APPENDIX TO CHAPTER 13

Solution of the problem 13.32

Initial data $I := 60 \quad d := 0.7 \quad x_A := -0.1 \quad x_B := 0.15$

$y_A := -0.15 \quad y_B := 0.2$

$$Hx(x,y) := \frac{-I}{2\cdot\pi}\cdot\left[\frac{2\cdot y}{(d+x)^2+y^2} - \frac{y}{(d-x)^2+y^2} + \frac{\sqrt{3}\cdot d - y}{\left(\sqrt{3}\cdot d - y\right)^2 + x^2}\right]$$

$$Hy(x,y) := \frac{I}{2\cdot\pi}\cdot\left[\frac{2\cdot(d+x)}{(d+x)^2+y^2} + \frac{d-x}{(d-x)^2+y^2} - \frac{x}{\left(\sqrt{3}\cdot d + y\right)^2 + x^2}\right]$$

$$H_A := \sqrt{Hx\left(x_A,y_A\right)^2 + Hy\left(x_A,y_A\right)^2} \qquad H_B := \sqrt{Hx\left(x_B,y_B\right)^2 + Hy\left(x_B,y_B\right)^2}$$

Answer for *H, A/m*

$$H := \begin{pmatrix} Hx\left(x_A,y_A\right) \\ Hy\left(x_A,y_A\right) \\ H_A \\ Hx\left(x_B,y_B\right) \\ Hy\left(x_B,y_B\right) \\ H_B \end{pmatrix} \qquad H = \begin{pmatrix} -1.644 \\ 42.328 \\ 42.36 \\ -8.663 \\ 35.915 \\ 36.945 \end{pmatrix}$$

$$U_M := \int_{x_A}^{x_B} Hx\left(x,y_A\right)\,dx + \int_{y_A}^{y_B} Hy\left(x_B,y\right)\,dy$$

[illegible] U_M, [illegible]

$U_M = 12.117$

Solution of the problem 13.33

Initial data $r1 := 0.004 \quad r2 := 0.002 \quad d1 := 0.5 \quad d2 := 0.2$

$h1 := 0.2 \quad h2 := 0.3 \quad \mu0 := 4\pi\cdot 10^{-7}$

Distances $r12 := h1 \qquad r12' := \sqrt{h2^2 + d2^2 - (h2-h1)^2}$

$$r1'2 := \sqrt{h1^2 + d1^2} \qquad r1'2' := \sqrt{h2^2 + \left[d1 - \sqrt{d2^2 - (h2-h1)^2}\right]^2}$$

Mutual inductance, *H/km*

$$M := \frac{\mu0}{2\cdot\pi}\cdot 10^3\cdot\ln\left(\frac{r12'\cdot r1'2'}{r12\cdot r1'2}\right) \qquad M = 7.109\times 10^{-5}$$

Solution of the problem 13.34

Initial data $h := 0.15$ $I := 3600$ $l := 0.5$ $ORIGIN := 1$

$\mu 0 := 4\cdot\pi\cdot 10^{-7}$ $W := 150$ $a := 0.004$ $b := 0.01$ $c := 0.005$

Curren density in the bus-bar, *A/mm^2* $\delta := \dfrac{-I}{2a\cdot h}$ $\delta = -3\times 10^{6}$

Poison's equation in Cartezian coordinate system

$$\frac{d^2}{dy^2}A = \begin{cases} -\mu 0\cdot\delta & \text{if } 0 \le y \le a \\ 0 & \text{otherwise} \end{cases}$$

Solution of Poison's equation $A1(y) = -0.5\cdot\mu 0\cdot\delta\cdot y^2 + C1\cdot y + C2$

Vector magnetic potential $A2(y) = C3\cdot y + C4$

Magnetic field intensity $H1(y) = \delta\cdot y - \dfrac{C1}{\mu 0}$ $H2(y) = \dfrac{-C3}{\mu 0}$

Equation system $A1(a) = A2(a)$ $H1(a) = H2(a)$ $H2(a) = \dfrac{-I}{2h}$ $C2 = 0$

Values for the first approximation $C1 := 0$ $C2 := 0$ $C3 := 0$ $C4 := 0$

Given $C2 = 0$

$$-0.5\cdot\mu 0\cdot\delta\cdot a^2 + C1\cdot a - C3\cdot a - C4 = 0 \qquad \delta\cdot a - \frac{C1}{\mu 0} + \frac{C3}{\mu 0} = 0 \qquad \frac{-C3}{\mu 0} = \frac{-I}{2\cdot h}$$

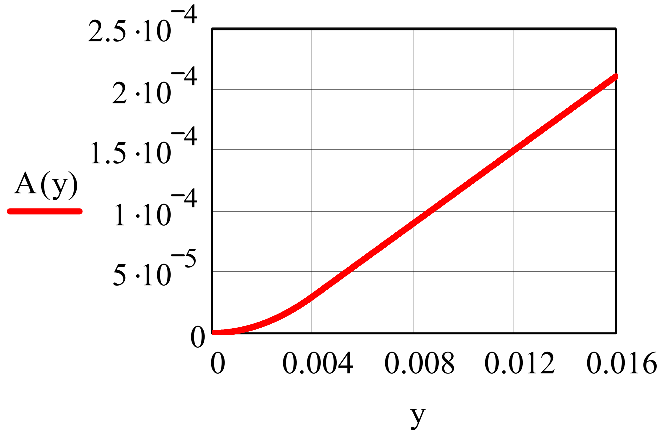
$C := \mathrm{Find}(C1, C2, C3, C4)$ $C^T = \begin{pmatrix} 0 & 0 & 0.015 & -3.016\times 10^{-5} \end{pmatrix}$

Answers for the vector magnetic potential, *Wb/m*

$A1(y) := -0.5\cdot\mu 0\cdot\delta\cdot y^2 + C_1\cdot y + C_2$ $A2(y) := C_3\cdot y + C_4$

$A1(y) \left|\begin{matrix}\text{simplify}\\ \text{float},4\end{matrix}\right. \to 1.885\cdot y^2$ $A2(y) \left|\begin{matrix}\text{simplify}\\ \text{float},4\end{matrix}\right. \to .1508e\text{-}1\cdot y - .3016e\text{-}4$

$A(y) := A1(y)\cdot(\Phi(y) - \Phi(y-a)) + A2(y)\cdot\Phi(y-a)$



Magnetic flux through the frame, *Wb*

$$F := l\cdot(A(b+c) - A(b))$$

$$F = 3.77\times 10^{-5}$$

Mutual inductance of the bus-bar and frame, *H* $\quad M := F\cdot\dfrac{W}{I} \qquad M = 1.571\times 10^{-6}$

Solution of the problem 13.35

Initial data $\quad h := 0.2 \qquad I := 600 \qquad l := 1 \qquad ORIGIN := 0$

$$\mu0 := 4\cdot\pi\cdot 10^{-7} \qquad \mu1 := 800 \qquad \mu2 := 400 \qquad a := 0.004$$

Bus-bar current density, *A/mm^2* $\quad \delta := \dfrac{I}{a\cdot h} \qquad \delta = 7.5\times 10^{5}$

Poison's equation in Cartezian coordinate system

$$\frac{d^2}{dx^2}A = \begin{cases} 0 & \text{if } x \le -2a \\ \mu0\cdot\mu1\cdot\delta & \text{if } -2a \le x \le -a \\ 0 & \text{if } -a \le x \le a \\ -\mu0\cdot\mu2\cdot\delta & \text{if } a \le x \le 2a \\ 0 & \text{otherwise} \end{cases}$$

Solution of Poison's equation
Vector magnetic potential

$$A0(x) = C0\cdot x + C1 \qquad A2(x) = C4\cdot x + C5 \qquad A4(x) = C8\cdot x + C9$$

$$A1(x) = 0.5\cdot\mu0\cdot\mu1\cdot\delta\cdot x^2 + C2\cdot x + C3 \qquad A3(x) = -0.5\cdot\mu0\cdot\mu2\cdot\delta\cdot x^2 + C6\cdot x + C7$$

Magnetic field intensity

$$H0(x) = \frac{-C0}{\mu0} \qquad H1(x) = -\delta\cdot x - \frac{C2}{\mu0} \qquad H2(x) = \frac{-C4}{\mu0}$$

$$H3(x) = \delta\cdot x - \frac{C6}{\mu0} \qquad H4(x) = \frac{-C8}{\mu0}$$

Equation system for integration constants

$$A0(-2a) = A1(-2a) \qquad H0(-2a) = H1(-2a) \qquad C0 = 0$$

$$A1(-a) = A2(-a) \qquad H1(-a) = H2(-a) \qquad C5 = 0$$

$$A2(a) = A3(a) \qquad A3(2a) = A4(2a) \qquad H2(a) = H3(a) \qquad H3(2a) = H4(2a)$$

Values for the first approximation

$C0 := 0 \quad C1 := 0 \quad C2 := 0 \quad C3 := 0 \quad C4 := 0$

$C5 := 0 \quad C6 := 0 \quad C7 := 0 \quad C8 := 0 \quad C9 := 0$

Given

$$C0 = 0 \qquad C0\cdot(-2a) + C1 = \left[0.5\cdot\mu0\cdot\mu1\cdot\delta\cdot(-2a)^2 + C2\cdot(-2a)\right] + C3$$

$$\frac{-C0}{\mu0} = -\delta\cdot(-2a) - \frac{C2}{\mu0\cdot\mu1} \qquad 0.5\cdot\mu0\cdot\mu1\cdot\delta\cdot(-a)^2 - C2\cdot a + C3 = -C4\cdot a + C5$$

$$-0.5\cdot\mu0\cdot\mu2\cdot\delta\cdot a^2 + C6\cdot a + C7 = C4\cdot a + C5 \qquad \frac{-C4}{\mu0} = -\delta\cdot(-a) - \frac{C2}{\mu0\cdot\mu1}$$

$$-0.5\cdot\mu0\cdot\mu2\cdot\delta\cdot(2a)^2 + C6\cdot 2\cdot a + C7 = C8\cdot 2\cdot a + C9 \qquad \frac{-C4}{\mu0} = \delta\cdot a - \frac{C6}{\mu0\cdot\mu2}$$

$$C5 = 0 \qquad \frac{-C8}{\mu0} = \delta\cdot 2\cdot a - \frac{C6}{\mu0\cdot\mu2}$$

$$C := \mathrm{Find}(C0, C1, C2, C3, C4, C5, C6, C7, C8, C9)$$

Answers for the vector magnetic potential, *Wb/m*

C =

	0
0	0
1	$-6.047\cdot10^{-3}$
2	6.032
3	0.018
4	$3.77\cdot10^{-3}$
5	0
6	3.016
7	$-9.033\cdot10^{-3}$
8	0
9	$3.031\cdot10^{-3}$

$$A0(x) := C_0\cdot x + C_1 \qquad A1(x) := 0.5\cdot\mu0\cdot\mu1\cdot\delta\cdot x^2 + C_2\cdot x + C_3$$

$$A2(x) := C_4\cdot x + C_5 \qquad A3(x) := -0.5\cdot\mu0\cdot\mu2\cdot\delta\cdot x^2 + C_6\cdot x + C_7$$

$$A4(x) := C_8\cdot x + C_9$$

$$A0(x)\ \Big|\ \text{simplify, float},4 \rightarrow -.6047e\text{-}2 \qquad A2(x)\ \Big|\ \text{simplify, float},4 \rightarrow .3770e\text{-}2\cdot x$$

$$A1(x)\ \Big|\ \text{simplify, float},4 \rightarrow 377.0\cdot x^2 + 6.032\cdot x + .1808e\text{-}1$$

$$A3(x)\ \Big|\ \text{simplify, float},4 \rightarrow (-188.5)\cdot x^2 + 3.016\cdot x - .9033e\text{-}2$$

$$A4(x)\ \Big|\ \text{simplify, float},4 \rightarrow .3031e\text{-}2$$

Answers for the magnetic intensity, *A/m*

$$H0(x) := \frac{-C_0}{\mu0} \qquad H1(x) := -\delta\cdot x + \frac{-C_2}{\mu0\cdot\mu1} \qquad H2(x) := \frac{-C_4}{\mu0}$$

$$H3(x) := \delta \cdot x + \frac{-C_6}{\mu 0 \cdot \mu 2} \qquad H4(x) := \frac{-C_8}{\mu 0}$$

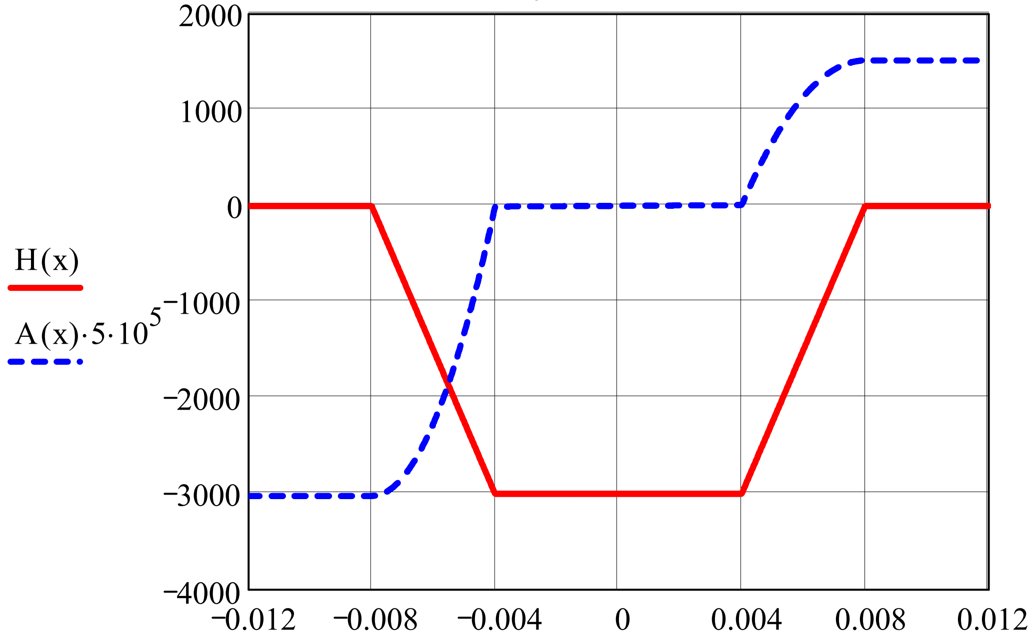
$$H0(x) \left| \begin{array}{l} simplify \\ float, 4 \end{array} \right. \rightarrow 0 \qquad H2(x) \left| \begin{array}{l} simplify \\ float, 4 \end{array} \right. \rightarrow -3000. \qquad H4(x) \left| \begin{array}{l} simplify \\ float, 4 \end{array} \right. \rightarrow 0$$

$$H1(x) \left| \begin{array}{l} simplify \\ float, 4 \end{array} \right. \rightarrow (-.7500e6) \cdot x - 6000. \qquad H3(x) \left| \begin{array}{l} simplify \\ float, 4 \end{array} \right. \rightarrow .7500e6 \cdot x - 6000.$$

$$\begin{aligned} A(x) := {} & A0(x) \cdot (1 - \Phi(x + 2a)) + A1(x) \cdot (\Phi(x + 2 \cdot a) - \Phi(x + a)) \ldots \\ & + A2(x) \cdot (\Phi(x + a) - \Phi(x - a)) + A3(x) \cdot (\Phi(x - a) - \Phi(x - 2a)) \ldots \\ & + A4(x) \cdot \Phi(x - 2a) \end{aligned}$$

$$\begin{aligned} H(x) := {} & H0(x) \cdot (1 - \Phi(x + 2a)) + H1(x) \cdot (\Phi(x + 2 \cdot a) - \Phi(x + a)) \ldots \\ & + H2(x) \cdot (\Phi(x + a) - \Phi(x - a)) + H3(x) \cdot (\Phi(x - a) - \Phi(x - 2a)) \ldots \\ & + H4(x) \cdot \Phi(x - 2a) \end{aligned}$$

Graphs *H(x)* and *A(x)*



Magnetic flux through the left bus-bar, *Wb* $\quad F := A(-a) - A(-2a) \qquad F = 6.032 \times 10^{-3}$

Magnetic flux through the right bus-bar, *Wb* $\quad FF := A(2a) - A(a) \qquad FF = 3.016 \times 10^{-3}$

Inner inductance, *H* $\quad L := \frac{F + FF}{I} \qquad L = 1.508 \times 10^{-5}$

Solution of the problem 13.36

Initial data $\quad \mu 1 := 10 \quad I := 1000 \quad ORIGIN := 0$

$\mu 0 := 4 \cdot \pi \cdot 10^{-7} \quad r1 := 0.1 \quad r2 := 0.15 \quad r3 := 0.2$

Current density in pipe, *A/mm^2*

$$\delta1 := \frac{I}{\pi\cdot\left(r2^2 - r1^2\right) + 1.5\cdot\pi\cdot\left(r3^2 - r2^2\right)}$$

$\delta1 = 8.214\times10^3$ $\delta2 := \delta1\cdot1.5$ $\delta2 = 1.232\times10^4$

Poison's equation in the cylindrical coordinate system

$$\frac{1}{r}\cdot\left[\frac{d}{dr}\left[r\cdot\left(\frac{d}{dr}A\right)\right]\right] = \begin{cases} 0 & \text{if } r \le r1 \\ -\mu0\cdot\mu1\cdot\delta1 & \text{if } r1 \le x \le r2 \\ -\mu0\cdot\delta2 & \text{if } r2 \le x \le r3 \\ 0 & \text{otherwise} \end{cases}$$

Solution of Poison's equation

Vector magnetic potential

$$A0(x) = C0\cdot\ln(r) + C1 \qquad A1(x) = -0.25\cdot\mu0\cdot\mu1\cdot\delta1\cdot r^2 + C2\cdot\ln(r) + C3$$

$$A2(x) = -0.25\cdot\mu0\cdot\delta2\cdot r^2 + C4\cdot\ln(r) + C5 \qquad A3(x) = C6\cdot\ln(r) + C7$$

Magnetic intensity $H0(x) = 0$

$$H1(x) = 0.5\delta1\cdot r - \frac{C2}{\mu0\cdot\mu1\cdot r} \qquad H2(x) = 0.5\delta2\cdot r - \frac{C4}{\mu0\cdot r} \qquad H3(x) = \frac{-C6}{\mu0\cdot r}$$

Equation system for integration constants

$$A0(0) = 0 \qquad C0 = 0 \qquad A0(r1) = A1(r1)$$

$$A1(r2) = A2(r2) \qquad H0(r1) = H1(r1)$$

$$A2(r3) = A3(r3) \qquad H1(r2) = H2(r2) \qquad H2(r3) = H3(r3)$$

Values for the first approximation

$$C0 := 0 \quad C1 := 0 \quad C2 := 0 \quad C3 := 0 \quad C4 := 0$$

$$C5 := 0 \quad C6 := 0 \quad C7 := 0$$

Given

$$C0 = 0 \qquad C1 = 0 \qquad -0.25\cdot\mu0\cdot\mu1\cdot\delta1\cdot r1^2 + C2\cdot\ln(r1) + C3 = C1$$

$$-0.25\cdot\mu0\cdot\mu1\cdot\delta1\cdot r2^2 + C2\cdot\ln(r2) + C3 = -0.25\cdot\mu0\cdot\delta2\cdot r2^2 + C4\cdot\ln(r2) + C5$$

$$-0.25\cdot\mu0\cdot\delta2\cdot r3^2 + C4\cdot\ln(r3) + C5 = C6\cdot\ln(r3) + C7$$

$$0 = 0.5\cdot\delta1\cdot r1 - \frac{C2}{\mu0\cdot\mu1\cdot r1} \qquad 0.5\cdot\delta1\cdot r2 - \frac{C2}{\mu0\cdot\mu1\cdot r2} = 0.5\cdot\delta2\cdot r2 - \frac{C4}{\mu0\cdot r2}$$

$$\frac{-C6}{\mu0\cdot r3} = 0.5\cdot\delta2\cdot r3 - \frac{C4}{\mu0\cdot r3}$$

$$C := \text{Find}(C0, C1, C2, C3, C4, C5, C6, C7)$$

$$C = \begin{pmatrix} 0 \\ 0 \\ 5.161 \times 10^{-4} \\ 1.446 \times 10^{-3} \\ 1.097 \times 10^{-4} \\ 1.819 \times 10^{-4} \\ -2 \times 10^{-4} \\ -4.714 \times 10^{-4} \end{pmatrix}$$

Answers for the vector magnetic potential, *Wb*

$$A0(r) := C_0 \cdot \ln(r) + C_1$$

$$A1(r) := -0.25 \cdot \mu0 \cdot \mu1 \cdot \delta1 \cdot r^2 + C_2 \cdot \ln(r) + C_3$$

$$A2(r) := -0.25 \cdot \mu0 \cdot \delta2 \cdot r^2 + C_4 \cdot \ln(r) + C_5$$

$$A3(r) := C_6 \cdot \ln(r) + C_7$$

$$A0(r) \;\Big|\; \begin{matrix} \text{simplify} \\ \text{float}, 4 \end{matrix} \rightarrow 0$$

$$A1(r) \;\Big|\; \begin{matrix} \text{simplify} \\ \text{float}, 4 \end{matrix} \rightarrow (-.2581e\text{-}1) \cdot r^2 + .5161e\text{-}3 \cdot \ln(r) + .1446e\text{-}2$$

$$A2(r) \;\Big|\; \begin{matrix} \text{simplify} \\ \text{float}, 4 \end{matrix} \rightarrow (-.3871e\text{-}2) \cdot r^2 + .1097e\text{-}3 \cdot \ln(r) + .1819e\text{-}3$$

$$A3(r) \;\Big|\; \begin{matrix} \text{simplify} \\ \text{float}, 4 \end{matrix} \rightarrow (-.2000e\text{-}3) \cdot \ln(r) - .4714e\text{-}3$$

$$A(r) := A0(r) \cdot (\Phi(r) - \Phi(r - r1)) + A1(r) \cdot (\Phi(r - r1) - \Phi(r - r2)) \; ... \\ + A2(r) \cdot (\Phi(r - r2) - \Phi(r - r3)) + A3(r) \cdot \Phi(r - r3)$$

Graph *A(r)*