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GENEREATION OF SPECIALY COORDINATED SURFASES APPLICABLE TO STRENGTH ANALYSIS OF SHELLS

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Abstract. The paper is devoted to geometrical research concerning engineering design of the thin construction. As the transformation of parametric line on concrete surface from general to special parameters is not the elementary the method of analytical generation of advanced specially coordinated surfaces is proposed.

Keywords: curvilinear coordinate, congruence, curvature line, special parameterization, thin construction.

1. Introduction

Any methods of strength analysis of thin constructions named shells require the representation of the surfaces using special coordination. Here two cases are possible [1].

First case. Line coordinates $\alpha = const,\ \beta = const$ on middle surfaces will be orthogonal.

Second case. Coordinate lines of middle surfaces will be curvature lines.

Hence the problem of special parameterization of a surface is relevant.

2. Terminology and any determinations on theory of curvilinear coordinates and congruences

Let curvilinear coordinates $u,\ v,\ w$ be introduced by the functions

$$x = f(u,\ v),\ y = g(u,\ v),\ z = h(u,\ v), \qquad (1)$$

where $x,\ y,\ z$ are Cartesian coordinates.

As we assume that we do not have $\frac{D(f,\ g,\ h)}{D(u,\ v,\ w)} = 0.$

According to Lane [1] and Eisenhart [2] the functions (1) may be applied for the representation of curve congruence. For this one of three coordinate $u,\ v,\ w$ will be point on congruence line.

Hencefunctions(1) represent not one but at once three congruences of the lines which are lines of system (1). In the domain where the coordination by functions (1) is regular it is possible to solve (1) as parametric equations relatively $u,\ v,\ w$.

Let

$$u = u(x,\ y,\ z),\ v = v(x,\ y,\ z),\ w = w(x,\ y,\ z), \qquad (2)$$

be obtained solutions.

In the case when one of three couple from coordinates $u,\ v,\ w$ is represented by fixed values and other rests variable the functions (1) are transformed to parametric equations of one three coordinate lines of the system (1). The Type of this line is determined by fixed values of the couple and name by the variable coordinate.

In the case when at three coordinate $u,\ v,\ w$ are variable the functions (1) may be considered as parametric equations of three coordinate lines of system (1). Let's namea congruence, a surfaces of it, a line of it as $u-,\ v-,\ w-$ congruence, $u-,\ v-,\ w-$ surface. Hence the theory of curvilinear coordinates and the theory of curve congruences are related by the factors resulting from the initial representation.

3. Principles on which generation of specially coordinated surfaces is based

The form of any surface may be determined as whole analytically by the forms of its parametric lines and by the structure of its parametric families.

It is possible to choose the structure of both parametric families and for one of them the form of its parametric lines by choice of the coordinate system (1) whose one of the three congruencies will be containing the parametric lines of the surface S.

The second parametric family of required surface and the form of its elements is determined by line m immersed in mentioned congruence of coordinate lines.

If none of known coordinate system is applicable for the generation of surface of the required forms it is necessary to make out the special coordinate system following from the structure of the parametric families and the forms of their elements of therequired surface S.

Let(1) be the functions introducing the coordinate system u, v, w and m be the line, represented by the parametric equations

$$x = p(\alpha),\ y = q(\alpha),\ z = s(\alpha) \tag{3}$$

by immersing of which in $u-$congruence the $u-$surface will be generated.

Thefunctions (1), second and third of the functions (2) and the functions (3) complete the base for generation of parametric equations of $u-$surface. The substitution of the functions (3) for second and third functions (2) gives the substituents for the functions (1). So the parametric equations of required$u-$surface, passing through the line m will be represented as superposition of the functions

$$\begin{aligned} x &= f\{u, v[p(\alpha), q(\alpha), s(\alpha)], w[p(\alpha), q(\alpha), s(\alpha)]\}, \\ y &= g\{u, v[p(\alpha), q(\alpha), s(\alpha)], w[p(\alpha), q(\alpha), s(\alpha)]\}, \\ z &= h\{u, v[p(\alpha), q(\alpha), s(\alpha)], w[p(\alpha), q(\alpha), s(\alpha)]\}. \end{aligned} \tag{4}$$

Curvilinear coordinates for $v-$surface are

$$u[p(\alpha), q(\alpha), s(\alpha)], v, w[p(\alpha), q(\alpha), s(\alpha)] \equiv v,\ \alpha$$

and for $w-$ surface

$$u[p(\alpha), q(\alpha), s(\alpha)], v[p(\alpha), q(\alpha), s(\alpha)], w \equiv w,\ \alpha.$$

Let us consider the cases when a surface (4) is specially coordinated according to demands of strength analysis of shells.

Theorem. For parametric lines $u = const, \alpha = const$ of $u-$surface (4) to be:

1) orthogonal the system of coordinates (1) will be orthogonal;

2) lines of curvature the coordinates surfaces $u = const$ of orthogonal coordinate system (1) should be the spheres or the planes.

Proof. In order to the theorem it is sufficient to make following consequence of arguments provided that the coordinate system (1) is orthogonal:

- the coordinate surface $u = const$ and lines of $u-$congruence we orthogonal;
- $u-$surface containing the family of lines belonging to $u-$congruence makes the right angles with each surface of family $u = const$;
- for $u-$surface(4) the lines $\alpha = const$ as lines of $u-$congruence and the lines $u = const$ as intersections of $u-$surface with surfaces of the family $u = const$ are orthogonal;
- any line on the sphere is its curvature line;
- providing that coordinate surfaces $u = const$ of system (1) are the spheres them intersections $u = const$ with $u-$surface (4) are the curvature lines in accordance with Ioachimsthal's theorem [4] both for spheres and $u-$surface. But the lines of different families $u = const, \alpha = const$ are orthogonal and lines $u = const$ are the curvature lines.

Hence the lines of the family $\alpha = const$ on $u-$surface are also the curvature lines. The theorem is proven.

By following corollary we discriminate the case which is very important for strength analysis of shells.

Corollary. For immersing line m belongs to parametric family of $u-$lines on $u-$surface (4) it will be situated on one of coordinate $u = const$ of the system (1).

Remark. Our considerations were applicable for the generation of $u-$surfaces. They are also applicable when of prefixes for $u-$surfaces and $w-$ surfaces are changed.

4. Introducing of cyclic system of coordinates

Let the coordinates u, v be the radii of conjugated circles pencils and w be the angle of pencils rotation around the circles centers of one of them. Make coinciding the axis of rotation with axis oz.The functions (1) introducing the coordinate system are:

$$\begin{aligned} x &= u\frac{u\sqrt{v^2 + c^2} + v\sqrt{u^2 - c^2}}{u^2 + v^2}\cos w, \\ y &= u\frac{u\sqrt{v^2 + c^2} + v\sqrt{u^2 - c^2}}{u^2 + v^2}\sin w, \\ z &= v\frac{u\sqrt{v^2 + c^2} + v\sqrt{u^2 - c^2}}{u^2 + v^2}, \end{aligned} \tag{5}$$

where $c \geq 0$ is constant, $k = -1, 0, 1 -$ coefficient.

<u>First case.</u> In this case $k = 1, u \geq c$ and axis oz is centers line of circles belonging to hyperbolic pencil. Introduced system is known as toroidal with other of u, v, w [5, 6].

<u>Second case.</u> In this case $k = -1,\ v \ge c$ and axis oz is centers line of circles belonging to elliptic pencil. Introduced system is known as bipolar [5, 6] with other content of $u,\ v,\ w$.

<u>Third limiting case.</u> In this case $k = 0,\ c = 0$. Both conjugated pencils of circles are parabolic.

Domain of the functions(5):

$$\left.\begin{array}{l} k=1,\quad c>0,\ c\le u<\infty,\ 0<v<\infty \\ k=-1,\ c>0,\ 0<u<\infty,\ c\le v<\infty \\ k=0,\quad c=0,\ 0<u<\infty,\ 0<v<\infty \end{array}\right\} 0\le w\le 2\pi$$

Coordinate surfaces of the system (5):

- $u = const -$ the spheres center of which are situated on axis oz;
- $v = const -$ toroidal surfaces of revolution the meridian of which are the arcs with centers on the plane xoy;
- $w = const -$ demi-planes of the pencil with axis oz.

Coordinate lines of the system (5):

- $u-$line – an arc of a circle with center on the plane xoy situated on a demi-plane of the pencil with axis oz;
- $v-$line – an arc of a circle with center on axis oz situated on a demi-plane of the pencil with axis oz;
- $w-$ line – a circle with center on axis oz situated on the plane parallel to the plane xoy.

As all three coordinate lines of the coordinate system (5) are the circles it is called cyclic.

Considering the functions(5) as parametric equations and resolving them relativeby $u,\ v,\ \cos w,\ \sin w$ us obtain:

$$u = \frac{1}{2z}\sqrt{\left(x^2+y^2+z^2-kc^2\right)^2+4z^2kc^2} \qquad (6)$$

$$v = \frac{1}{2\sqrt{x^2+y^2}}\sqrt{\left(x^2+y^2+z^2+kc^2\right)^2-4\left(x^2+y^2\right)kc^2} \qquad (7)$$

$$\cos w = \frac{x}{\sqrt{x^2+y^2}},\ \sin w = \frac{y}{\sqrt{x^2+y^2}} \qquad (8)$$

The domain off functions (6),(7), (8) is $\left(x\ne 0 \cap y\ne 0\right)\cup z\ne 0.$

5. Applications

Let us demonstrate by several examples the proposed method of specially coordinated surfaces generation.

Example 1. To generate the specially coordinated $u-,\ v-$ and $w-$surfaces basing on cyclic coordinate system (5) by immersing the helices

$$x = p(\alpha) = r\cos\alpha,\ y = q(\alpha) = r\sin\alpha,\ z = s(\alpha) = b\alpha \qquad (9)$$

in $u-,\ v-$ and $w-$ congruence's in turn.

Solutions. a) As the coordinate system (5) is orthogonal and its coordinate surfaces $u = const$ are the spheres we obtain the parametric equations of $u-$surface as the superposition of the functions received according to the scheme: $(9)\frac{\square\ (7)\ \square}{\square\ (8)\ \square}(5)$.

The parametric lines of $u-$surface represented by form (4) are: $u = const -$ spherical lines of curvature, $\alpha = const - u -$ circles which are the line of curvature;

b) the scheme for representation of the parametric equations of $v-$surface is following: $(9)\frac{\square\ (6)\ \square}{\square\ (8)\ \square}(5)$.

As the coordinate surface $v = const$ of system (4) are tori (not spheres or planes) the $v-$surface will be represented by parametric lines $v = const$ on the surface of tori and $\alpha = const - v -$ circles of the coordinate system (5). The parameterization of $v-$surface is orthogonal;

c) as coordinate, surface $w = const$ are the planes of $w-$ surface will be the curvature lines the scheme for obtaining of parametric equations of $w-$surface is following: $(9)\frac{\square\ (6)\ \square}{\square\ (7)\ \square}(5)$.

In general case $w-$surface is one of revolution in this particular case it is cylinder of revolution.

It will be remarked that in all three cases the immersing line m does not belongs to the parametric families of demanded surface.

The computer aided representations of $u-,\ v-$ and $w-$surface are given in figure 1.

a) b) c)

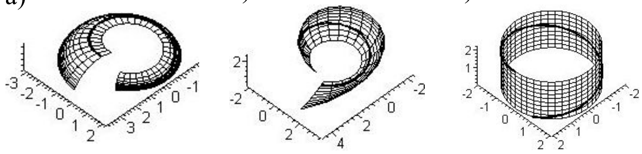

Fig. 1. $U-,\ v-$ and $w-$ surfaces passing through helices $a = 2,\ b = 0.3,\ c = 1$

a) $k = 1,\quad 0.4 \le u \le 8$

b) $k = -1,\ 0.5 \le v \le 2.5$

c) $k = 0,\quad 0 \le w \le 2\pi$

Example 2. To generate $u-$surface by immersing in $u-$congruence of system (5) of the curve given by equation

$$t = t_1 + b\cos\left(n\alpha\right), \qquad (10)$$

where $t-$ variable, $t_1 -$ fixed angle measured in horizontal plane, $\alpha -$ variable angle in vertical plane, $b-$ amplitude, $n-$ number of periods.

The equation(10) is internal relatively to parametric equations

$$x = p(t,\alpha) = u\cos t\cos\alpha,\ y = q(t,\alpha) = u\cos t\sin\alpha,$$
$$z = s(t,\alpha) = \sqrt{u^2 - kc^2} + u\sin t, \tag{11}$$

which represent one of coordinate sphere $u = const$ of the coordinate system (5).

*Solutions.*The parametric equations of required surface relatively to the curvature lines we obtain according to following scheme: $(10) \rightarrow (11) \xrightarrow[(8)]{(7)} (5)$.

As immersing line m given by equations (10), (11) belongs to one coordinate surface $u = const$ which is the sphere it is an element of the parametric family $u = const$ of $u -$surface.

The figure 2 shows the required$u -$surface.

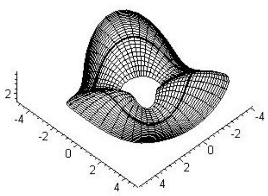

Fig. 2.U – surface passing through line, belonging to coordinate sphere

6. Conclusion

The proposed method of surface generation gives the possibilities to foresee its form, to correct it using computer added visualization, to use the methods of thin constructions having a form of a middle surface, generating by proposed method.

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