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V. V. Romanuke (cand. of tech. sc.)
Khmelnytskyy National University
romanukevadimv@mail.ru

CONTROLLING PROGRAMMATICALLY THE REGULAR PROJECTOR OPTIMAL STRATEGY IN THE FOUR-SUPPORTS-CONSTRUCTION WITH PARTIALLY UNDETERMINED COMPRESSIVE FORCES

A program Matlab-tool has been designed for determining the projector optimal strategy in the four-supports-construction, meaning the most appropriate four cross-sections squares, which are compressively loaded with partially undetermined forces. This optimal strategy had been called regular as it is determined under the special condition, and examples of getting its regular components are being laid in screenshots.

Keywords: building construction, support construction, cross-sections, compressive forces, partial indeterminancy, continuous antagonistic game, projector optimal strategy, regular components, program Matlab-tool.

A problem introduction

A problem of rationally sharing available resources comes up within any common activity. Such a problem is pretty natural for structural mechanics, and especially in its building support constructions. There always is a dilemma, where it is necessary to ensure reliability, safety, and security with expending additionally metallic alloys, nonorganic composites, wood, timbers and some, standing against saving the building resources and finances, as well as minimizing the total weight and massiveness of the support construction. Certainly, there must not be any compromises, because this is a matter of entire reliability [1]. So, decisions with probabilities or math expectances are inadmissible here. In building the N-support constructions there is a problem of evaluating the load or compressive forces [2, 3], acting on it or its supports (pillars, bars or other identical geometrical form supports). Often the load or compressive forces are determined only partially [4, 5], that is, after having unit-normed the acting load, the i-th support is compressed with force x_i by

$$x_i \in [a_i; b_i] \subset (0; 1) \subset [0; 1] \quad \forall\, i = \overline{1, N} \tag{1}$$

and

$$\mu_{\mathbb{R}}\left([a_i; b_i]\right) > 0 \quad \forall\, i = \overline{1, N}. \tag{2}$$

Obviously, that the total load on the N-support construction is unit, what lets stating

$$\sum_{i=1}^{N} x_i = 1. \tag{3}$$

So, having nonzero $[a_i; b_i]$-undetermined compression on the i-th support $\forall\, i = \overline{1, N}$, the main problem is to distribute the unit-normed total cross-section square among the N supports into their cross-section squares $\{y_i\}_{i=1}^{N}$, that the maximum of the statements

$$\{T_i(x_i, y_i)\}_{i=1}^{N} = \left\{\alpha \frac{x_i}{y_i^2}\right\}_{i=1}^{N} \tag{4}$$

would be minimal [2, 3, 6], where $\alpha > 0$ is a mechanical factor. Here, obviously,

$$y_i \in [a_i; b_i] \subset (0; 1) \subset [0; 1] \quad \forall\, i = \overline{1, N} \tag{5}$$

and it is unknown the relationship between x_i and y_i^2, though, if it were at $x_i = y_i^2 \;\; \forall\, i = \overline{1, N}$, then the problem would have been disappeared.

Analysis of investigations on optimal projection models

As it has been said, optimal projection models in building construction must give solutions with guaranteed result, ensuring safety fixedly. To solve the described problem there is a learned antagonistic model in the form of the continuous antagonistic game [6]. And for the further being spoken classic four-supports-construction this game kernel

$$T(\mathbf{X}, \mathbf{Y}) = T(x_1, x_2, x_3; y_1, y_2, y_3) = \max\left\{\{T_i(x_i, y_i)\}_{i=1}^{4}\right\} =$$
$$= \max\{T_1(x_1, y_1), T_2(x_2, y_2), T_3(x_3, y_3), T_4(x_4, y_4)\} =$$
$$= \max\left\{\alpha \frac{x_1}{y_1^2}, \alpha \frac{x_2}{y_2^2}, \alpha \frac{x_3}{y_3^2}, \alpha \frac{1 - x_1 - x_2 - x_3}{(1 - y_1 - y_2 - y_3)^2}\right\} =$$
$$= \alpha \max\left\{\frac{x_1}{y_1^2}, \frac{x_2}{y_2^2}, \frac{x_3}{y_3^2}, \frac{1 - x_1 - x_2 - x_3}{(1 - y_1 - y_2 - y_3)^2}\right\} \tag{6}$$

is defined on the Cartesian product

$$\boldsymbol{X}\times\boldsymbol{Y}=\left\{[a_1;\,b_1]\times[a_2;\,b_2]\times[a_3;\,b_3]\right\}\times\left\{[a_1;\,b_1]\times[a_2;\,b_2]\times[a_3;\,b_3]\right\}=$$
$$=\prod_{p=1}^{2}[a_1;\,b_1]\times[a_2;\,b_2]\times[a_3;\,b_3]=$$
$$=\prod_{p=1}^{2}\left(\prod_{d=1}^{3}[a_d;\,b_d]\right)\subset\prod_{k=1}^{6}(0;\,1)\subset\prod_{k=1}^{6}[0;\,1]\subset\mathbb{R}^6 \qquad (7)$$

of two cubes

$$\boldsymbol{X}=[a_1;\,b_1]\times[a_2;\,b_2]\times[a_3;\,b_3]=$$
$$=\prod_{d=1}^{3}[a_d;\,b_d]\subset\prod_{d=1}^{3}(0;\,1)\subset\prod_{d=1}^{3}[0;\,1]\subset\mathbb{R}^3 \qquad (8)$$

and

$$\boldsymbol{Y}=[a_1;\,b_1]\times[a_2;\,b_2]\times[a_3;\,b_3]=$$
$$=\prod_{d=1}^{3}[a_d;\,b_d]\subset\prod_{d=1}^{3}(0;\,1)\subset\prod_{d=1}^{3}[0;\,1]\subset\mathbb{R}^3 \qquad (9)$$

as the subset of the unit six-dimensional hypercube, where (3) and the clearest

$$\sum_{i=1}^{N}y_i=1 \qquad (10)$$

just have been applied for reduction of the eight variables $\{x_i,\,y_i\}_{i=1}^{4}$ to six $\{x_d,\,y_d\}_{d=1}^{3}$. As it easy to see, the game with kernel (6) on the hypercube (7) is conditionally strictly convex,

$$\frac{\partial^2}{\partial y_d^2}\left(\frac{x_d}{y_d^2}\right)=\frac{\partial}{\partial y_d}\left(-\frac{2x_d}{y_d^3}\right)=$$
$$=\frac{6x_d}{y_d^4}>0\quad\forall\;x_d\in[a_d;\,b_d]\;\text{ and }\;\forall\;y_d\in[a_d;\,b_d]\;\text{ at }\;d=\overline{1,\,3}, \qquad (11)$$

$$\frac{\partial^2}{\partial y_d^2}\left(\frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2}\right)=\frac{\partial}{\partial y_d}\left(\frac{2(1-x_1-x_2-x_3)}{(1-y_1-y_2-y_3)^3}\right)=$$

$$=\frac{6(1-x_1-x_2-x_3)}{(1-y_1-y_2-y_3)^4}>0 \quad \forall\, x_d\in[a_d;b_d] \text{ and } \forall\, y_d\in[a_d;b_d] \text{ at } d=\overline{1,3}, \quad (12)$$

that is almost everywhere

$$\frac{\partial^2}{\partial y_d^2}T(x_1,x_2,x_3;y_1,y_2,y_3)>0 \quad \forall\, x_d\in[a_d;b_d]$$
$$\text{and } \forall\, y_d\in[a_d;b_d] \text{ at } d=\overline{1,3}, \quad (13)$$

and so the second player, having been personified by the projector, may have here the single optimal pure strategy

$$\mathbf{Y}_*=\begin{bmatrix} y_1^* & y_2^* & y_3^* \end{bmatrix}\in[a_1;b_1]\times[a_2;b_2]\times[a_3;b_3]=\prod_{d=1}^{3}[a_d;b_d]=\boldsymbol{Y}. \quad (14)$$

And there only should be found the components of (14) to control them programmatically. Then this optimal projection model, ensuring safety fixedly with minimaxed cross-section squares $\left\{y_d^*\right\}_{d=1}^{3}$ and $y_4^*=1-\sum_{d=1}^{3}y_d^*$, will be acceptable.

Paper object and assignment

The started paper object is to develop a module for controlling programmatically the projector optimal strategy (14) in the having been spoken four-supports-construction with $\left\{[a_d;b_d]\right\}_{d=1}^{3}$-undetermined compressive forces. To accomplish it there ought to be determined the strategy (14), and to be designed the corresponding program tool.

Regular components of the projector optimal strategy (14)

For the conditionally strictly convex continuous antagonistic game with kernel (6) on the hypercube (7) have

$$\max_{\mathbf{X}\in\boldsymbol{X}} T(\mathbf{X}, \mathbf{Y}) = \alpha \max_{\mathbf{X}\in\boldsymbol{X}} \left\{ \max\left\{ \frac{x_1}{y_1^2}, \frac{x_2}{y_2^2}, \frac{x_3}{y_3^2}, \frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2} \right\} \right\} =$$

$$= \alpha \max\left\{ \max_{x_1\in[a_1;\,b_1]}\left\{\frac{x_1}{y_1^2}\right\}, \max_{x_2\in[a_2;\,b_2]}\left\{\frac{x_2}{y_2^2}\right\}, \max_{x_3\in[a_3;\,b_3]}\left\{\frac{x_3}{y_3^2}\right\}, \max_{\mathbf{X}\in\boldsymbol{X}}\left\{\frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2}\right\} \right\} =$$

$$= \alpha \max\left\{ \frac{b_1}{y_1^2}, \frac{b_2}{y_2^2}, \frac{b_3}{y_3^2}, \frac{1-a_1-a_2-a_3}{(1-y_1-y_2-y_3)^2} \right\}. \quad (15)$$

Due to [6, p. 140 — 143] the optimal game value v_* may be reached at each component under maximum sign in (15), that is by

$$[y_1 \quad y_2 \quad y_3] = [y_1^* \quad y_2^* \quad y_3^*]. \quad (16)$$

May this chance be called regularity with regular components [7] in $\mathbb{R}^3$-point (16). So having

$$v_* = \alpha\frac{b_1}{(y_1^*)^2} = \alpha\frac{b_2}{(y_2^*)^2} = \alpha\frac{b_3}{(y_3^*)^2} = \alpha\frac{1-a_1-a_2-a_3}{(1-y_1^*-y_2^*-y_3^*)^2}, \quad (17)$$

there are components

$$y_d^* = \sqrt{\frac{\alpha b_d}{v_*}} \quad \forall\ d = \overline{1, 3} \quad (18)$$

and the fourth square

$$y_4^* = 1 - y_1^* - y_2^* - y_3^* = \sqrt{\frac{\alpha(1-a_1-a_2-a_3)}{v_*}}. \quad (19)$$

Summing (18) and (19) properly on their side parts, get

$$1 = \sqrt{\frac{\alpha b_1}{v_*}} + \sqrt{\frac{\alpha b_2}{v_*}} + \sqrt{\frac{\alpha b_3}{v_*}} + \sqrt{\frac{\alpha(1-a_1-a_2-a_3)}{v_*}} =$$

$$= \sqrt{\alpha}\,\frac{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}}{\sqrt{v_*}} =$$

$$= \sqrt{\alpha} \frac{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}}{\sqrt{v_*}} . \quad (20)$$

The statement (20) gives the optimal game value in (17)

$$v_* = \alpha \left(\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3} \right)^2 =$$

$$= \alpha \left(\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d} \right)^2 \quad (21)$$

to substitute it into (18) and get finally the regular components of the projector optimal strategy (14):

$$y_i^* = \frac{\sqrt{b_i}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} =$$

$$= \frac{\sqrt{b_i}}{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}} , \; i = \overline{1, 3} . \quad (22)$$

At that the obvious condition of regularity is

$$\frac{\sqrt{b_i}}{\sum_{d=1}^{3} \sqrt{b_d} + \sqrt{1 - \sum_{d=1}^{3} a_d}} \in [a_i ; b_i] \quad \forall \; i = \overline{1, 3} \quad (23)$$

simultaneously, as at least one of the three compulsory memberships in (23) may be violated.

Tool for controlling the regular projector optimal strategy (14)

For developing the program tool will apply the programmable environment Matlab [8, 9]. May a Matlab-function, being further the tool for controlling the regular projector optimal strategy (14) with its components (22), be named “optregsqr4sc” (as a several words lettered abbreviation of the sentence “optimal regular squares in four-supports-construction”). The first assigning line and the header comment (needed when the help is called) in its code are on figure 1. The tool has a single input argument, what is the undetermined forces, included into 3×2-matrix with left and right ends of the segments $\left\{\left[a_d; b_d\right]\right\}_{d=1}^{3}$. The output argument is optimal squares set $\left\{y_i^*\right\}_{i=1}^{4}$. Typing on, there comes the check whether the input is a 3×2-matrix at all (figure 2). If the input is a 3×2-matrix, then its elements are being renamed (figure 3) for more convenient use.

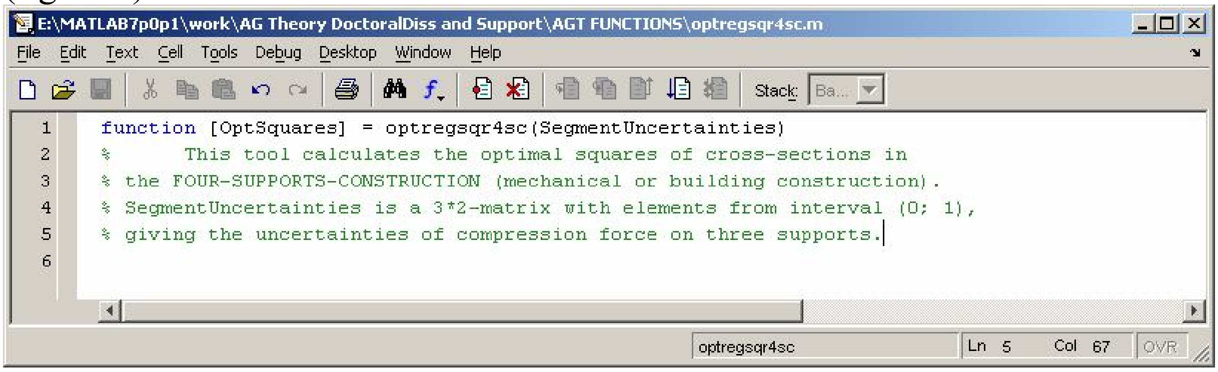

Figure 1 — Starting creation of Matlab-tool “optregsqr4sc” with its first assigning line and the header comment in lines 2 — 5

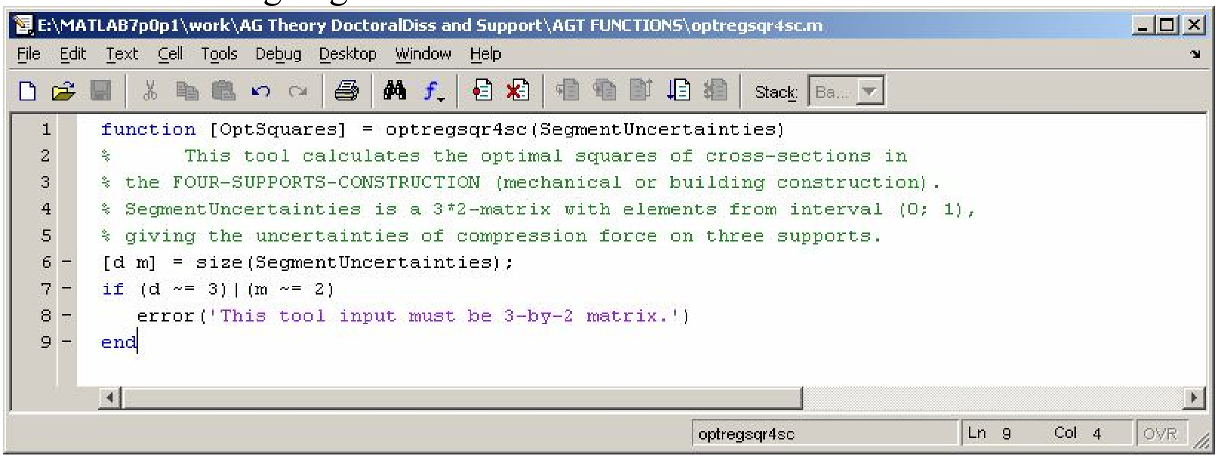

Figure 2 — Checking whether the input is a 3×2-matrix at all (lines 6 — 9)

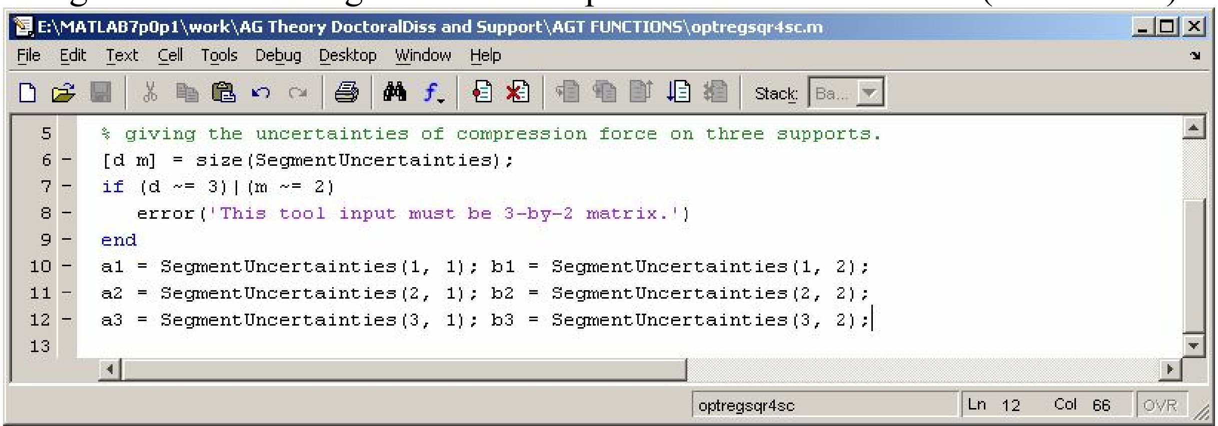

Figure 3 — Renaming the elements of the input 3×2-matrix into $\left\{a_d\right\}_{d=1}^{3}$

and $\{b_d\}_{d=1}^{3}$ (lines 10 — 12)

As those pretty preluding checks and renames are carried out, there goes turn to confirm that numbers $\{a_d\}_{d=1}^{3}$ and $\{b_d\}_{d=1}^{3}$ are relevant to (1) or (5) and (2). In other words, lines 13 — 19 verify whether (figure 4)

$$a_d > 0,\; a_d < b_d,\; b_d < 1 \quad \forall\, d = \overline{1, 3}. \tag{24}$$

Verification of the condition

$$\sum_{d=1}^{3} b_d < 1 \tag{25}$$

is in lines 20 — 24 (figure 5). The condition (24) is equivalent to (1) or (5) and (2) without $\forall\, i = \overline{1, N}$. The condition (25) completes the check of (1) or (5) and (2), making the fourth compressive force be nonzero.

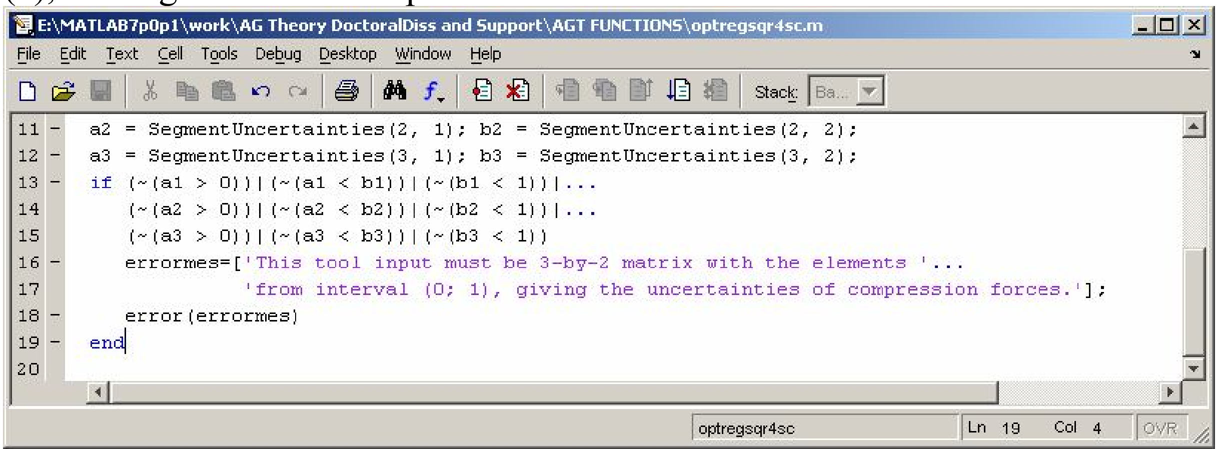

Figure 4 — Verifying whether (24) is true (lines 13 — 19)

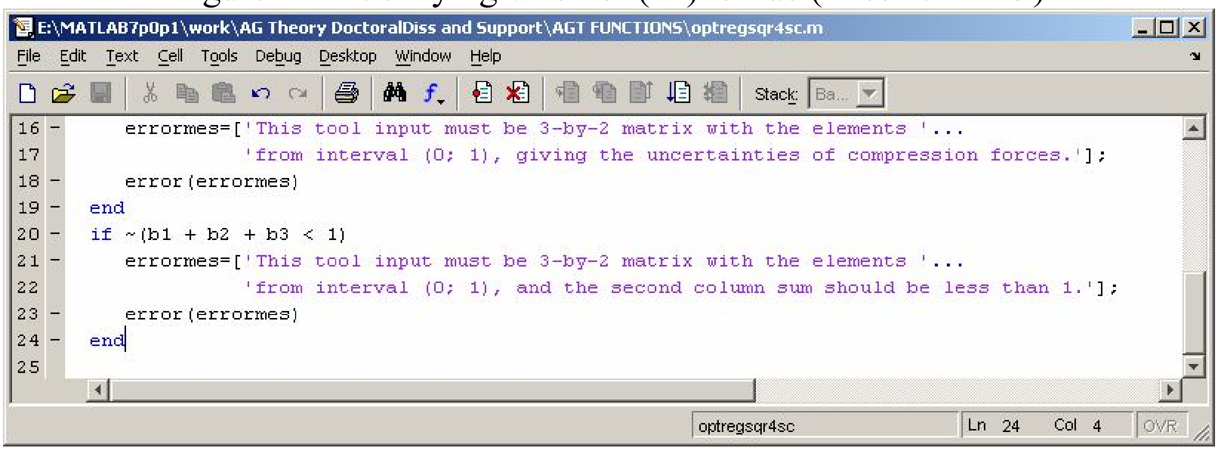

Figure 5 — Verifying whether (25) is true (lines 20 — 24)

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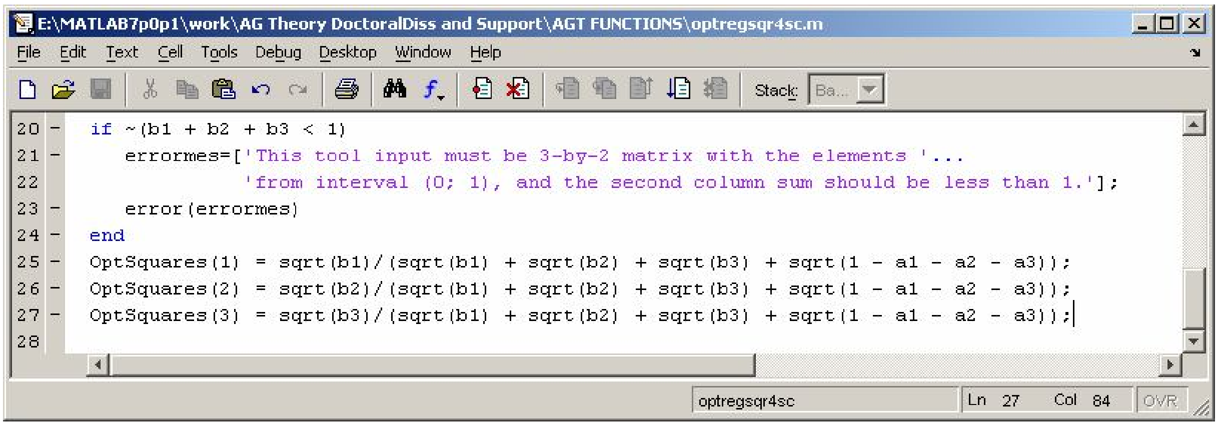
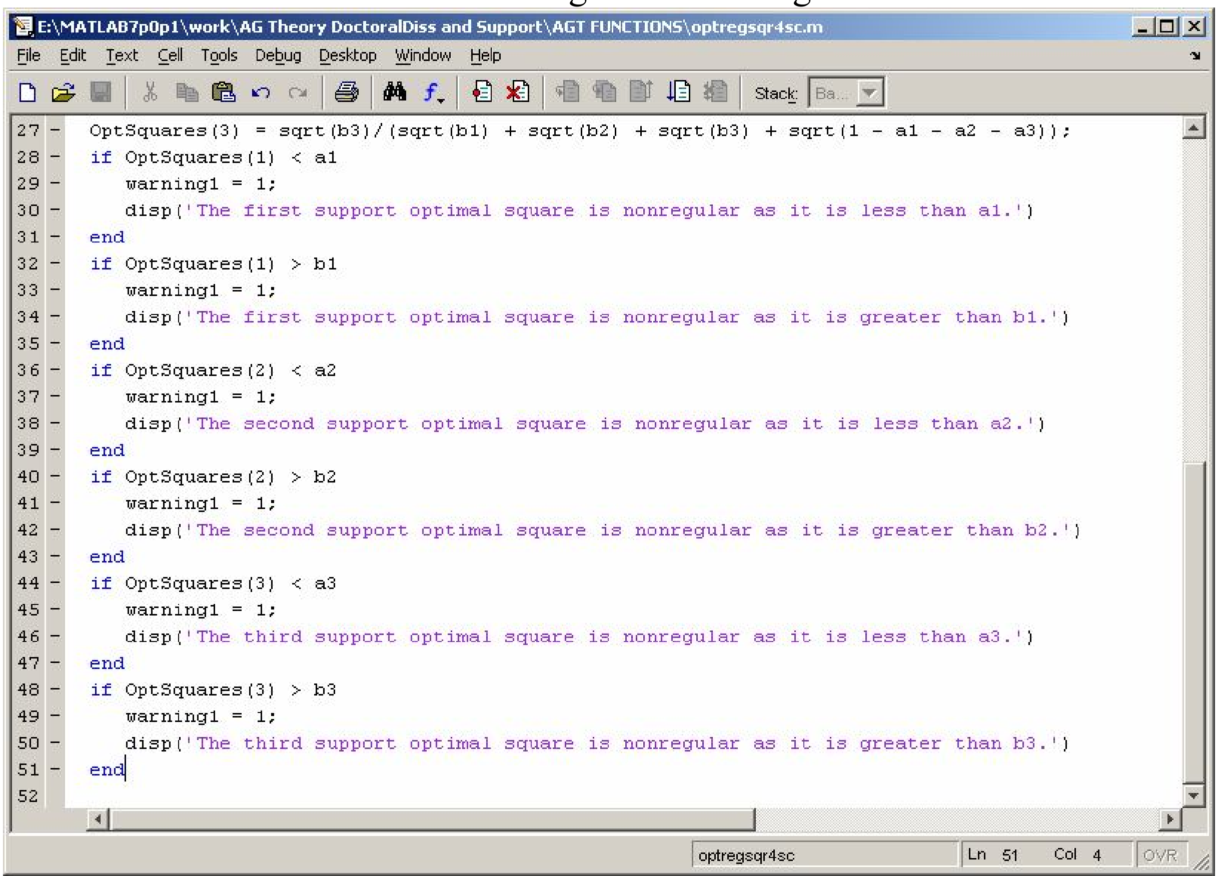
After that there are lines 25 — 27 with computing the components (22), and their regularity is going to be checked down deeper (figure 6). Actually, it is being accomplished in lines 28 — 51, shown on figure 7. If the computed strategy is regular, that is its components (22) satisfy (23), then the fourth optimal square is computed (lines 52 — 56 on figure 8). Otherwise the error message will appear when the tool runs. Before the general error message there is displayed a message on what exact of the components $\left\{y_d^*\right\}_{d=1}^{3}$ is less than the one of the three ends $\left\{a_d\right\}_{d=1}^{3}$ or is greater than the one of the three ends $\left\{b_d\right\}_{d=1}^{3}$.




```
20 - if ~(b1 + b2 + b3 < 1)
21 -    errormes=['This tool input must be 3-by-2 matrix with the elements '...
22              'from interval (0; 1), and the second column sum should be less than 1.'];
23 -    error(errormes)
24 - end
25 - OptSquares(1) = sqrt(b1)/(sqrt(b1) + sqrt(b2) + sqrt(b3) + sqrt(1 - a1 - a2 - a3));
26 - OptSquares(2) = sqrt(b2)/(sqrt(b1) + sqrt(b2) + sqrt(b3) + sqrt(1 - a1 - a2 - a3));
27 - OptSquares(3) = sqrt(b3)/(sqrt(b1) + sqrt(b2) + sqrt(b3) + sqrt(1 - a1 - a2 - a3));
28
```

Figure 6 — Computing the components (22) in lines 25 — 27 before checking them to be regular




```
27 - OptSquares(3) = sqrt(b3)/(sqrt(b1) + sqrt(b2) + sqrt(b3) + sqrt(1 - a1 - a2 - a3));
28 - if OptSquares(1) < a1
29 -    warning1 = 1;
30 -    disp('The first support optimal square is nonregular as it is less than a1.')
31 - end
32 - if OptSquares(1) > b1
33 -    warning1 = 1;
34 -    disp('The first support optimal square is nonregular as it is greater than b1.')
35 - end
36 - if OptSquares(2) < a2
37 -    warning1 = 1;
38 -    disp('The second support optimal square is nonregular as it is less than a2.')
39 - end
40 - if OptSquares(2) > b2
41 -    warning1 = 1;
42 -    disp('The second support optimal square is nonregular as it is greater than b2.')
43 - end
44 - if OptSquares(3) < a3
45 -    warning1 = 1;
46 -    disp('The third support optimal square is nonregular as it is less than a3.')
47 - end
48 - if OptSquares(3) > b3
49 -    warning1 = 1;
50 -    disp('The third support optimal square is nonregular as it is greater than b3.')
51 - end
52
```

Figure 7 — Checking regularity of the strategy (14) components (lines 28 — 51)

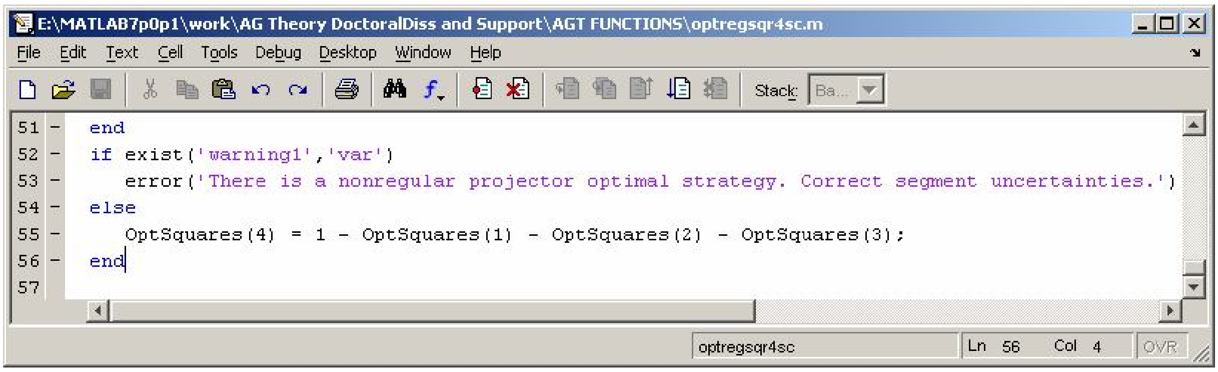


Figure 8 — The fourth optimal square is computed only in the case with all regular strategy (14) components (lines 52 — 56), and then there is no any message

Finally, the tool "optregsqr4sc" should visualize the optimal cross-sections of the four supports, that will help the projector in orienting on the construction. The lines 57 — 59 are for plotting such visualization in the form of a rectangular (figure 9), where the left lower node corresponds to the first optimal cross-section square, the left upper node corresponds to the second, the right upper node corresponds to the third, and the right lower node corresponds to the fourth optimal cross-section square.

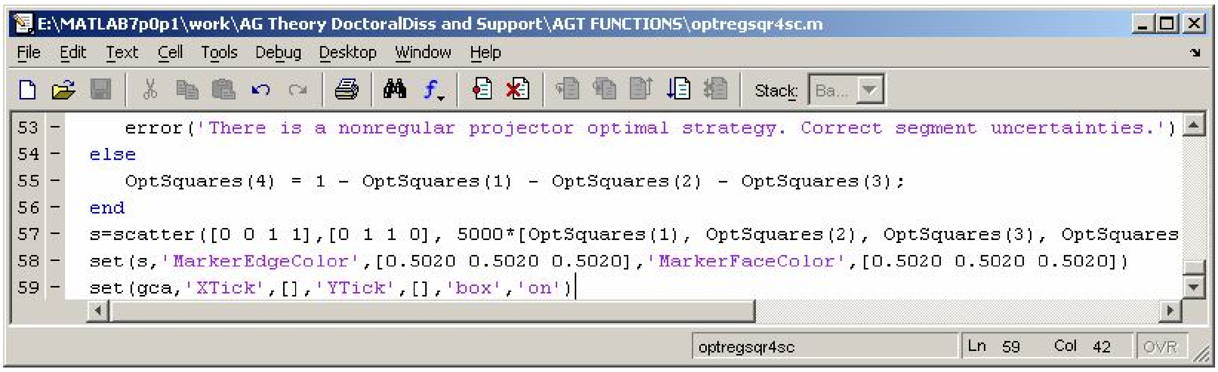


Figure 9 — Visualizing the optimal cross-sections of the four supports in the form of a rectangular (lines 57 — 59)

At now it remains to exemplify the developed Matlab-tool, what will provide users with the corresponding guidance.

Exemplification in Matlab Command Window

To exemplify the developed Matlab-tool assume, that cross-sections indeterminancies have been concluded within the following segments:

$$[a_1; b_1] = [0.1; 0.2],\ [a_2; b_2] = [0.15; 0.25],\ [a_3; b_3] = [0.4; 0.5]. \qquad (26)$$

After having typed the data (26) in the Matlab Command Window prompt line (figure 10), run the tool "optregsqr4sc" and get the optimal squares (figure 11). As it is clear there must be corrected the third indeterminancy segment to try getting the regular strategy (14). Letting $a_3 = 0.3$ gives the desired regularity (figure 12 and figure 13 with the screenshot of the four supports optimal cross-sections visualization).

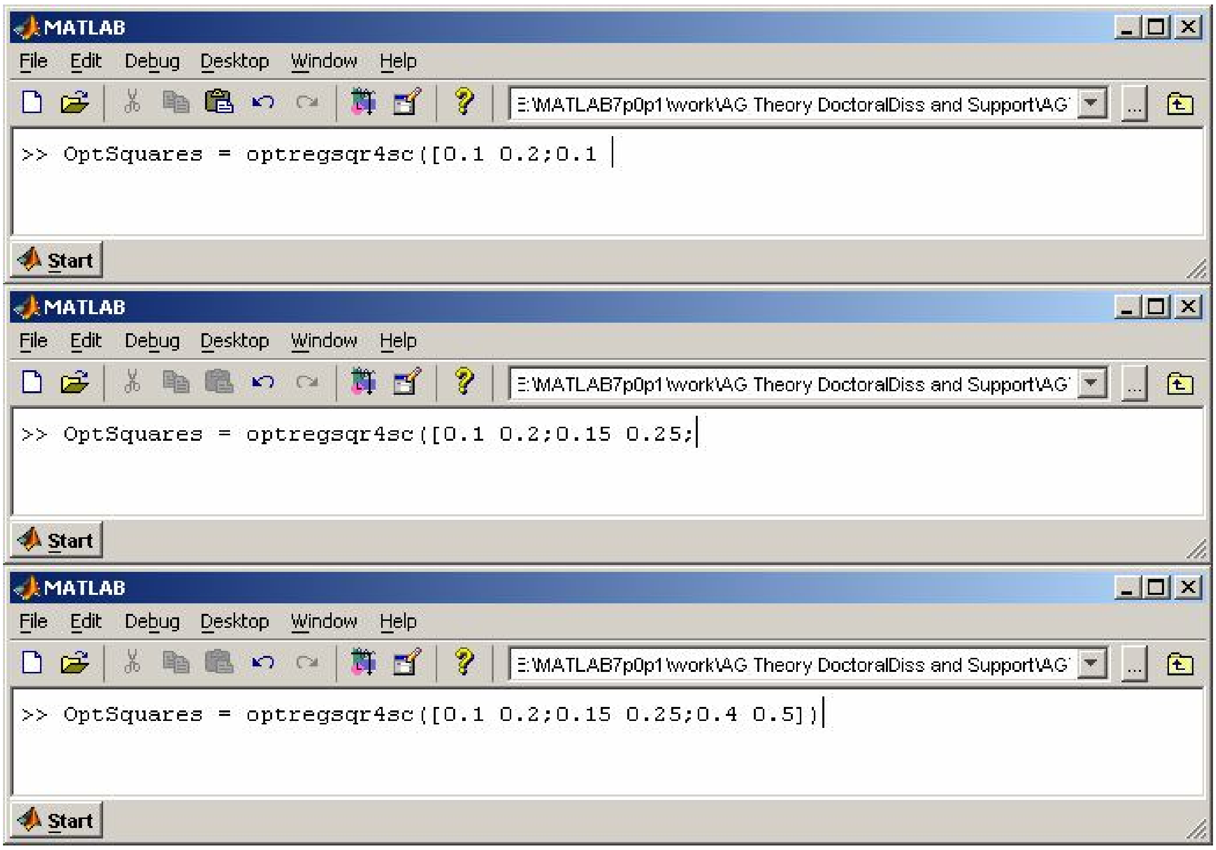


Figure 10 — Typing the data (26) in the Matlab Command Window prompt line

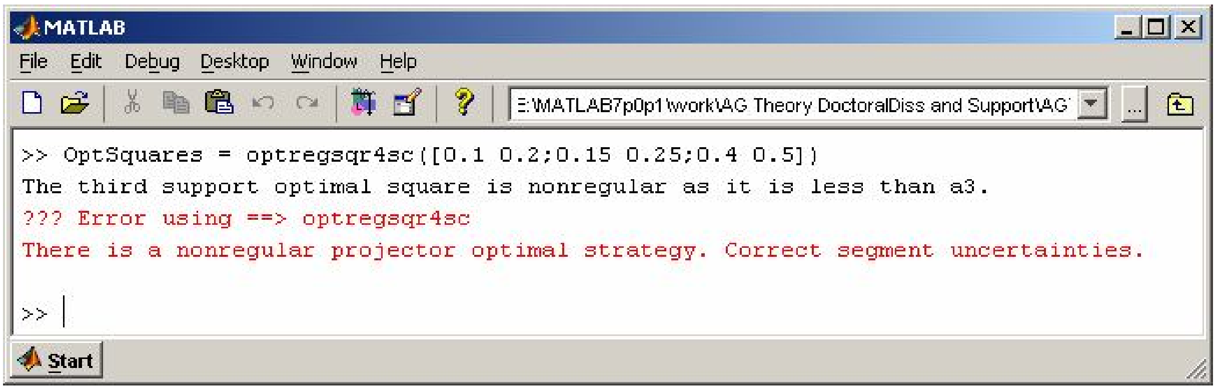


Figure 11 — The tool "optregsqr4sc" had been run: the strategy (14) has appeared to be nonregular as y_3^* by (22) is less than a_3
(there is an appropriate message on it and the general error message)

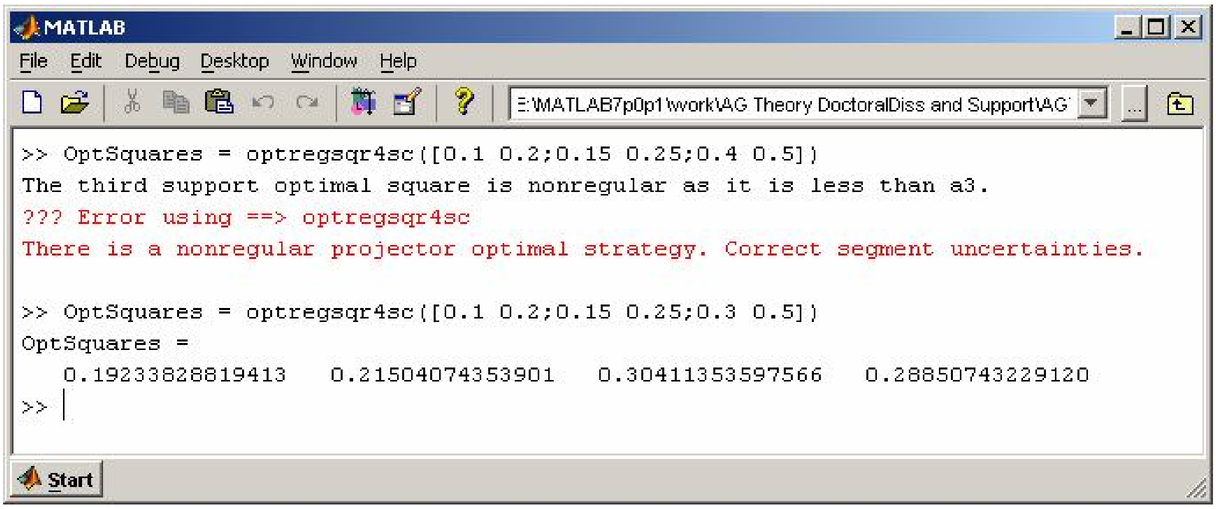


Figure 12 — Running the tool “optregsqr4sc” once again
with only changed value $a_3 = 0.3$

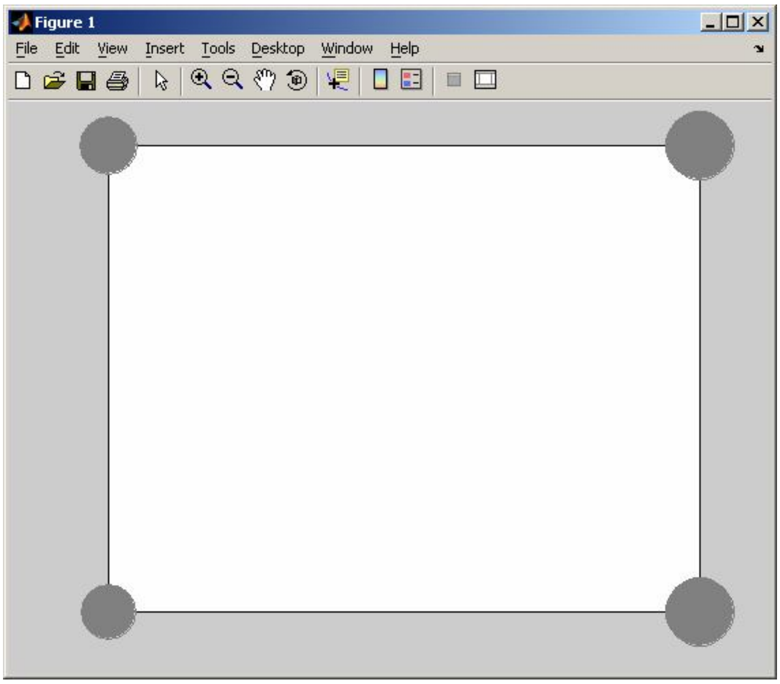


Figure 13 — Four supports optimal cross-sections visualization,
corresponding to (26) by the changed $a_3 = 0.4$ down to $a_3 = 0.3$

Conclusion and outlook for keeping the investigation on

The designed tool “optregsqr4sc” is an only slight reflection of those needs and wants, arising in building support constructions. A scheme like the one on figure 13 is for helping the projector to evaluate the cross-section visually, and, possibly, to re-arrange the construction in the case of nonregularity of one of the components $\left\{ y_d^* \right\}_{d=1}^{3}$. A nonregular component can be regularized [3, 5, 7], but there is a hazard to have the regularized component, being equal to one of the end of the indeterminancy segment, what may cause superfluous marginality in supporting. And the regular values $\left\{ y_i^* \right\}_{i=1}^{4}$ ensure the needed reliability. An outlook for keeping the investigation on is to explore the question of regularization as profound as it will be needed.

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Рецензент: канд. техн. наук, доц. Красник М.Ю.

В. В. Романюк
Хмельницький національний університет

Програмний контроль регулярної оптимальної стратегії проектувальника у чотирьохопорній конструкції з частково невизначеними стискаючими зусиллями. Розроблено програмний Matlab-засіб для визначення оптимальної стратегії проектувальника у чотирьохопорній конструкції, що означає найбільш підходящі площі чотирьох поперечних перерізів, котрі навантажуються стисненням з частково

невизначеними зусиллями. Ця оптимальна стратегія була названа регулярною, оскільки вона визначається за спеціальної умови, і приклади отримання її регулярних компонент наводяться у скріншотах.
Ключові слова: будівельна конструкція, опорна конструкція, поперечні перерізи, стискаючі зусилля, часткова невизначеність, неперервна антагоністична гра, оптимальна стратегія проектувальника, регулярні компоненти, програмний Matlab-засіб.

В. В. Романюк
Хмельницкий национальный университет

Программный контроль регулярной оптимальной стратегии проектировщика в четырёхопорной конструкции с частично неопределёнными сжимающими усилиями. Разработано программное Matlab-средство для определения оптимальной стратегии проектировщика в четырёхопорной конструкции, что означает наиболее подходящие площади четырёх поперечных сечений, которые нагружаются сжатием с частично неопределёнными усилиями. Эта оптимальная стратегия была названа регулярной, поскольку она определяется по специальному условию, и примеры получения её регулярных компонент приводятся в скриншотах.
Ключевые слова: строительная конструкция, опорная конструкция, поперечные сечения, сжимающие усилия, частичная неопределённость, непрерывная антагонистическая игра, оптимальная стратегия проектировщика, регулярные компоненты, программное Matlab-средство.