THE SET OF FORMAL TOOLS TO SYNTHESIS OF THE TORSIONALLY VIBRATING MECHATRONIC SYSTEMS

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In this paper the modeling by different category graphs and analysis of vibrating clamped – free mechatronic system by the approximate method called Galerkin's method has been presented. The frequency - modal analysis and assignment of amplitude -frequency characteristics of the mechatronic system is considered. The aim was to nominate the relevance or irrelevance between the characteristics obtained by exact - only for shaft - and approximate method. Such formulation especially concerns the relevance the relevance of the natural frequencies-poles of characteristics both of mechanical subsystem and the discrete – continuous clamped – free vibrating mechatronic system. This approach is a fact, that approximate solutions fulfill all conditions for vibrating mechanical and/or mechatronic systems and can be an introduction to synthesis of these systems modeled by different category graphs. Using of the hypergraph methods of modeling and synthesis methods of torsionally vibrating bars to the synthesis of discrete-continuous mechatronic systems is originality of such formulation problems.

1. Dynamical Characteristic of Discrete-Continuous Torsionally Vibrating Mechatronic System

The set of equations of considered torsional vibrating mechatronic system is following (the meaning of symbols was passed in e.g. [8])

$$\begin{cases} \ddot{\varphi} - a^2\varphi_{xx} - bU\left[\delta(x-x_1) - \delta(x-x_2)\right] = cM\delta(x-l), \\ \dot{U} + \alpha_1 U + \alpha_2\dot{\varphi}(l_p,t) = 0. \end{cases} \tag{1}$$

After transformations [8-10] dynamic flexibility is obtain

$$Y_{xl}^{(n)} = \frac{c\delta(x-l)\left(\omega + \dfrac{\alpha_1}{e^{i\frac{\pi}{2}}}\right)}{\sin kx\left[a^2\left(\dfrac{\pi}{2l}\right)^2 - \omega^2\right]\left(\omega + \dfrac{\alpha_1}{e^{i\frac{\pi}{2}}}\right) - \dfrac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\delta(x)\alpha_2\omega\sin kl_p} \, . \tag{2}$$

Absolute value of dynamic flexibility for mechanical subsystem of mechatronic system is equal [7,8]

$$\left|Y_{ll}\right| = \left|\frac{1}{c_\varphi \lambda}\operatorname{tg}\lambda l\right| . \tag{3}$$

The transients of absolute value of flexibilities (2) - after further formal transformations and after putting of the numerical values of parameters and when $x=l$, that is $\left|Y_{ll}\right|$ - and (3) for three first vibration modes - - it was showed in Fig. 1.

2. Transformations of Characteristics of Torsionally Vibrating Subsystems of Mechatronic Systems

A characteristic - dynamical flexibility of mechanical subsystem of mechatronic system is given in form

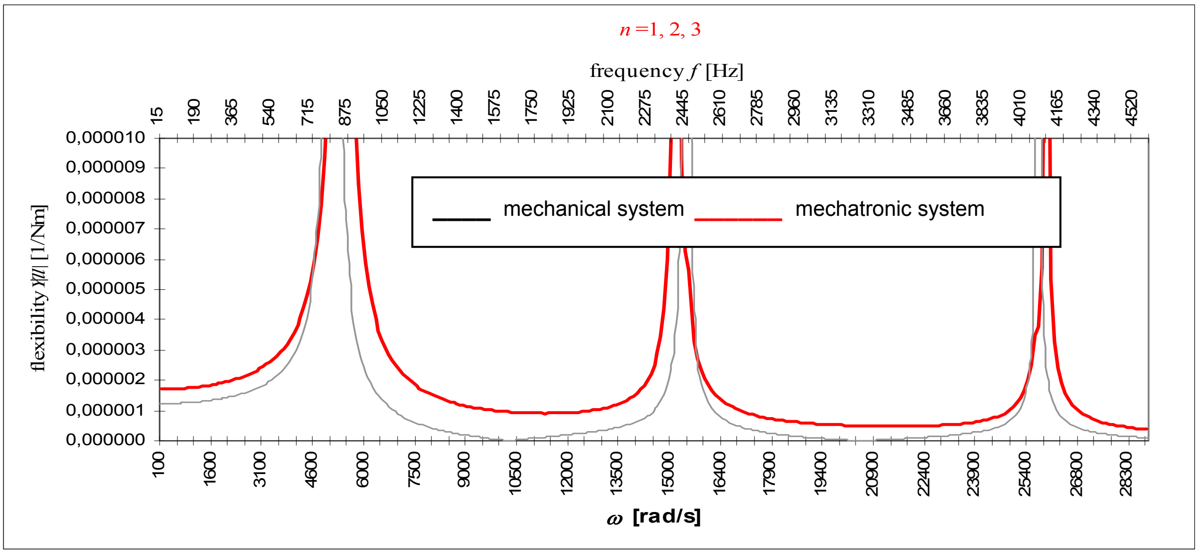

Fig. 1. Transient of the sum for n=1, 2, 3 vibration mode

$$Y(s)=\frac{\sum_{i=0}^{k} c_i th^j \Gamma s}{s\sum_{j=0}^{l} d_j th^j \Gamma s}. \tag{4}$$

After transformations $V(s)=sY(s)$ and Richards' transformation $r=\mathrm{th}\Gamma s$ [3-6] the mobility (4) has been obtained as

$$V(r)=\frac{\sum_{i=0}^{k} c_i r^i}{\sum_{j=0}^{l} d_j r^j}, \tag{5}$$

3. Modeling the Considered Subsystems of Mechatronic Systems by Means the Different Category Graphs

Weighted graphs and hypergraphs have been applied to modelling of the considered mechanical or/and mechatronic systems [3-6].

A following couple (using the symbols introduced in papers [1-3])

$$X=\left({}_1X,\ {}_2X\right) \tag{6}$$

is called a *graph*.

The *hypergraph* is called a couple

$${}^kX=\left({}_1X,\ {}_2^kX\right). \tag{7}$$

where: ${}_1X$ is the set as in (6), and ${}_2^kX=\left({}_2^kX^{(i)}/i\in\mathrm{N}\right)$, ($k$=2,3, ... ∈N) is a family of subsets of set ${}_1X$; the family ${}_2^kX$ is called a *hypergraph* over ${}_1X$ as well, and ${}_2^kX=\left\{{}_2^kX^{(1)},\ {}_2^kX^{(2)},\ \ldots,\ {}_2^kX^{(m)}\right\}$ is a set of edges [1,2], called *hyperedges* or *blocks.*

4. The Synthesis of the Mechanical and/or Mechatronic Systems Represented by Different Category Graphs

In this paper is showed how the method which were applied in order to synthesize the dynamical characteristic of the torsional vibrating mechanical system may be applied to synthesis the mechatronic system with cascade structure as well. This is the cascade method of distribution of characteristic represented by different category graphs [3-6].

5. Algorithm of Synthesis of Mechanical Subsystem by the Recurrent Cascade Method as Necessary Condition of Synthesis of Mechatronic System

The equation enabling the synthesis of the mobility function and, at the same time, its inversion is introduced. In order to obtain these function, the third category graph [3-6], as a model of torsional vibration mechnical and/or mechatronic system.

To carry out the synthesis of the mobility $V(p)$ the form of (5) or its inversions $U(p)$ (comp. with [3-6]) by the cascade method it necessary to:

1^0 Assume that

$$V(p)=V^{(1)}(p)\,. \tag{8}$$

2^0 Determine values of parameters $(GJ)^{(i)}$ and $(\rho J)^{(i)}$ from equations

$$\begin{cases}(GJ)^{(i)}=\dfrac{1}{\beta V^{(i)}(1)},\\[2mm] (\rho J)^{(i)}=\dfrac{(GJ)^{(i)}}{G}\rho,\end{cases} \tag{9}$$

assuming p=1 and i=1, $\beta=\sqrt{\dfrac{\rho}{G}}$.

3^0 Determine $V^{(2)}(p)$ of other part of system containing segments from i=2 to n, from Richards' theorem

$$V^{(i+1)}(p)=V^{(i)}(1)\frac{V^{(i)}(p)-pV^{(i)}(1)}{V^{(i)}(1)-pV^{(i)}(p)}\,. \tag{10}$$

4^0 Divide the numerator and denominator of mobility $V^{(2)}(p)$ by (p^2-1); this is the condition of the physical realization of calculated mobility $V^{(2)}(p)$.

5^0 Repeat step 2, assuming i=2.

6^0 Carry out step 3^0 in order to calculate $V^{(3)}(p)$.

7^0 Check step 4^0 by dividing the numerator and denominator of $V^{(3)}(p)$ by (p^2-1).

8^0 Repeat steps 2^0, 3^0, 4^0, ... successively to determine formulas $V^{(4)}(p)$, $V^{(5)}(p)$, ... , $V^{(n)}(p)$.

The algorithm described above is to be continued until type p or $\frac{1}{p}$ of mobility $V^{(n)}(p)$ is achieved - multiplied by real constant H - and it is not possible to carry out step 3^0 after step 2^0 in order to determine $(GJ)^{(n)}$ and $(\rho J)^{(n)}$. This is the end of the synthesizing process.

6. Last remarks

Applied method and received results can make up the introduction to the synthesis of torsionaly vibrating mechatronic systems with constant changeable cross-section. The problems will be presented in future works.

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