Tetralogic and Tetracodes: an effective Method for Information Coding

A. Anopriyenko
Donetsk State Technical University, Computer Science Department
Artyoma Str. 58, UA-340000 Donetsk, Ukraine
e-mail: anoprien@dstu.donetsk.ua

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ABSTRACT

A variant of expansion of classical binary logic and binary notation is presented, which allows to essentially increase their information capacity and efficiency for description of dynamic processes and structures. One of the most efficient application areas for the tetracodes is image encoding. As an example the results of experimental research of efficiency of tetracodes for binary image encoding are presented.

2D logical Space

The majority of known modifications of binary and multivalued logic are in essence one-dimensional: the logic values 0 ("false", "no") and 1 ("true", "yes") are set on one axis, and all logic operations are carried out within the limits of this axis [2, 3, 4]. But the variety of human knowledge is such, that the surrounding world can not be always unequivocally described by such a simplified scheme. Mainly because, there are at least two situations, which play an important part in the system of human knowledge: "Don't know" and "Know so much, that cannot answer definitely" [1]. An orthogonal arrangement of "False" and "True" axes permits to include the indicated situations in the formal logic system (Fig. 1). In such two-dimensional logic plane various systems of computer logic can be constructed.

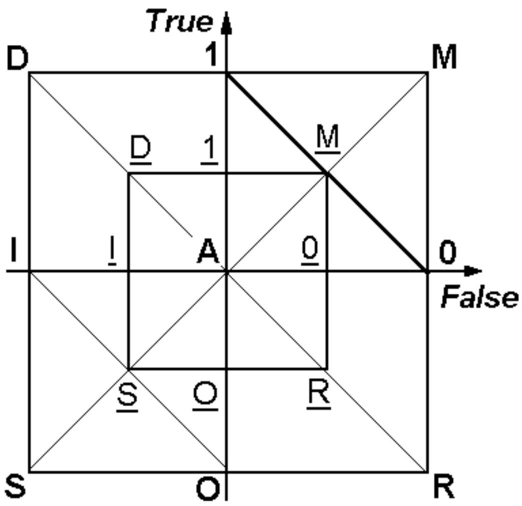

Figure 1. 2D logical space

The point A of absolute uncertainty is considered as the origin of coordinates.
The points 1 and 0 are traditional logic significance's "true" and "false".
M can be interpreted as "multivalue" or "true and false".
<u>M</u> can be interpreted as "equiprobability of true and false".
S - "symmetry" of M.
I and O - "symmetry" of 1 and 0 respectively.
D and R - two other variants of the "multivalue".
Various logical systems in this space can be specified as $L_N^K = \{x_1, x_2, ..., x_K\}$ with K= 1, 2, 3, 4, ... , where K is an order of the logic; $x_1, x_2, ..., x_K$ are various logical values from the 2D logical space; N is a number of logic of order K.

Tetralogic

The variants of logic systems with four conditions present the heaviest theoretical and practical interest. Such systems can be named as "tetralogic". The tetralogic, for example, can include besides two traditional logic values “true” and “false” two additional values:

$L_1^4 = \{1, 0, A, M\}$ (it was proposed in [1]), $L_2^4 = \{1, 0, \underline{M}, M\}$,

$L_3^4 = \{1, 0, S, M\}$, $L_4^4 = \{1, 0, A, \underline{M}\}$.

The introduction of the suggested logic conditions allows to essentially expand the opportunities of classical binary logic and to adapt it for the features of human thinking.

Tetracode

Binary logic being a basis of binary code, tetralogic can be considered as a basis for tetracode. Each position of a tetracode represents a two-bit combination, corresponding to one of the four values of tetralogic. Thus, total amount of bits for the representation of numbers increases 2 times, but there is an important qualitative change of the code: from a point (i.e. zero-dimensional) number it turns in one-dimensional number, using all space of a numerical axis.

Thus, a number can encode not only an individual value, but also some set of values, rhythmically distributed along a numerical axis. Moreover, depending on the variant of tetracode used, symmetric infringements and uncertainty can be allowed in the set of values. Various tetracodes can be defined as follows (similar tetralogics):

$C_1^4 = \{1, 0, A, M\}$, $C_2^4 = \{1, 0, \underline{M}, M\}$, $C_3^4 = \{1,0, S,M\}$, $C_4^4 = \{1,0,A,\underline{M}\}$.

A principles of tetracoding can be demonstrated on some examples. We use in the examples three positions of the tetracode for the representation of corresponding values on the numerical axis with symbols “+” (definite value), “-” (no value for this point), “~” (value for this point is not defined), “=” (one of the alternative values), “$\underline{a}$”, “$\underline{b}$”... (one of the alternative values a, b...) for eight points of the axis:

000 = [+ - - - - - - -]; 001= [- + - - - - - -], ...

Tetracode with M and S generates 2^m values, where m is an amount of positions with M and S:

00M = [+ + - - - - - -];	0M0 = [+ - + - - - - -];	1MM = [- - - - + + + +].
0S1 = [- + + - - - - -];	S01 = [- + - - - - + -];	SM0 = [+ + - - - - + +].
1$\underline{M}$1 = [- - - - - = - =];	$\underline{M}$0$\underline{M}$ = [= = - - = = - -];	M$\underline{MM}$ = [$\underline{a}$ $\underline{a}$ $\underline{a}$ $\underline{a}$ $\underline{b}$ $\underline{b}$ $\underline{b}$ $\underline{b}$].
1AA= [- - - - ~ ~ ~ ~];	01A$\rightarrow$ 01$\underline{M}$ = [- - = = - - - -].	

Symbols I and O can be interpreted as “all values except...”:

OOO = [- + + + + + + +]; 1I I = [- - - - + + + -]; O10 = [- - + - + + + +].

Symbols D and R can be interpreted as “multivalue with inversion” and “symmetry with inversion” respectively:

D01 = [- + - - + - + +]; R01 = [- + - - + + - +].

The degree of information capacity of tetracode can grow at transition from numerical axis to spaces of greater dimensions.

So, for example, two-dimensional tetracode with total amount of bits sufficient in the case of usual binary coding only for two points, can describe much more complex families of objects on plane (like regular and quasiregular lattices and other structures). Four-dimensional tetracode, for example, allows to obtain an exceedingly compact descriptions of rhythmical evolution in time of a complex multi-dimensional object. A possibility of representation and use of tetracodes in floating point format is considered as well.

Image encoding

One of the most efficient application areas for the tetracode is image encoding. As an example the results of experimental research of efficiency of tetracode for two types of test binary image (Fig. 3) encoding are presented.
Various algorithms of binary image encoding (Fig. 2) on the basis of tetracode were developed and investigated. The main results are as follows:

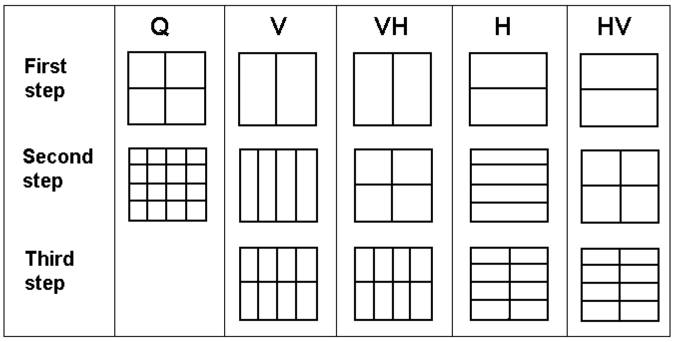

Figure 2. Different variants of space devision for image encoding on the basis of 2D tetracode.

- the compression of information is more efficient in the majority of cases than in case of using such a well known format as GIF (Fig. 4, 5);
- the codes obtained contain an important generalised information on some essential attributes of the images: degree of regularity, orientation of basic elements, distribution of elements by sizes, etc.;
- the codes obtained can be effectively used for further analysis and processing of images.

The concept of tetralogic and tetracodes seems to be quite fruitful both from theoretical point of view and from the point of view of future practical applications in simulation systems, image recognition and computer graphics - the most closely adjoining with problems of extreme information complexity of the surrounding world.

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