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INVERSION OF SURFACES REFERRED TO THE LINES OF CURVATURE

Reference of surfaces to the lines of curvature plays an important role in the theory of covers and at determination of optimum trajectories of processing on NC machine tools. Transition from any parameterization to the special one, at which the coordinate grid coincides with the grid of lines of curvature, is possible only for some surfaces. It is connected with the fact that finding of the lines of curvature on a surface is reduced to the solution of the differential equations which can be integrated in some cases only. Considering this designing of the surfaces and finding the families of the lines of curvature on them is a topical task.

One of the most widespread mathematical apparatuses of designing of lines and surfaces are geometrical transformations. They allow to receive new images, keeping certain useful properties of prototypes. The transformation of the known surfaces referred to the families of the lines of curvature will be carried out using the inversion. As far as this transformation is conformal, the orthogonality of the lines remains constant and the lines of curvature of the initial surface turn into the lines of curvature of the new surface [1].

If $$\bar{x} = \bar{x}(t,u), \quad \bar{y} = \bar{y}(t,u), \quad \bar{z} = \bar{z}(t,u) \tag{1}$$

are the parametrical equations of the prototype surface, the parametrical equations of the image surface, received by application of the transformation of the inversion [2], are

$$\begin{aligned} x &= \frac{R^2(\bar{x}+\bar{x}_0)}{(\bar{x}+\bar{x}_0)^2+(\bar{y}+\bar{y}_0)^2+(\bar{z}+\bar{z}_0)^2}, \\ y &= \frac{R^2(\bar{y}+\bar{y}_0)}{(\bar{x}+\bar{x}_0)^2+(\bar{y}+\bar{y}_0)^2+(\bar{z}+\bar{z}_0)^2}, \\ z &= \frac{R^2(\bar{z}+\bar{z}_0)}{(\bar{x}+\bar{x}_0)^2+(\bar{y}+\bar{y}_0)^2+(\bar{z}+\bar{z}_0)^2}, \end{aligned} \tag{2}$$

where $\bar{x}_0\ \bar{y}_0\ \bar{z}_0$ are the coordinates of the center of inversion,

R is the radius of inversion.

Thus, if t=const, u=const are the lines of the curvature of the surface (1), they are the lines of curvature and for the surface (2) too.

Let's note a number of properties useful to the applied formation of characteristics which the inversion transformation has:

- the image of the sphere is the sphere (we will remind that the plane is the sphere too and has the infinite radius);
- the image of the plane which passes through the center of inversion is the plane which passes through the center too;
- the image of the plane which doesn't pass through the center of inversion is the sphere which passes through the center of inversion;
- the image of the sphere which doesn't pass through the center of inversion is the sphere which doesn't pass through the center of inversion;
- the image of the sphere which passes through the center of inversion is the plane which doesn't pass through the center of inversion;
- the transformation by inversion is conformal: it keeps corners.

For example, tor surface of the rotation (fig. 1), which equation is

$$\bar{x} = a + (R + r\cos u)\cos t, \qquad \bar{y} = (R + r\cos u)\sin t, \qquad \bar{z} = r\sin u \tag{3}$$

where R is the radius of the circle of the centers of one-parametrical family of the forming circles, r is the radius of the forming circle;

by transformation of the inversion (2) is raised to the equation of the cyclide of Dupin of the fourth order without conic points

$$x = \frac{c^2[(R + r\cos u)\cos t + a]}{2a(R + r\cos u)\cos t + 2Rr\cos u + R^2 + r^2 + a^2},$$

$$y = \frac{c^2(R + r\cos u)\sin t}{2a(R + r\cos u)\cos t + 2Rr\cos u + R^2 + r^2 + a^2}, \tag{4}$$

$$z = \frac{c^2 r\sin u}{2a(R + r\cos u)\cos t + 2Rr\cos u + R^2 + r^2 + a^2}.$$

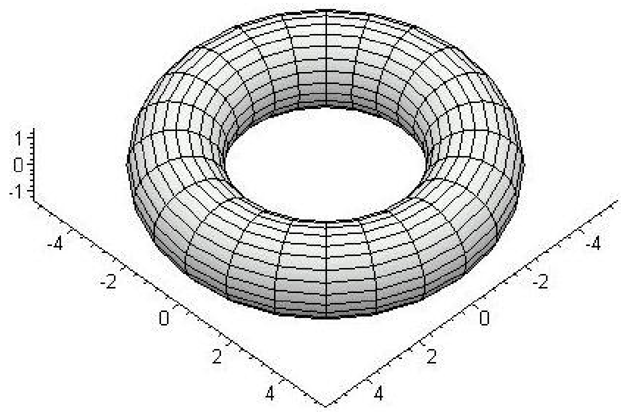

Fig. 1 – The prototype surface

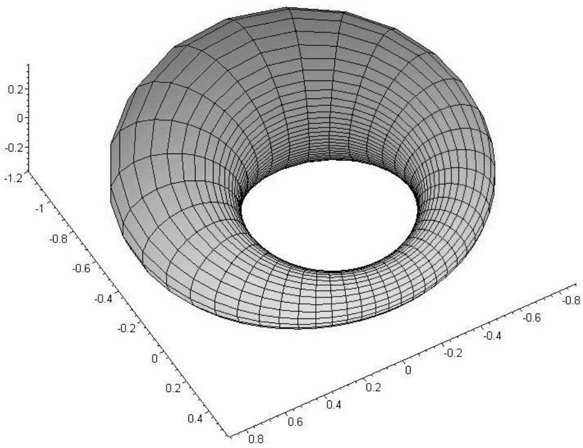

Fig. 2 – The image surface

In fig. 2 this cyclide constructed on the equation (4) at R=4, r=1.3, a=2.7, c=1, , $0 \le t \le 2\pi$, $0 \le u \le 2\pi$ is shown.

Dupin's cyclide of the fourth order with two conic points is possible to be received using the inversion of the rotation cone provided that the center of the inversion is assumed not to belong to its surface [3].

The parametrical equations of the cone-prototype will be written down as

$$\overline{x} = u^3 \sin\alpha \cos t, \qquad \overline{y} = u^3 \sin\alpha \sin t, \qquad \overline{z} = u^3 \cos\alpha + b \tag{5}$$

Let's substitute (5) to (2) and receive

$$\overline{x} = \frac{c^2 u^3 \sin\alpha \cos t}{2u^3 b\cos\alpha + b^2 + u^6}, \; \overline{y} = \frac{c^2 u^3 \sin\alpha \sin t}{2u^3 b\cos\alpha + b^2 + u^6}, \overline{z} = \frac{c^2 (u^3 \cos\alpha + b)}{2u^3 b\cos\alpha + b^2 + u^6} \tag{6}$$

the parametrical equations of the cyclide of the fourth order with two conic points (fig. 3) at $\alpha = \frac{\pi}{6}$ - a tilt angle forming the cone-prototype to its axis, b = - 1 - distance from the center of inversion to cone prototype's top, c=4, $0 \le t \le 2\pi$, $-10 \le u \le 10$.

At last, the cyclide of the fourth order with one conic point is possible to be received using the inversion of rotation of the cylinder concerning the sphere which center is assumed not to belong to its surface [3]. The parametrical equations of the cylinder are

$$\overline{x} = r\cos t, \qquad \overline{y} = r\sin t, \qquad \overline{z} = u^3. \tag{7}$$

let's substitute to (2) and receive

$$\bar{x} = \frac{c^2 r \cos t}{r^2 + u^6}, \qquad \bar{y} = \frac{c^2 r \sin t}{r^2 + u^6}, \qquad \bar{z} = \frac{c^2 u^3}{r^2 + u^6} \tag{8}$$

the parametrical equations of the cyclide of the fourth order with one conic point.

This cyclide is shown in fig. 4 at $r=6$, $c=5$, $0 \le t \le 2\pi$, $-10 \le u \le 10$..

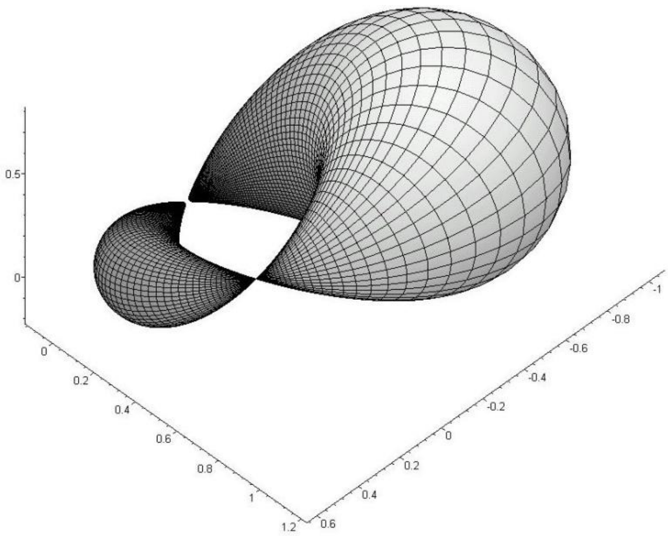

Fig. 1 – Dupin's cyclide of the fourth order with two conic points

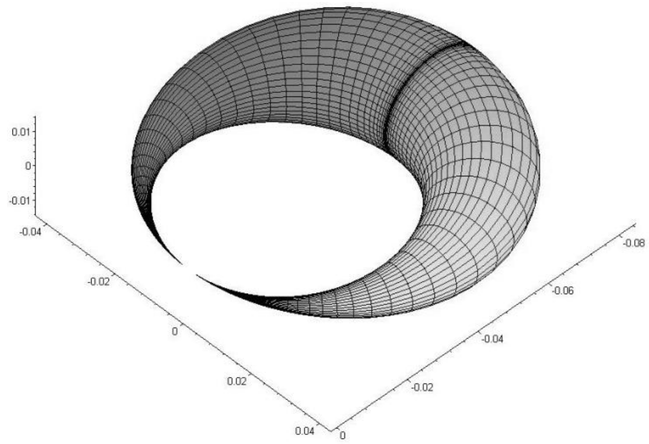

Fig. 2 – Dupin's cyclide of the fourth order with one conic point

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