APPENDIX TO CHAPTER 11

Solution of the problem 11.42

Initial data

$E0 := 12\cdot 10^{3}$ $\quad d := 0.5$ $\quad \varepsilon 0 := 8.86\cdot 10^{-14}$

$a := 25$ $\quad b := 100$ $\quad \varepsilon := 4$

Electric intensity, *V/cm*

$$E(x) := E0\cdot\left(1 - \frac{x^2}{2\cdot d^2}\right) \qquad E(x) \;\Big|\; \begin{matrix}\text{simplify}\\ \text{float},4\end{matrix} \rightarrow .1200e5 - .2400e5\cdot x^2$$

Volume density of free charge between plates, *C/cm*³

$$\rho(x) := \varepsilon\cdot\varepsilon 0\cdot\left(\frac{d}{dx}E(x)\right) \qquad \rho(x) \;\Big|\; \begin{matrix}\text{simplify}\\ \text{float},4\end{matrix} \rightarrow (-.1701e\text{-}7)\cdot x$$

Summary charge between plates, *C*

$$Q := \int_0^d a\cdot b\cdot\rho(x)\,dx \qquad Q = -5.316\times 10^{-6}$$

Voltage between plates, *V* $\quad U := \int_0^d E(x)\,dx \qquad U = 5\times 10^{3}$

Solution of the problem 11.43

Initial data

$R1 := 0.2$ $\quad R2 := 0.4$ $\quad \varepsilon := 4$ $\quad \varepsilon 0 := 8.86\cdot 10^{-12}$

$\rho := 5\cdot 10^{-6}$ $\quad ORIGIN := 1$

$$\phi(R, A1, A2) := \frac{-\rho}{6\varepsilon\cdot\varepsilon 0}\cdot R^2 + \frac{-A1}{R} + A2 \qquad E(R, A1, A2) := \frac{\rho}{3\varepsilon\cdot\varepsilon 0}\cdot R - \frac{A1}{R^2}$$

Values for the first approximation $\quad A1 := 0 \quad A2 := 0$

Equation system $\quad$ Given $\quad \phi(R2, A1, A2) = 0 \quad E(R1, A1, A2) = 0$

$X := \text{Find}(A1, A2)$ $\qquad X = \begin{pmatrix} 376.223 \\ 4.703\times 10^{3} \end{pmatrix}$

Ancwers: A1 [*V/m*], A2 [*B*] $\quad A1 := X_1 \quad A2 := X_2$

$A1 = 376.223 \qquad A2 = 4.703\times 10^{3}$

Solution of the problem 11.44

Initial data

$R1 := 0.04$ $R2 := 0.1$ $\varepsilon := 4$ $\varepsilon0 := 8.86\cdot 10^{-12}$

$\rho := 10^{-4}$ $U := 40000$ $ORIGIN := 0$

$$\phi(R,q,A1,A2) := \frac{-\rho}{6\varepsilon\cdot\varepsilon0}\cdot R^2 + \frac{-A1}{R} + A2$$

$$E(R,q,A1,A2) := \frac{\rho}{3\varepsilon\cdot\varepsilon0}\cdot R - \frac{A1}{R^2}$$

Values for the first approximation $q := 0$ $A1 := 0$ $A2 := 0$

Equation system Given $\phi(R2,q,A1,A2) = 0$

$\phi(R1,q,A1,A2) = U$ $E(R1,q,A1,A2) = \dfrac{q}{4\pi\cdot\varepsilon\cdot\varepsilon0\cdot R1^2}$

$X := Find(q,A1,A2)$

$$X = \begin{pmatrix} 1.097\times 10^{-6} \\ -2.403\times 10^{3} \\ -1.933\times 10^{4} \end{pmatrix}$$

Answers: q[*C*], A1 [*V*/*m*], A2 [*V*]

$q := X_0$ $A1 := X_1$ $A2 := X_2$

$q = 1.097\times 10^{-6}$ $A1 = -2.403\times 10^{3}$ $A2 = -1.933\times 10^{4}$

Solution of the problem 11.45

Initial data

$r1 := 0.01$ $r2 := 0.1$ $\varepsilon := 4$ $\varepsilon0 := 8.86\cdot 10^{-12}$

$\rho := 10^{-4}$ $l := 100$ $ORIGIN := 1$

$$\phi(r,A1,A2) := \frac{-\rho}{4\varepsilon\cdot\varepsilon0}\cdot r^2 + A1\cdot\ln(r) + A2 \qquad E(r,A1,A2) := \frac{\rho}{2\varepsilon\cdot\varepsilon0}\cdot r - \frac{A1}{r}$$

Values for the first approximation $A1 := 0$ $A2 := 0$

Equation system Given

$\phi(r2,A1,A2) = 0$ $E(r1,A1,A2) = 0$

$X := Find(A1,A2)$

$$X = \begin{pmatrix} 141.084 \\ 7.379\times 10^{3} \end{pmatrix}$$

$$\phi(r) := \frac{-\rho}{4\varepsilon\cdot\varepsilon 0}\cdot r^2 + X_1\cdot\ln(r) + X_2$$

$$\phi(r) \left| \begin{array}{l} \text{simplify} \\ \text{float},4 \end{array} \right. \rightarrow (-.7054e6)\cdot r^2 + 141.1\cdot\ln(r) + 7379.$$

Sought energy determined in two ways, *J*

$$W1 := \int_{r1}^{r2} \rho\cdot\pi\cdot l\cdot\phi(r)\cdot r\,dr \qquad W1 = 0.533$$

$$E(r) := \frac{\rho}{2\varepsilon\cdot\varepsilon 0}\cdot r - \frac{X_1}{r} \qquad E(r) \left| \begin{array}{l} \text{simplify} \\ \text{float},4 \end{array} \right. \rightarrow 141.1\cdot\frac{.1000e5\cdot r^2 - 1.}{r}$$

$$W2 := \int_{r1}^{r2} \varepsilon\cdot\varepsilon 0\cdot\pi\cdot l\cdot E(r)^2\cdot r\,dr \qquad W2 = 0.533$$

$$W := \frac{\rho^2\cdot\pi\cdot l}{2\cdot\varepsilon\cdot\varepsilon 0}\cdot\left[\frac{3\cdot r1^4 + r2^4}{8} + (-0.5)\cdot r1^2\cdot r2^2 + (-0.5)\cdot r1^4\cdot\ln\left(\frac{r1}{r2}\right)\right]$$

Verification $W = 0.533$

Solution of the problem 11.46

Initial data

$R1 := 0.01 \quad R2 := 0.03 \quad R3 := 0.04 \quad R4 := 0.05$

$\varepsilon 1 := 2 \quad \varepsilon 2 := 1 \quad \varepsilon 0 := 8.86\cdot 10^{-12} \quad ORIGIN := 0$

$U := 1000 \quad \rho := 10^{-4}$

Formulae for potential in two layers of dielectric determined with the of Gauss' law of flux

$$\phi 1(q,R,A1,A2) := \frac{q - \rho\cdot\frac{4\pi\cdot R1^3}{3}}{4\pi\cdot\varepsilon 1\cdot\varepsilon 0\cdot R} + \frac{-\rho\cdot R^2}{6\cdot\varepsilon 1\cdot\varepsilon 0} + A1$$

$$\phi 2(q,R,A1,A2) := \frac{q + \rho\cdot\frac{4\cdot\pi\cdot\left(R2^3 - R1^3\right)}{3}}{4\cdot\pi\cdot\varepsilon 2\cdot\varepsilon 0\cdot R} + A2$$

Values for the first approximation $\quad q := 0 \quad A1 := 0 \quad A2 := 0$

Equation system

Given

$$\phi 2(q, R4, A1, A2) = 0$$

$$\phi 1(q, R1, A1, A2) = U$$

$$\phi 1(q, R2, A1, A2) = \phi 2(q, R3, A1, A2)$$

$$X := \mathrm{Find}(q, A1, A2)$$

$$X = \begin{pmatrix} -3.373 \times 10^{-10} \\ 1.434 \times 10^{3} \\ -1.896 \times 10^{3} \end{pmatrix}$$

Answers: A1 $[B]$, A2 $[B]$, q $[C]$

$q := X_0$ $A1 := X_1$ $A2 := X_2$

$q = -3.373 \times 10^{-10}$ $A1 = 1.434 \times 10^{3}$ $A2 = -1.896 \times 10^{3}$

Solution of the problem 11.47

Initial data $U := 220 \cdot 10^{3}$ $\varepsilon 0 := 8.86 \cdot 10^{-12}$ $d := 2$ $r0 := 0.01$

Capacitance of two-wire line, F/m

$$C0 := \frac{\pi \cdot \varepsilon 0}{\ln\left(\frac{d - r0}{r0}\right)} \qquad C0 = 5.258 \times 10^{-12}$$

The line charge per unit length, C/m

$$\tau := C0 \cdot U \qquad \tau = 1.157 \times 10^{-6}$$

Electric intensity components from left and right wires, respectively

$$E1(x) := \frac{\tau}{2\pi \cdot \varepsilon 0 \cdot (0.5d + x)}$$

$$E2(x) := \frac{\tau}{2\pi \cdot \varepsilon 0 \cdot |0.5d - x|}$$

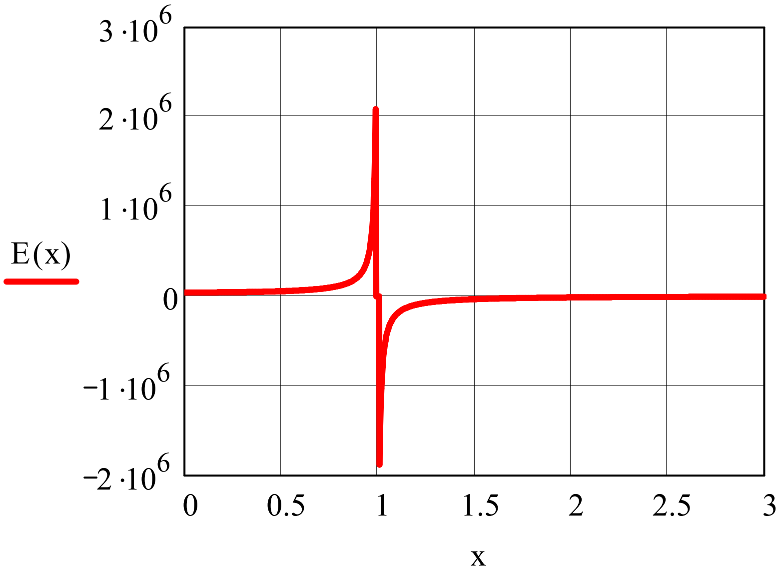
Electric field intensity E against

$$E(x) := (E1(x) + E2(x)) \cdot (\Phi(x) - \Phi(x - 0.5 \cdot d + r0)) \ldots + (E1(x) - E2(x)) \cdot (\Phi(x - 0.5 \cdot d - r0))$$

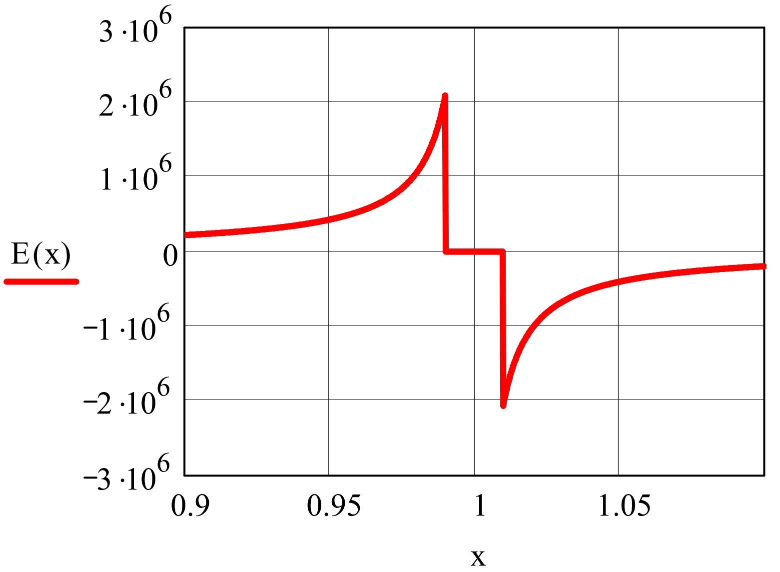
Maximum values of the electric intensity, V/m

$$Emax := E(0.5 \cdot d - r0)$$

$$Emax = 2.089 \times 10^{6}$$



a)



b) The same dependence as a), however, in interval of x from 0.9m to 1.1 m.

Solution of the problem 11.48

Initial data

$E1 := 5\cdot10^3$ $E2 := 2\cdot10^3$ $\varepsilon0 := 8.86\cdot10^{-12}$

$d12 := 3$ $d23 := 2$ $r0 := 0.01$

$h_1 := 5$ $h_2 := 7$ $h_3 := 6$ $ORIGIN := 1$

Distance between wires, m

$$a12 := \sqrt{(h_1 - h_2)^2 + d12^2} \qquad a12 = 3.606$$

$$a13 := \sqrt{(h_1 - h_3)^2 + (d12 + d23)^2} \qquad a13 = 5.099$$

$$a23 := \sqrt{(h_2 - h_3)^2 + d23^2} \qquad a23 = 2.236$$

Distance between wires and their copies, *m*

$$b12 := \sqrt{(h_1 + h_2)^2 + d12^2} \qquad b12 = 12.369$$

$$b13 := \sqrt{(h_1 + h_3)^2 + (d12 + d23)^2} \qquad b13 = 12.083$$

$$b23 := \sqrt{(h_2 + h_3)^2 + d23^2} \qquad b23 = 13.153$$

Potential coefficients, *m/F*

$$i := 1..3 \qquad \alpha_{i,i} := \frac{1}{2\pi \cdot \varepsilon 0} \cdot \ln\left(\frac{2 \cdot h_i}{r0}\right)$$

$$\alpha_{1,2} := \frac{1}{2\pi \cdot \varepsilon 0} \cdot \ln\left(\frac{b12}{a12}\right) \quad \alpha_{1,3} := \frac{1}{2\pi \cdot \varepsilon 0} \cdot \ln\left(\frac{b13}{a13}\right) \quad \alpha_{2,3} := \frac{1}{2\pi \cdot \varepsilon 0} \cdot \ln\left(\frac{b23}{a23}\right)$$

$$\alpha_{2,1} := \alpha_{1,2} \qquad \alpha_{3,1} := \alpha_{1,3} \qquad \alpha_{3,2} := \alpha_{2,3}$$

$$\alpha = \begin{pmatrix} 1.241 \times 10^{11} & 2.214 \times 10^{10} & 1.55 \times 10^{10} \\ 2.214 \times 10^{10} & 1.301 \times 10^{11} & 3.183 \times 10^{10} \\ 1.55 \times 10^{10} & 3.183 \times 10^{10} & 1.274 \times 10^{11} \end{pmatrix}$$

The first approximation

$$\phi 1 := 0 \quad \phi 2 := 0 \quad \phi 3 := 0 \qquad \tau 1 := 0 \quad \tau 2 := 0 \quad \tau 3 := 0$$

Given

Equation system

The first group of Maxwell's formulae

$$\phi 1 = \alpha_{1,1} \cdot \tau 1 + \alpha_{1,2} \cdot \tau 2 + \alpha_{1,3} \cdot \tau 3$$

$$\phi 2 = \alpha_{1,2} \cdot \tau 1 + \alpha_{2,2} \cdot \tau 2 + \alpha_{2,3} \cdot \tau 3$$

$$\phi 3 = \alpha_{1,3} \cdot \tau 1 + \alpha_{2,3} \cdot \tau 2 + \alpha_{3,3} \cdot \tau 3$$

Additional conditions

$$\phi 1 - \phi 2 = E1 \qquad \phi 3 = E2 \qquad \tau 1 + \tau 2 = 0$$

Solution of the equation system: ϕ [*V*], τ [*C/m*]

$$X := \mathrm{Find}(\phi 1, \phi 2, \phi 3, \tau 1, \tau 2, \tau 3)$$

Ancwers

$$X = \begin{pmatrix} 2.872 \times 10^{3} \\ -2.128 \times 10^{3} \\ 2 \times 10^{3} \\ 2.529 \times 10^{-8} \\ -2.529 \times 10^{-8} \\ 1.895 \times 10^{-8} \end{pmatrix}$$

Matrix of capacitive coefficients, *F/m*

$$\beta := \alpha^{-1}$$

$$\beta = \begin{pmatrix} 8.37 \times 10^{-12} & -1.252 \times 10^{-12} & -7.057 \times 10^{-13} \\ -1.252 \times 10^{-12} & 8.372 \times 10^{-12} & -1.94 \times 10^{-12} \\ -7.057 \times 10^{-13} & -1.94 \times 10^{-12} & 8.422 \times 10^{-12} \end{pmatrix}$$

Capacitances, *F/m*

$$k := 1 .. 3 \qquad i := 1 .. 3$$

$$C_{i,k} := -\beta_{i,k} \qquad C_{i,i} := \sum_{k=1}^{3} \beta_{i,k}$$

$$C = \begin{pmatrix} 6.413 \times 10^{-12} & 1.252 \times 10^{-12} & 7.057 \times 10^{-13} \\ 1.252 \times 10^{-12} & 5.18 \times 10^{-12} & 1.94 \times 10^{-12} \\ 7.057 \times 10^{-13} & 1.94 \times 10^{-12} & 5.777 \times 10^{-12} \end{pmatrix}$$