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TO THE QUESTION OF FORMING BIHARMONIC OSCILLATIONS IN TWO-MASSES NONLINEAR VIBRATING MACHINES UNDER IDEAL HARMONIC EXCITATION

The principle scheme of two-masses vibrating machine with polynomial characteristics of elastic ties and ideal harmonic excitation is considered. With the help of harmonic balance method its dynamics is studied in the frequency zone located between two natural ones. For certain parameters of the system existence of superharmonic resonance 2:1 is discovered, the influence of nonlinearity and asymmetry of elastic characteristics and dissipation factor upon the behavior of the system and number of its regimes are described. The possibility of forming of practically significant biharmonic oscillations is demonstrated.
Key words: *vibromachine, antiresonance, polyharmonic vibration, harmonic balance method, bifurcation diagram, numerical analysis*

1. Statement of the problem

In industry there are used vibrating machines with biharmonic oscillations of working organs. Traditionally such oscillations are realized with the help of two exciters having different frequencies [1]. But it is well known that in nonlinear systems the oscillations sometimes become quite complicated and may have pronounced polyharmonic character [2]. Usually these phenomena take place for certain parameters of the dynamical system when, the so-called, combination resonances take place. By this reason, the purpose of this article is to investigate principle possibility of forming practically important oscillations in two masses vibrating machine having polynomial elastic ties and harmonic excitation.

2. Principle scheme and mathematical model

The principle scheme of such vibrating machine is shown in Figure 1, where m_1 – mass of a frame, m_2 – mass of a working organ, m_0 – unbalanced mass, r – eccentricity of an exciter, ω – frequency of an vibroexciter, k_0 – stiffness coefficient and $\mu k_0'$ – coefficient of viscous resistance of shock absorbers, $k(x) = k_1 + k_2 x + k_3 x^2$ – stiffness coefficient and $\mu k'(x)$, where $k'(x) = k_1' + k_2' x + k_3' x^2$,– resistance coefficient of the elastic ties connecting frame and working organ, $F_e = k_1 x + k_2 x^2 + k_3 x^3$ – its elastic characteristic, k_1, k_2, k_3 – parameters of the elastic ties and k_1', k_2', k_3',– of dissipation, μ – coefficient of inelastic resistance of absorbers and elastic ties, $P(t) = m_0 r \omega^2 \cos\omega t$ – constraining force of inertial vibroexciter.

As generalized coordinates we take x_1, – displacement of the frame and x_2 , – displacement of the working organ.

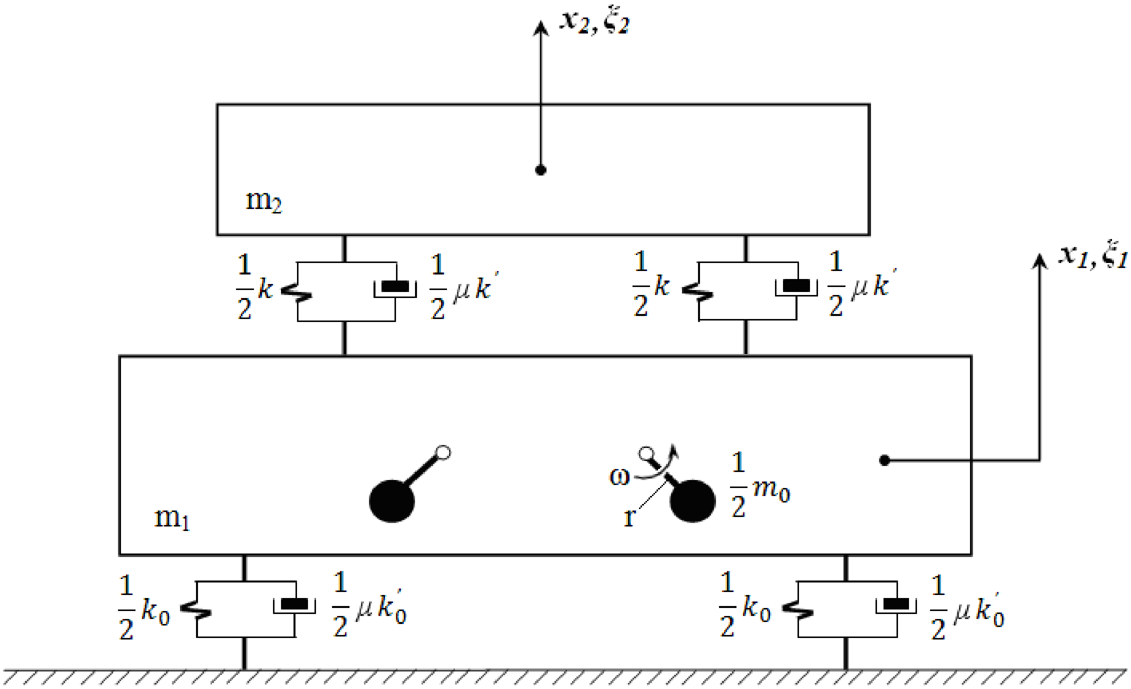

Figure 1 – The principal scheme of a vibrating machine

Using Lagrange equations [3] we get the equations of the motion

$$\begin{cases} m_1' \ddot{x}_1 + k_0 x_1 - k_1 (x_2 - x_1) - k_2 (x_2 - x_1)^2 - k_3 (x_2 - x_1)^3 + \\ \quad + \mu k_0' \dot{x}_1 - \mu \left(k_1' + k_2' (x_2 - x_1) + k_3' (x_2 - x_1)^2\right)(\dot{x}_2 - \dot{x}_1) = m_0 r \omega^2 \cos \omega t, \\ m_2 \ddot{x}_2 + k_1 (x_2 - x_1) + k_2 (x_2 - x_1)^2 + k_3 (x_2 - x_1)^3 + \\ \quad + \mu \left(k_1' + k_2' (x_2 - x_1) + k_3' (x_2 - x_1)^2\right)(\dot{x}_2 - \dot{x}_1) = 0. \end{cases}$$

Then subtracting one of them from the other one, denoting $x = x_2 - x_1$, turning to the variable $\tau = \omega_1 t$, where ω_1 – the first natural frequency of a vibromachine and introducing non-dimensional variables $\xi_1 = x_1 / \Delta$, $\xi = x / \Delta$, where $\Delta = 10^{-3}$ m we represent mathematical model of the vibromachine in the form

$$\begin{cases} \frac{d^2 \xi_1}{d\tau^2} + b_{10} \frac{d\xi_1}{d\tau} + b_{11} \frac{d\xi}{d\tau} + b_{12} \xi \frac{d\xi}{d\tau} + b_{13} \xi^2 \frac{d\xi}{d\tau} + \\ \qquad + k_{10} \xi_1 + k_{11} \xi + k_{12} \xi^2 + k_{13} \xi^3 = P_1 \cos \eta \tau, \\ \frac{d^2 \xi}{d\tau^2} + b_{20} \frac{d\xi_1}{d\tau} + b_{21} \frac{d\xi}{d\tau} + b_{22} \xi \frac{d\xi}{d\tau} + b_{23} \xi^2 \frac{d\xi}{d\tau} + \\ \qquad + k_{20} \xi_1 + k_{21} \xi + k_{22} \xi^2 + k_{23} \xi^3 = P_2 \cos \eta \tau, \end{cases} \tag{1}$$

where $b_{10} = \frac{\mu k_0'}{m_1' \omega_1}$, $b_{11} = -\frac{\mu k_1'}{m_1' \omega_1}$, $b_{12} = -\frac{\mu k_2' \Delta}{m_1' \omega_1}$, $b_{13} = -\frac{\mu k_3' \Delta^2}{m_1' \omega_1}$, $b_{20} = -\frac{\mu k_0'}{m_1' \omega_1}$,

$b_{21} = \frac{\mu (m_1' + m_2) k_1'}{m_1' m_2 \omega_1}$, $b_{22} = \frac{\mu (m_1' + m_2) k_2' \Delta}{m_1' m_2 \omega_1}$, $b_{23} = \frac{\mu (m_1' + m_2) k_3' \Delta^2}{m_1' m_2 \omega_1}$, $k_{10} = \frac{k_0}{m_1' \omega_1^2}$,

$k_{11}=-\frac{k_1}{m_1' \omega_1^2}$, $k_{12}=-\frac{k_2 \Delta}{m_1' \omega_1^2}$, $k_{13}=-\frac{k_3 \Delta^2}{m_1' \omega_1^2}$, $k_{20}=-\frac{k_0}{m_1' \omega_1^2}$, $k_{21}=\frac{k_1(m_1'+m_2)}{m_1' m_2 \omega_1^2}$,

$k_{22}=\frac{k_2(m_1'+m_2)\Delta}{m_1' m_2 \omega_1^2}$, $k_{23}=\frac{k_3(m_1'+m_2)\Delta^2}{m_1' m_2 \omega_1^2}$, $P_1=\frac{m_0 r}{m_1' \Delta}\eta^2$, $P_2=-P_1$, $m_1'=m_0+m_1$,

$\eta=\omega/\omega_1$.

Values of the physical parameters of the vibromachine are $m_0=50$ kg, $m_1=700$ kg, $m_2=550$ kg, $k_0=0.12\cdot 10^6$ N/m, $k_1=5.5\cdot 10^6$ N/m, $r=0.088$ m, $\mu=0.0008$ s, $k_0'=k_0$, $k_1'=k_1$ and the working frequency of the engine $\omega=100$ rad/s . We suppose below the possibility of changing η, k_2, k_3, k_2', k_3' only.

3. Method of investigation

The steady motions of the machine we find by the harmonic balance method [4]. According to it solutions of the system (1) we find in the form of finite complex expansions

$$\xi_1(\tau)=\sum_{n=-N}^{N} c_n^{(1)} e^{in\eta\tau}, \quad \xi(\tau)=\sum_{n=-N}^{N} c_n e^{in\eta\tau}, \tag{2}$$

where N is a number of harmonics taken into consideration. It is supposed that the trigonometric view of the solutions of (1) is $\sum_{j=0}^{N} A_j \cos(j\eta\tau-\varphi_j)$, where the amplitude $A_j=2\sqrt{c_j c_{-j}}$ and initial phase $\varphi_j\in[-\pi,\pi)$, i.e. $\varphi_j=\arccos\frac{c_j+c_{-j}}{2\sqrt{c_j c_{-j}}}$ and $\varphi_j=-\arccos\frac{c_j+c_{-j}}{2\sqrt{c_j c_{-j}}}$ if $\left(\Im c_{-j}=0 \wedge \Re c_{-j}<0\right) \vee \Im c_{-j}<0$.

After substituting (2) into (1) and equating coefficients of equal powers of $e^{in\eta\tau}$ one may get the algebraic system of equations with respect to c_n

$$\begin{cases} (k_{10}+b_{10}i\eta n-\eta^2 n^2)c_n^{(1)}+(k_{11}+b_{11}i\eta n)c_n+ \\ \quad +\sum_{j=-N}^{N} c_j c_{n-j}(k_{12}+b_{12}i\eta(n-j))+ \\ \quad +\sum_{j=-N}^{N}\sum_{m=-N}^{N} c_j c_m c_{n-j-m}(k_{13}+b_{13}i\eta(n-j-m))=\begin{cases} P_1/2, & n=\pm 1 \\ 0, & n\neq\pm 1 \end{cases}, \\ (k_{20}+b_{20}i\eta n)c_n^{(1)}+(k_{21}+b_{21}i\eta n-\eta^2 n^2)c_n+ \\ \quad +\sum_{j=-N}^{N} c_j c_{n-j}(k_{22}+b_{22}i\eta(n-j))+ \\ \quad +\sum_{j=-N}^{N}\sum_{m=-N}^{N} c_j c_m c_{n-j-m}(k_{23}+b_{23}i\eta(n-j-m))=\begin{cases} P_2/2, & n=\pm 1 \\ 0, & n\neq\pm 1 \end{cases}, \end{cases} \tag{3}$$

where $n, n-j, n-j-m\in[-N,N]$. Dimension of this system equals $2(2N+1)$. Then consequently changing one of the parameters of the system (1) and solving the system (3) one may find bifurcation diagrams of the system, in particular, the amplitude- (AFC) and phase-frequency characteristics (PFC). Computations are fulfilled for five harmonic components in (2) were taken into account, i.e. $N=5$. The corresponding software are worked out as the toolbox of the program MATLAB and described in [5].

4. Results

Here we considered the frequency zone located between the natural ones (see Figure 2) and studied the pure resonances of lower order. The rectangular symbol in figure shows the present working frequency of the machine. One may mention that this is the frequency of antiresonance, – very small oscillations of the frame and quite sufficient motions of the working organ.

The main practical results are the following.

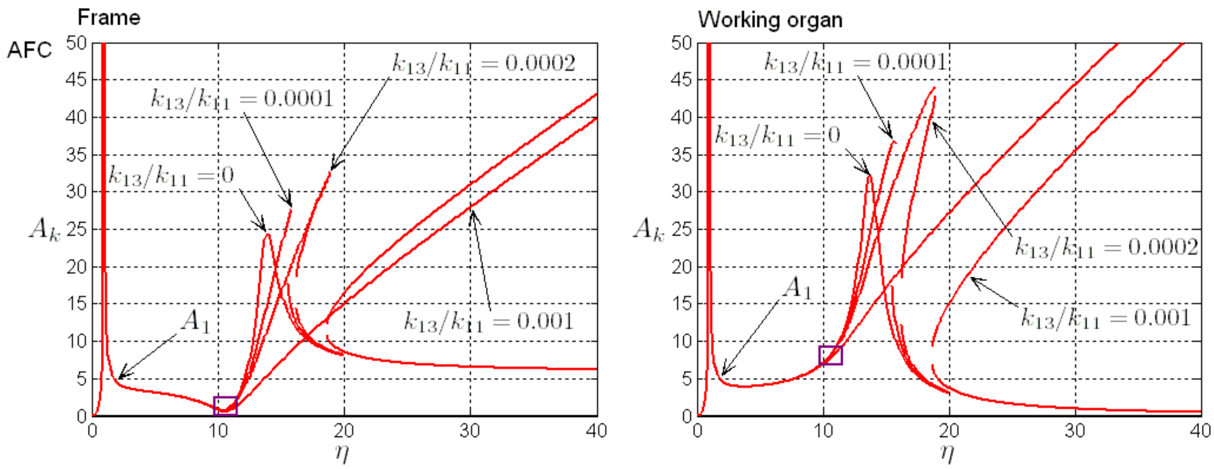

Figure 2 – AFC for different values of the degree of nonlinearity of the elastic characteristics

Using correlation $p\omega \approx |p_1|\omega_1 + |p_2|\omega_2$ [6], where $p, p_1, p_2 \in Z$ in this zone with the help of our software we succeeded to find only the resonances of $3:1$, $2:1$ and $1:3$. For linear dissipation ($k_2' = k_3' = 0$) the corresponding bifurcation diagrams (AFC and PFC) are shown in Figures 3–5.

By our opinion the superharmonic resonance $2:1$ (see Figure 4) is the most interesting among it for practical purposes, it is rather intensive and gives an opportunity to form biharmonic oscillations which are necessary for practice. Really, one may compare, for example, motions that are recommended for concentration tables [1] and oscillations generated in superharmonic zone for $\eta = 20$ (see Figure 6). The relative value of the second harmonic A_2 / A_1 in displacement of the working organ forms approximately $12 \div 15\,\%$ and skew of it approximately equals $2\varphi_2 - \varphi_1 \approx 0.08\pi$ (see Figure 4). But one of the essential faults of superharmonic resonances of even order is the existence of two opposite regimes (see regimes 1 and 2 in Figure 6), that may create certain difficulties when you start the engine.

Trying to strengthen one of the superharmonic regimes we introduce asymmetry into elastic ties, this attempt is demonstrated in Figure 7. From the view of the PFC it may be stated that two opposite regimes continue to exist, but the practical results of this measure hardly may be interpreted surely without getting their basins of attraction.

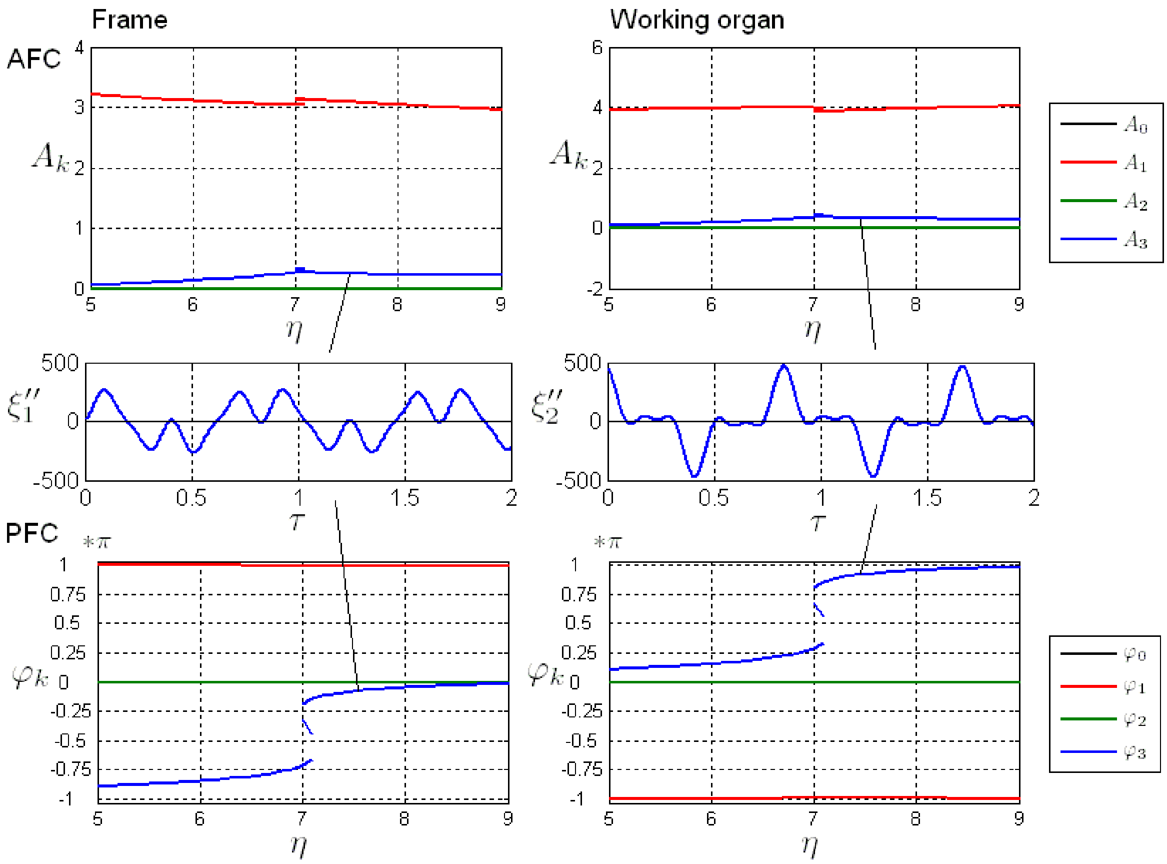

Figure 3 – Superharmonic resonance of order $3:1$

It is also necessary to mention that the discovered resonance is quite stable to level of dissipation in the system. This fact is demonstrated in Figure 8 where the bifurcation diagrams (the amplitude and phase frequency response) are given for asymmetric elastic ties and nonlinear resistance. It is interesting that in this case the view of PFC already shows the existence of a single polyharmonic regime for $\eta = 17$, for example. But, truly saying, some certain changes also take place in the amplitude-phase correlations (see diagrams of accelerations in Figure 8). Thus, shift phase of the second harmonic with respect to the first one equals already $2\varphi_2 - \varphi_1 \approx 0.25\pi$.

It is also important to notice that the resonance $2:1$ in this vibrating machine may be realized without essential changes in its construction and for the same angle velocity of the engine $\omega = 100$ rad/s by choosing only the parameters of the main elastic ties. Calculations which were performed on the base of correlations (1) and expression for the first natural frequency

$$\omega_1^2 = \frac{\omega^2}{\eta^2} = \frac{k_{10} + k_{21} - \sqrt{(k_{10} - k_{21})^2 + 4 k_{11} k_{20}}}{2}$$

show that for getting the value of non-dimensional frequency $\eta = 17$ the stiffness of the linear part of the main elastic ties must be taken $k_1 = 0.36 \cdot 10^6$ N/m instead of its initial value equal $k_1 = 5.5 \cdot 10^6$ N/m.

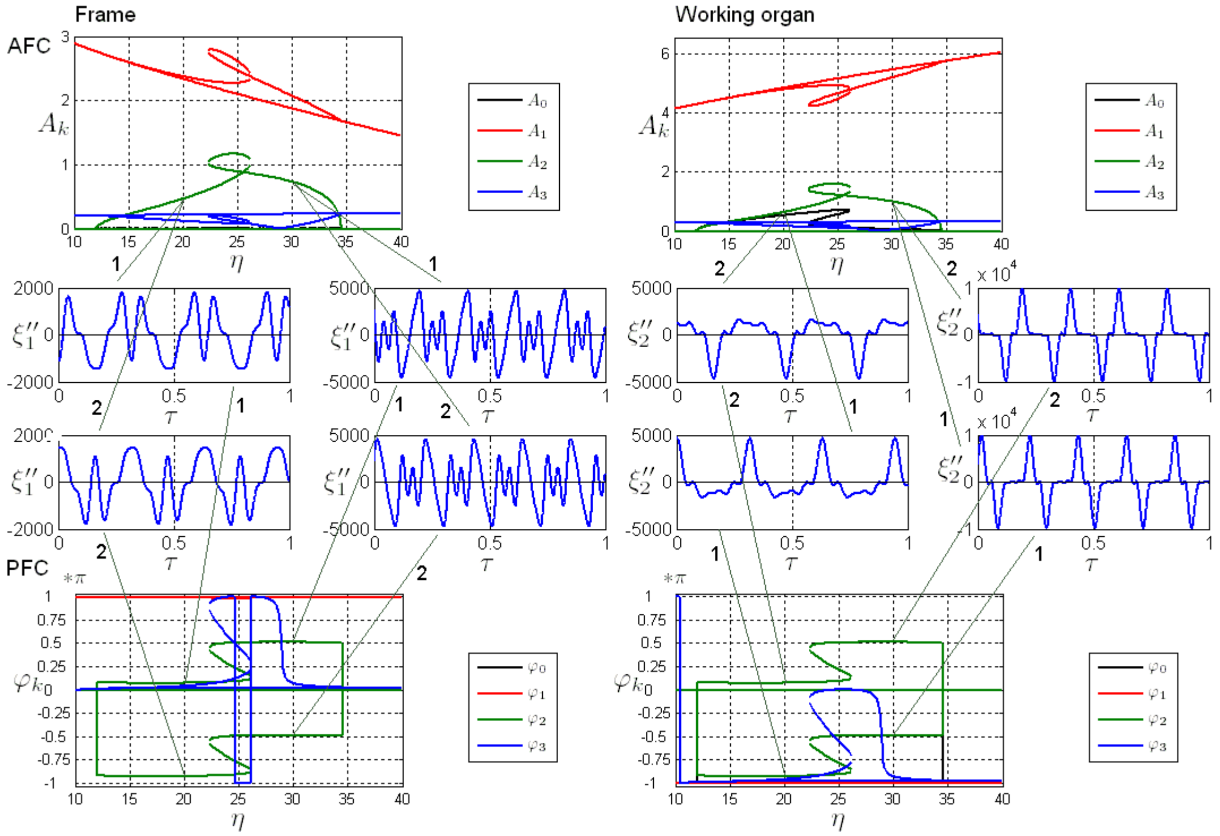

Figure 4 – Superharmonic resonance of order *2 : 1*

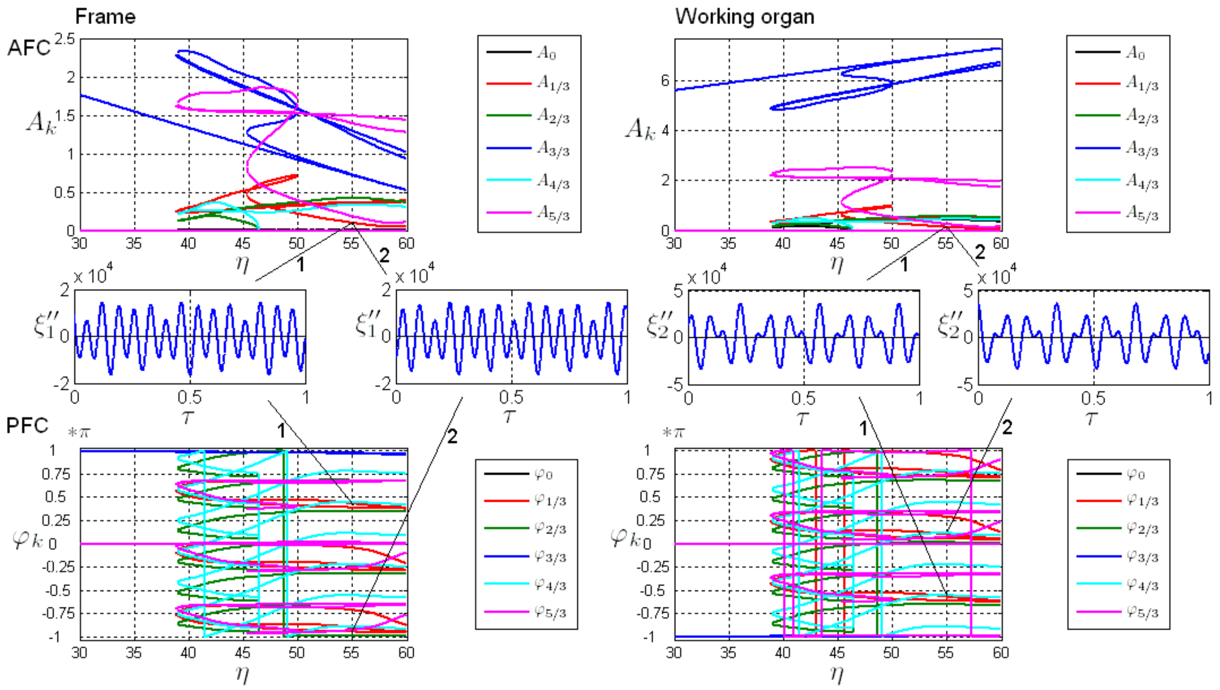

Figure 5 – Subharmonic resonance of order *1 : 3*

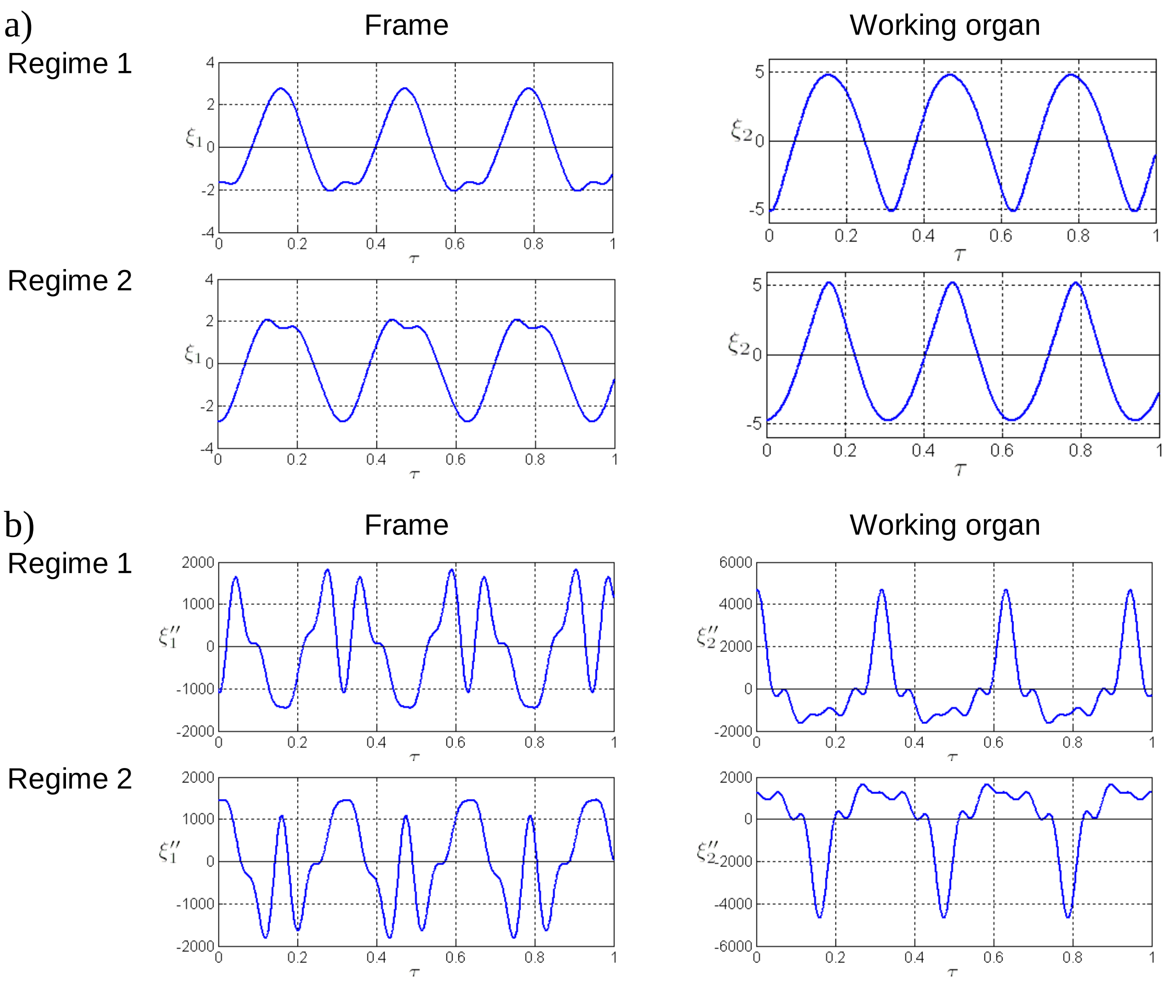

Figure 6 – Diagrams of displacements (a) and accelerations (b) of vibromachine for superharmonic resonance $2:1$ and $\eta = 20$

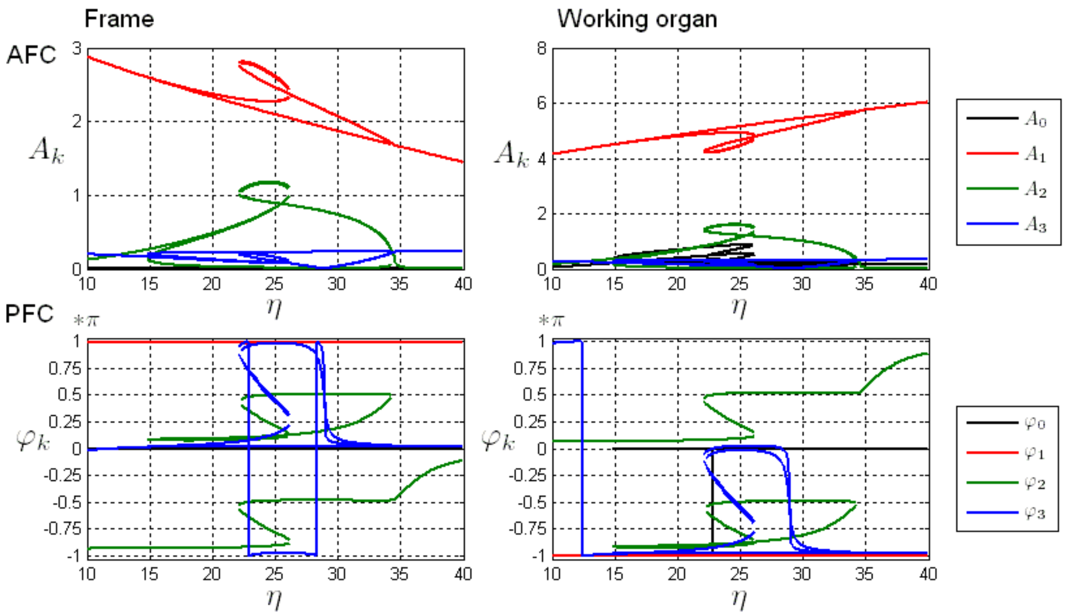

Figure 7 – Superharmonic resonance $2:1$ for linear dissipation and $k_{13}/k_{11} = 1$, $k_{12}/k_{11} = -0.5$

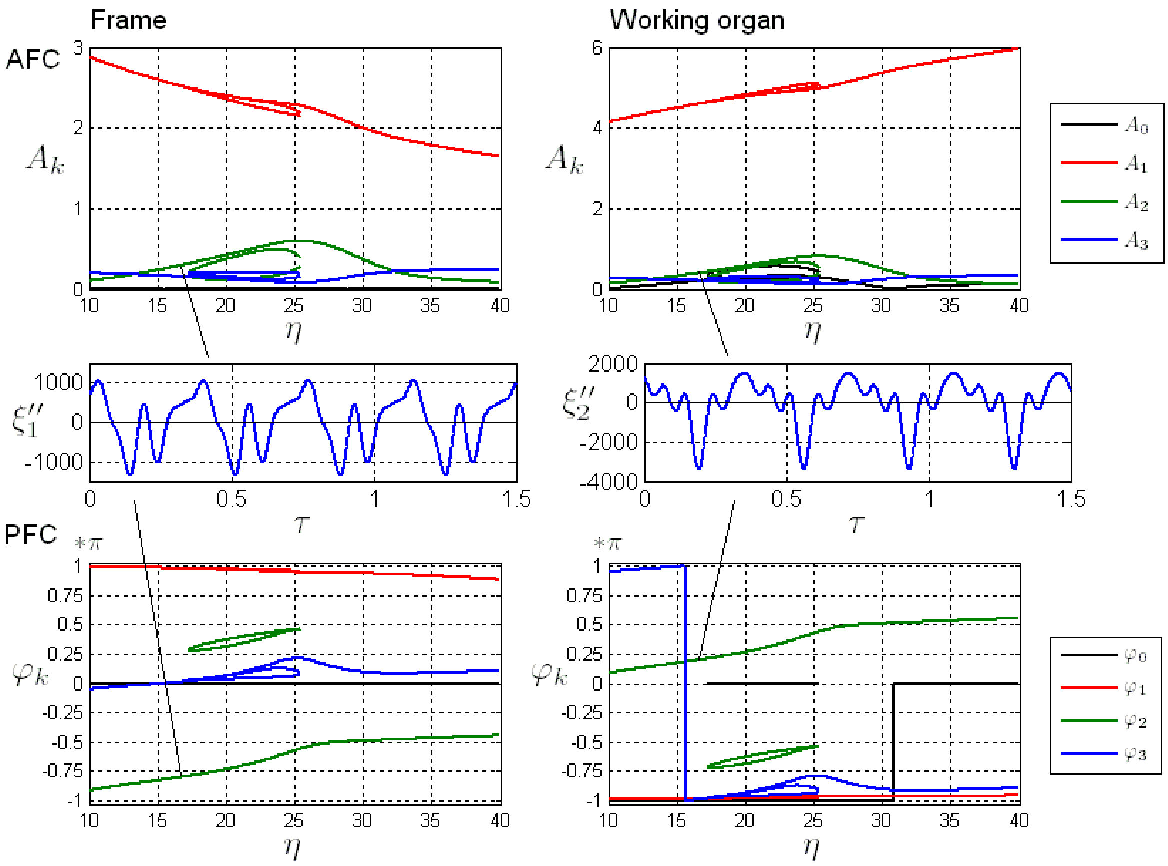

Figure 8 – Superharmonic resonance $2:1$ for nonlinear dissipation ($k_2' = k_2$, $k_3' = k_3$) and $k_{13}/k_{11}=1$, $k_{12}/k_{11}=-0.5$

5. Conclusions

So, the carried out investigations demonstrate the principal possibility of forming practically significant polyharmonic oscillations of the working organ of the vibrating machines by use of nonlinear elastic ties and realization of the superharmonic resonance of the second order. These oscillations exist for broad range of the parameters and quite stable to the level of dissipation. But one needs to keep in mind that the frame of the machine is also pulled into these motions and as a result quality of foundation insulation has become worse.

To the number of advantages of such way of forming polyharmonic vibrations one may include the presence only one harmonic exciter and the necessity to make just small changes in the machine construction. But some problems connected with the design of such ties still remain.

The limited capacity of the engine is also one of the interesting and important factors which can make adjustments upon the process of forming polyharmonic vibrations and, by this reason, must be taken into account too.

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К ВОПРОСУ О ФОРМИРОВАНИИ БИГАРМОНИЧЕСКИХ КОЛЕБАНИЙ В ДВУХМАССНЫХ НЕЛИНЕЙНЫХ ВИБРАЦИОННЫХ МАШИНАХ ПРИ ИДЕАЛЬНОМ ГАРМОНИЧЕСКОМ ВОЗБУЖДЕНИИ

Рассматривается принципиальная схема двухмассной вибрационной машины с полиномиальной характеристикой упругих связей и идеальным гармоническим возбуждением. С помощью метода гармонического баланса изучается ее динамика в частотной зоне, расположенной между двумя основными частотами. Для некоторых параметров системы обнаружено существование супергармонического резонанса 2:1, описано влияние нелинейности, асимметрии упругой характеристики и диссипации на поведение системы, описано число ее режимов. Продемонстрирована возможность формирования практически значимых бигармонических колебаний.

Ключевые слова: *вибромашина, антирезонанс, полигармоническая вибрация, метод гармонического баланса, бифуркационная диаграмма, численный анализ*

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ДО ПИТАННЯ ПРО ФОРМУВАННЯ БІГАРМОНІЙНИХ КОЛИВАНЬ В ДВОМАСНИХ НЕЛІНІЙНИХ ВІБРАЦІЙНИХ МАШИНАХ ПРИ ІДЕАЛЬНОМУ ГАРМОНІЙНОМУ ЗБУДЖЕННІ

Розглядається принципова схема двомасної вібраційної машини з поліноміальною характеристикою пружних зв'язків і ідеальним гармонійним збудженням. За допомогою методу гармонійного балансу вивчається її динаміка в частотній зоні, розташованій між двома основними частотами. Для деяких параметрів системи виявлено існування супергармонійного резонансу 2:1, описаний вплив нелінійності, асиметрії пружної характеристики і дисипації на поведінку системи, описано число її режимів. Продемонстрована можливість формування практично значимих бігармонійних коливань.

Ключеві слова: *вібромашина, антірезонанс, полігармонійна вібрація, метод гармонійного балансу, біфуркаційна діаграма, чисельний аналіз*

Надійшла до редколегії ____________________