for the criterion stating that wear is proportional to specific power of friction the following substantive assumption have formed the basis of model of wear process:

- the total wear $I_z = I_{zp} + I_{zw}$ (mm) in the direction of the normal to a contact of two contacting teeth is proportional to the amount of specific power expended on the overcoming of the friction forces.
- the wear of each of the contacting teeth (of the pinion I_{zp} and the wheel I_{zw}) is proportional to the way of sliding and inversely proportional to the hardness of the contact surface.

On the strength of these assumptions it is possible to find the values of the wear in the contact point for each of the contacting teeth

$$\left.\begin{aligned} I_{zw} &= \frac{I_z}{1+\alpha_{zI}} \\ I_{zp} &= I_{zw}\alpha_{zI}\alpha_u \end{aligned}\right\}, \tag{9}$$

where: α_{zI} =Distribution factor of wear between the pinion and the wheel in the contact point;

α_u =Factor of influence of tooth frequency on the tooth wear.

From the fact that in a unit of time a tooth of pinion enters into engagement in a gear ratio of times more often than a tooth of wheel it is necessary to expect a corresponding increase of wear of the pinion tooth. However, the experimental data on the determination of volumetric wear of teeth do not confirm it. Therefore, the correction factor α_u is entered. As a first approximation this factor can be accepted as equal to the ratio of the volumetric wear tooth of the pinion to the volumetric wear tooth of the wheel for a certain period of operation of the transmission working in the conditions similar to the conditions of the researched toothed gear.

According to the accepted assumptions the values α_{zI} and I_z are determined as follows

- distribution factor of wear between the pinion and wheel surfaces in the contact point

$$\alpha_{zI} = \frac{I_{zp}}{I_{zw}} = \frac{S_p}{S_w}\frac{HRN_p}{HRN_w} = \frac{\upsilon_{typ}}{\upsilon_{tyw}}\frac{HRN_p}{HRN_w}, \tag{10}$$

where: $S_p = 2b_H\dfrac{\upsilon_{sz}}{\upsilon_{typ}}$, $S_w = 2b_H\dfrac{\upsilon_{sz}}{\upsilon_{tyw}}$ = Ways of sliding of the contact zone on the pinion and wheel tooth shapes, (m).

- total wear of teeth in the contact point

$$I_z = i_z\alpha_T\alpha_{Bl}P_z = i_z\alpha_T\alpha_{Bl}\alpha_{Wil}f_z s_c\sqrt{K_d K_\beta}\cdot\upsilon_{sz}, \tag{11}$$

where: i_z = Specific intensity of teeth wear process (mm^3/W); α_T = The factor of variation of specific intensity of wear process, taking into account the change i_z while in the operation, (it is determined experimentally); α_{Bl} = The factor, which is taking into account the influence of instant temperature (of Blok) at the contact of teeth; α_{Wil} = Factor, which is taking into account the distribution of contact stress at the edge contact of teeth.

The factor of distribution of loading on the length of a tooth K_β was determined according to the standard ISO/DIS 6336/I [38].

Influence of instant contact temperature on teeth wear. The factor α_{Bl} which is taking into account the influence of instant temperature of tooth contact, is accepted equal to the ratio of temperature of flash in the zone of teeth contact, (transformed to fit the calculations of steel wheels) to the critical value of temperature of flash

$$\alpha_{Bl} = \frac{0.68 f_z \upsilon_{sz} \sqrt[4]{\frac{W_n}{\rho_{red}}}}{(\sqrt{\upsilon_{typ}} + \sqrt{\upsilon_{tyw}}) C_p \nu_{50^0}^{0,06}} \geq 1, \tag{12}$$

where: ν_{50^0} = Kinematic viscosity of oil at a temperature of 50^0 (mm^2/s); C_p = The factor which is taking into account the properties of lubricant oil: $C_p = 1{,}7$ - usual oils; $C_p = 2{,}2$ [5] - oils with an anti-scuffing additive.

If the factor $\alpha_{Bl} < 1$, it is necessary to accept: $\alpha_{Bl} = 1$.

The coefficient of friction for the conditions of hardly loaded contact was determined under the Drozdov's formula [5]. This formula is intended for the determination of the coefficient of friction for the case when friction surfaces are rolled one on another with sliding for the regime of boundary friction. Drozdov made a number of the experiments in the rolling friction mode with sliding on tooth gearings. The range of applicability of the Drozdov formula completely coincides with the range of parameters of tooth gearings experimentally investigated in the given work. The Drozdov formula looks as follows

$$f_z = \frac{4.5 \cdot 10^{-3} s_c^{0.02} \left[10 + \lg\left(\frac{BHN \cdot R_a}{E_{red} \rho_{red}} \right) \right]}{\nu^{0.07} (\upsilon_{typ} + \upsilon_{tyw})^{0.12} \upsilon_{sz}^{0.2}} \leq f_{\max}, \tag{13}$$

where ν = Kinematic viscosity of oil at a working temperature of teeth; [m^2/s];

BHN = Hardness of the less hard one of the two contacting materials [MPa];

R_a = Arithmetical mean of the profile deviation of roughness of the harder one of the two contact surfaces [m];

$f_{\max}$ = Maximal value of friction factor for the given conditions of friction, accepted on the basis of experimental data ($f_{\max} = 0{,}12$).

The range of applicability of the Drozdov formula: $\nu > 10^{-6}$ m^2/s, $s_c > 300\text{MPa}$, BHN >500MPa, $\rho_{red} > 0{,}005$ m.

Account for the edge contact of teeth. Determination of the factor α_{Wil} which is taking into account the distribution of normal contact stress in the edge contact of teeth, is based on the application of the Wellauer principle of imposing [39]. The principle is based on the balance of the moments of external and internal forces. The essence of the principle of imposing consists in the following: at the application of a concentrated force at a final distance from the edge of a beam (smaller than the length of zone of distributions of the moment on the length of an indefinitely wide beam) the cut off part of the zone is imposed on the rest of the zone, and therefore, the area of zone distributions of the moment on length remains constant.

It is obvious that the principle of imposing can be applied likewise to the case of edge teeth contact when the point of contact is at a distance, smaller than a half Hertzian width of contact from the top of the tooth. That is despite the incomplete width of a half Hertzian width of contact, the area of zone of distributions of normal contact stress should remain constant to counterbalance the enclosed external normal force. Thus it is possible to use the known parabolic law of distribution of normal contact stress in function of the distance t from the point of application of external force in the full contact of teeth

[5] By results of researches of the author

$$s_{ct} = s_c \sqrt{1 - \left(\frac{t}{b_H}\right)^2} \,. \tag{14}$$

The scheme of determination of normal stress in the point of contact in the immediate proximity to the top of the tooth with application of the Wellauer principle of imposing is shown in Fig. 3. Suppose, a certain point of the contact k is at the distance t_e from the top of the tooth e. The normal contact stress at the point k is equal to

$$s_{cw} = s_c + s_{ct} = s_c + s_c \sqrt{1 - \left(\frac{2t_e}{b_H}\right)^2} \,. \tag{15}$$

The factor, which is taking into account the distribution of contact stress at the edge contact of teeth, equals to

$$\alpha_{Wil} = \frac{s_{cw}}{s_c} = 1 + \sqrt{1 - \left(\frac{2t_e}{b_H}\right)^2} \,. \tag{16}$$

The factor α_{Wil} is determined only for the values of the distance from the edge of the tooth up to the point of contact, taking place within the limits of $0 \le t_e \le 0{,}5 b_H$. If $t_e > 0{,}5 b_H$, the normal contact stress is equal to s_c.

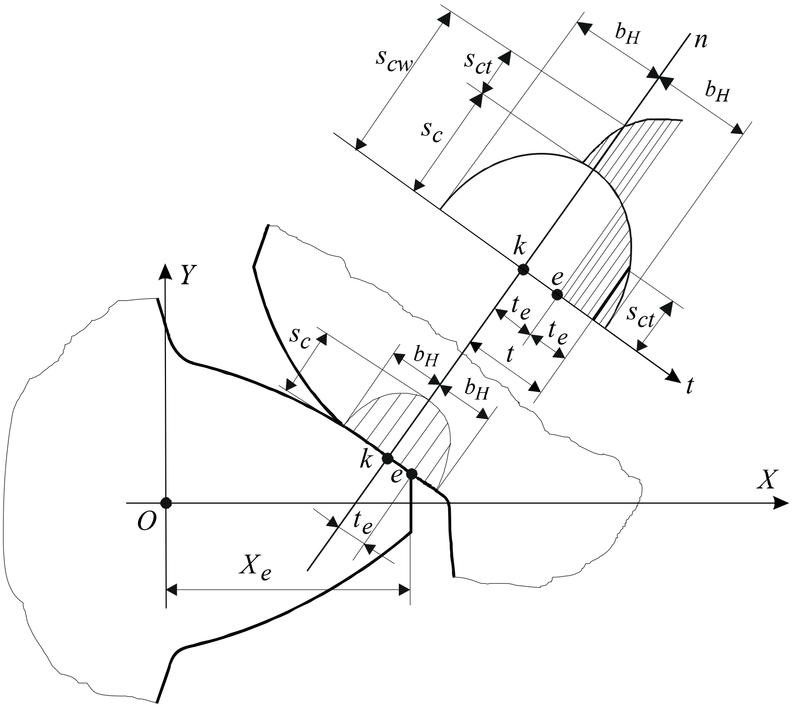


Fig. 3. The scheme of determination of normal contact stress at the edge teeth contact

Account for the eccentricity of pitch circle. From the expression (11) it follows that in the zone of pitch point is necessary to expect a zero value of the tooth wear as the sliding velocity in this zone equals to zero. However, according to the experimental data [16,18,26,27] in the zone of pitch point the wear, smaller as it is in comparison with other parts of the tooth always takes place. It is obvious, that an additional factor having influence on the wear in the circumpolar zone takes place. As such factor, a relative sliding of pinion and wheel teeth in the direction of a centers line because of their eccentricities of pitch circles was accepted.

It is well known that owing to inevitable errors of teeth wheels manufacturing the discrepancy between the centers of pitch circles and the axes of rotation takes place. This error influences the kinematic accuracy and is estimated by the size of the admitted eccentricities determined by a set accuracy of manufacturing.

The total teeth wear I_e as a result of tooth sliding because of the eccentricity of the pitch circle and its distribution between the contacting teeth (of the pinion I_{ep} and the wheel I_{ew}) can be found from the expression, similar to (9)

$$\left.\begin{aligned} I_{ew} &= \frac{I_e}{1+\alpha_{eI}} \\ I_{ep} &= I_{ew}\alpha_{eI}\alpha_u \end{aligned}\right\}, \tag{17}$$

where: α_{eI} = Factor of distribution of wear between contacting surfaces in the contact point.

The factor α_{eI} is determined in the same way as α_{zI} (see Fig. 2)

$$\alpha_{eI} = \frac{I_{ep}}{I_{ew}} = \frac{\upsilon_{ew}}{\upsilon_{ep}}\frac{RHN_w}{RHN_p}, \tag{18}$$

where: υ_{ep} and υ_{ew} = Velocities of the zone of contact on pinion and wheel tooth shapes owing to the eccentricity of pitch circle (m/s).

Similarly to the previous case, the total wear of teeth in the contact point is determined as follows

$$\left.\begin{aligned} I_e &= i_e\alpha_T P_e \\ P_e &= \alpha_{Wil} f_e s_c\sqrt{K_d K_\beta}\cdot\upsilon_{se} \end{aligned}\right\}, \tag{19}$$

where: i_e = Specific intensity of wear process at teeth sliding, owing to the eccentricity of pitch circle (mm^3/W); P_e = Specific capacity of friction forces in contact at a relative sliding of teeth (W/mm^2); f_e = Factor of a sliding friction owing to the eccentricity of pitch circle; $\upsilon_{se} = \upsilon_{ep} - \upsilon_{ew}$ = Speed of sliding while moving owing to the eccentricity of pitch circle.

Because of the small values of the sliding velocity υ_{se} the influence of temperature flash in this case was not taken into account.

As the sliding friction between the teeth at their relative sliding because of the eccentricity of pitch circle has a reciprocating character, the factor of friction was determined according to the formulas for slide ways [40]

$$f_e = f_m + 0{,}04\cdot\left(\frac{s_c\sqrt{K_d K_\beta}}{BHN}\right)^{0{,}25}, \tag{20}$$

where: f_m = Molecular component of the friction factor, accepted as equal to the minimal value of the factor of friction for the researched modes of friction (for hard steel surfaces usually $f_m = 0{,}08...0{,}10$).

The velocities of the zone of contact on the pinion and wheel tooth shapes owing to the eccentricity of pitch circle are determined from the average value of the radial pulsation during one revolution of a toothed wheel

$$\upsilon_{ei} = \frac{0{,}5F_{ei}\cos\delta_{av}}{T_i}\cos\alpha_\omega, \qquad (i = p, w), \tag{21}$$

where: F_{ei} = Tolerance of eccentricity of pitch circle of the pinion and wheel determined under the national standards depending on a degree of accuracy of tooth gearing manufacturing (m); $T_i = \dfrac{2\pi}{\omega_i}$ = Time of one revolution of the pinion and wheel (s); ω_i = Angular velocity of the pinion and wheel (rd/s); δ_{av} = Average value of angle between a vector of radial pulsation of pitch circle and a line of centers (before accumulation of experimental data it is possible to accept as equal to $\delta_{av} = 45^0$); α_ω = Pressure angle.

Finally, after the summation of corresponding values of the expressions (9) and (17), the wear of contacting teeth in the contact point in the direction of a normal to tooth shapes will be equal to

$$\left.\begin{aligned} I_p &= I_{zp} + I_{ep} \\ I_w &= I_{zw} + I_{ew} \end{aligned}\right\}. \tag{22}$$

5. Model of synthesis of the teeth outlines

The obtained sets of new coordinates of the teeth outlines are subject to processing with the purpose of their description by the approximating functions.

As was shown in the analysis of distribution of contact points on the worn tooth shapes, this distribution is not uniform and significantly depends on a degree of the tooth wear. It proves to be true by the pattern of change of rolling velocities of worn teeth according to their engagement [10]. After the distortion of teeth outlines the rolling velocities considerably differ from the theoretical values - there are parts with zero value, which means an instant stop, and parts with negative values of rolling velocities, which means the movement of the contact point in the opposite direction. An example is given in Fig. 4, where the hachure diagram of distribution of contact points on the height of a tooth of unworn teeth (a) and worn teeth (b) of the pinion and wheel is shown.

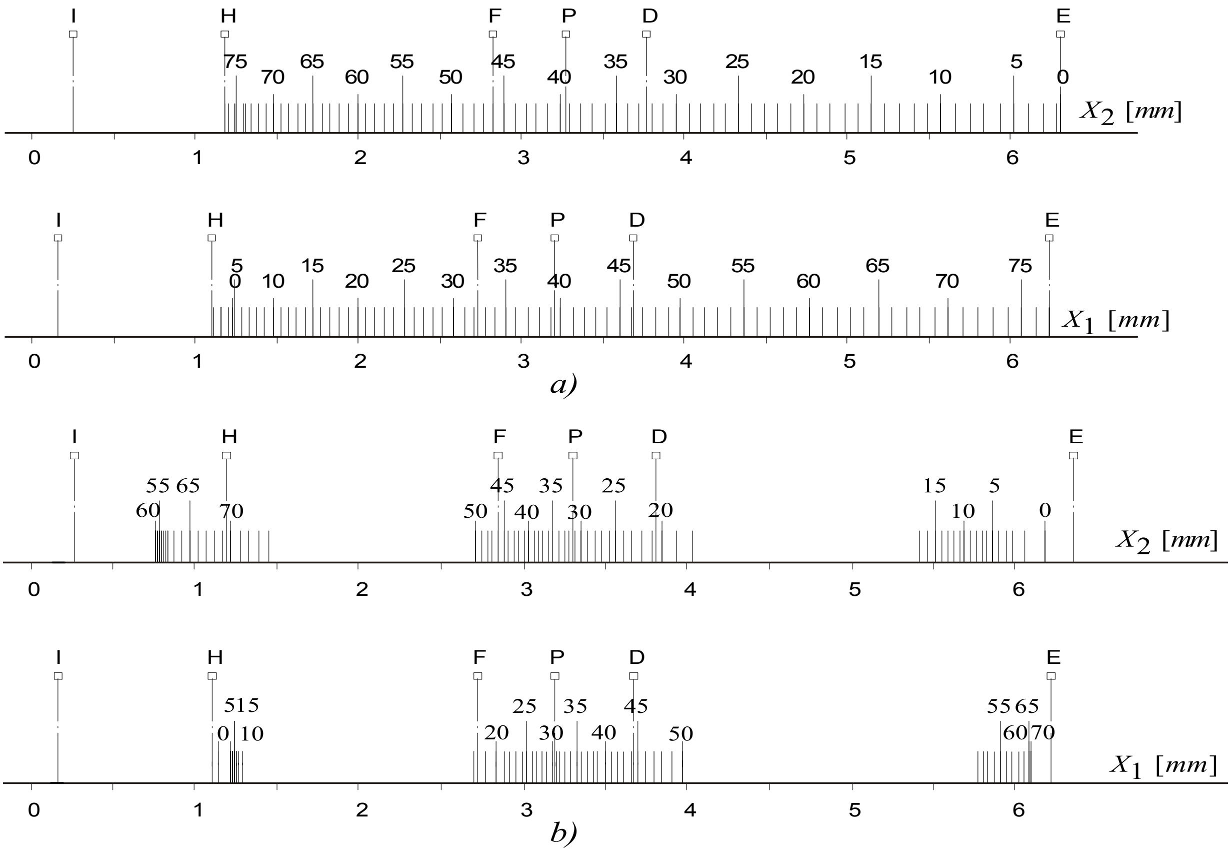


Fig. 4. Hachure diagram of the distribution of contact points on the height of a tooth of the pinion(X_1) and wheels (X_2); a) not worn teeth; b) worn teeth

The figures above the hachure designation of the abscissas of the contact points in Fig. 4 mean a consecutive ordinal number of contact. The distances between the letters represent the following: HF and DE - zone of two-pair gearing, FD - zone of one-pair gearing, P – pitch point. It is necessary to pay attention to the fact that in the beginning of the worn teeth engagement the zone of contact practically does not move on the wheel tooth, being at its top. Whereas on the flank of the pinion tooth the zone of contact moves, but in the opposite direction. For the teeth outlines distorted by wear the infringements of a continuity of gearing after the points 17 and 50 are observed, which corresponds to the jumps on the line of action [27]. The character of such moving is also reflected in the diagram of the rolling velocities.

However, when considering a problem of approximation of the worn teeth outlines, by far the most important circumstance is that on the parts of tooth shapes after the points 17 and 50 there is no teeth contact and, hence, their wear cannot take place either. Thus the length of these parts considerably exceeds the width of the Hertzian contact.

In these conditions the direct application of the polynomial approximation method can result in some significant errors. The polynomial approximation with a degree, smaller than the number of points, results in the good smoothing of the form of the worn teeth outlines. But the approximating function will pass between the points, and then the part of the profile, which does not show any wear, will be deformed, which contradicts the physical sense.

On the other hand, the spline approximation provides a passage of the curve through all the points that provides an invariance of the part of the tooth shapes that has not been affected by the wear. However, the spline approximation does not smooth well the parts of greater extent.

Therefore, the method of combined approximations carried out in two stages has been applied here.

At the first stage a set of abscissas of the contact points was analyzed in order to reveal the parts of the worn tooth outlines, for which the distance between the neighboring points does not exceed the double width of the Hertzian contact strip. With the purpose of excluding sharp changes of the form of tooth outlines on the borders of such parts, the unworn parts were partly, by the width of the Hertzian contact, included in them. For the allocated segments a smoothing approximation with an automatic choice of the degree of the polynomial was carried out.

At the second stage all the segments, including those not worn out, were united in one set and a cubic spline approximation was made. As a result, the discrete set $C_{x,y}$ of pairs of the points coordinates describing the new synthesized worn tooth outlines has been obtained

$$C_{x,y} = \{(x_0, y_0),(x_1, y_1),...,(x_N, y_N),(x_{N+1}, y_{N+1})\}, \tag{23}$$

where x_{N+1}, y_{N+1} - coordinates of a certain fictitious point used for the determination of the radius of curvature of the worn tooth outline at its top.

The necessity of introduction of a fictitious point is dictated by the following important circumstance. As is clear from Fig. 3 the contact of the tooth's top (that is of one point with a very small, due to the affinity between the top of the tooth and the fictitious point, radius of curvature) with the dedendum of the tooth of contacting wheel with a significantly greater value of radius of curvature is possible. In this case the solution to the equation of compatibility of tangents to curves in the point of contact can be impossible. To exclude this opportunity and to preserve the point of contact in the immediate proximity to the top of the tooth the method of local rounding off of the top of the tooth was offered. Its essence consists in the following.

At the first stage the subset $c_{x,y}$ consisting of the last five points with numbers *N-3, N-2, N-1, N, N+1* from set $C_{x,y}$ is allocated, then in the points of this subset, except for the extreme ones, the curvature of tooth shapes was calculated. In the points *N-2, N-1* the radius of curvature was determined by the method accepted in the kinematic model.

The radius of curvature in the point *N* of the subset $c_{x,y}$ was determined on the basis of the following assumptions:

- in view of a small radius of curvature in the point *N* in comparison with the radius of curvature of the contacting surface it is possible to count it equal to the reduced radius of curvature in the given contact point;

- theoretically indefinitely big contact stresses at the point contact are limited (as a first approximation) by the limit of stretching the strain σ_{ss} of the material (MPa)

According to the accepted assumptions the curvature in the point *N* will be equal to

$$K_N = \frac{1}{Rred} = \frac{1}{W_n}\left(\frac{\sigma_{ss}}{C_p}\right)^2, \tag{24}$$

where:

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_w^2}{E_w}\right)}} \text{ - elastic coefficient.} \tag{25}$$

At the second stage, having accepted, that within the limits of the subset $c_{x,y}$ the curvature K changes according to the parabolic law, the differential equation of the curve, which pass through the points of the subset $c_{x,y}$, is formed out

$$K = \frac{\frac{d^2y}{dz^2}}{\left(1+\left(\frac{dy}{dz}\right)^2\right)^{1,5}} = K_{N-2} + a_1 z + a_2 z^2 . \tag{26}$$

where: $z = x - x_{N-2}$ - relative ordinate of a subset $c_{x,y}$.

Since $\frac{dy}{dz} << 1$, the expression for K can be simplified considerably:

$$\frac{d^2y}{dz^2} = K_{N-2} + a_1 z + a_2 z^2 . \tag{27}$$

The values of the factors a_1 and a_2 can be found with the help of the known values of the curvature in the points *N-1* and *N* from the following expressions

$$\left.\begin{aligned} K_{N-1} &= K_{N-2} + a_1 z_{N-1} + a_2 z_{N-1}^2 \\ K_N &= K_{N-2} + a_1 z_N + a_2 z_N^2 \\ z_{N-1} &= x_{N-1} - x_{N-2} \\ z_N &= x_N - x_{N-2} \end{aligned}\right\} . \tag{28}$$

The solution of this system will make the formulas for determination of the factors a_1 and a_2 look like

$$\left.\begin{aligned} a_1 &= \frac{z_N^2(K_{N-1} - K_{N-2}) - z_{N-1}^2(K_N - K_{N-2})}{z_N z_{N-1}(z_N - z_{N-1})} \\ a_2 &= \frac{z_{N-1}(K_N - K_{N-2}) - z_N(K_{N-1} - K_{N-2})}{z_N z_{N-1}(z_N - z_{N-1})} \end{aligned}\right\} . \tag{29}$$

Integrating the differential equation twice, and, considering that in the point with *z=0* the coordinates of tooth shapes and the first derivative remain constant, we can obtain the equation of the tooth profile from its top within the limits of the subset $c_{x,y}$

$$\left.\begin{array}{l} y = y_{N-2} + \dot{y}_{N-2} z + \dfrac{K_{N-2}}{2} z^2 + \dfrac{a_1}{6} z^3 + \dfrac{a_2}{12} z^4 \\ z = x - x_{N-2} \end{array}\right\} . \tag{30}$$

When the abscissas from the subset $c_{x,y}$ and the calculations of corresponding values of ordinates are introduced in the expression for y, the final correction of coordinates of the set $c_{x,y}$ is made.

On the obtained values of the ordinates of the worn tooth shapes the volumetric tooth wear for the j-th cycle (Fig. 1) of the tooth wear is calculated

$$V_j = b_{\min} \sum_{i=1}^{N} \frac{(y_{j,i} - y_{j-1,i}) + (y_{j,i-1} - y_{j-1,i-1})}{2} \cdot (x_i - x_{i-1}) . \tag{31}$$

6. Basic parameters of the model of prognostication of the teeth shape of toothed gearing

In connection with the complexity of the model of prognostication of the quality characteristic of toothed gearing it is necessary to select the parameters of model, using the method of consecutive approaching by comparing the obtained results with the experimental data of the toothed gearing working in similar conditions.

For the initial adjustment of blocks of the model it is possible to use the following approaches based on the experience of the use of computer programs of modeling of prognostication of the tooth shape of toothed gearing.

The maximum number of steps of iteration should be accepted within the limits of $j_{\max} = 80...100$. It is possible to set conditions which, when fulfilled, will make the iterative process automatically stop. For example: the wear exceeds the thickness of carbonization film, the contact stress exceeds the allowable values, the error of rotation is beyond the scope of the allowable limits, etc.

Then one cycle of iteration will be equivalent to a certain size of the worked product of the machine. The worked product is usually understood as the quantity of production made by a machine for a period of operation. For example, for the coal combines it is understood as the quantity of coal extracted by a machine for the period of work, for the lorries – the quantity of kilometer-tons, etc. Most often the worked product is measured by the operating time of a machine. In this case the scale factor of a cycle of iteration will be equal to

$$\mu_T = \frac{T_e}{j_{\max}} , \tag{32}$$

where: T_e = - time of operation of machine (hour).

The computer time of one cycle of iteration, if a personal computer with clock frequency 100 *MHz* is used, makes about 70 *sec*. Hence, the long period of operation of toothed gearing which in industrial conditions may take several years, with the help of the program of prognostication is reproduced during 1,5…2,5 hours, which allows to estimate the degree of influence of various factors on the qualitative characteristics of the projected toothed gearing in a brief space of time and also to pick up a transmission with optimum parameters.

The factor of variation of specific intensity of wear process in the absence of the experimental data can be accepted as equal to: $\alpha_T = 1$.

The factor, which is taking into account the influence of the frequency of input of teeth in the gearing before the accumulation of experimental data, can be accepted as equal to

$$\alpha_u = 0,5u , \tag{33}$$

With the availability of the experimental data on teeth wear of transmissions similar to the modeled, it is possible to approximately determine the average total linear wear for one cycle of iteration with the formula

$$I_{zav} = \frac{V_p + V_w}{2mb_{\min} j_{\max}} \tag{34}$$

where: V_p and V_w = Volumetric wear of teeth of the pinion and wheel for the period of operation, (mm^3). Then the minimum value of the specific intensity of teeth wear will be equal to

$$i_z = \frac{I_{zav}}{f_z s_c \upsilon_{sz}} \tag{35}$$

where: υ_{sz} = Average value of sliding velocity.

The specific intensity of wear process, taking into consideration the influence of eccentricity of pitch circle, can be determined by the following formulas

$$\left.\begin{aligned} i_e &= 0,2\frac{I_{zav}}{f_e s_c \upsilon_{eav}} \\ \upsilon_{eav} &= 0.05\omega_p\left(F_{ep} - \frac{F_{ew}}{u}\right) \end{aligned}\right\}. \tag{36}$$

For lack of experimental data about teeth wear it is possible to tentatively accept $I_{zav} = 0,01...0,02$ mm.

The analysis of experimental data has allowed us to establish that the volumetric wear of the pinion tooth after 15 % of the term of operation almost three times exceeds the error of manufacturing. In the further work of the transmission this difference increases in a still greater degree. Taking into account this circumstance and, besides, a significant disorder in the experimental data on the teeth wear, at the given stage of research the influence of teeth manufacturing errors can be neglected.

7. Numerical experiment on prognostication of worn tooth shapes

The scheme of performance of the numerical experiment consists in the following.

- Selection of the researched toothed gearing from the category of spur gear transmissions of power machines.
- Measurements of teeth wear of toothed gears after the completion of their operation.
- Specification of parameters of adjustment of the model of prognostication of form of tooth profiles on the basis of the information on teeth wear.
- Introduction of parameters of the researched transmission to the model of prognostication of the form of tooth profiles and obtaining the theoretically expected tooth shapes.

Selection of researched gear transmissions. The investigation of engagement of the worn teeth is carried out on the basis of a pair of closed cylindrical spur gears of the drives of mining machines (coal-combines) often used in the mines of the Donbass region in Ukraine. Twelve coal-combines, with the time of operation in the limits from 3000 hours to 12000 hours and the worked product from 150000 up to 600000 tons of extracted coal were observed. Twenty two toothed wheels from 11 coal combines have been investigated. The main parameters of the investigated spur gear transmission are given in Table 2.

Table 2
Main parameters of investigated transmission

Module	m=8 mm
Pressure angle of basic rack (form of the rack corresponds to rack 20^o Full-Depth Involute standardized by AGMA)	α_b =20^o
Number of teeth	z_p=16, z_w=51
Shift profile coefficients	x'_p=0.8, x'_w=1.175
Center distance and face width	a=281.57 mm, $b_{\min}$=75 mm
Normal backlash	j_n=0.22 mm
Tolerance of eccentricity of pitch circle of pinion and wheel	F_{ep}=0,15 mm, F_{ew}=0,20 mm
Average arithmetic deviation of a roughness of tooth shapes contacting materials	R_a=0,0063 mm
Limits of stretching strain of materials of the toothed wheels	σ_{ss}=1300 N/mm^2
Heat treatment	carbonization, hardening to 58-62 RHN
Modulus of elasticity and the Poisson's ratio for the materials of pinion and wheel	E=210000 N/mm^2, ν = 0,3
Nominal angular velocity of pinion	ω_p =53 rd/s
Input torque and output torque	T_p=1605 Nm, T_w=4911 Nm
Kinematic viscosity of oil I-45 at $t=80^0$ C	ν_{80^0} =16 sSt (mm^2/s)

While determining the inertial and elastic characteristics of the dynamic model, its elements (shaft, teeth wheels, shaft coupling, etc.) have been found analytically. The damping parameter was determined experimentally (in view of a hysteresis), when testing the combine, which is similar to the examined one. On the basis of these data the parameters of the dynamic model with the number of degrees of freedom equal to 4 have been obtained (Table 3).

Table 3
Parameters of the dynamic model of a tooth drive of the investigated coal combine

Polar mass of inertia	
Reduced from the motor to a tooth wheel preceding the pinion	$J_{d,p-1}$ = 9,95 kgm^2
Reduced from the working mechanism to the tooth wheel nearest to the researched one	$J_{r,w+1}$= 5,84 kgm^2
Of pinion and wheel	J_p =0,21 kgm^2, J_w=0,88 kgm^2
Reduced rigidity of shafts	
From the motor to the pinion	$c_{d,p}$= 595500 Nm/rd
From the working mechanism to the wheel	$c_{r,w}$ =418700 Nm/rd
Coefficient of damping	$\phi = 0,24$

Measurement of teeth wear of toothed gears after the completion of its operation. The co-ordinates of real profiles are determined by the experimental method. The information about the real co-ordinates of the tooth outlines was obtained by the method of replicas. It consists in the comparison of the replica of tooth-space with its theoretical profile with a 20-fold increase[6]. The number of measure points was accepted so that the distance between them should not be bigger than a half Hertzian width. Generally for this purpose it is enough to have 30-40 points of the measurement. The precision of the measurement method equals to $\Delta I = 0.05$ mm which allows to measure the normal wear in the diapason of 0,1…0,7 mm to sufficient accuracy. Such a large wear is the consequence of dynamic character of loading and penetration of abrasion from the environment of a coal combine into the lubricating system of its drives. For each coal-combine the conventional time of operation, proportional to the quantity of coal extracted by the machine, was registered.

Specification of the parameters of adjustment of the model of prognostication of the form of tooth profiles on the basis of the information on teeth wear. In the researched coal-combine the asynchronous electric motor with the value of maximum torque equal to 1050 Nm for the average conditions of rigidity of electricity supply network was applied. At the same time the registered value of the steady moment of resistance reduced at the shaft of electric motor is equal to 600 Nm. As the maximum moment of the motor greatly exceeds the rating value of the moment of resistance, it is possible to count the mechanical characteristic of the motor as rigid enough and, when considering the engagement of a pair of teeth, the driving moment can be accepted as constant.

The analysis of frequency characteristics of the external moment of resistance of coal-combines has shown that the greatest frequency of its change is equal to 25 Hz. This frequency is much less than the tooth frequency of the researched transmission which is equal to 135 Hz. On this basis, when considering the engagement of a pair of teeth, the moment of resistance has been accepted as constant. The additional data needed to ensure the functioning of the model of teeth wear in the examined toothed gearing are presented in Table 4.

Table 4.
Additional data to ensure functioning of the model of tooth wearing of the examined gear

Maximum value of a worked product	Q_{max}=600000 ton
Maximum real time of the operation	T_e=12000 hour
Number of cycles of the iterations	j_{max}=80
Factor of the properties of lubricant oil (f. 4)	$C_p = 1{,}7$
Specific intensity of teeth wear process	i_z=0,001 mm^3/W
Specific intensity of wear process at sliding of teeth	i_e=0,200 mm^3/W
Scale factor of a cycle of iteration in the function:	
of time of the operation	μ_T =150 hour/cycle
of the worked product	μ_Q = 7500 ton/cycle

The factor of influence of tooth frequency on teeth wear α_u and the factor of variation of specific intensity of wear process α_T have been determined by the results of the analysis of information on teeth wear process in operation. On the obtained data the values of the volumetric wear of the teeth of pinion and wheel as well as their total value (31) have been calculated. The error of calculation of volumetric wear was determined by the error of measurement of wear $\Delta I = 0.05$ mm - $\Delta V = 2mb_{\min}\Delta I = 2 \cdot 8 \cdot 75 \cdot 0{,}5 = 60$ mm^3. The obtained values of the volumetric wear have been approximated by a power polynomial of the third degree, which has allowed to find a corresponding functional dependence of the volumetric wear of a tooth on the

[6] This method improved by the application of computer technology is described in the work [41].

time of operation T of the transmission: $V_p = V_p(T)$, $V_w = V_w(T)$ and $V_{p,w} = V_{p,w}(T)$, m^3.

The factor α_u has been accepted, as a first approximation, as equal to the relation of values tooth volumetric wear of the pinion and wheel

$$\alpha_u = \frac{V_p(T)}{V_w(T)}. \tag{36}$$

The factor α_T has been accepted as simply proportional to the speed of change of total volumetric wear

$$\alpha_T = \frac{\dot{V}_{p,w}(T)}{\dot{V}_{p,w}(0)}, \tag{37}$$

where: $\dot{V}_{p,w}(T) = \dfrac{dV_{p,w}}{dT}$ = the first derivative of function $V_{p,w}(T)$, m^3/s; $\dot{V}_{p,w}(0)$ = Value of function $\dot{V}_{p,m}(T)$ if $T = 0$.

The results of calculations by the formulas (36) and (37) are shown in Fig. 5. From Fig. 5 it is evident that the volumetric wear of the pinion tooth in α_u times greater than the wear of the wheel tooth, but is always smaller than the gear ratio of the transmission. Besides, on the basis of the obtained results it is possible to draw the conclusion that the intensity of wear process at the initial stage of operation decreases, which proves the availability of natural grind effect, then the process is stabilized and after a certain period it increases again, which can be explained by the amplification of influence of the distortions of form of tooth profiles as a result of their non-uniform wear.

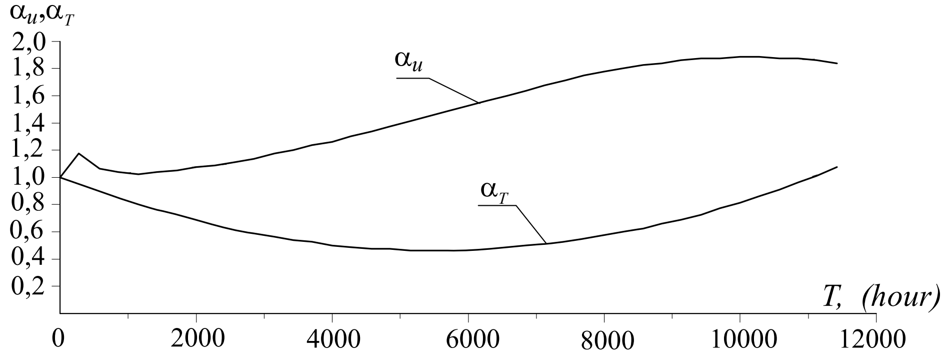


Fig. 5. Determination of the factor of variation of specific intensity of wear process α_T and the factor of influence of teeth frequency on teeth wear α_u.

Thus the parameters of kinematic and elastic dynamic models, the model of wearing of teeth, and the model of synthesis of outlines of the worn teeth have been found. These parameters have been introduced into the model of prognostication of the form of tooth profiles and obtaining the theoretically expected tooth shapes of the transmission. As a result of operation of a complex of computer programs the parameters of investigated transmission depending on the time of operation, which were sought for, have been obtained.

8. Comparison of results of the numerical experiment on prognostication of the tooth shapes with the experimental data

The principles of estimation of a degree of conformity of results of prognostication of teeth wear with the experimental data. The correspondence of the results of numerical experiment and the experimental data was evaluated on the basis of comparison of volumetric wear and linear wear of the teeth. The estimation of the degree of this correspondence was based on the probability of disposition of the experimental data in the limits of the width of confidence interval that depends on the values of the tooth wear by the results of prognostication. Besides, the correspondence of the form of the field of correlation and that of the confidence zone was taken into consideration. Thus the following assumptions have been accepted:

- the difference of the values of the theoretical wear and experimental wear belong to one and the same general group (after the elimination of blunders);
- the distribution of these differences as a first approximation abides by the normal law;
- the width of the confidence interval is proportional to the measuring error of wear - ΔV, ΔI.

For example, when estimating the volumetric wear, the probability of the fact that the interval of the half-width $\alpha\Delta V$ will cover the experimental data is determined by the formula

$$P = 2\Phi - 1 = 2 \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt - 1, \tag{38}$$

where: Φ = Normal distribution function; $t = \frac{\alpha \Delta V}{\sigma_V}$ = Quintile of normal distribution (the relative width of interval); $\Delta V = V_{ti} - V_{ei}$ = Measuring error of the volumetric wear, mm^3; V_{ti} = Volumetric wear by the results of numerical experiment; V_{ei} = Volumetric wear by the results of physical experiment: α = Coefficient of the width of confidence interval, determined by the requirement that at least 80 % of experimental points will be inside the interval; $\sigma_V = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \Delta V^2}$ = Average quadratic deviation, mm^3; N = Number of experimental points of measurement of tooth wear.

For the calculation of the required confidence probability it is more convenient to make use of the following formula obtained through the approximation of the tabulated values of the normal distribution function by a polynomial of the third degree

$$P = 0{,}03242t^3 - 0{,}3089t^2 + 0{,}9736t - 0{,}01728. \tag{39}$$

The formula allows to evaluate P with an error less than 0,7 % (with reference to the tabulated values) and is valid at $0{,}2 \le t \le 3{,}5$.

The meaning $P > 0{,}65 \ldots 0{,}70$ may be regarded as the level of confidence probability testifying to the adequacy of simulated and actual processes.

Comparison of the theoretical wear of tooth shapes with the experimental data.

To estimate the degree of correspondence of the results obtained by the numerical experiment with the results of physical experiments, two basic parameters were used: the volumetric wear of a tooth and the form of a tooth profile.

The values of the **volumetric tooth wear** obtained as a result of the numerical experiment of simulation of wear process and the relevant experimental data are presented in Fig. 6.

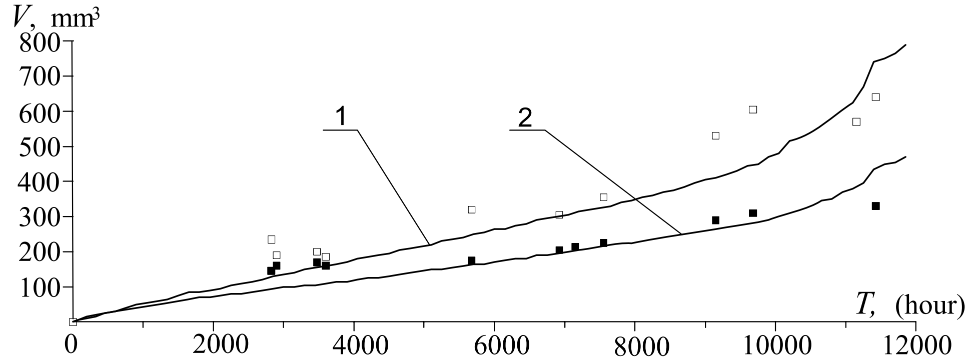


Fig. 6. Volumetric wear of the tooth transmission of coal combine
theoretical data: 1 – tooth of pinion, 2 – tooth of wheel
experimental data: □ – tooth of pinion, ■ – tooth of wheel

Because of the considerable disorder of experimental data the coefficient of width of the confidence interval has been taken as equal to α=1,4. At such width of interval the confidence probability of disposition of 80 % experimental data in this interval is equal to: for the pinion - P_p = 0,72, for the wheel - P_w = 0,91. Hence, with the probability more than 72 % it is possible to state that the values of volumetric tooth wear, obtained as a result of operation of the coal combine's gear transmission, are close enough to the value, obtained as a result of simulation of the wear process of wearing of teeth for analogous operation conditions.

The comparative estimation of the **form of the tooth profile** is produced by averaging of the experimental data of teeth wear as follows.

The investigated combines have been divided into three groups according to the coefficient of degree of operation

$$\beta_T = \frac{T_i}{T_{\max}}, \tag{40}$$

where: T_i = Operation time; T_e = Maximal real time of the operation (Table 5).

In each group 3…5 combines were included so that the volumetric wear of the wheel tooth in the group could differ from the average wear of the group not exceeding the measurement error of wear. The field of correlation, the approximated curve, the confidence interval (α=1,25) and the prognosticated form of the worn tooth profile are presented in Fig. 7 for β_{T_1} =0,42; β_{T_2} =0,50 and β_{T_3} =0,86.

Due to the great volume of tested points the confidence intervals were determined with respect to the curve approximating the experimentally determined values of the teeth wear.

The values of the confidence probability and the averaged volumetric wear values for each of the groups of coal-combines are presented in Table 5.

Table 5.
Values of confidence probability and averaged volumetric teeth wear

Groups coal combine	β_T	Volumetric wear of a tooth, mm^3				Confidence probability	
		Wheel		Pinion			
		Experiments	Model	Experiments	Model	Pinion	Wheel
1	0.42	195	203	168	167	0.94	0.97
2	0.50	322	325	204	202	0.82	0.88
3	0.86	572	581	309	285	0.72	0.80

The comparison of theoretical and experimental data on tooth wearing shows that the prognosticated form of the worn tooth profile (the unbroken line in Fig. 7) is inside the confidence interval practically for all variants, except a small field for the pinion with β_{T_3} = 0,86. The prognosticated curve will basically match the form of the curve approximating experimental data, especially at the small and average values of the teeth wear.

Hence, the results of numerical experiment show that the character of wear pattern of the model will match to the character of teeth wear in operation fairly well. For all groups of the inspected combines with the probability of 0.72…0.97 the confidence interval covers the prognosticated curve of tooth wear, and, taking into account that the experimental data of tooth wear are obtained under production conditions, it is possible to speak about a sufficiently high level of reflection of reality by the offered model of prognosticating of gear drive's tooth wear in some heavily loaded machines.

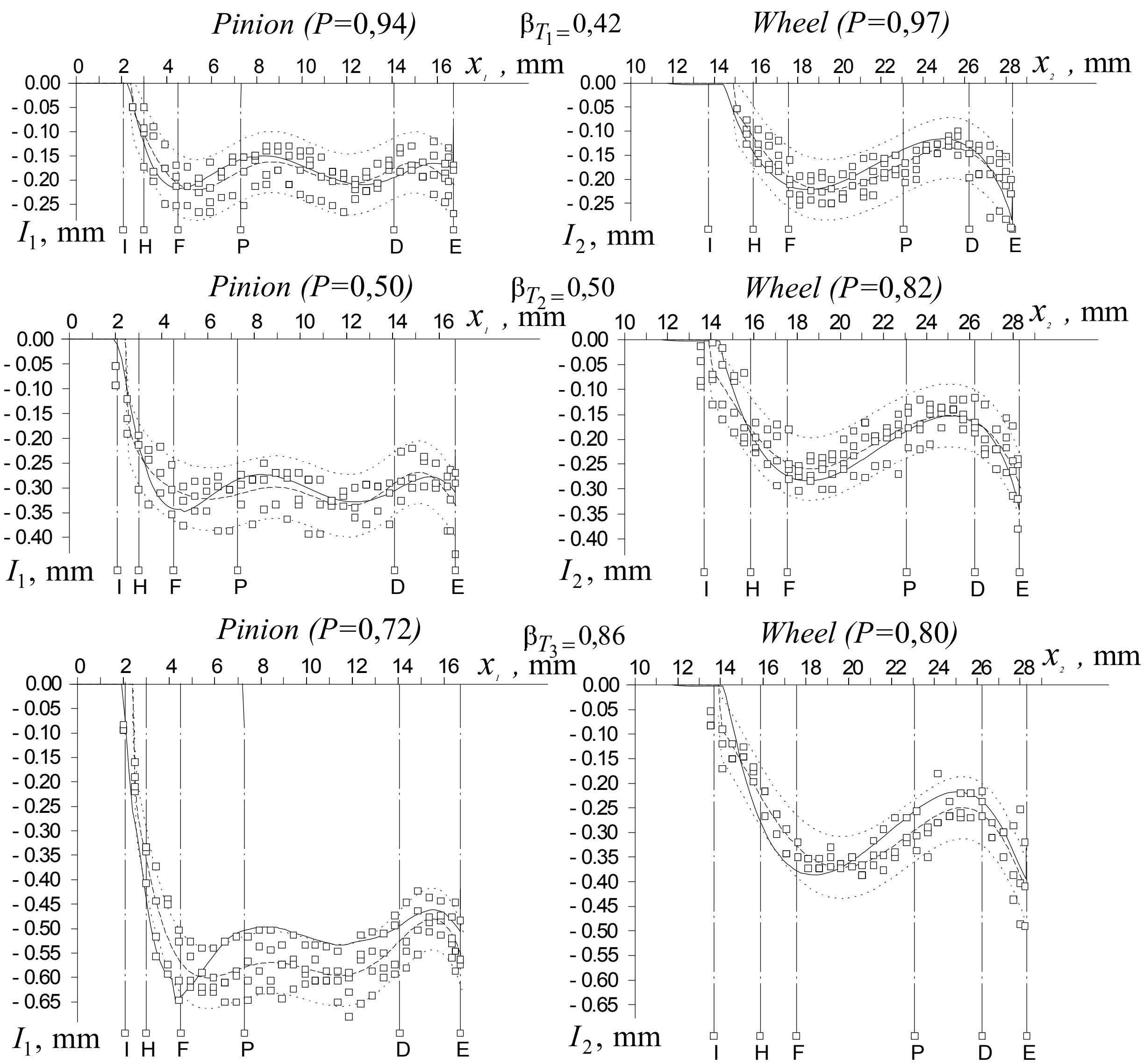


Fig. 7. Comparison of theoretical and experimental data of tooth wearing

——— - results of prognostication of the teeth outlines;

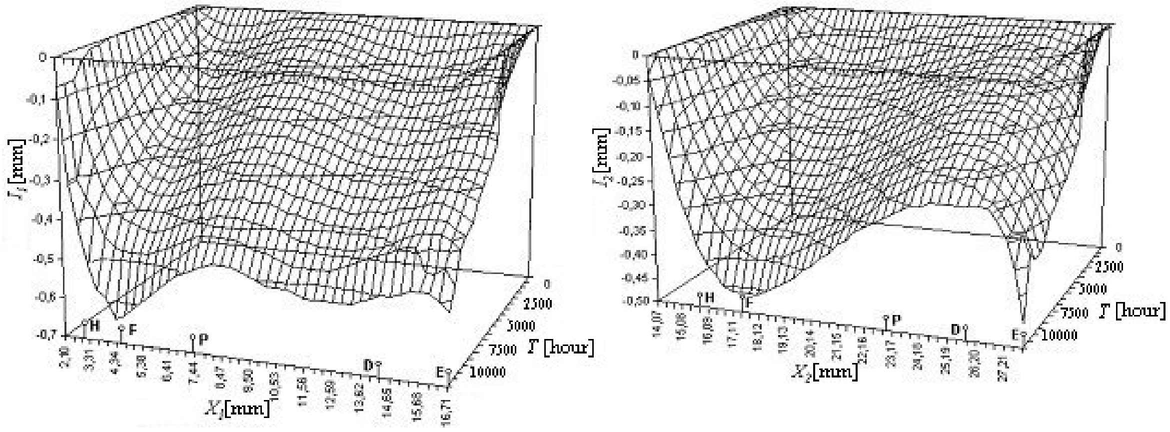
------- - line of approximation of the experimental data;

··········· - limits of confidence intervals for the probability P

9. Change of gear transmission's qualitative characteristics as a result of teeth wear

For the purpose of the visual demonstration of the character of change of qualitative characteristics of engagement of the transmission with worn teeth relevant spatial diagrams have been drawn. The character of change of tooth wear of the pinion and wheel of the investigated transmission is presented in Fig. 8. This figure, as a visual proof, clearly shows that as a result of tooth wear the profile is distorted and also that these distortions are non-uniform along the height of tooth.

The non-uniformity of tooth wear causes the variability of instantaneous value of the gear ratio (Fig. 9). In the outcome, even at the constant value of the angular velocity of the pinion, the wheel rotates with an angular acceleration. Accordingly, there is an additional inertial reaction estimated with the help of the dynamic factor K_d (Fig. 10).



a) *b)*

Fig. 8. Character of change of tooth wear of the pinion (a) and of the wheel (b) in the function of the time of operation

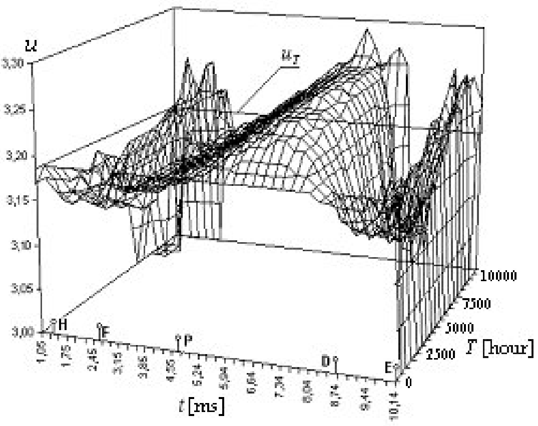


Fig. 9. Character of change of the gear ratio (u_T - theoretical value)

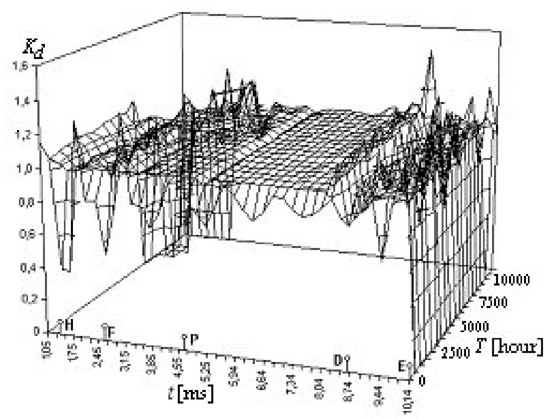
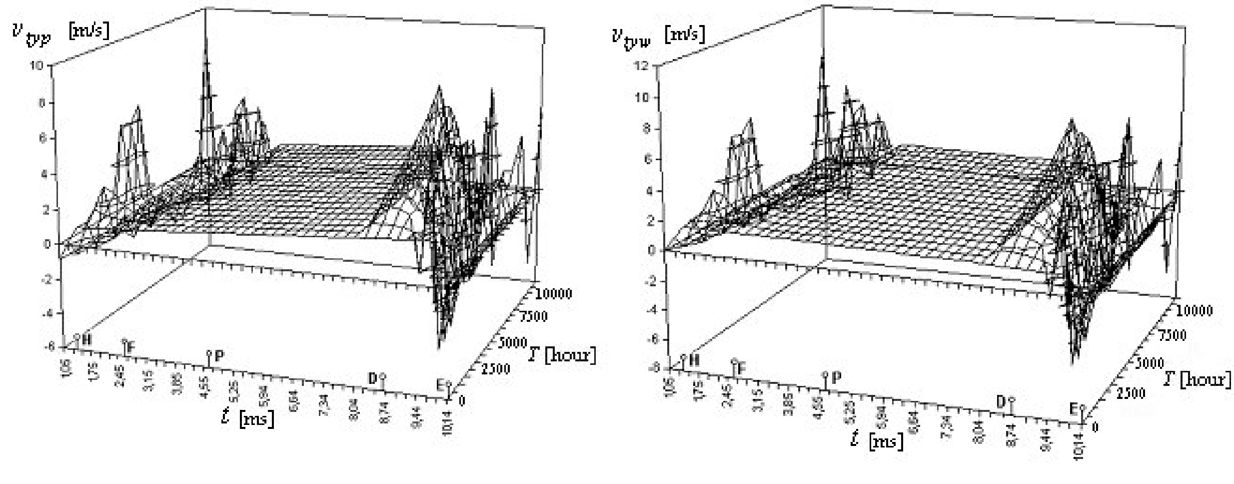


Fig. 10. Character of change of the dynamic factor K_d

As a result of the tooth wear, the rolling velocities of moving of the contact zone of the pinion and wheel tooth shapes change considerably, especially in the beginning and the ending of engagement (Fig. 11).



a) *b)*

Fig. 11. Character of change of a rolling velocity of the teeth of pinion (*a*) and of wheel (*b*)

The sliding velocity between the teeth practically does not vary except for the zones of two-pair engagement of mating teeth, and that only at a considerable wear of the teeth (Fig. 12).

In a fuller measure the distortions of the teeth profiles cause the change of normal contact stress since in determining these stresses both the change of radiuses curvature of profiles' and the change of normal loading (in view of the dynamic factor) are taken into account. The character of change of normal contact stress is presented in Fig. 13.

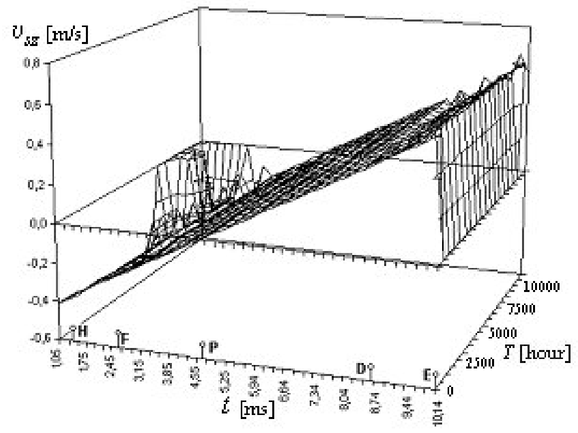


Fig. 12. Character of change of sliding velocity between the teeth

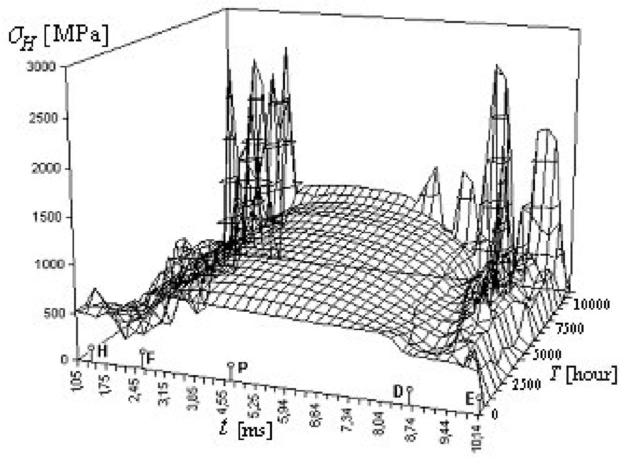


Fig. 13. Character of change of normal contact stress (of Hertz)

From the suggested results of the numerical experiment on modeling of process of operation of tooth gears it follows that the distortions of profiles caused by the tooth wear render a considerable influence on the qualitative characteristics of the gearing. The application of the designed computer technology of prognostication of face and flank teeth' condition allows us to estimate the quality of the designed transmission and to trace the influence of different constructive and technological factors on the parameters of transmission after a particular operation period with the purpose of choosing the most optimal transmission characteristics.

10. Conclusions

Based on the analysis of character of tooth wear of the gear power transmission the following conclusions have been done:

- the distortions of tooth profile are non-uniform along its height and this non-uniformity considerably (more than an order) exceeds tolerances on an error of the profile manufacture,
- in the zone of theoretical pitch point, despite the sliding velocity being equal to zero, a considerable wear takes place,
- in the process of the gearing operation the intensity of tooth wear process is not constant,
- the ratio of the volumetric wear of the pinion tooth to the volumetric wear of the wheel tooth is not equal to the gear ratio.

It is proved that the estimation of tooth gearing longevity according to their involute profile parameters is approximate and it doesn't reflect the fact that due to tooth wear the profile's form is becoming different from the involute one. As a result, the loaded and kinematic parameters of contact are changed and then they change, in their turn, the tempo of wear and the form of tooth profile. The theoretical principles have been developed and the model of worn teeth engagement has been created. This model allows to determine:

- the position of the point of contact in a fixed system and mobile system of coordinates;
- the rolling velocities and sliding velocity;
- the instant gear ratio;
- the normal load and normal contact stress.

All these parameters were determined having taken into consideration the real curvature of profiles, the flexible and contact pliability, the distribution of loading between pairs of teeth.

It is shown that the distortions of tooth profiles, caused by their deformation and wear, influence the line of action, which is becoming curvilinear. Thus, the time of engagement is reduced. The distortion of the line of action causes the change of the instant gear ratio, preserving its average value equal to the theoretical one.

The elastic dynamic model with 4 degrees of freedom is based theoretically and its computer realization is created. This model allows to estimate the dynamic load of contact, taking into consideration the inertial and dissipative characteristics of gearing and of drive, the instant gear ratio, the friction forces and the normal lateral clearance.

It is proved that the deformation and errors of manufacture of teeth profile influence the dynamic loading only at the initial stage of transmission work. The distortion of teeth profile caused by its wear influence the dynamic loading in engagement much greater due to increasing of operation time.

The model of working teeth surface wear has been worked out. This model takes into consideration the conditions of machine operation, corresponding tribological theories, the eccentricity of pitch circumference of gear wheels, the account for edge contact of teeth and the instant temperature in the contact.

The model of prognostication of tooth gearing characteristics has been determined. It takes into account the continuous influence of the profile form on the contact parameters and the influence of the parameters of contact on the profile form. The model is done in the form of a package of computer programs and includes the kinematic model of teeth engagement with any form of profiles, the elastic dynamic model, the model of teeth wear and the model of synthesis of wear tooth profile.

The computer technology created on the base of the suggested theory may be recommended as an instrument for designing of the tooth gearing of working machines drives with optimal parameters.

Acknowledgements

The author thanks the Engineering Library of the University of Auckland, Engineering Faculty, New Zealand for their bibliographic aid.

References

[1] Wu S., Cheng H. S., “A Friction Model of Partial-EHL Contacts and its Application to Power Loss in Spur Gears”, Tribology Transactions, Vol. 34, No. 3, pp.398-407, 1991.

[2] Трение, износ и смазка (трибология и триботехника) / А.В. Чичинадзе, Э.М. Берлинер, Э.Д. Браун и др. Под ред. А.В. Чичинадзе. – М.: Машиностроение, 2003. – 576 с.

[3] Misharin, Y. A., “Influence of The Friction Condition on The Magnitude of The Friction Coefficient in The Case of Rollers with Sliding”, Proc. Int. Conf. On Gearing, 1958, Inst. Mech. Eng., London, pp. 159-164, 1958.

[4] Drozdov, Y. N. and Gavrikov, Y. A., “Friction and Scoring Under The Conditions of Simultaneous Rolling and Sliding of Bodies”, Wear, pp. 291-302, 1967.

[5] Дроздов Ю.Н., Павлов В.Г., Пучков В.Н. Трение и износ в экстремальных условиях: Справочник. Москва, «Машиностроение», 1986. 224 с.

[6] Wu Shifeng, Cheng, H.S. Sliding wear calculation in spur gears (1993) *Journal of Tribology*, 115 (3), pp. 493-500.

[7] H. Xu, A. Kahraman, D.R. Houser “A Model to Predict Friction Losses of Hypoid Gears”, Ohio State University, Department of Mechanical Engineering 500 Montgomery St., Suite 350 Alexandria Virginia, 22314, AGMA October, 2005, 14 p.

[8] Ahmet Kahraman, Donald R. Houser, Hai Xu Development of a generalized mechanical efficiency prediction methodology for gear pairs, The Ohio State University, Department of Mechanical Engineering 650 Ackerman Road, Columbus, OH 43202, October 15, 2005, 163 p.
[9] Bochet B. Nouvelles Recherches Expérimentales sur le frottement et Glissement. «Annuels, Mines Carbures». 1861, No. 37, vol. 19, pp. 27-120.
[10] Franke J. Über die Abhängigkeit der gleitenden Reibung von der Geschwindigkeit. „Civiling". 1882.
[11] Крагельский И.В., Добычин М.Н., Комбалов В.С. Основы расчетов на трение и износ. Москва, «Машиностроение», 1977. – 526 стр.
[12] Wang C. C., On Analytical Evaluation of Gear Dynamic Factors Based on Rigid Body Dynamics, Transactions of the ASME, J. of Mechanism, Transmissions and Automation in Design, Vol.107, June 1985, 301-311.
[13] Wojnarowski J., Oniszczenko W. Wpływ zmiennego przełożenia na dynamiczne nadwyżki przekładni zębatej napędu kombajnu węglowego // Problemy Maszyn Roboczych, Z.7, Warszawa (Polska), 1996. - S.29-48.
[14] Kasuba R. and Evans J. W. "An extended model for determining dynamic loads in spur gearing", ASME Journal of Mechanical Design, vol. 103, No. 2, April 1981, pp. 398-409.
[15] Wilk A. "Wpływ parametrów technologicznych i konstrukcyjnych na dynamikę przekładni o zębach prostych". Zeszyty naukowe Politechniki Śląskiej, «Mechanika» Nr 679, Gliwice, 1981, 108 s. PL.
[16] Oniszczenko Walentyn. Modelowanie procesu zużycia zębów w przekładniach maszyn roboczych // XI Konferencja Naukowa „Problemy rozwoju maszyn roboczych", Komitet Budowy Maszyn Polskiej Akademii Nauk. Wydawnictwo PolSl, Gliwice-Zakopane (Polska), Z.2, 1998. - S. 293-301.
[17] Kuang J.H. and Lin A. D., 1999, "The effect of tooth wear on the vibration spectrum of a spur gear pair", Proceedings of DETC99 ASME Design Engineering Technical Conference, paper DETC99/VIB-8102.
[18] Валентин Онищенко Прогнозирование долговечности тяжелонагруженных зубчатых передач на основе моделирования износа зубьев. – Gliwice (PL): Politechnika Śląska Z. 131 (1410) Mechanika, 1999. – 199 c.
[19] Bajpai P., Kahraman A., Anderson N.E. A surface wear prediction model for parallel-axis gear pairs (2003) *Proceedings of the ASME Design Engineering Technical Conference*, 4 B, pp. 817-826.
[20] Bajpai P., Kahraman A., Anderson N.E. A surface wear prediction methodology for parallel-axis gear pairs (2004) *Journal of Tribology*, 126 (3), pp. 597-605.
[21] Kahraman, A., Bajpai, P., Anderson, N.E. Influence of tooth profile deviations on helical gear wear (2005) *Journal of Mechanical Design, Transactions of the ASME*, 127 (4), pp. 656-663.
[22] Yuksel C, and Kahraman, A., ''Dynamic Tooth Loads of Planetary Gear Sets Having Tooth Profile Wear,'' Mech. Mach. Theory, Elsevier Science Ltd, Oxford, UK. Volume 39 (2004), pp 695-715.
[23] Flodin, A., and Andersson, S., 1997, ''Simulation of Mild Wear in Spur Gears,'' Wear, 207, pp. 123-128.
[24] Archard J. F., 1953, ''Contact of Rubbing Flat Surfaces,'' J. Appl. Phys., 24, pp. 981–988.
[25] Литвин Ф.Л. Теория зубчатых зацеплений. - Москва: Наука, 1968. - 584 с.
[26] Jozef Wojnarowski and Valentin Onishchenko Tooth wear effects on spur gear dynamics // Mechanism and Machine Theory, Elsevier Science Ltd, Oxford, UK. Volume 38, Issue 2, February 2003, Pages 161-178.
[27] Jozef Wojnarowski and Valentin Onishchenko Influence of the Wear Teeth on Parameters of the Engagement of Gear Power Transmissions // Proceeding of the 11th

World Congress in Mechanism and Machine Science, vol. 2, p. 830-834, 2004, Tianjin, China. China Machine Press.
[28] Онищенко В.П. Оценка жесткости контакта при анализе динамики зацепления изношенных зубьев // Вибрации в технике и технологиях, Всеукраїнський науково технічний журнал. – 2001. - №2(18) Вінниця: Вінницький державний аграрний університет. – С. 58-65.
[29] Коднир Д.С. Контактная гидродинамика деталей машин: Учебное пособие. Куйбышевский авиационный институт. Куйбышев, 1979. – 116с.
[30] Решетов Д.Н. Детали машин: Учебник для студентов. – 4-е изд. – Москва: «Машиностроение», 1989. – 496 с.
[31] Чудаков Е.А. Новый метод расчета шестерни. Ленинград, Изд-во АН СССР, 1934.
[32] Кетов Х.Ф., Колчин Н.И. Теория механизмов и машин. Машгиз, 1937.
[33] Крагельский И.В. Трение и износ. М., изд-во «Машиностроение», 1968, с. 189-200.
[34] Fleischer G. Energetische Methode der Bestimmung des Verschleiβes. Schmierungstechnik, Band 4, 1973, s.9.
[35] Глаголев Н.И. Работа сил трения и износ перекатываемых тел. Труды III Всесоюзной конференции по трению и износу в машинах, т.2. М., Изд-во АН СССР, 1960, с. 34-54.
[36] Иосилевич Г.Б. Детали машин. М., «Машиностроение», 1988, с. 56.
[37] Молдавский Л.А., Онищенко В.П., Симонов А.Л.: О расчете на износ зубьев зубчатых колес угольных комбайнов. “Детали машин” Республиканский межведомственный научно-технический сборник, №18, 1974. “Техніка”, Киев, с.59.
[38] Calculation of Load Capacity of Spur and Helical Gears, Calculation of Surface Durability (Pitting). ISO/DIS 6336/I, 1998.
[39] Wellauer E. J., Seireg A. Bending strength of gear teeth by cantilever - plate theory. «Transactions of the ASME»,1959, NrA-50.
[40] Крагельский И.В., Михин Н.М. Узлы трения машин. Справочник. – Москва: «Машиностроение», 1984. – 280 с.
[41] Онищенко В.П., Голдобин В.А. Применение компьютерных технологий для определения износа зубьев зубчатых колес // Вісник Національного Технічного університету “ХПІ”. Збірник наукових праць, тематичний випуск “Проблеми механічного привода”, Харків: НТУ “ХПІ”. – 2004, №30. – С. 176-184.