6. LINEAR CIRCUITS AT PERIODICAL NON-SINUSOIDAL VOLTAGES AND CURRENTS

6.1. EXPANSION OF A PERIODIC FUNCTION INTO THE FOURIER SERIES

6-1 (6.1). The dependence *u(t)* is shown in fig. 6.1,a and simultaneously set by table 6.1 (but for the first quarter of period), it is symmetrical as regard to both the coordinate origin (the dependence being odd) and the abscissa axis ($u(t) = -u(-t) = -u(t+T/2)$).

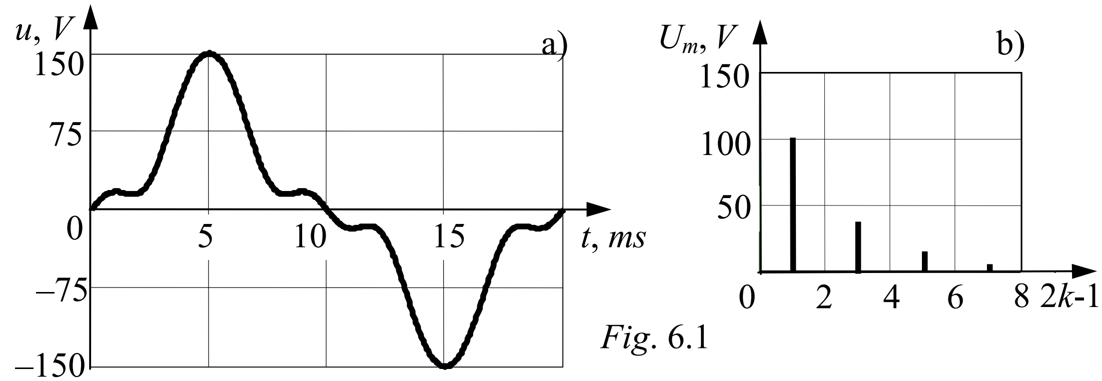


Fig. 6.1

Expand the dependence *u(t)* into the Fourier series and construct its linear frequency spectrum.

Table 6.1. The values of the function *u(t)* for the first quarter of a period at $\Delta t = 0.5$ *ms*

n	1	2	3	4	5	6	7	8	9	10
t, ms	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
u_n, *V*	12.35	17.53	15.89	16.09	28.15	54.93	89.78	121.7	142.7	149.8

Solution. The function *u(t)* possesses simultaneously two kinds of symmetry. It is an odd one and at the same time it is symmetrical as regard to the abscissa axis. That's why its series expansion includes only sinusoids with odd serial number, while the integral to determine the harmonic amplitude of the number $(2k-1)$ is calculated for the quarter of a period with subsequent multiplication of the result by 4. Then the value of the amplitude U_{m2k-1} is determined under the expression:

$$U_{m\,2k-1} = \frac{2}{T}\int_0^T u(t)\sin((2k-1)\omega\cdot t)dt = \frac{8}{T}\int_0^{T/4} u(t)\sin((2k-1)\omega\cdot t)dt. \qquad (6.1)$$

As the function is set by the table, it is expanded into the Fourier series by the grapho-analytical method. While using an approximate integration, the function period is divided into the equal intervals (in the given example, their number is $N = 40$) and then the substitution $dt = T/N = T/40$ is performed. However, as the function has two different values at the ends of two symmetrical intervals equal to a quarter of the period, then in order to obtain the better result owing to compensation of the positive inaccuracy of one period by the negative inaccuracy of another symmetrical interval the approximate integration is to be performed for the half period. That's why let's extend the table 6.1 to the half of period.

Continuation of table 6.1. The function values *u(t)* for the 2nd quarter of a period

n	11	12	13	14	15	16	17	18	19	20
t, ms	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
u_n, *V*	142.7	121.7	89.78	54.93	28.15	16.09	15.89	17.53	12.35	0

Then the latter expression (6.1) is reduced to the following view (summation for the half period):

$$U_{m\,2k-1} \approx \frac{4}{T}\sum_{n=1}^{20} u_n\, sin((2k-1)\omega \cdot t_n)\frac{T}{40} = \frac{1}{10}\sum_{n=1}^{20} u_n\, sin((2k-1)\omega \cdot t_n), \qquad (6.2)$$

where $T = 0.02\ s$ – a period of function $u(t)$;
$n = 1\ldots20$ – the number of an interval of the approximate integration at $\Delta t = T/40$.

Using the data from table 6.1 we calculate the amplitudes of the first decade of the harmonic components (taking into account only the odd harmonics) in accordance with the expression (6.2), and receive the result:

$U_{m1} = 100\ V$; $U_{m3} = -40\ V$; $U_{m5} = 15\ V$; $U_{m7} = 5\ V$; $U_{m9} = -0{,}19\ V$.

The ninth harmonic is negligible and can be excluded from the further calculations.

So, the instantaneous value of the the expansion of the function $u(t)$ into the Fourier series(odd harmonics 1…9) is as follows:

$$u(t) = \sum_{k=1}^{9} U_{mk}\, sin(k\omega t) = 100 sin(\omega t) - 40 sin(3\omega t) + 15 sin(5\omega t) + 5 sin(7\omega t)\ V.$$

The plots of function $u(t)$ and amplitude frequency spectrum are in fig. 6.1.

6-2 (6.2). Expand a periodic saw-tooth voltage described in the interval $0 < \omega t < 2\pi$ by the expression $u(\omega t) = \dfrac{\omega t}{2\pi}$ into the Fourier series.

Solution. Direct-current component (zero harmonic) is as follows:

$$U_0 = \frac{1}{2\pi}\int_0^{2\pi} u(\omega t)d(\omega t) = \frac{1}{4\pi^2}\int_0^{2\pi} \omega t d(\omega t) = \frac{1}{2}.$$

By the way, it may also be found from the formula of the area of a triangle.

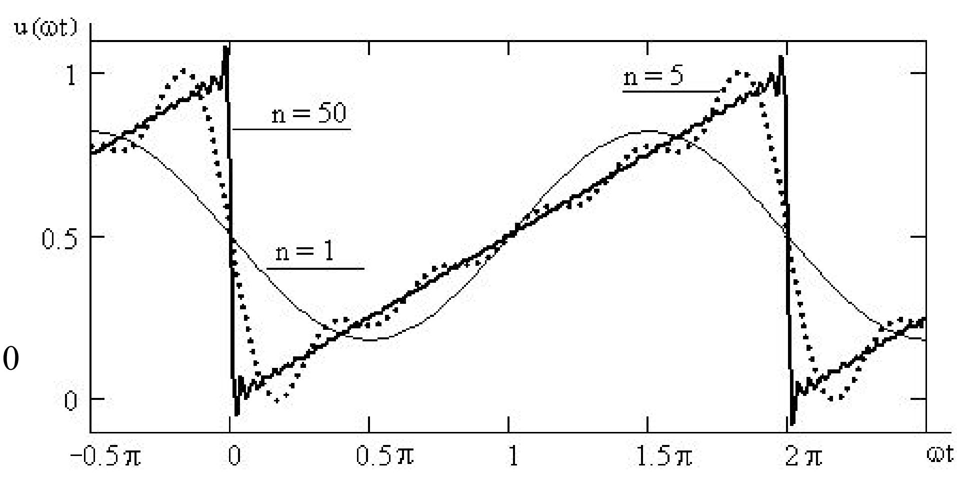
The coefficients of *sin*-components of k^{th}-harmonic are set by the following expression:

$$U'_k = \frac{1}{2\pi^2}\int_0^{2\pi} \omega t\, sin(k\omega t)d(\omega t) = \frac{sin(k2\pi) - k2\pi\, cos(k2\pi)}{2(k\pi)^2} = -\frac{1}{k\pi}.$$

Similarly, for the coefficients of *cos*-components:

$$U''_k = \frac{1}{2\pi^2}\int_0^{2\pi} \omega t\, cos(k\omega t)d(\omega t) = \frac{k2\pi\, sin(k2\pi) + cos(k2\pi) - cos(0)}{2(k\pi)^2} = 0.$$

Fig. 6.2. Plots of function $u(\omega t)$. $n = 1$ – the 1st harmonic of expansion (6.3); $n = 5$ – sum of 5 harmonics; $n = 50$ – sum of 50 harmonics.



So, there are but *sin*-components in the Fourier series expansion of the function $u(\omega t)$, the expansion having a view: $u(\omega t)=\frac{1}{2}-\frac{1}{\pi}\cdot\sum_{k=1}^{n}\frac{sin(k\omega t)}{k}$. (6.3)

In fig. 6.2 there are plots of the function $u(\omega t)$ obtained in accordance with (6.3) for different number of harmonics. Obviously, the increase of the number of harmonics improves the performance accuracy for the original function. However, there are specific spikes in the ordinary jumps of function (Gibbs phenomenon).

6-3 (6.5). This problem illustrates the application of the complex transfer function, which was first considered in problem 5.13.

Calculate and plot the output voltage $u_2(t)$ of the scheme with the complex transfer function $\underline{Z}(j\omega)=\frac{83333}{j\omega+66.67}$ *Ohm* under the condition that the source of the non-sinusoidal current acts at the circuit input, its plot is presented in fig. 6.3.

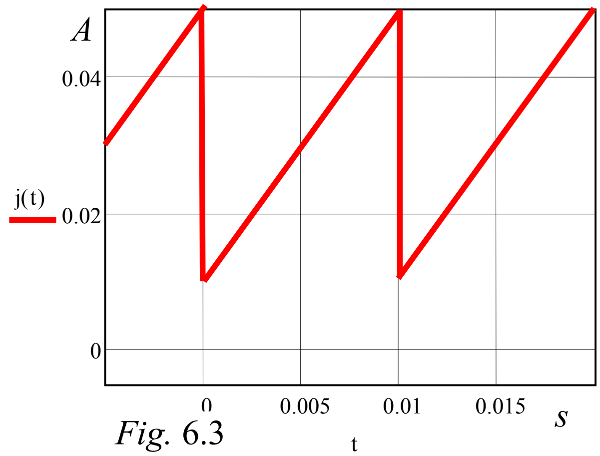


Fig. 6.3

Solution. Let's perform the expansion of the plot fig. 6.3 into the Fourier series. The period and angular velocity of the fundamental harmonic are:

$$T=0.01\ s,\quad \omega=2\pi/T=628\ rad/s.$$

The source current in the range $0\le t\le T$ can be analytically described as:

$$j(t) := 0.01+4\cdot t\ A.$$

Let the quantity of harmonics under consideration equal to $m := 10$, it means the harmonic numbers change in the range $k := 1\ ..\ m$.

The steady component is: $j0(t) := \frac{1}{T}\cdot\int_0^T j(t)dt$ $\quad j0(0.01)=0.03$,

i.e. $Bm_0 := 0.03,\ Cm_0 := 0$.

The amplitudes of the *sin*- and *cos*-components are:

$$Bm_k := \frac{2}{T}\cdot\int_0^T j(t)\cdot sin(k\cdot\omega\cdot t)dt \qquad Cm_k := \frac{2}{T}\cdot\int_0^T j(t)\cdot cos(k\cdot\omega\cdot t)dt.$$

$Bm^T =$

	0	1	2	3	4	5
0	0.03	-0.013	$-6.366\cdot10^{-3}$	$-4.244\cdot10^{-3}$	$-3.183\cdot10^{-3}$	$-2.546\cdot10^{-3}$

$Cm^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	10^{-14}	0	0	0	10^{-14}	0

Apparently, there are no *cos*-components in the expansion.

The total amplitudes and the initial phases of the harmonics are:

$$Jm_0 := j0(0.01)\quad Jm_k := \sqrt{(Bm_k)^2+(Cm_k)^2}\quad \psi_k := if\left(Bm_k>0, atan\left(\frac{Cm_k}{Bm_k}\right), \pi+atan\left(\frac{Cm_k}{Bm_k}\right)\right)$$

$$\psi_0 := \pi/2$$

$\frac{\psi^T}{deg}$ =	0	1	2	3	4	5	6	7	8	9
0	90	180	180	180	180	180	180	180	180	180

Jm^T =	0	1	2	3	4	5
0	0.03	0.013	$6.366\cdot10^{-3}$	$4.244\cdot10^{-3}$	$3.183\cdot10^{-3}$	$2.546\cdot10^{-3}$

The individual harmonics and the source total current are:

$j1(t) := Jm_1\cdot sin(\omega\cdot t + \psi_1)$ $\quad$ $j2(t) := Jm_2\cdot sin(2\cdot\omega\cdot t + \psi_2)$

$j3(t) := Jm_3\cdot sin(3\cdot\omega\cdot t + \psi_3)$ $\quad$ $j4(t) := Jm_4\cdot sin(4\cdot\omega\cdot t + \psi_4)$

$j5(t) := Jm_5\cdot sin(5\cdot\omega\cdot t + \psi_5)$ $\quad$ $j6(t) := Jm_6\cdot sin(6\cdot\omega\cdot t + \psi_6)$

$$j(t) := j0(t) + \sum_{k=1}^{m}\left(Jm_k\cdot sin(k\cdot\omega\cdot t + \psi_k)\right).$$

The answers with numerical data are: $j_0(t) = 0.03\ A$,

$j_1(t) = 0.013 sin(\omega\cdot t + 180°)\ A$, $\quad$ $j_2(t) = 0.00637 sin(2\cdot\omega\cdot t + 180°)\ A$,

$j_3(t) = 0.00424 sin(3\cdot\omega\cdot t + 180°)\ A$, $\quad$ $j_4(t) = 0.00138 sin(4\cdot\omega\cdot t + 180°)\ A$,

$j_5(t) = 0.00255 sin(5\cdot\omega\cdot t + 180°)\ A$, $\quad$ $j_6(t) = 0.00212 sin(6\cdot\omega\cdot t + 180°)\ A$.

The plots of the direct-current component, the first five harmonics and the total current with account of the first 11 harmonics are presented in fig. 6.4.

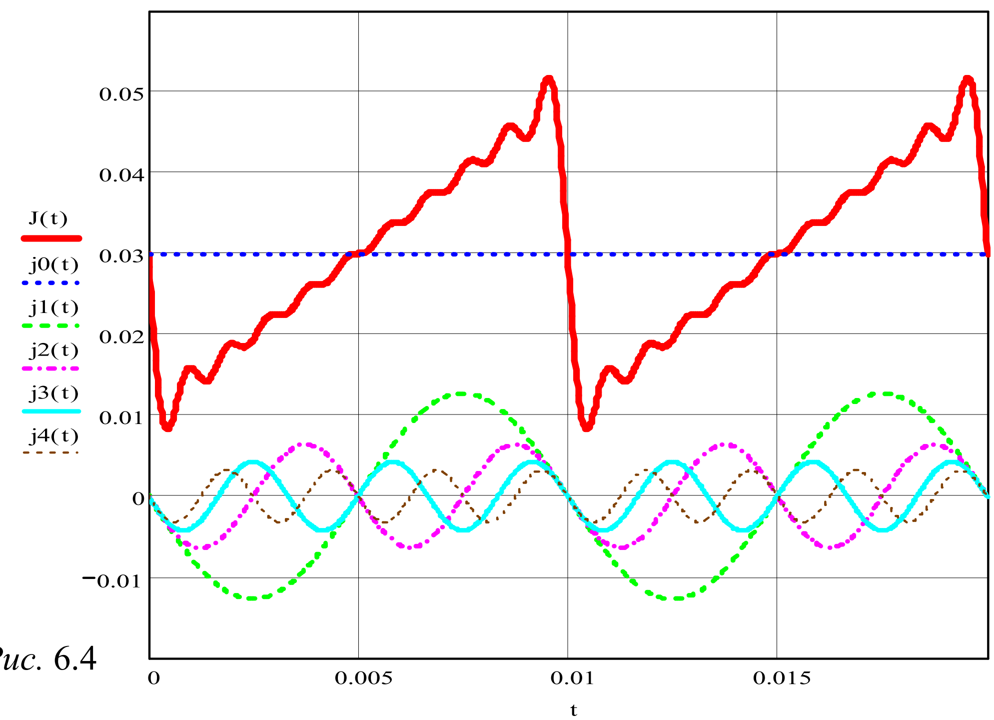


Рис. 6.4

With the aid of the complex transfer impedance or more exactly of the amplitude frequency and phase-frequency characteristics obtained when solved problem 5.13

$$\underline{Z}(j\omega) = \frac{83333}{j\omega + 66.67}\ Ohm, \quad Z(\omega) = \sqrt{\frac{83333^2}{\omega^2 + 66.67^2}}\ Ohm, \quad \varphi(\omega) = -arctg\frac{\omega}{66.67},$$

we determine the harmonics of the output voltage:

$u20(t) := Z(0)\cdot j0(t)$ $\quad$ $U2m_k := Z(k\cdot\omega)\cdot Jm_k$ $\quad$ $\psi u2_k := \psi_k + \varphi(k\cdot\omega)$

$U2m^T$ =	0	1	2	3	4	5	6	7	8	9
0	37.5	1.679	0.422	0.188	0.106	0.068	0.047	0.034	0.026	0.021

$\frac{\psi u2^T}{deg} =$

	0	1	2	3	4	5	6	7
0	0	96.057	93.037	92.026	91.519	91.216	91.013	90.868

$u_{20}(t) = 37.5\ V$,

$u_{21}(t) = 1.679 sin(\omega \cdot t + 96.06°)\ V$, $u_{22}(t) = 0.422 sin(2 \cdot \omega \cdot t + 93.03°)\ V$,

$u_{23}(t) = 0.188 sin(3 \cdot \omega \cdot t + 92.03°)\ V$, $u_{24}(t) = 0.106 sin(4 \cdot \omega \cdot t + 91.52°)\ V$,

$u_{25}(t) = 0.068 sin(5 \cdot \omega \cdot t + 91.22°)\ V$, $u_{26}(t) = 0.047 sin(6 \cdot \omega \cdot t + 91.01°)\ V$.

The individual harmonics and the total output voltage are:

$u21(t) := U2m_1 \cdot sin(\omega \cdot t + \psi u2_1)$ $u22(t) := U2m_2 \cdot sin(2 \cdot \omega \cdot t + \psi u2_2)$

$u23(t) := U2m_3 \cdot sin(3 \cdot \omega \cdot t + \psi u2_3)$ $u24(t) := U2m_4 \cdot sin(4 \cdot \omega \cdot t + \psi u2_4)$

$u25(t) := U2m_5 \cdot sin(5 \cdot \omega \cdot t + \psi u2_5)$ $u26(t) := U2m_6 \cdot sin(6 \cdot \omega \cdot t + \psi u2_6)$

$$u2(t) := u20(t) + \sum_{k=1}^{m} \left(U2m_k \cdot sin(k \cdot \omega \cdot t + \psi u2_k) \right).$$

The plots of the direct-current component, the first four harmonics and the full output voltage with account of the 11 harmonics are presented in fig. 6.5.

It is seen from the graph, that amplitude of the first harmonic of the voltage does not exceed 2 *V*, while the amplitudes of the subsequent harmonics are less than 0.5 *V*. Thus, the variations of the output voltage are not significant, it is smoothed owing to the application of an electric low-pass filter.

6.2. SINGLE-PHASE CIRCUITS OF THE NONSINUSOIDAL CURENT

6-4 (6.6). The series circuit $r = 40\ Ohm$ and $C = 40\ \mu F$ is supplied with the voltage which changes according to the law (see problem 6-1):

$$u(t) = Um^{(1)} sin(\omega t) + Um^{(3)} sin(3\omega t) + Um^{(5)} sin(5\omega t) + \dots, \quad (6.4)$$

where $Um^{(1)} = 100\ V$, $Um^{(3)} = -40\ V$, $Um^{(5)} = 15\ V$, $Um^{(7)} = 5\ B$, $Um^{(9)} = -0.19\ V$.

In a common coordinate system, plot the voltages across the resistance, the capacitance, as well as the source voltage. Determine their effective values, coefficients of distortion, harmonics; verify the power balance.

Solution. We solve the problem with the aid of MathCAD. Information input: arrays of amplitudes and initial phases of the source voltage, its period *T*, angular velocity *ω*, the circuit parameters *r*, *C* and a ranked variable *k*:

$$T := 0.02 \quad r := 40 \quad C := 40 \cdot 10^{-6} \quad k := 1..5$$

$$Um := \begin{pmatrix} 100 \\ 40 \\ 15 \\ 5 \\ 0.19 \end{pmatrix} \quad \psi := \begin{pmatrix} 0 \\ \pi \\ 0 \\ 0 \\ \pi \end{pmatrix} \quad \omega := \frac{2 \cdot \pi}{T}$$

In order to determine the arrays of the complex amplitudes of the source voltage, input impedances, the complex amplitudes of the currents and voltages across the capacitance we create the user functions taking into account that expansion (6.4) contains but the odd harmonics and the harmonic numbers $2 \cdot k - 1 = 1, 3, 5, \dots$ correspond to the variable $k = 1, 2, 3 \dots$, respectively:

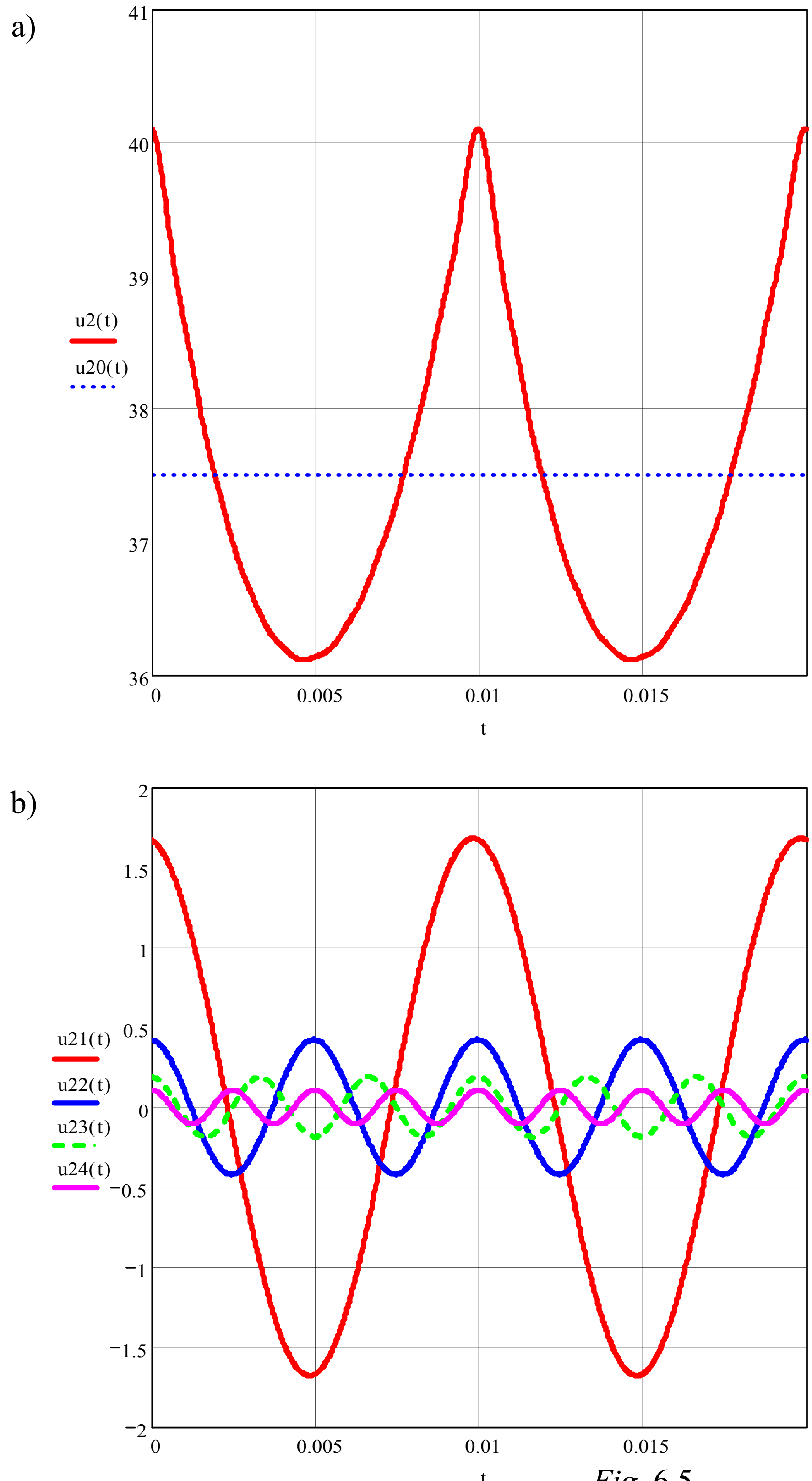

a)
41
40
39
38
37
36
u2(t)
u20(t)
0
0.005
0.01
0.015
t
b)
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
u21(t)
u22(t)
u23(t)
u24(t)
0
0.005
0.01
0.015
t

Fig. 6.5

$$Um_k := Um_k \cdot e^{j \cdot \psi_k} \qquad Z_k := r + \frac{1}{j \cdot (2 \cdot k - 1) \cdot \omega \cdot C} \qquad Im_k := \frac{Um_k}{Z_k}$$

$$Ucm_k := Im_k \cdot \frac{1}{j \cdot (2 \cdot k - 1) \cdot \omega \cdot C}$$

The data output is:

$$Um = \begin{pmatrix} 100 \\ -40 \\ 15 \\ 5 \\ -0.19 \end{pmatrix} \qquad \overrightarrow{|Im| \cdot r} = \begin{pmatrix} 44.911 \\ 33.336 \\ 13.937 \\ 4.810 \\ 0.186 \end{pmatrix} \qquad \overrightarrow{|Ucm|} = \begin{pmatrix} 89.348 \\ 22.107 \\ 5.545 \\ 1.367 \\ 0.041 \end{pmatrix}$$

Now, determine the effective values of the source voltage U, current I, voltage across resistor $I \cdot r$, voltage across the capacitance Uc:

$$U := \sqrt{\sum_{k=1}^{5} \frac{(Um_k)^2}{2}} \qquad I := \sqrt{\sum_{k=1}^{5} \frac{(|Im_k|)^2}{2}} \qquad Uc := \sqrt{\sum_{k=1}^{5} \frac{(|Ucm_k|)^2}{2}}$$

$$U = 76.974 \qquad I = 1.023 \qquad I \cdot r = 40.901 \qquad Uc = 65.209$$

Determine powers of individual harmonics, powers of source and consumer, power factor χ :

$$P_{Ek} := \mathrm{Re}\left(\frac{1}{2} \cdot Um_k \cdot \overline{Im_k}\right) \qquad P_{E\Sigma} := \sum_{k=1}^{5} P_{Ek} \qquad P_C := I^2 \cdot r \qquad P_E = \begin{pmatrix} 25.12 \\ 13.891 \\ 2.428 \\ 0.289 \\ 4.302 \cdot 10^{-4} \end{pmatrix}$$

$$P_{E\Sigma} = 41.821 \qquad P_C = 41.821 \qquad \chi := \frac{P_{E\Sigma}}{U \cdot I} \qquad \chi = 0.531$$

Determine the distortion coefficient k_i and coefficient of harmonics k_g for three voltages (source, across resistor, across capacitance):

$$k_g := \begin{bmatrix} \frac{\sqrt{U^2 - \left(\frac{Um_1}{\sqrt{2}}\right)^2}}{U} \\ \frac{\sqrt{I^2 - \left(\frac{|Im_1|}{\sqrt{2}}\right)^2}}{I} \\ \frac{\sqrt{Uc^2 - \left(\frac{|Ucm_1|}{\sqrt{2}}\right)^2}}{Uc} \end{bmatrix} \qquad k_i := \begin{pmatrix} \frac{Um_1}{U \cdot \sqrt{2}} \\ \frac{|Im_1|}{I \cdot \sqrt{2}} \\ \frac{|Ucm_1|}{Uc \cdot \sqrt{2}} \end{pmatrix} \qquad k_g = \begin{pmatrix} 0.395 \\ 0.631 \\ 0.248 \end{pmatrix} \qquad k_i = \begin{pmatrix} 0.919 \\ 0.776 \\ 0.969 \end{pmatrix}$$

Instantaneous values of the circuit voltage and current are:

$$u(t) := \left[\sum_{k=1}^{5} Um_k \cdot \sin[(2 \cdot k - 1) \cdot \omega \cdot t]\right] \qquad i(t) := \left[\sum_{k=1}^{5} |Im_k| \cdot \sin[(2 \cdot k - 1) \cdot \omega \cdot t + \arg(Im_k)]\right]$$

$u_r(t) := i(t)\cdot r \qquad u_C(t) := u(t) - u_r(t)$

The circuit voltages curves are presented in fig. 6.6.

Comments to the curves in fig. 6.6:

1. Higher harmonics being in the expansion of the source voltage intensively show themselves in the resistance voltage (hence, in the current curve) and are negligible in the capacitance voltage.

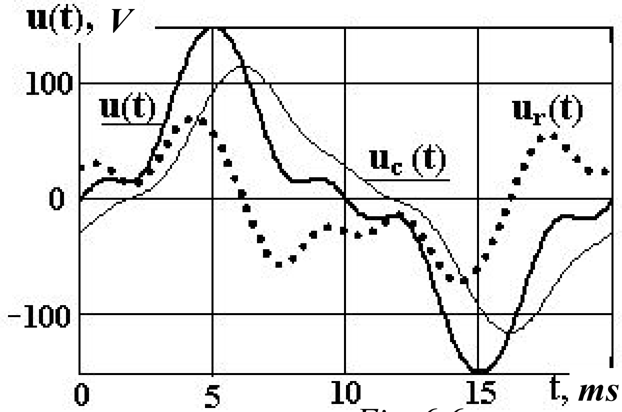
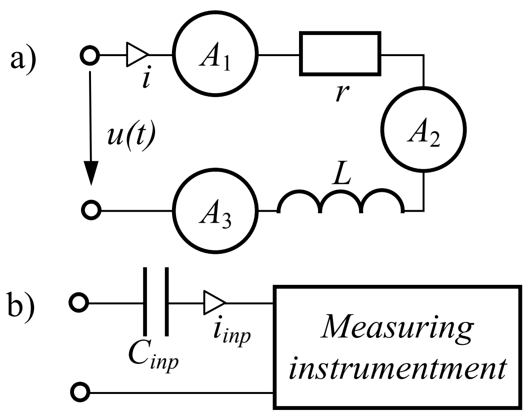


Fig. 6.6

2. Comparing the distortion coefficients, we see the capacitance voltage curve (k_i = 0.969) is hardly distorted in comparison with the current curve (k_i = 0.776).

3. Harmonic coefficient k_g gives evidence of the higher harmonics character. The coefficient is the lowest for the capacitance voltage curve (k_g = 0.248) while it is high for the current curve k_g = 0.631.

Conclusion. It is typical for the circuit $r - C$ that the current (or the resistor voltage) absorbs the high-frequency oscillations making the capacitance voltage free of them.



A_1 – d'Arsonval system,
A_2 – electromagnetic system,
A_3 – digital volt-ohm-milliampere meter

Fig. 6.7

6-5 (6.11). Non-sinusoidal current $i(\omega t) = 5 - 10\sin\omega t + 7\cos 2\omega t + 4\sin 3\omega t$ *A* flows in a circuit *r-L* (fig. 6.7,a).

Determine the instrument readings (for A_3, find the reading for two positions of the current kind selector).

Solution. 1) Both A_1 and A_3 with the selector in the position "direct current" (=) react upon the direct-current component. In this case they give identical readings: A_1 and A_3 $\rightarrow$ $I_0 = 5$ *A*.

2) A_2 measures the effective value of both sinusoidal and non-sinusoidal currents; its reading is $A_2 \;\rightarrow\; I = \sqrt{I_0^2 + \sum_{k=1}^{n} \frac{I_{km}^2}{2}} = \sqrt{5^2 + \frac{10^2}{2} + \frac{7^2}{2} + \frac{4^2}{2}} = 10.34$ *A*.

3) When the selector is in position "alternating current ~", A_3 eliminates the direct-current component owing to the input capacitor (fig. 6.7,b); the current through the measuring element consists of but alternating components

$$i_{inp} = -10\cdot\sin\omega t + 7\cdot\cos 2\omega t + 4\cdot\sin 3\omega t \; A.$$

This current curve is presented in fig. 6.8. The moments $\omega t_1 = 36.2°$, $\omega t_2 = 143.8°$

when the current changes its sign are determined by the numerical method.

Average of the absolute current value is $I_{inp\ av} = \dfrac{1}{2\pi}\int_0^{2\pi} |i_{inp}(\omega t)|\, d\omega t = 7.532\ A.$

The ammeter reading when the selector is in the position (~):

$$A_3 \ \rightarrow \ I_{inp\ av} \cdot k_f = 7.532 \cdot 1.11 = 8.36\ A,$$

where $k_f = 1.11$ – sinusoid shape coefficient.

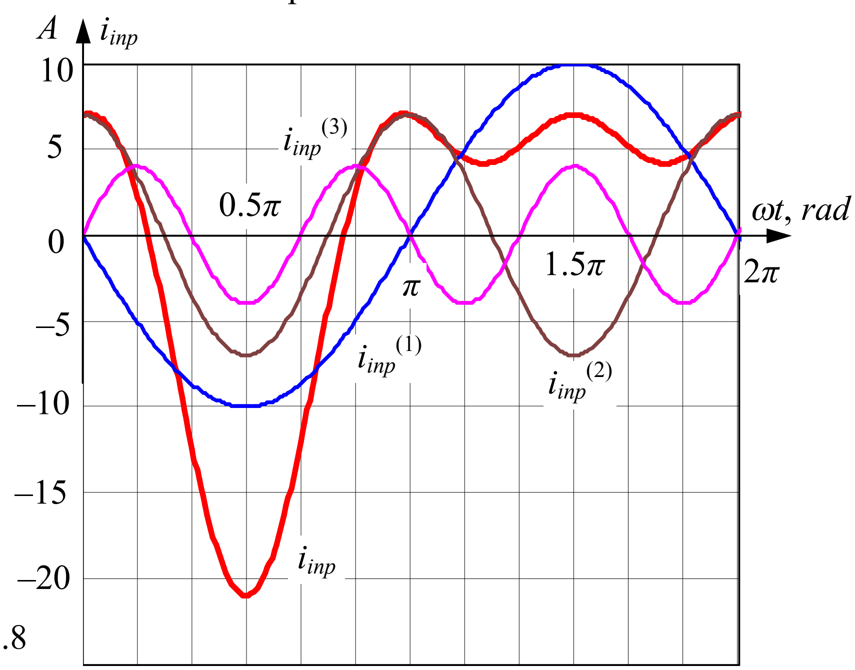


Fig. 6.8

6-6 (6.13). In fig. 6.9,a there is a circuit with a source of nonsinusoidal voltage

$$u(t) = \frac{U_m}{2} - \frac{U_m}{\pi} \cdot \sum_{k=1}^{n} \frac{\sin(k\omega t)}{k},$$

its plot being presented in fig. 6.9,b.

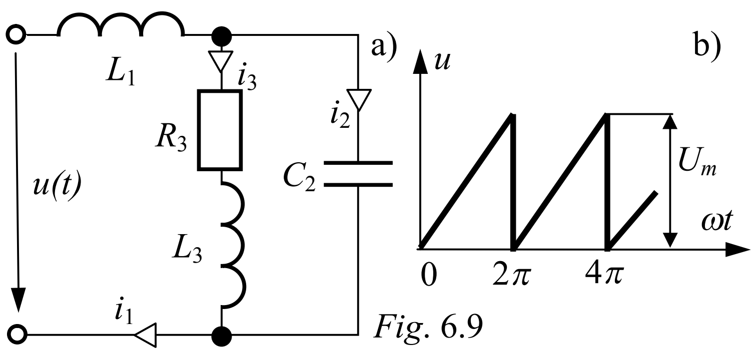


Fig. 6.9

Numerical data: $U_m = 20\ V$, $\omega = 1000\ rad/s$, $L_1 = 0.004\ H$, $L_3 = 0.002\ H$, $R_3 = 10\ Ohm$, $C_2 = 50\ \mu F$.

Using only five components of the expansion into the Fourier series, do the following:

1. Write down the formula of the emf instantaneous value substituting amplitude and angular velocity of the fundamental harmonic into the given formula; determine its effective value.
2. Calculate the effective currents, write down the instantaneous values of the first current and construct its plot.
3. Calculate the circuit power factor. Work out the balance of the active and reactive powers.

Solution. 1. The nstantaneous voltage is:

$$u(t)=\frac{20}{2}-\frac{20}{\pi}\cdot\left(\frac{sin(1000t)}{1}+\frac{sin(2000t)}{2}+\frac{sin(3000t)}{3}+...\right)\approx$$

$$\approx 10 + 6.366\cdot sin(1000t + 180°) + 3.183\cdot sin(2000t + 180°) +$$
$$+ 2.122\cdot sin(3000t + 180°) + 1.592\cdot sin(4000t + 180°)\ V.$$

2. The effective voltage is:

$$U=\sqrt{(U^{(0)})^2+\sum_{k=1}^{\infty}\frac{(U_m^{(k)})^2}{2}}=\sqrt{10^2+0.5\cdot(6.366^2+3.183^2+2.122^2+1.592^2)}=$$
$$= 11.35\ V.$$

3. Calculate the direct-current components:

$i_2^{(0)} = 0$; $i_1^{(0)} = i_3^{(0)} = U^{(0)}/R_3 = 10/10 = 1\ A$; $P^{(0)} = U^{(0)}\cdot i_1^{(0)} = 10\cdot 1 = 10\ W$.

4. Calculate the harmonic components by the following formulae:

- reactances: $x_1^{(k)} = k\cdot\omega\cdot L_1$, $x_2^{(k)} = 1/(k\cdot\omega\cdot C_2)$, $x_3^{(k)} = k\cdot\omega\cdot L_3$;
- impedances: $\underline{Z}_1^{(k)} = j\cdot x_1^{(k)}$; $\underline{Z}_2^{(k)} = -j\cdot x_2^{(k)}$;
  $\underline{Z}_3^{(k)} = R_3 + j\cdot x_3^{(k)}$; $\underline{Z}_{23}^{(k)} = \underline{Z}_2^{(k)}\cdot\underline{Z}_3^{(k)}/(\underline{Z}_2^{(k)} + \underline{Z}_3^{(k)})$; $\underline{Z}^{(k)} = \underline{Z}_1^{(k)} + \underline{Z}_{23}^{(k)}$.
- complex amplitudes of the source voltage: $\underline{U}_m^{(1)} = 6.366\cdot e^{j\cdot 180°}$;
  $\underline{U}_m^{(2)} = 3.183\cdot e^{j\cdot 180°}$; $\underline{U}_m^{(3)} = 2.122\cdot e^{j\cdot 180°}$; $\underline{U}_m^{(4)} = 1.592\cdot e^{j\cdot 180°}$;
- complex amplitudes of the currents of k-th harmonic:

$$\underline{I}_{m1}^{(k)} = \underline{U}_m^{(k)}/\underline{Z}^{(k)};\quad \underline{I}_{m2}^{(k)} = \underline{I}_{m1}^{(k)}\frac{\underline{Z}_3^{(k)}}{\underline{Z}_2^{(k)}+\underline{Z}_3^{(k)}};\quad \underline{I}_{m3}^{(k)} = \underline{I}_{m1}^{(k)}\frac{\underline{Z}_2^{(k)}}{\underline{Z}_2^{(k)}+\underline{Z}_3^{(k)}}.$$

- complex power of the source for the k-th harmonic: $\underline{S}^{(k)} = ½\cdot\underline{U}_m^{(k)}\cdot\overset{*}{\underline{I}}_{m1}^{(k)}$.

5. The effective currents are calculated by the formula:

$$I_q=\sqrt{(I_q^{(0)})^2+\sum_{k=1}^{\infty}\frac{(I_{mq}^{(k)})^2}{2}};$$

powers in separated branches of the circuit are: $P_q = R_q\cdot(I_q)^2$; $Q_q^{(k)} = \pm x_q^{(k)}\cdot(I_q^{(k)})^2$.

Check up the power balance:

- active, reactive powers and volt-amperes of the source are:

$$P = \text{Re}(\sum_k \underline{S}^{(k)}) = 12.966\ W;\quad Q = \text{Im}(\sum_k \underline{S}^{(k)}) = 0.789\ VAr;$$
$$S = U\cdot I_1 = 11.35\cdot 1.17 = 13.28\ VA.$$

The calculation results are tabulated in tables 6.2, 6.3 and 6.4.

Attention: $S^2 = 176.4 > P^2 + Q^2 = 168.8\ VA$.

- active and reactive powers of the consumers:

$$P_C = P_1 + P_2 + P_3 = 0 + 0 + 13 = 13\ W;$$
$$Q_C = Q_1 + Q_2 + Q_3 = 2.284 - 2.307 + 0.811 = 0.788\ VAr.$$

Power balance $P = P_C$ and $Q = Q_C$ is true.

6. The instantaneous value of the first current and its plot (fig. 6.10):

$$i_1(t) = 1 + 0.671\cdot sin(1000\cdot t + 174.1°) + 0.411\cdot sin(2000\cdot t + 161.8°) +$$
$$+ 0.296\cdot sin(3000\cdot t + 128.2°) + 0,151\cdot sin(4000\cdot t + 102.5°)\ A.$$

Table 6.2

№ of harmonic	1	2	3	4
$x_1^{(k)}$, *Ohm*	4	8	12	16
$x_2^{(k)}$, *Ohm*	20	10	6,67	5
$x_3^{(k)}$, *Ohm*	2	4	6	8
$\underline{Z}_1^{(k)}$, *Ohm*	$j\cdot 4$	$j\cdot 8$	$j\cdot 12$	$j\cdot 16$
$\underline{Z}_2^{(k)}$, *Ohm*	$-j\cdot 20$	$-j\cdot 10$	$-j\cdot 6.67$	$-j\cdot 5$
$\underline{Z}_3^{(k)}$, *Ohm*	$10.2 \angle 11.3°$	$10.8 \angle 21.8°$	$11.66 \angle 31.0°$	$12.81 \angle 38.7°$
$\underline{Z}_{23}^{(k)}$, *Ohm*	$9.91 \angle -17.7°$	$9.24 \angle -37.2°$	$7.76 \angle -55.2°$	$6.13 \angle 68.0°$
$\underline{Z}^{(k)}$, *Ohm*	$9.49 \angle 5.9°$	$7.74 \angle 18.2°$	$7.16 \angle 51.8°$	$10.56 \angle 77.5°$
$\underline{U}_m^{(k)}$, *V*	–6.366	–3.183	–2.122	–1.592
$\underline{I}_{m1}^{(k)}$, *A*	$0.671 \angle 174.1°$	$0.411 \angle 161.8°$	$0.296 \angle 128.2°$	$0.151 \angle 102.5°$
$\underline{I}_{m2}^{(k)}$, *A*	$0.322 \angle -114.7°$	$0.380 \angle -145.4°$	$0.345 \angle 163.0°$	$0.185 \angle -124.5°$
$\underline{I}_{m3}^{(k)}$, *A*	$0.652 \angle 145.0°$	$0.353 \angle 102.8°$	$0.197 \angle 42.0°$	$0.072 \angle 4.2°$
$\underline{S}^{(k)}$, *VA*	$2.125 + j0.22$	$0.62 + j0.205$	$0.195 + j0.247$	$0.026 + j0.117$

Table 6.3

I_1, *A*	I_2, *A*	I_3, *A*	P_1, *W*	P_2, *W*	P_3, *W*
1.17	0.45	1.14	0	0	13.0

Table 6.4

№ of harmonic	1	2	3	4	Q_Σ
Q_1, *VAr*	0.9	0.676	0.526	0.82	2.284
Q_2, *VAr*	–1.102	–0.722	–0.397	–0.086	–2.307
Q_3, *VAr*	0.425	0.249	0.116	0.021	0.811

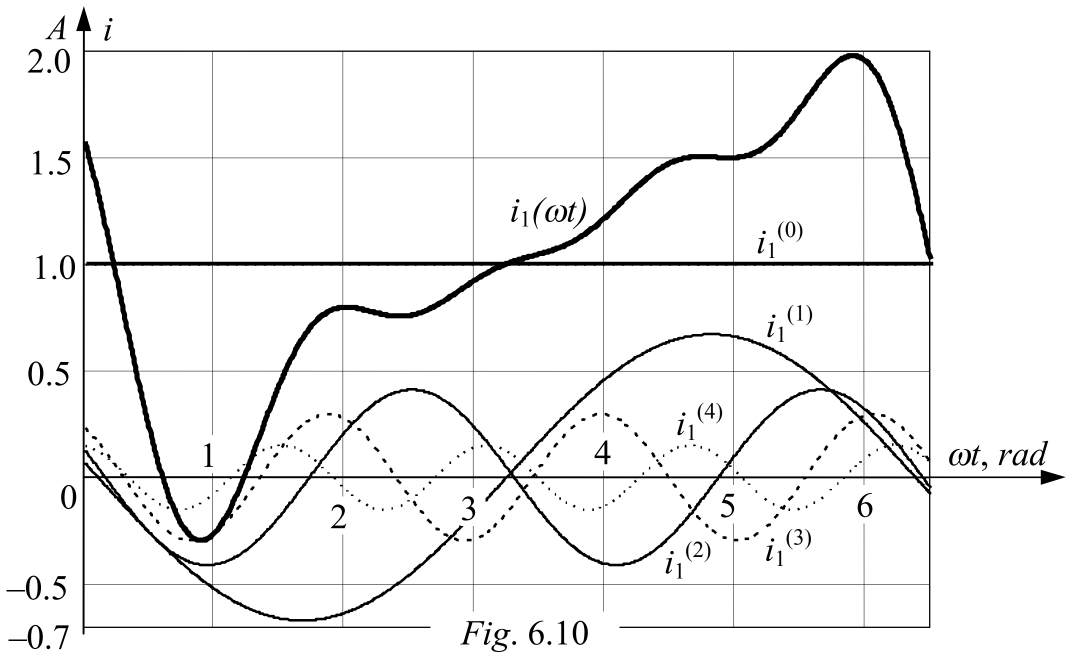


Fig. 6.10

6-7 (6.14). Voltage

$u = 50 + 200sin(\omega t + 45°) + 100sin(3\omega t + 60°)\ V$

is supplied to the circuit input fig. 6.11. Impedances of the circuit elements to the currents of the fundamental harmonic are as follows $r_1 = r_2 = r_3 = 8\ Ohm$;

$\dfrac{1}{\omega C_1} = 15\ Ohm;\ \ \omega L_2 = 3\ Ohm;\ \ \dfrac{1}{\omega C_3} = 15\ Ohm.$

Determine the instantaneous and effective values of all the currents as well as voltages u_{12} and u_{23}. Calculate the circuit power factor and plot the current $i_1(t)$, if the mains frequency is $f = 50\ Гц$.

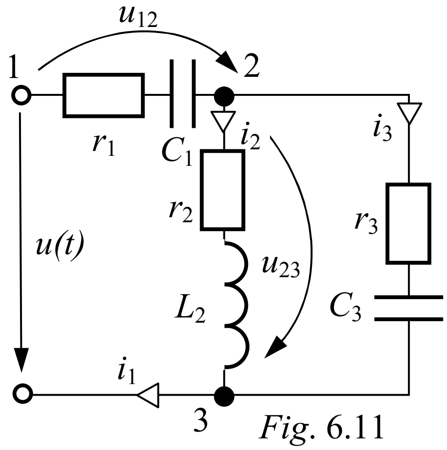


Fig. 6.11

Solution. Calculation is performed under the superposition method for each harmonic separately.

1. Calculation of the direct-current component. As $\dfrac{1}{\omega C_1} = \infty$, the currents of the harmonic under consideration cannot flow, the source full voltage being impressed to the capacitor C_1: $I_1^{(0)} = 0,\ \ I_2^{(0)} = 0,\ \ I_3^{(0)} = 0,\ \ U_{12}^{(0)} = 50\,V,\ \ U_{23}^{(0)} = 0.$

2. Calculation of the first (fundamental) harmonic.

Determine the branch and input circuit impedances

$$\underline{Z}_1^{(1)} = r_1 - \frac{j}{\omega C_1} = 8 - j15\ Ohm; \qquad \underline{Z}_2^{(1)} = r_2 + j\omega L_2 = 8 + j3\ Ohm;$$

$$\underline{Z}_3^{(1)} = r_3 - \frac{j}{\omega C_3} = 8 - j15\ Ohm;$$

$$\underline{Z}^{(1)} = \underline{Z}_1^{(1)} + \frac{\underline{Z}_2^{(1)} \cdot \underline{Z}_3^{(1)}}{\underline{Z}_2^{(1)} + \underline{Z}_3^{(1)}} = 15.24 - j15.57 = 21.79e^{-j45.6°}\ Ohm.$$

Complex amplitudes of the currents and voltages are:

$$\underline{I}_{1m}^{(1)} = \frac{\underline{U}_m^{(1)}}{\underline{Z}^{(1)}} = \frac{200e^{j45}}{21.79e^{-j45.6}} = 9{,}18e^{j90.6}\quad A,$$

$$\underline{I}_{2m}^{(1)} = \underline{I}_{1m}^{(1)} \frac{\underline{Z}_3^{(1)}}{\underline{Z}_2^{(1)} + \underline{Z}_3^{(1)}} = 7.8e^{j65.6}\quad A,$$

$$\underline{I}_{3m}^{(1)} = \underline{I}_{1m}^{(1)} \frac{\underline{Z}_2^{(1)}}{\underline{Z}_2^{(1)} + \underline{Z}_3^{(1)}} = 3.92e^{j148}\quad A,$$

$$\underline{U}_{12m}^{(1)} = \underline{I}_{1m}^{(1)} \cdot \underline{Z}_1^{(1)} = 156.1e^{j28.7}\quad V,$$

$$\underline{U}_{23m}^{(1)} = \underline{I}_{2m}^{(1)} \cdot \underline{Z}_2^{(1)} = 66.7e^{j86.1}\quad V.$$

3. Calculation of the third harmonic.

Determine the branch and input circuit impedances

$$\underline{Z}_1^{(3)} = r_1 - \frac{j}{3\omega C_1} = 8 - j5\ Ohm; \qquad \underline{Z}_2^{(3)} = r_2 + j3\omega L_2 = 8 + j9\ Ohm;$$

$$\underline{Z}_3^{(3)} = r_3 - \frac{j}{3\omega C_3} = 8 - j5\ Ohm;$$

$$\underline{Z}^{(3)} = \underline{Z}_1^{(3)} + \frac{\underline{Z}_2^{(3)} \cdot \underline{Z}_3^{(3)}}{\underline{Z}_2^{(3)} + \underline{Z}_3^{(3)}} = 14.9 - j4.72 = 15.61e^{-j17.6^\circ}\ Ohm.$$

Complex amplitudes of the currents and voltages are:

$$\underline{I}_{1m}^{(3)} = \frac{\underline{U}_m^{(3)}}{\underline{Z}^{(3)}} = \frac{100e^{j60}}{15.61e^{-j17.6}} = 6.41e^{j77.6}\quad A,$$

$$\underline{I}_{2m}^{(3)} = \underline{I}_{1m}^{(3)} \frac{\underline{Z}_3^{(3)}}{\underline{Z}_2^{(3)} + \underline{Z}_3^{(3)}} = 3.66e^{j31.6}\quad A,$$

$$\underline{I}_{3m}^{(3)} = \underline{I}_{1m}^{(3)} \frac{\underline{Z}_2^{(3)}}{\underline{Z}_2^{(3)} + \underline{Z}_3^{(3)}} = 4.68e^{j111.9}\quad A,$$

$$\underline{U}_{12m}^{(3)} = \underline{I}_{1m}^{(3)} \cdot \underline{Z}_1^{(3)} = 60.42e^{j45.6}\quad V,$$

$$\underline{U}_{23m}^{(3)} = \underline{I}_{2m}^{(3)} \cdot \underline{Z}_2^{(3)} = 44.12e^{j79.9}\quad V.$$

4. Calculate the effective values of currents and voltages

$$I_1 = \sqrt{\left(I_1^{(0)}\right)^2 + \frac{\left(I_{1m}^{(1)}\right)^2 + \left(I_{1m}^{(3)}\right)^2}{2}} = 7.92\ A,$$

$$I_2 = \sqrt{\left(I_2^{(0)}\right)^2 + \frac{\left(I_{2m}^{(1)}\right)^2 + \left(I_{2m}^{(3)}\right)^2}{2}} = 6.1\ A,$$

$$I_3 = \sqrt{\left(I_{3m}^{(0)}\right)^2 + \frac{\left(I_{3m}^{(1)}\right)^2 + \left(I_{3m}^{(3)}\right)^2}{2}} = 4.32\ A,$$

$$U_{12} = \sqrt{\left(U_{12}^{(0)}\right)^2 + \frac{\left(U_{12m}^{(1)}\right)^2 + \left(U_{12m}^{(3)}\right)^2}{2}} = 128.5\,V,$$

$$U_{23} = \sqrt{\left(U_{23}^{(0)}\right)^2 + \frac{\left(U_{23m}^{(1)}\right)^2 + \left(U_{23m}^{(3)}\right)^2}{2}} = 56.5\,V.$$

Instantaneous currents and voltages are:

$$i_1(t) = i_1^{(0)} + i_1^{(1)} + i_1^{(3)} = 9.18\sin(314t + 90.6^\circ) + 6.41\sin(942t + 77.6^\circ)\ A,$$

$$i_2(t) = i_2^{(0)} + i_2^{(1)} + i_2^{(3)} = 7.8\sin(314t + 65.6^\circ) + 3.66\sin(942t + 31.6^\circ)\ A,$$

$$i_3(t) = i_3^{(0)} + i_3^{(1)} + i_3^{(3)} = 3.92\sin(314t + 148^\circ) + 4.68\sin(942t + 111.9^\circ)\ A,$$

$$u_{12}(t) = u_{12}^{(0)} + u_{12}^{(1)} + u_{12}^{(3)} = 50 + 156.1\sin(314t + 28.7^\circ) + 60.42\sin(942t + 45.6^\circ)\ V,$$

$$u_{23}(t) = u_{23}^{(0)} + u_{23}^{(1)} + u_{23}^{(3)} = 66.7\sin(314t + 86.1^\circ) + 44.12\sin(942t + 79.9^o)\ V.$$

5. In order to determine the circuit power factor, let's previously calculate the powers

$P = P^{(0)} + P^{(1)} + P^{(3)} =$

$$=U^{(0)}I_1^{(0)}+Re[\underline{U}^{(1)}\underline{I}_1^{*(1)}]+Re[\underline{U}^{(3)}\underline{I}_1^{*(3)}]=0+642.1+305.3=947.4\ W,$$

$$S=UI_1=\sqrt{\left(U^{(0)}\right)^2+\frac{\left(U_m^{(1)}\right)^2+\left(U_m^{(3)}\right)^2}{2}}\cdot I_1=1313\ VA,\qquad \cos\theta=\frac{P}{S}=0.722.$$

6. The current $i_1(t)$ is plotted in fig. 6.12.

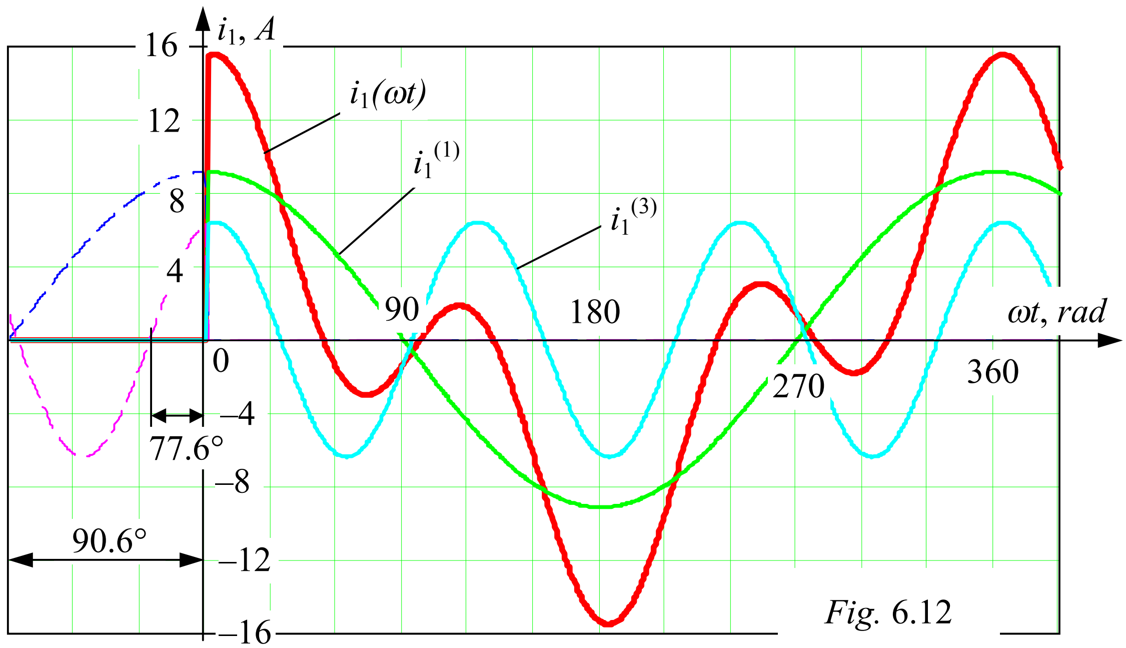


Fig. 6.12

6.3. THREE-PHASE CIRCUITS OF NON-SINUSOIDAL CURRENT

6-8 (6.15). The phase voltage of the *Y*-connected generator under no-load condition contains the first U_1 and the third U_3 harmonics.

1. Determine the effective voltages of the mentioned harmonics based on the known readings of the voltmeters of the phase voltage $U_{ph}=125\ V$ and the line voltage $U_l=210\ V$.

2. What is the calculation inaccuracy if there is an additional fifth harmonic in the voltage which comprises not more than 10% of the fundamental harmonic?

Solution. 1. The voltmeters readings expressed through the effective values of two harmonic components are described by the following equation system:

$$\sqrt{U_1{}^2+U_3{}^2}=U_{ph};\qquad \sqrt{3}\cdot U_1=U_l,$$

its solution is $U_1=121.2\ V,\ U_3=30.4\ V.$

2. The voltmeters readings expressed through the effective values of three harmonic components are as follows:

$$\sqrt{U_1{}^2+U_3{}^2+0.1^2\cdot U_1{}^2}=U_{ph};\qquad \sqrt{3}\cdot\sqrt{U_1^2+0{,}1^2\cdot U_1^2}=U_l,$$

the solution is $U_1=120.6\ V,\ U_3=30.4\ V.$

Comparing this result with the previous one, we conclude the measurement inaccuracy does not exceed 0.5% (furthermore, there is no inaccuracy in the third harmonic at all).

6-9 (6.16). Symmetrical generator with the phase voltage

$$u_A(\omega t)=310\cdot\sin(\omega t-30°)+93\cdot\sin(3\omega t+45°)\ V$$

supplies Y-connected nonbalanced load with the following phase impedances offered to the currents of fundamental harmonic $\underline{Z}_A^{(1)} = 15\ Ohm$, $\underline{Z}_B^{(1)} = j15\ Ohm$, $\underline{Z}_C^{(1)} = -j15\ Ohm$ (fig. 6.13); the neutral impedance is $\underline{Z}_N^{(1)} = 2 + j2\ Ohm$. Determine the readings of the moving-iron instruments for the cases:

a) switches S_1 and S_2 are closed;

b) S_1 is opened, while S_2 is closed;

c) both of them are opened.

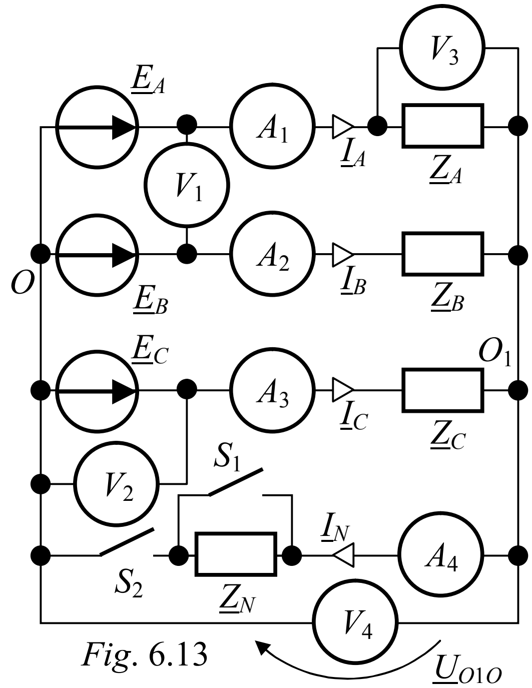


Fig. 6.13

Solution. a). *Switches are closed*.

1. Calculate the currents and voltages of the first harmonic.

Effective values of the complex phase voltages of a generator are

$$\underline{U}_A^{(1)} = \frac{U_{Am}^{(1)} \cdot e^{j\psi 1}}{\sqrt{2}} = 220e^{-j30}\ V,$$

$$\underline{U}_B^{(1)} = \underline{U}_A^{(1)} \cdot e^{-j120} = 220e^{-j150}\ V,$$

$$\underline{U}_C^{(1)} = \underline{U}_A^{(1)} \cdot e^{j120} = 220e^{j90}\ V.$$

As there is a neutral conductor, there is no voltage of the neutral displacement $\underline{U}_{O1O}$; that's why the phase voltages of the consumer are equal to the corresponding phase voltages of the source; the line currents being determined under Ohm's law:

$$\underline{I}_A^{(1)} = \frac{\underline{U}_A^{(1)}}{\underline{Z}_A^{(1)}} = 12.7 - j7.33 = 14.67e^{-j30}\ A,$$

$$\underline{I}_B^{(1)} = \frac{\underline{U}_B^{(1)}}{\underline{Z}_B^{(1)}} = -7.33 + j12.7 = 14.67e^{j120}\ A,$$

$$\underline{I}_C^{(1)} = \frac{\underline{U}_C^{(1)}}{\underline{Z}_C^{(1)}} = -14.67\ A.$$

The neutral conductor current of the fundamental harmonic is

$$\underline{I}_N^{(1)} = \underline{I}_A^{(1)} + \underline{I}_B^{(1)} + \underline{I}_C^{(1)} = -9.3 + j5.37 = 10.74e^{j150}\ A.$$

2. Calculate the currents and voltages of the third harmonic. The third harmonic forms a system of zero phase sequence, then the complexes of the generator phase voltages are:

$$\underline{U}_A^{(3)} = \underline{U}_B^{(3)} = \underline{U}_C^{(3)} = \frac{U_{Am}^{(3)} \cdot e^{j\psi 3}}{\sqrt{2}} = \frac{93e^{j45}}{\sqrt{2}} = 65.76e^{j45}\ V.$$

There is no voltage of the neutral displacement and the line currents are determined under Ohm's law:

$$\underline{I}_A^{(3)} = \frac{\underline{U}_A^{(3)}}{\underline{Z}_A^{(3)}} = \frac{56.76e^{j45}}{15} = 3.2 - j3.1 = 4.38e^{j45}\ A,$$

$$\underline{I}_B^{(3)} = \frac{\underline{U}_B^{(3)}}{\underline{Z}_B^{(3)}} = \frac{56.76e^{j45}}{j3\cdot 15} = 1.03 - j1.03 = 1.46e^{-j45}\ A,$$

$$\underline{I}_C^{(3)} = \frac{\underline{U}_C^{(3)}}{\underline{Z}_C^{(3)}} = \frac{56.76e^{j45}}{-j15/3} = -9.3 + j9.3 = 13.15e^{j135}\ A.$$

The neutral conductor current of the third harmonic is

$$\underline{I}_N^{(3)} = \underline{I}_A^{(3)} + \underline{I}_B^{(3)} + \underline{I}_C^{(3)} = -5.17 + j11.37 = 12.49e^{j114.4}\ A.$$

3. Determine the instrument readings.

The line voltage of the generator (reading of the voltmeter V_1) is equal to (the line voltages do not contain harmonics divisible by 3):

$$U_{AB} = U_{AB}^{(1)} = \sqrt{3}U_A^{(1)} = \sqrt{3}\cdot 220 = 380\,V.$$

The phase voltage U_C of the generator (reading of the voltmeter V_2) contains all the given harmonics $U_C = \sqrt{\left(U_C^{(1)}\right)^2 + \left(U_C^{(3)}\right)^2} = 229.6\,V.$

The phase voltage U'_A of the consumer (reading of the voltmeter V_3) is

$$U'_A = \sqrt{(I_A^{(1)}Z_A^{(1)})^2 + (I_A^{(3)}Z_A^{(3)})^2} = 229.6\,V.$$

Voltage of the neutral displacement (reading of the voltmeter V_4) is $\underline{U}_{O1O} = 0$.

The effective currents in the wires are

$$I_A = \sqrt{\left(I_A^{(1)}\right)^2 + \left(I_A^{(3)}\right)^2} = 15.31\,A \qquad \text{(reading of the ammeter } A_1\text{);}$$

$$I_B = \sqrt{\left(I_B^{(1)}\right)^2 + \left(I_B^{(3)}\right)^2} = 14.74\,A \qquad \text{(reading of the ammeter } A_2\text{);}$$

$$I_C = \sqrt{\left(I_C^{(1)}\right)^2 + \left(I_C^{(3)}\right)^2} = 19.7\,A \qquad \text{(reading of the ammeter } A_3\text{);}$$

$$I_N = \sqrt{\left(I_N^{(1)}\right)^2 + \left(I_N^{(3)}\right)^2} = 16.47\,A \qquad \text{(reading of the ammeter } A_4\text{).}$$

b). *The neutral conductor contains the impedance* $\underline{Z}_N$.

1. Calculation of the currents and voltages of the first harmonic.

Determine the neutral displacement voltage

$$\underline{U}_{O_1O}^{(1)} = \frac{\underline{U}_A^{(1)}\cdot\frac{1}{\underline{Z}_A^{(1)}} + \underline{U}_B^{(1)}\cdot\frac{1}{\underline{Z}_B^{(1)}} + \underline{U}_C^{(1)}\cdot\frac{1}{\underline{Z}_C^{(1)}}}{\frac{1}{\underline{Z}_A^{(1)}} + \frac{1}{\underline{Z}_B^{(1)}} + \frac{1}{\underline{Z}_C^{(1)}} + \frac{1}{\underline{Z}_N^{(1)}}} = 26.15\cdot e^{-j171.71^\circ} = -26.24 - j3.82\ V.$$

The line currents are determined by Ohm's law

$$\underline{I}_A^{(1)} = \frac{\underline{U}_A^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_A^{(1)}} = \frac{240.6\cdot e^{-j26.08}}{15} = 16.04\cdot e^{-j26.08^\circ} = 14.41 - j7.05\ A,$$

$$\underline{I}_B^{(1)} = \frac{\underline{U}_B^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_B^{(1)}} = \frac{194.8\cdot e^{-j147.11}}{j15} = 12.99\cdot e^{-j237.11^\circ} = -7.05 + j10.91\ A,$$

$$\underline{I}_C^{(1)} = \frac{\underline{U}_C^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_C^{(1)}} = \frac{224.6 \cdot e^{j83.29}}{-j15} = 14.97{\cdot}e^{j173.29°} = -14.87 + j1.75\ A,$$

$$\underline{I}_N^{(1)} = \frac{\underline{U}_{O_1O}^{(1)}}{\underline{Z}_N^{(1)}} = \frac{26.52 \cdot e^{-j171.71}}{2.828 \cdot e^{j45}} = 9.38{\cdot}e^{-j216.71°} = -7.52 + j5.60\ A.$$

Verification:

$$\underline{I}_A^{(1)} + \underline{I}_B^{(1)} + \underline{I}_C^{(1)} = 14.41 - j7.05 - 7.05 + j10.91 - 14{,}87 + j1.75 = -7.51 + j5.61 = \underline{I}_N^{(1)}.$$

2. Calculation of the currents and voltages of the third harmonic.

Determine the neutral displacement voltage

$$\underline{U}_{O_1O}^{(3)} = \frac{\underline{U}_A^{(3)} \cdot \frac{1}{\underline{Z}_A^{(1)}} + \underline{U}_B^{(3)} \cdot \frac{1}{\underline{Z}_B^{(1)}} + \underline{U}_C^{(3)} \cdot \frac{1}{\underline{Z}_C^{(3)}}}{\frac{1}{\underline{Z}_A^{(3)}} + \frac{1}{\underline{Z}_B^{(3)}} + \frac{1}{\underline{Z}_C^{(3)}} + \frac{1}{\underline{Z}_N^{(3)}}} = 104.1{\cdot}e^{j101.05°} = -19.96 + j102.2\ V.$$

The line currents are determined by Ohm's law

$$\underline{I}_A^{(3)} = \frac{\underline{U}_A^{(3)} - \underline{U}_{O_1O}^{(3)}}{\underline{Z}_A^{(3)}} = \frac{86.7 \cdot e^{-j39.96}}{15} = 5.78{\cdot}e^{-j39.96°} = 4.43 - j3.71\ A,$$

$$\underline{I}_B^{(3)} = \frac{\underline{U}_B^{(3)} - \underline{U}_{O_1O}^{(3)}}{\underline{Z}_B^{(3)}} = \frac{86.7 \cdot e^{-j39.96}}{j45} = 1.93{\cdot}e^{-j129.96°} = -1.24 - j1.48\ A,$$

$$\underline{I}_C^{(3)} = \frac{\underline{U}_C^{(3)} - \underline{U}_{O_1O}^{(3)}}{\underline{Z}_C^{(1)}} = \frac{86{,}7 \cdot e^{-j39.96}}{-j5} = 17.34{\cdot}e^{j50.04°} = 11.14 + j13.29\ A,$$

$$\underline{I}_N^{(3)} = \frac{\underline{U}_{O_1O}^{(3)}}{\underline{Z}_N^{(3)}} = \frac{104.1 \cdot e^{j101.05}}{2 + j6} = 16.46{\cdot}e^{j29.49°} = 14.33 + j8.10\ A.$$

Verification:

$$\underline{I}_A^{(3)} + \underline{I}_B^{(3)} + \underline{I}_C^{(3)} = 4.43 - j3.71 - 1.24 - j1.48 + 11.14 + j13.29 = 14.33 + j8.10 = \underline{I}_N^{(3)}.$$

3. Determine the instrument readings.

The line and the phase voltages of the generator (reading of the voltmeters V_1 and V_2, respectively) remain the same as in the first case.

The phase voltage U'_A of the consumer (reading of the voltmeter V_3) is:

$$U'_A = \sqrt{\left(U'^{(1)}_A\right)^2 + \left(U'^{(3)}_A\right)^2} = \sqrt{240.6^2 + 86.7^2} = 255.7\ V.$$

The voltage of the neutral displacement (reading of the voltmeter V_4) is:

$$U_{O_1O} = \sqrt{\left(U_{O_1O}^{(1)}\right)^2 + \left(U_{O_1O}^{(3)}\right)^2} = \sqrt{26.15^2 + 104.1^2} = 107.4\ V.$$

The effective currents in the wires are:

$$I_A = \sqrt{\left(I_A^{(1)}\right)^2 + \left(I_A^{(3)}\right)^2} = 17.05\ A \quad \text{(reading of the ammeter } A_1\text{)};$$

$$I_B = \sqrt{\left(I_B^{(1)}\right)^2 + \left(I_B^{(3)}\right)^2} = 13.13\ A \quad \text{(reading of the ammeter } A_2\text{)};$$

$$I_C = \sqrt{\left(I_C^{(1)}\right)^2 + \left(I_C^{(3)}\right)^2} = 22.91\ A \quad \text{(reading of the ammeter } A_3\text{);}$$

$$I_N = \sqrt{\left(I_N^{(1)}\right)^2 + \left(I_N^{(3)}\right)^2} = 18.94\ A \quad \text{(reading of the ammeter } A_4\text{).}$$

c). *Neutral conductor is disconnected.*

1. Calculation of the currents and voltages of the first harmonic.

Determine the neutral displacement voltage

$$\underline{U}_{O_1O}^{(1)} = \frac{\underline{U}_A^{(1)} \cdot \frac{1}{\underline{Z}_A^{(1)}} + \underline{U}_B^{(1)} \cdot \frac{1}{\underline{Z}_B^{(1)}} + \underline{U}_C^{(1)} \cdot \frac{1}{\underline{Z}_C^{(1)}}}{\frac{1}{\underline{Z}_A^{(1)}} + \frac{1}{\underline{Z}_B^{(1)}} + \frac{1}{\underline{Z}_C^{(1)}}} = 161e^{j150^\circ} = -\,139.5 + j80.5\ V.$$

The line currents are determined by Ohm's law

$$\underline{I}_A^{(1)} = \frac{\underline{U}_A^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_A^{(1)}} = 22 - j12.7 = 25.4e^{-j30}\ A,$$

$$\underline{I}_B^{(1)} = \frac{\underline{U}_B^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_B^{(1)}} = -12.7 + j3.4 = 13.15e^{j165}\ A,$$

$$\underline{I}_C^{(1)} = \frac{\underline{U}_C^{(1)} - \underline{U}_{O_1O}^{(1)}}{\underline{Z}_C^{(1)}} = -9.3 + j9.3 = 13.1e^{j135}\ A.$$

Verification: $\underline{I}_A^{(1)} + \underline{I}_B^{(1)} + \underline{I}_C^{(1)} = 0.$

2. Calculation of the currents and voltages of the third harmonic.

There are no line currents as in order to close the zero-phase-sequence current (currents of the harmonics divisible by 3) there should be a neutral conductor:

$$\underline{I}_A^{(3)} = \underline{I}_B^{(3)} = \underline{I}_C^{(3)} = 0.$$

The neutral displacement voltage is equal to the phase voltage of the generator

$$\underline{U}_{O_1O}^{(3)} = \underline{U}_A^{(3)} = 65.76e^{j45}\ V.$$

3. Determine the instrument readings.

The line and the phase voltages of the generator (reading of the voltmeters V_1 and V_2, respectively) remain the same as in the first case.

The phase voltage U'_A of the consumer (reading of the voltmeter V_3) is:

$$U'_A = I_A^{(1)} Z_A^{(1)} = 380\ V.$$

The voltage of the neutral displacement (reading of the voltmeter V_4) is:

$$U_{O_1O} = \sqrt{\left(U_{O_1O}^{(1)}\right)^2 + \left(U_{O_1O}^{(3)}\right)^2} = 174\ V.$$

The effective currents in the wires are

$I_A = I_A^{(1)} = 25.4\ A$ (reading of the ammeter A_1),

$I_B = I_B^{(1)} = 13.15\ A$ (reading of the ammeter A_2),

$I_C = I_C^{(1)} = 13.15\ A$ (reading of the ammeter A_3),

There is no current in the neutral conductor $I_N = 0$ (reading of the ammeter A_4).

6-10 (6.17). Solve the problem 6.9 under condition the phase impedances are identical: $\underline{Z}^{(1)} = 15 + j10$ *Ohm*. The generator voltages and the neutral impedance are of the same value.

Solution. As the load is balanced, the calculation may be performed for but a single phase. The voltages and currents of the first harmonic form the symmetric system with a positive phase sequence, that's why there is no neutral displacement voltage of the first harmonic at any state of the neutral conductor; it means the voltages and currents of the first harmonic do not depend on the position of switches, they remain the same regardless the switches position. A single-line diagram for calculation is presented in fig. 6.14,a. The voltages and currents of the third harmonic form the symmetric system as well, however, that being of the zero phase sequence. This time, a single-line diagram takes into account the triple impedance of the neutral conductor (see «Three-phase circuits. Method of symmetric components»). The scheme is presented in fig. 6.14,b.

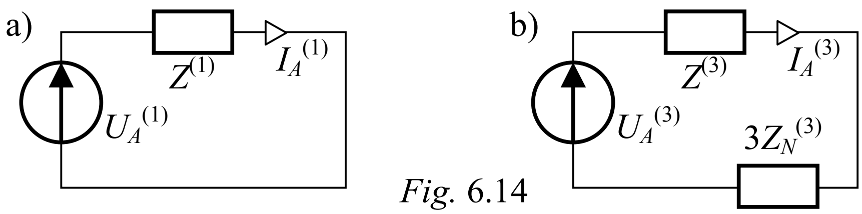


Fig. 6.14

Calculate the first harmonic.

The complex of the source phase voltage is $\underline{U}_A^{(1)} = \dfrac{U_{Am}^{(1)} \cdot e^{j\psi 1}}{\sqrt{2}} = 220e^{-j30}$ *V*.

The complex of the line current is $\underline{I}_A^{(1)} = \dfrac{\underline{U}_A^{(1)}}{\underline{Z}^{(1)}} = \dfrac{220e^{-j30}}{15 + j10} = 12.16 \cdot e^{-j63.69°}$ *A*.

There is no neutral current of the first harmonic.

Calculate the third harmonic for different cases. The complex of the source phase voltage is $\underline{U}_A^{(3)} = \dfrac{U_{Am}^{(3)} \cdot e^{j\psi 3}}{\sqrt{2}} = \dfrac{93e^{j45}}{\sqrt{2}} = 65.76e^{j45}$ *V*.

The single-phase impedance of the load to currents of the third harmonic is

$$\underline{Z}^{(3)} = 15 + j3 \cdot 10 = 15 + j30 = 33.54 \cdot e^{j63.44°} \ Ohm.$$

a). *Neutral conductor is connected.*

The neutral impedance is zero, there is no neutral displacement voltage, the voltages across the load phases coincide with those of the source. This time, the current of phase *A* in accordance with Ohm's law is: $\underline{I}_A^{(3)} = \dfrac{\underline{U}_A^{(3)}}{\underline{Z}^{(3)}} = \dfrac{65.8e^{j45}}{33.54e^{j63.44}} = 1.96 \cdot e^{-j18.44°}$ *A*.

The neutral current is equal to the sum of three identical line currents:

$$\underline{I}_N^{(3)} = 3\,\underline{I}_A^{(3)} = 5.88 \cdot e^{-j18.44°} \ A.$$

Let's find the instruments readings.

The reading of the voltmeters V_1 and V_2 are the same as in problem 6.9:

$$V_1 \to U_{AB} = U_{AB}^{(1)} = \sqrt{3}U_A^{(1)} = \sqrt{3} \cdot 220 = 380 \ V;$$

$$V_2 \to \ U_C = U_A = \sqrt{\left(U_A^{(1)}\right)^2 + \left(U_A^{(3)}\right)^2} = 229{,}6 \ V.$$

The phase voltage U'_A of the consumer (reading of the voltmeter V_3) in the presence of the neutral conductor coincides with the phase voltage of the source, that's why

$$U_A' = U_A = 229.6\ V.$$

The reading of the voltmeter V_4 (voltage of the neutral displacement) is equal to zero.

The effective currents in the line wires are identical (reading of the ammeters $A_1 \div A_3$):

$$I_A = \sqrt{\left(I_A^{(1)}\right)^2 + \left(I_A^{(3)}\right)^2} = \sqrt{12.16^2 + 1.96^2} = 12.32\ A.$$

In the neutral conductor (reading of the ammeter A_4), there is only the current of the third harmonic: $I_N = I_N^{(3)} = 5.88\ A$.

b). *Neutral conductor contains the impedance $\underline{Z}_N$.* Perform the circuit calculation for the third harmonic.

$$\underline{I}_A^{(3)} = \frac{\underline{U}_A^{(3)}}{\underline{Z}^{(3)} + 3\underline{Z}_N^{(3)}} = \frac{65.8e^{j45}}{33.54e^{j63,44} + 6 + j18} = 1.26 \cdot e^{-j21.37°} = 1.17 - j0.46\ A.$$

$$\underline{I}_N^{(3)} = 3\,\underline{I}_A^{(3)} = 3.77 \cdot e^{-j21.37°} = 3.51 - j1.37\ A.$$

Note, calculation of the third harmonic may be performed according to the commonly used strategy as well:

- voltage of the neutral displacement:

$$\underline{U}_{O_1O}^{(3)} = \frac{\underline{U}_A^{(3)} \cdot \dfrac{3}{\underline{Z}^{(3)}}}{\dfrac{3}{\underline{Z}^{(3)}} + \dfrac{1}{\underline{Z}_N^{(3)}}} = 23.82 \cdot e^{j50.19°} = 15.25 + j18.30\ V.$$

- currents in the line conductor A and in the neutral under Ohm's law:

$$\underline{I}_A^{(3)} = \frac{\underline{U}_A^{(3)} - \underline{U}_{O_1O}^{(3)}}{\underline{Z}_A^{(3)}} = \frac{42.1 \cdot e^{j42.06}}{15 + j30} = 1.26 \cdot e^{-j21.37°} = 1.17 - j0.46\ A.$$

$$\underline{I}_N^{(3)} = \frac{\underline{U}_{O_1O}^{(3)}}{\underline{Z}_N^{(3)}} = \frac{23.82 \cdot e^{j50.19}}{2 + j6} = 3.77 \cdot e^{-j21.37°} = 3.51 - j1.37\ A.$$

The verification is: $\underline{I}_A^{(3)} + \underline{I}_B^{(3)} + \underline{I}_C^{(3)} = 3 \cdot \underline{I}_A^{(3)} = 3.51 - j1.38 = \underline{I}_N^{(3)}$.

The instruments readings are: $V_1 \rightarrow U_A = 380\ V$; $V_2 \rightarrow U_C = 229.6\ V$;

$$V_3 \rightarrow U_A' = \sqrt{\left(U_A'^{(1)}\right)^2 + \left(U_A'^{(3)}\right)^2} = \sqrt{220^2 + 42.1^2} = 223.2\ V;$$

$$V_4 \rightarrow U_N = U_N^{(3)} = 23.82\ V.$$

$$A_1,\ A_2,\ A_3 \rightarrow I_A = \sqrt{\left(I_A^{(1)}\right)^2 + \left(I_A^{(3)}\right)^2} = \sqrt{12.16^2 + 1.26^2} = 12.22\ A;$$

$$A_4 \rightarrow I_N = I_N^{(3)} = 3.77\ A.$$

c). *Neutral conductor is disconnected.*

There are no neutral currents because the flow of zero-sequence currents (currents of harmonics divisible by 3) requires a neutral conductor: $\underline{I}_A^{(3)} = \underline{I}_B^{(3)} = \underline{I}_C^{(3)} = 0$.

The neutral displacement voltage is equal to the phase voltage of the generator

$$\underline{U}_{O_1O}^{(3)} = \underline{U}_A^{(3)} = 65.76 e^{j45} \quad B.$$

The instruments readings are: $V_1 \rightarrow U_{AB} = 380\ V$; $V_2 \rightarrow U_C = 229.6\ V$;

$V_3 \rightarrow U_A' = U_A'^{(1)} = 220\ V$; $V_4 \rightarrow U_{O_1O} = U_{O_1O}^{(3)} = 65.76\ V$.

The effective currents in the line wires are $A_1, A_2, A_3 \rightarrow I_A = I_A^{(1)} = 12.16\ A$. There is no current in the neutral conductor $I_N = 0$ (reading of the ammeter A_4).

7. TRANSIENT PROCESSES IN LINEAR CIRCUITS WITH LUMPED PARAMETERS

7.1. CLASSIC METHOD OF TRANSIENTS ANALYSIS

7.1.1. Transients in circuits with a single storage element

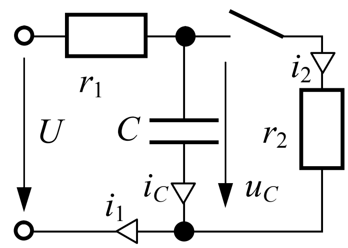


Fig. 7.1

7-1 (7.1). In scheme fig. 7.1, calculate the voltage across the capacitor as well as the currents of the transient process. The circuit parameters are: $U = 100\ V$, $r_1 = 60\ Ohm$, $r_2 = 40\ Ohm$, $C = 10\ \mu F$. Plot the capacitor voltage.

Solution. After the commutation, the circuit is described by the following equation system under Kirchhoff's laws concerning the instantaneous currents and voltage across the capacitor:

$$\begin{cases} i_1 - i_2 - i_C = 0, \\ i_1 \cdot r_1 + u_C = U, \\ i_2 \cdot r_2 - u_C = 0. \end{cases}$$

The additional equation is the coupling equation between the current and voltage of the capacitor: $i_C = C\dfrac{du_C}{dt}$.

The equation system is solved by the substitution method – all the currents are expressed through the capacitor voltage and are incorporated into the first equation of the system. As a result, the equation system is reduced to one linear non-homogeneous differential equation of the first order with constant coefficients. It is advisable to add, the equation order is determined by the quantity of the circuit storage elements. This time, there is a single storage element – a capacitor; that's why the differential equation happens of the first order.

$$i_1 = \frac{U - u_C}{r_1}; \quad i_2 = \frac{u_C}{r_2}; \quad i_C = C\frac{du_C}{dt}; \quad \frac{U - u_C}{r_1} - \frac{u_C}{r_2} - C\frac{du_C}{dt} = 0.$$

$$\frac{du_C}{dt} + \frac{r_1 + r_2}{C r_1 r_2} u_C = \frac{U}{C r_1}.$$

The equation solution concerning $u_C(t)$ is found as a sum of particular solution of the non-homogeneous differential equation and general solution of the corresponding homogeneous differential equation. They are termed, respectively, steady-state and transient components: $u_C(t) = u_{Cs}(t) + u_{Ct}(t)$. Such a method of the transient processes analysis is called a *classical method*. The view of the steady-state component depends on the view of the right side of the equation, i.e. on the source character. In the case in question, there is a D-C source, the steady-state component of the capacitor voltage is constant as well, correspondingly its derivative is

$$\frac{du_{Cs}}{dt} = 0: \quad u_{Cs} = \frac{C r_1 r_2}{r_1 + r_2} \cdot \frac{U}{C r_1} = \frac{r_2}{r_1 + r_2} \cdot U = \frac{40}{60 + 40} \cdot 100 = 40\ V.$$

A transient component form depends on the number and view of the roots of the characteristic equation. That's why we draw and solve the characteristic equation. Here, the derivative of u_C is replaced by p, quantity u_C itself – by 1, the right side is equated with zero:

$$p + \frac{r_1 + r_2}{C r_1 r_2} = 0.$$

The equation solution: $p = -\frac{r_1 + r_2}{C r_1 r_2} = -\frac{60 + 40}{10^{-5} \cdot 60 \cdot 40} = -4167\ s^{-1}.$

If there is one negative root of the characteristic equation, the transient component takes the form: $u_{Ct}(t) = A \cdot e^{pt}$. The integration constant A is found through the initial conditions. The voltage across the capacitor before the commutation is: $u_C(0_-) = U = 100\ V$. In accordance with the second commutation law, we have $u_C(0_+) = u_C(0_-) = 100\ V$. Thus, the integration constant is

$$A = u_{Ct}(0) = u_C(0) - u_{Cs}(0) = 100 - 40 = 60\ V.$$

Finally, we have: $u_C(t) = 40 + 60 \cdot e^{-4167t}\ V.$

The branch currents are:

$$i_1(t) = \frac{U - u_C}{r_1} = \frac{100 - 40 - 60 \cdot e^{-4167t}}{60} = 1 - 1 \cdot e^{-4167t}\ A,$$

$$i_2(t) = \frac{u_C}{r_2} = \frac{40 + 60 \cdot e^{-4167t}}{40} = 1 + 1.5 \cdot e^{-4167t}\ A,$$

$$i_C(t) = i_1(t) - i_2(t) = -2.5 \cdot e^{-4167t}\ A.$$

In order to plot $u_C(t)$, let's calculate in addition:

- the circuit time constant $\tau = 1/|p| = 1/4167\ s = 0.24\ ms$,
- practical duration of the transient process $T_{tp} = (3 \div 5)\tau = 4 \cdot \tau = 0.96\ ms$.

The graph $u_C(t)$ is drawn using the components: separately shown are steady-state and transient components, they being added graphically. The graph is presented in fig. 7.2.

7-2 (7.2). Determine the coil current and voltage across the inductance (fig. 7.3), if $U = 200\ V$, $r_c = 10\ Ohm$, $L = 25\ mH$.

Plot $i(t)$ and $u_L(t)$.

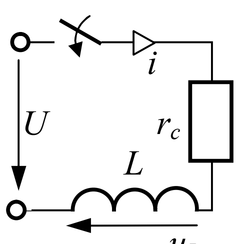


Fig. 7.3

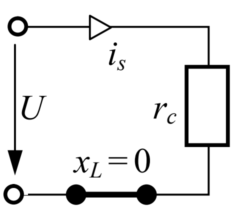


Fig. 7.4

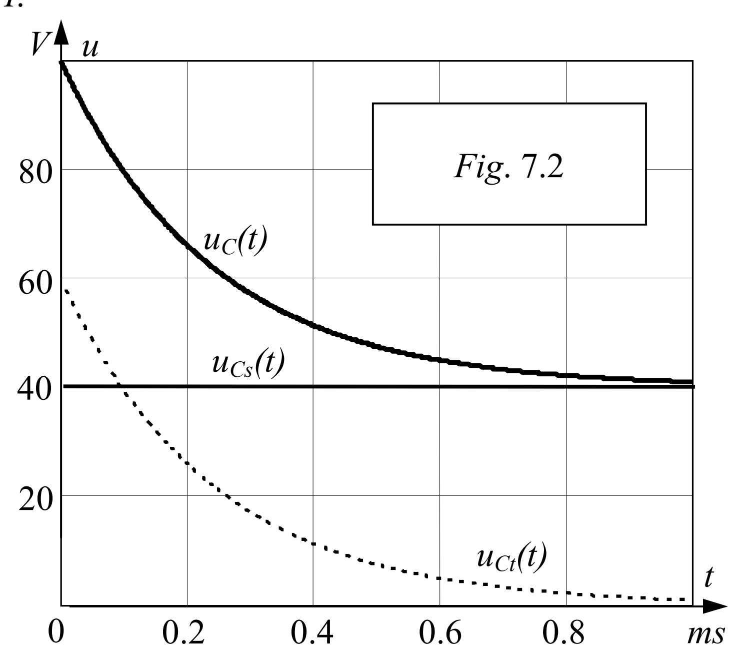


Fig. 7.2

Comments and answers.

1. The independent initial condition is: $i(0_+) = i(0_-) = 0$.
2. The steady-state mode computation is based on the scheme fig. 7.4:

$$i_s = 20\ A; \qquad u_{Ls} = 0.$$

3. The characteristic equation and its root are: $r_c + pL = 0,\ p = -400\ s^{-1}$.
4. The transient components are: $i_t = Ae^{pt}$; $u_{Lt} = Be^{pt}$.
5. The initial conditions are: $i_t(0_+) = i(0_+) - i_s = -20\ A$;

$$u_{Lt}(0_+) = u_L(0_+) = U - r_c i(0_+) = 200\ V.$$

6. The integration constants are: $A = i_t(0_+) = -20$; $B = u_{Lt}(0_+) = 200$.
7. The complete quantities are: $i(t) = 20 - 20e^{-400t}\ A$; $u_L(t) = 200e^{-400t}\ V$.
8. The circuit time constant and duration of the transients are:

$$\tau = \frac{1}{|p|} = \frac{L}{r_c} = 2.5 \cdot 10^{-3}\ s; \quad T_{tp} = 4 \cdot \tau = 0.01\ s.$$

The plots $i(t)$, $u_L(t)$ are in fig. 7.5.

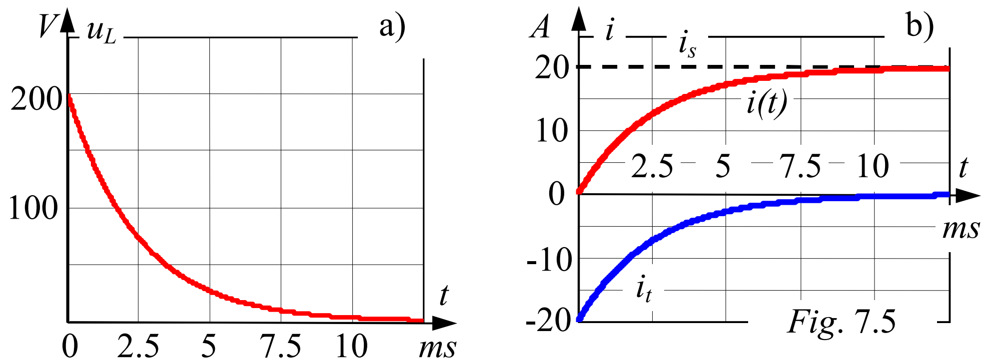


Fig. 7.5

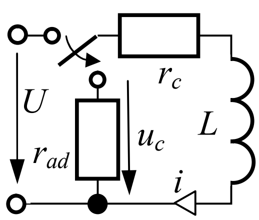


Fig. 7.6

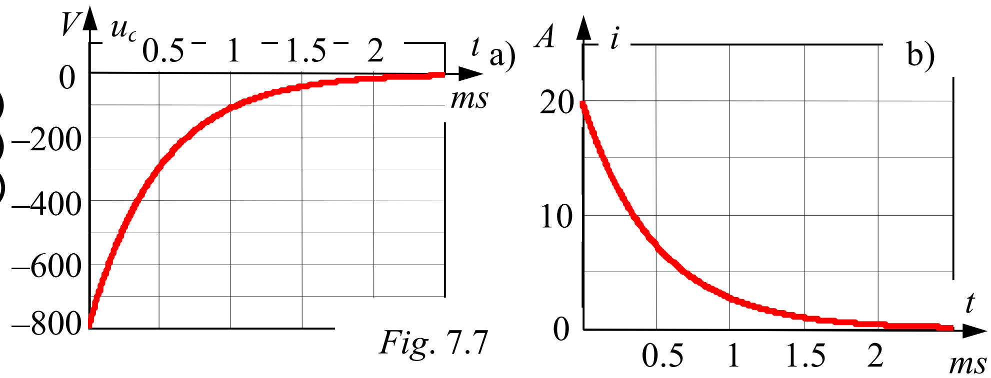


Fig. 7.7

7-3 (7.3). Determine the current and voltage of the coil when switching it to the additional resistance r_{ad} (fig. 7.6), if $U = 200\ V$, $r_c = 10\ Ohm$, $L = 25\ mH$, $r_{ad} = 40\ Ohm$.

Plot $i(t)$, $u_c(t)$.

Comments and answers.

1. The independent initial condition is: $i(0_+) = i(0_-) = \dfrac{U}{r_c} = 20\ A$.
2. The steady-state components are: $i_s = 0$; $u_{cs} = 0$.
3. The characteristic equation and its root are:

$$pL + (r_{ad} + r_c) = 0, \qquad p = -2000\ s^{-1}.$$

4. The transient components are: $i_t = Ae^{pt}$; $u_{ct} = Be^{pt}$.
5. The initial conditions are: $i_t(0_+) = i(0_+) - i_s = 20\ A$;

$u_L(0_+) = -i(0_+) \cdot (r_{ad} + r_c) = -1000\ V$ and $u_{ct}(0_+) = u_c(0_+) = r_c i(0_+) + u_L(0_+) = -800\ V$.

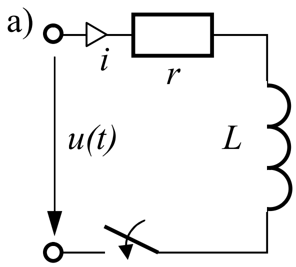
6. The integration constants are $A = i_t(0_+) = 20$; $B = u_{ct}(0_+) = -800$.

7. The complete quantities are: $i(t) = 20e^{-2000t}\ A$; $u_c(t) = -800e^{-2000t}\ V$.

8. The circuit time constant and duration of the transients are:

$$\tau = \frac{1}{|p|} = 0.5\cdot 10^{-3}\ s = 0.5\ ms;\quad T_{tp} = 2\ ms.$$

The plots $i(t)$, $u_c(t)$ are in fig. 7.7.



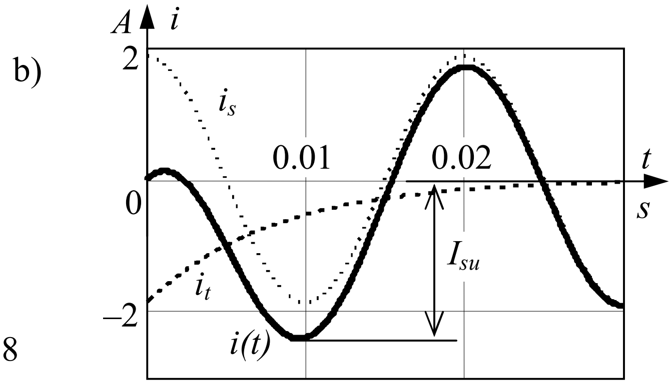


Fig. 7.8

7-4 (7.4). In fig. 7.8,a there is a scheme to compute the transients, the transient process can be observed wneh a transformer is switched to a no-load condition. Furthermore, $u(t) = 100\cdot sin(314t + \psi_u)\ V$, $r = 20\ Ohm$, $L = 0.159\ H$. Compute ψ_u to reach the “hardest” and the “easiest” condition. Plot the current of the “hardest” condition. Determine the surge current.

Solution. 1. This time, there is a zero independent initial condition – $i(0_+) = i(0_-) = 0$.

2. Perform the current computation by the classical method. The steady-state component has the following view:

$$i_s(t) = I_m\cdot sin(314t + \psi_i) = \frac{U_m}{Z}\cdot sin(314t + \psi_u - \varphi).$$

Here $Z = \sqrt{r^2 + (\omega L)^2} = \sqrt{20^2 + (314\cdot 0.159)^2} = \sqrt{20^2 + 50^2} = 53.9\ Ohm$,

$$\varphi = arctg\frac{\omega L}{r} = arctg\frac{50}{20} = 68.1°,\qquad I_m = \frac{U_m}{Z} = \frac{100}{53.9} = 1.86\ A.$$

Thus, $i_s(t) = 1.86\cdot sin(314t + \psi_u - 68.1°)\ A$.

The initial value of the current steady-state component is

$i_s(0) = 1.86\cdot sin(\psi_u - 68.1°)\ A$.

3. The characteristic equation and its root are:

$pL + r = 0$, $p = -r/L = 20/0.159 = -125.8\ s^{-1}$.

The circuit time constant and practical duration of the transients are:

$\tau = 1/|p| = 1/125.8 = 0.008\ s$, $T_{tp} = (3\div 5)\tau = (24\div 40)\ ms$.

The steady-state component oscillation period is: $T = \dfrac{2\pi}{\omega} = 20\ ms$.

4. In case of a single root of the characteristic equation, the transient component takes a view: $i_t(t) = A\cdot e^{pt}$.

The integration constant is $A = i_t(0) = i(0) - i_s(0) = -1.86\cdot sin(\psi_u - 68.1°)$.

5. The “hardest” transient process (the biggest value of the transient component) is observed at $\psi_u - 68.1° = \pm 90°$. It means $\psi_u = 158.1°$ or $\psi_u = -21.9°$.

The “easiest” process (when there is no transient component) is at $\psi_u - 68.1° = 0°$ or $\pm 180°$. It means $\psi_u = 68.1°$ or $\psi_u = -111.9°$.

If $\psi_u - 68.1° = 90°$, then $\psi_u = 158.1°$. The instantaneous current is as follows

$$i(t) = i_s(t) + i_t(t) = 1.86 \cdot \sin(314t + 90°) - 1.86 \cdot e^{-125{,}8t}\ A.$$

6. The diagrams of the current and its components are presented in fig. 7.8,b.

The maximum instantaneous value of the current during a transient process is termed "*surge current*". As it is seen from the diagram, the highest current value is $I_{su} = 2.4\ A$ and it occurs at the time moment $t = 9.7\ ms$.

7-5 (7.5). Compute the current of the transient process when switching the coil to sinusoidal voltage $u(t) = U_m \cdot \sin(\omega t + \psi_u)$ (fig. 7.8,a), if
$U_m = 200\ V$, $\omega = 1000\ rad/s$, $\psi_u = -30°$, $r = 10\ Ohm$, $L = 25\ mH$.

Plot $i(t)$.

Answer: $i(t) = 7.43\sin(1000t - 98.2°) + 7.35e^{-400t}\ A$;
the curve $i(t)$ is in fig. 7.9.

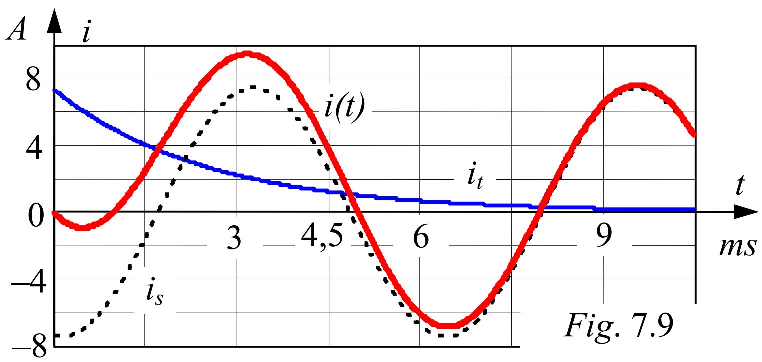


Fig. 7.9

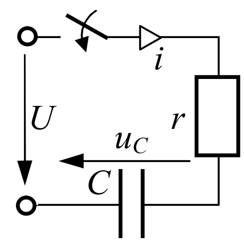


Fig. 7.10

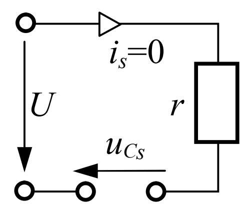


Fig. 7.11

7-6 (7.6). Compute the transient current and voltage across the capacitor (fig. 7.10), if $U = 200\ V$, $r = 100\ Ohm$, $C = 100\ \mu F$.

Plot $i(t)$, $u_C(t)$.

Comments and answers.

1. The independent initial condition is: $u_C(0_+) = u_C(0_-) = 0$.
2. The steady-state mode is computed based on the scheme fig. 7.11:

$$i_s = 0; \qquad u_{Cs} = U = 200\ V.$$

3. The characteristic equation and its root are: $r + \dfrac{1}{pC} = 0, \quad p = -\dfrac{1}{rC} = -100\ s^{-1}$.
4. The transient components are: $i_t(t) = Ae^{pt}$; $\quad u_{Ct}(t) = Be^{pt}$.
5. The initial conditions are: $u_{Ct}(0_+) = u_C(0_+) - u_{Cs} = -200\ V$.

$$i_t(0_+) = -\frac{u_{Ct}(0_+)}{r} = 2\ A.$$

6. The integration constants are $A = i_t(0_+) = 2$; $\quad B = u_{Ct}(0_+) = -200$.

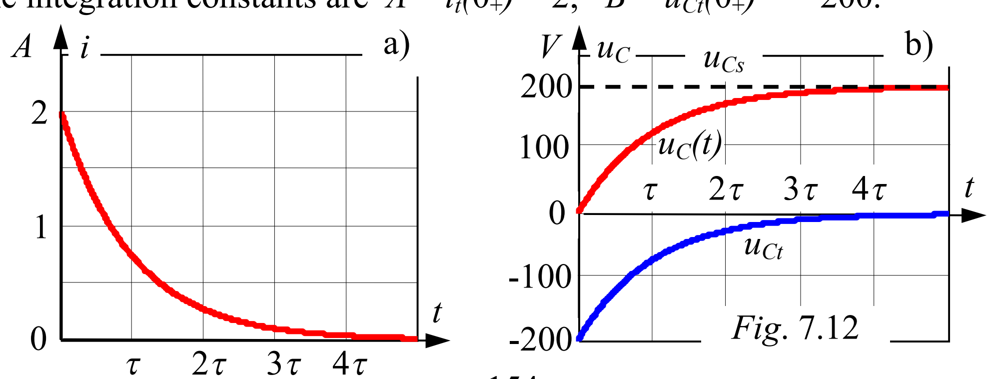


Fig. 7.12

7. The complete quantities are: $i(t) = 2e^{-100t}\ A$; $u_C(t) = 200 - 200e^{-100t}\ V$.

8. The circuit time constant and practical duration of the transients are:

$$\tau = \frac{1}{|p|} = 0.01\ s;\quad T_{tp} = 0.04\ s.$$

$i(t)$, $u_C(t)$ curves are in fig. 7.12.

7-7 (7.7). Determine the current and voltage across the capacitor when switching the circuit to the additional resistance r_{ad} (fig. 7.13), if

$$U = 200\ V,\ r = 100\ Ohm,\ C = 100\ \mu F,\ r_{ad} = 400\ Ohm.$$

Plot $i(t)$, $u_C(t)$.

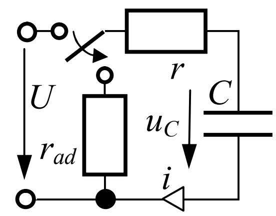


Fig. 7.13

Comments and answers. 1. The independent initial condition is:

$$u_C(0_+) = u_C(0_-) = U = 200\ V.$$

2. The steady-state components are: $i_s = 0$; $u_{Cs} = 0$.

3. The sharacteristic equation and its root are:

$$(r_{ad} + r) + \frac{1}{pC} = 0,\quad p = -\frac{1}{(r_{ad} + r)C} = -20\ s^{-1}.$$

4. The transient components are: $i_t = Ae^{pt}$; $u_{Ct} = Be^{pt}$.

5. The initial conditions are: $u_{Ct}(0_+) = u_C(0_+) - u_{Ct} = 200\ V$;

$$i_t(0_+) = -\frac{u_{Ct}(0_+)}{r_{ad} + r} = -0.4\ A.$$

6. The integration constants are $A = i_t(0_+) = -0.4$; $B = u_{Ct}(0_+) = 200$.

7. The complete quantities are: $i(t) = -0.4e^{-20t}\ A$, $u_C(t) = 200e^{-20t}\ V$.

8. The circuit time constant and practical duration of the transients are:

$$\tau = \frac{1}{|p|} = 0.05\ s,\qquad T_{tp} = 4\tau = 0.2\ s.$$

$i(t)$, $u_C(t)$ curves are in fig. 7.14.

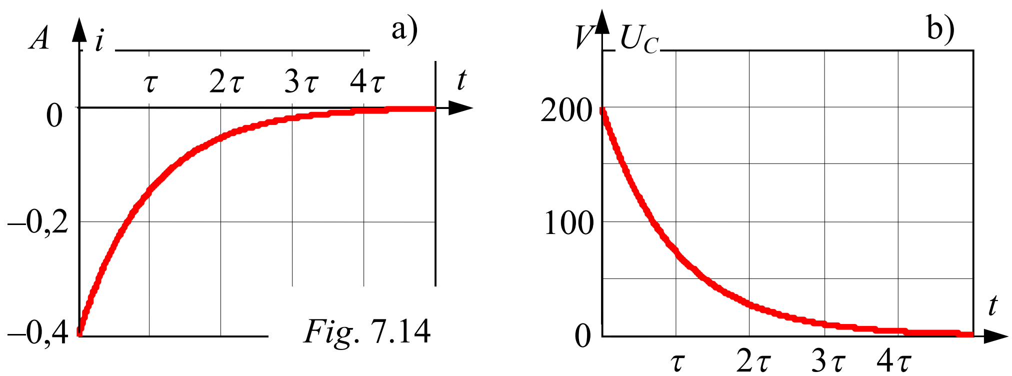


Fig. 7.14

7-8 (7.8). Determine the current $i(t)$ and the voltage across the capacitor $u_C(t)$ (fig. 7.15), if $r = 100\ Ohm$, $C = 10\ \mu F$, $E_0 = 300\ V$, $e(t) = 100sin(1000t - 90°)\ V$.

Plot $i(t)$.

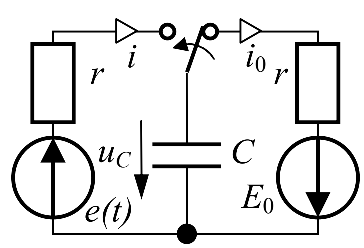


Fig. 7.15

Solution. Analyzing the scheme before the commutation, let's determine the capacitor voltage under Kirchhoff's voltage law: $ri_0(0_-) - u_C(0_-) = E_0$.

$i_0(0_-) = 0$, accordingly, $u_C(0_-) = -E_0 = -300\ V$.

In accordance with the second commutation law $u_C(0_+) = u_C(0_-) = -300\ V$.

After the commutation, scheme is described by the following linear differential equation $ri(t) + u_C(t) = e(t)$, where $u_C(t) = \frac{1}{C}\int idt$,

and its solution is: $i = i_s + i_t$; $u_C = u_{Cs} + u_{Ct}$.

In order to compute the steady-state mode we apply the complex-notation method.

$$\underline{E}_m = 100e^{-j90°}\ V,\quad x_C = \frac{1}{\omega C} = \frac{10^6}{1000 \cdot 10} = 100\ Ohm,$$

$$\underline{Z} = r - jx_C = 100 - j100 = 100\sqrt{2}\,e^{-j45°}\ Ohm,$$

$$\underline{I}_{ms} = \frac{\underline{E}_m}{\underline{Z}} = \frac{100e^{-j90}}{100\sqrt{2}e^{-j45}} = 0.5\sqrt{2}\,e^{-j45°} = 0.707e^{-j45°}\ A,$$

$$\underline{U}_{Cms} = -jx_C \cdot \underline{I}_{ms} = 100e^{-j90°} \cdot 0.5\sqrt{2}\,e^{-j45°} = 50\sqrt{2}\,e^{-j135°} = 70.7e^{-j135°}\ V.$$

The instantaneous values of the steady-state components are:

$i_s(t) = 0.707 sin(1000t - 45°)\ A$, $\quad u_{Cs}(t) = 70.7 sin(1000t - 135°)\ V$.

The characteristic equation is written upon the ground of the differential equation

$$r + \frac{1}{pC} = 0,\quad p = -\frac{1}{rC} = -\frac{10^6}{100 \cdot 10} = -1000\ s^{-1}.$$

Hence, the transient components are of the following form:

$i_t(t) = Ae^{-1000t}$; $\quad u_{Ct}(t) = Be^{-1000t}$.

The integration constants are determined from the initial conditions.

At $t = 0_+$ we have: $i_t(0_+) = A$; $\quad u_{Ct}(0_+) = B = u_C(0_+) - u_{Cs}(0_+)$,

where $u_{Cs}(0_+) = 70.7 sin(-135°) = -50\ V$,

$u_{Ct}(0_+) = B = -300 - (-50) = -250\ V$.

The dDifferential equation for the transient components at $t = 0_+$ has a view:

$ri_t(0_+) + u_{Ct}(0_+) = 0$.

$$i_t(0_+) = A = \frac{-u_{Ct}(0_+)}{r} = \frac{250}{100} = 2.5\ A, \text{ i.e.}$$

$i_t(t) = 2.5e^{-1000t}\ A$; $\quad u_{Ct}(t) = -250e^{-1000t}\ V$.

The required quantities have a final view:

$u_C(t) = 70.7 sin(1000t - 135°) - 250e^{-1000t}\ V$,

$i(t) = 0.707 sin(1000t - 45°) + 2.5e^{-1000t}\ A$.

In order to construct the diagrams (fig. 7.16) we determine the circuit time constant and duration of the transients: $\tau = \frac{1}{|p|} = \frac{1}{|-1000|} = 0.001\ s$, $\quad T_{tp} = 4\tau = 0.004\ s$.

The sinusoid period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1000} = 0.00628\ s = 6.28\ ms$.

The calculation results are tabulated in table 7.1.

Table 7.1

t, ms	0	0.75	1.5	2.25	3	3.75	4.5	5.25	6	6.75
i_s, A	–0.5	–0.02	0.46	0.7	0.57	0.12	–0.38	–0.7	–0.62	–0.22
i_t, A	2.5	1.18	0.56	0.26	0.12	0.06	0.03	0.01	0.006	0.003
i, A	2.0	1.16	1.02	0.96	0.69	0.18	–0.35	–0.68	–0.614	–0.217

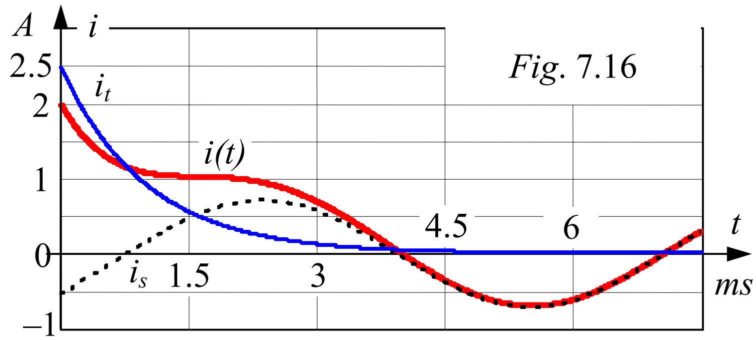


Fig. 7.16

7-9 (7.9). Compute the transient currents and voltage across the inductance in scheme fig. 7.17, if $E = 150\ V$, $r_1 = r_2 = 10\ Ohm$, $r_3 = r_4 = 5\ Ohm$, $L = 20\ mH$.

Plot $i_2(t)$, $u_L(t)$.

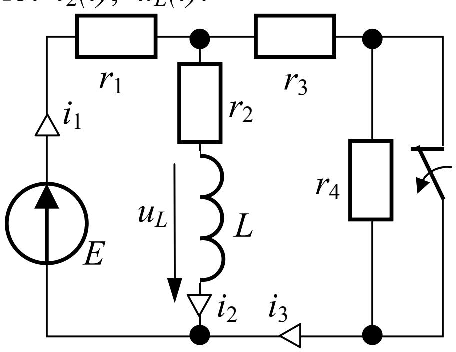


Fig. 7.17

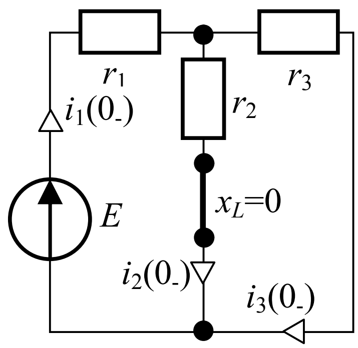


Fig. 7.18

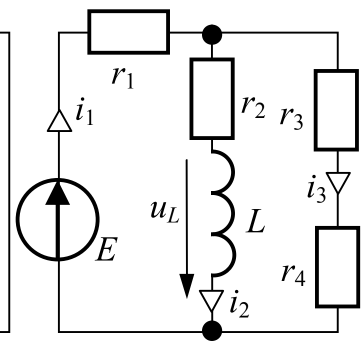


Fig. 7.19

Solution. 1. Analyzing the scheme before the commutation, let's determine the independent initial condition, this time it being the current through inductance – $i_2(0_-)$. Scheme before the commutation has a view fig. 7.18. The inductance current is

$$i_2(0_-) = \frac{E}{\dfrac{r_2 r_3}{r_2 + r_3} + r_1} \cdot \frac{r_3}{r_2 + r_3} = \frac{150}{\dfrac{10 \cdot 5}{10+5} + 10} \cdot \frac{5}{10+5} = 3.75\ A.$$

In accordance with the first commutation law, we have $i_2(0_+) = i_2(0_-) = 3.75\ A$.

2. The scheme after the commutation lools like in fig. 7.19, it being described by the system of the linear differential equations under Kirchhoff's laws:

$$\begin{cases} i_1(t) = i_2(t) + i_3(t), \\ r_1 i_1 + r_2 i_2 + L\dfrac{di_2}{dt} = E, \\ r_1 i_1 + (r_3 + r_4)\cdot i_3 = E, \end{cases}$$

and its solution having view $i_q(t) = i_{qs}(t) + i_{qt}(t)$; $u_L(t) = u_{Ls}(t) + u_{Lt}(t)$.

3. The steady-state mode is:

$$i_{1s} = \frac{E}{\dfrac{r_2(r_3 + r_4)}{r_2 + r_3 + r_4} + r_1} = \frac{150}{\dfrac{10 \cdot (5+5)}{10+5+5} + 10} = 10\ A;$$

$$i_{2s} = i_{1s} \cdot \frac{r_3 + r_4}{r_2 + r_3 + r_4} = 10 \cdot \frac{5+5}{10+5+5} = 5\ A;$$

$$i_{3s} = i_{1s} - i_{2s} = 10 - 5 = 5\ A, \qquad u_{Ls} = 0.$$

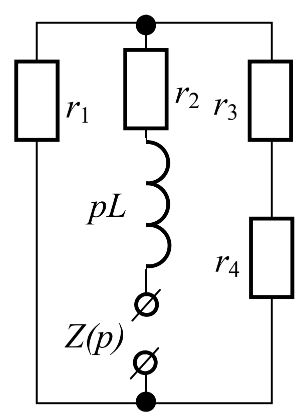


Fig. 7.20

4. Calculation of the transient components.

Let's draw the characteristic equation through the presentation of the circuit input impedance in operational form (2nd way): $\frac{(r_3+r_4)(r_2+pL)}{r_3+r_4+r_2+pL}+r_1=0.$

However, a characteristic equation may also be drawn concerning the branch with the storage element, the source being replaced by its inner impedance (fig. 7.20). This time, the equation is simpler:

$$\frac{(r_3+r_4)r_1}{r_3+r_4+r_1}+r_2+pL=0; \qquad \frac{(5+5)\cdot 10}{5+5+10}+10+20\cdot 10^{-3}\cdot p=0; \qquad p=-750\ s^{-1}.$$

Then $i_{1t}=A_1e^{pt}=A_1e^{-750t}$; $i_{2t}=A_2e^{-750t}$; $i_{3t}=A_3e^{-750t}$; $u_{Lt}=Be^{-750t}$.

The integration constants are determined at $t=0_+$.

Ist way of solution

The scheme after the commutation for moment $t=0_+$ has the view fig. 7.21. According to the method of two nodes, we have

$$u_{ab}(0_+)=\frac{E/r_1-i_2(0_+)}{\frac{1}{r_1}+\frac{1}{r_3+r_4}}=\frac{150/10-3.75}{\frac{1}{10}+\frac{1}{5+5}}=56.25\ V.$$

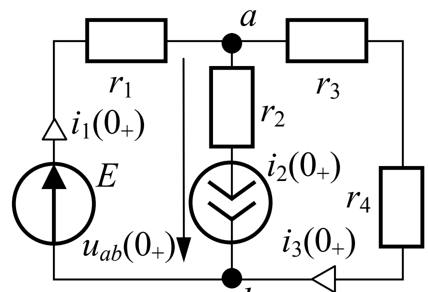


Fig. 7.21

The currents at the moment of commutation are

$$i_1(0_+)=\frac{E-u_{ab}(0_+)}{r_1}=\frac{150-56.25}{10}=9.375\ A,$$

$$i_3(0_+)=\frac{u_{ab}(0_+)}{r_3+r_4}=\frac{56.25}{10}=5.625\ A \quad \text{or} \quad i_3(0_+)=i_1(0_+)-i_2(0_+)=5.625\ A,$$

$u_L(0_+)=u_{ab}(0_+)-r_2i_2(0_+)=56.25-10\cdot 3.75=18.75\ V.$

Let's write down the transient components for moment $t=0_+$:

$i_{1t}(0_+)=A_1=i_1(0_+)-i_{1s}=9.375-10=-0.625\ A,$

$i_{2t}(0_+)=A_2=i_2(0_+)-i_{2s}=3.75-5=-1.25\ A,$

$i_{3t}(0_+)=A_3=i_3(0_+)-i_{3s}=5.625-5=0.625\ A,$

$u_{Lt}(0_+)=B=u_L(0_+)-u_{Ls}=18.75-0=18.75\ V.$

Thus: $i_{1t}(t)=-0.625e^{-750t}\ A,\quad i_1(t)=10-0.625e^{-750t}\ A,$

$i_{2t}(t)=-1.25e^{-750t}\ A,\quad i_2(t)=5-1.25e^{-750t}\ A,$

$i_{3t}(t)=0.625e^{-750t}\ A,\quad i_3(t)=5+0.625e^{-750t}\ A,$

$u_L(t)=u_{Lt}(t)=18.75e^{-750t}\ V.$

IInd way of solution

Integration constant A_2 may be determined at once, for the current i_2 obeys the first commutation law:

$A_2=i_2(0_+)-i_{2s}=3.75-5=-1.25\ A,$

$i_{2t}(t)=A_2e^{-750t}=-1.25e^{-750t}\ A,\qquad i_2(t)=i_{2s}(t)+i_{2t}(t)=5-1.25e^{-750t}\ A.$

Let's determine the voltage across the inductance

$$u_L(t)=L\frac{di_2}{dt}=20\cdot 10^{-3}\cdot(-1.25)(-750)e^{-750t}=18.75e^{-750t}\ V.$$

The junction voltage is

$$u_{ab}(t)=r_2i_2(t)+u_L(t)=10\cdot(5-1.25e^{-750t})+18.75e^{-750t}=50+6.25e^{-750t}\ V.$$

The currents are $i_3(t) = \dfrac{u_{ab}(t)}{r_3 + r_4} = \dfrac{50 + 6.25e^{-750t}}{5+5} = 5 + 0.625e^{-750t}\ A,$

$$i_1(t) = i_2(t) + i_3(t) = 5 - 1.25e^{-750t} + 5 + 0.625e^{-750t} = 10 - 0.625e^{-750t}\ A.$$

5. Let's plot $i_2(t)$ and $u_L(t)$. The transient process duration is

$$T_{tp} = 4\tau = \frac{4}{|p|} = \frac{4}{|-750|}\ s = 5.33\ ms.$$

The calculation results are presented in table 7.2.

Table 7.2

t, ms	0	1.33	2.67	4	5.33
i_{2t}, A	– 1.25	– 0.46	– 0.17	– 0.06	– 0.02
i_2, A	3.75	4.54	4.83	4.94	4.98
u_L, V	18.75	6.9	2.54	0.93	0.34

The curves are presented in fig. 7.22.

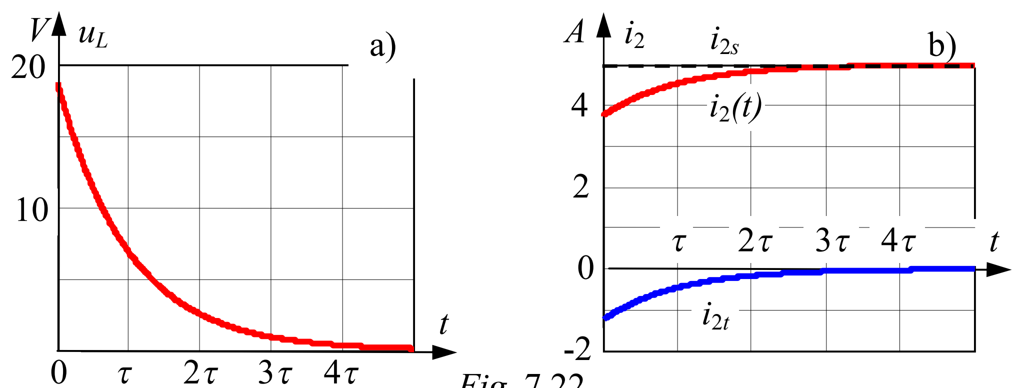


Fig. 7.22

7-10 (7.10). Calculate the transient currents in scheme fig. 7.23,a by classical method. The circuit parameters are: $U = 50\ V$, $r_1 = r_3 = 100\ Ohm$, $r_2 = 50\ Ohm$, $C = 100\ \mu F$.

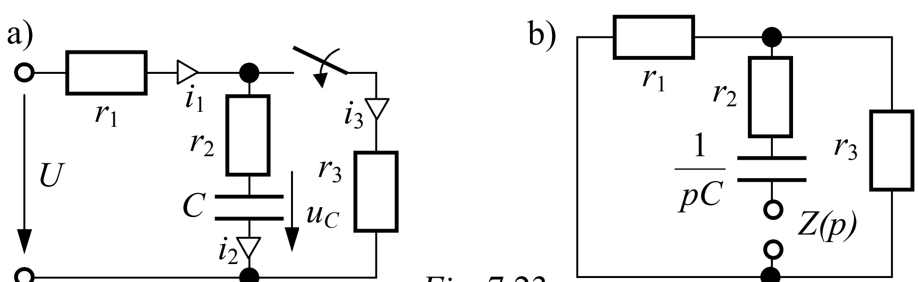


Fig. 7.23

Solution. 1. The circuit state before the commutation is: $i_1(t_-) = i_2(t_-) = 0$, $u_C(t_-) = U = 50\ V$. In accordance with the second commutation law, the independent initial condition is as follows – $u_C(0_+) = u_C(0_-) = 50\ V$.

2. In accordance with the classical method, the required transient currents are presented as the sum of steady-state and transient components:

$$i_1 = i_{1s} + i_{1t}, \quad i_2 = i_{2s} + i_{2t}, \quad i_3 = i_{3s} + i_{3t}.$$

3. Let's calculate the steady-state components of the currents:

$$i_{2s} = 0; \quad i_{1s} = i_{3s} = \frac{U}{r_1 + r_3} = \frac{50}{100+100} = 0.25\ A.$$

3. We draw the characteristic equation through the input impedance in the operational form:
$$Z(p) = \frac{1}{pC} + r_2 + \frac{r_1 r_3}{r_1 + r_3} = 0.$$

The root of the characteristic equation is:
$$p = -\frac{r_1 + r_3}{C(r_1 r_2 + r_1 r_3 + r_2 r_3)} = -\frac{100 + 100}{10^{-4}(100 \cdot 50 + 100 \cdot 100 + 50 \cdot 100)} = -100\ s^{-1}.$$

4. The transient components of the currents in case of a single root of the characteristic equation are as follows: $i_{1t} = A \cdot e^{pt}$, $i_{2t} = B \cdot e^{pt}$, $i_{3t} = D \cdot e^{pt}$.

5. The integration constants A, B, D are found with the aid of the initial conditions, which, however, may be obtained in different ways. Let's consider some of them.

a) *the 1st way*. The equation system under Kirchhoff's laws is generated for the post-commutation condition for the initial time moment:
$$\begin{cases} i_1(0_+) - i_2(0_+) - i_3(0_+) = 0, \\ i_1(0_+) \cdot r_1 + i_2(0_+) \cdot r_2 + u_C(0_+) = U, \\ i_1(0_+) \cdot r_1 + i_3(0_+) \cdot r_3 = U. \end{cases}$$

With the numerical data:
$$\begin{cases} i_1(0_+) - i_2(0_+) - i_3(0_+) = 0, \\ i_1(0_+) \cdot 100 + i_2(0_+) \cdot 50 + 50 = 50, \\ i_1(0_+) \cdot 100 + i_3(0_+) \cdot 100 = 50. \end{cases}$$

The system solution is: $i_1(0_+) = 0.125\ A$, $i_2(0_+) = -0.25\ A$, $i_3(0_+) = 0.375\ A$.

The integration constants are:
$$A = i_{1t}(0_+) = i_1(0_+) - i_{1s} = 0.125 - 0.25 = -0.125;$$
$$B = i_{2t}(0_+) = i_2(0_+) - i_{2s} = -0.25 - 0 = -0.25;$$
$$D = i_{3t}(0_+) = i_3(0_+) - i_{3s} = 0.375 - 0.25 = 0.125.$$

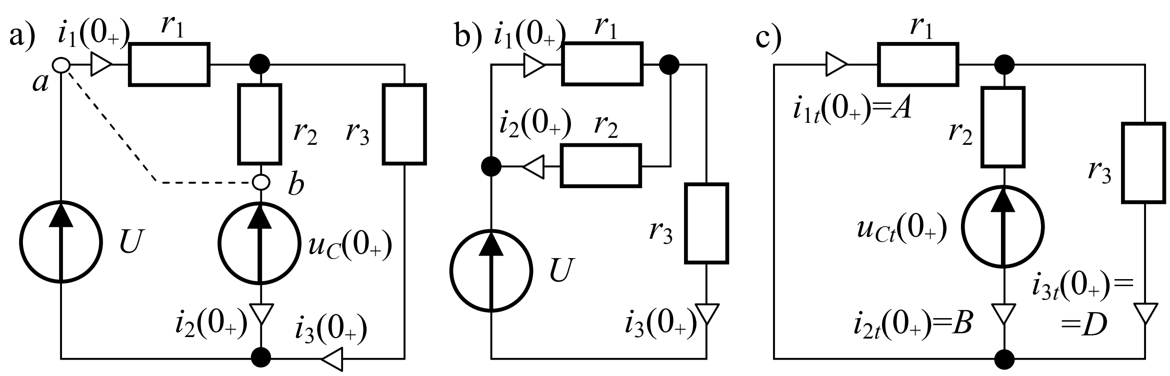


Fig. 7.24

b) *the 2nd way*. Calculation is performed under the equivalent scheme, it being drawn for the initial time moment. This time, we use the consequence of the commutation laws: an inductance at the commutation moment behaves as a current source $i_L(0_+)$, a capacitance behaves as a voltage source $u_C(0_+)$. Correspondingly, for the initial time moment, we obtain a scheme fig. 7.24,a. Taking into account that in the circuit there are two identical sources $u_C(0_+) = U$, we may claim that the potentials of the points a and b are equal; that's why they may be connected by a jumper. We obtain the scheme fig. 7.24,b; from which the initial currents are found:
$$i_3(0_+) = \frac{U}{r_3 + \frac{r_1 r_2}{r_1 + r_2}} = \frac{50}{100 + \frac{50 \cdot 100}{50 + 100}} = 0.375\ A,$$

$$i_1(0_+) = i_3(0_+)\cdot\frac{r_2}{r_1+r_2} = 0.375\cdot\frac{50}{50+100} = 0.125\ A,$$

$$i_2(0_+) = i_1(0_+) - i_3(0_+) = 0.125 - 0.375 = -0.25\ A.$$

Further, the integration constants are determined like in the 1st way.

c) *the 3rd way*. Calculation is performed under the equivalent scheme for the initial time moment concerning only the transient components (fig. 7.24,c). Let's determine the required initial value of the transient component of the voltage across capacitor. The steady-state component is: $u_{Cs} = i_{3s}\cdot r_3 = 0.25\cdot 100 = 25\ V$.

Then, $u_{Ct}(0_+) = u_C(0_+) - u_{Cs} = 50 - 25 = 25\ V$.

The integration constants are: $$B = \frac{-u_{Ct}(0)}{r_2 + \frac{r_1 r_3}{r_1+r_3}} = \frac{-25}{50+\frac{100}{2}} = -0.25;$$

$$A = B\cdot\frac{r_3}{r_1+r_3} = -0.25\cdot\frac{100}{100+100} = -0.125,$$

$$D = -B\cdot\frac{r_1}{r_1+r_3} = 0.25\cdot\frac{100}{100+100} = 0.125.$$

6. Write down the final expressions for currents:

$i_1(t) = 0.25 - 0.125\cdot e^{-100t}\ A$; $i_2(t) = -0.25\cdot e^{-100t}\ A$; $i_3(t) = 0.25 + 0.125\cdot e^{-100t}\ A$.

7. In order to plot $i_1(t)$, let's calculate:

- the circuit time constant $\tau = 1/|p| = 1/100\ s = 10\ ms$,
- the practical duration of the transient process $T_{tp} = (3\div5)\tau = 4\cdot\tau = 40\ ms$.

The diagram is presented in fig. 7.25.

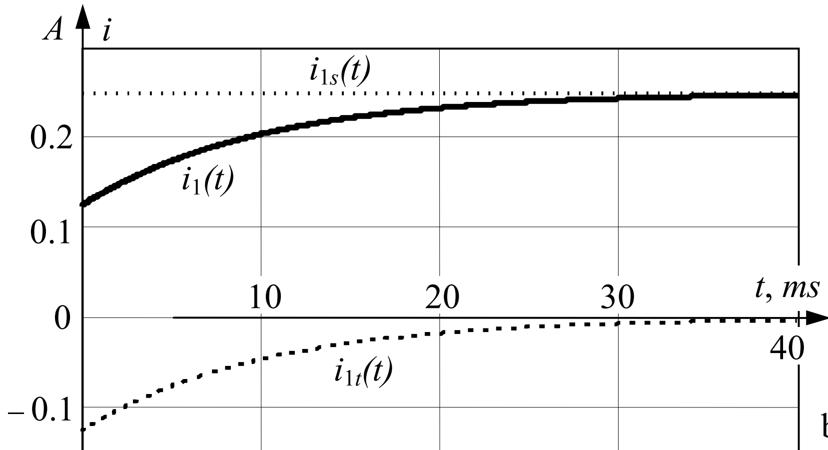


Fig. 7.25

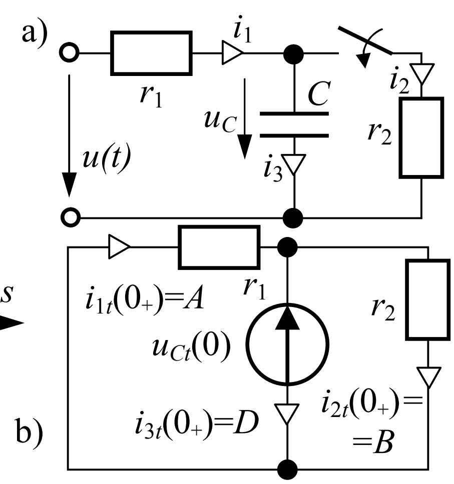


Fig. 7.26

7-11 (7.15). Calculate the transient currents in scheme fig. 7.26,a, by the classical method. The circuit parameters are as follows: $r_1 = 173\ Ohm$, $r_2 = 100\ Ohm$, $C = 100\ \mu F$, $u(t) = 100\cdot sin(100t + 30°)\ V$.

Solution. 1. Work out the independent initial condition by analyzing the circuit before the commutation: $i_2(t_-) = 0$; $i_1(t_-) = i_3(t_-) = I_{1m}\cdot sin(100t + \psi_{i1})$;

$$I_{1m} = \frac{U_{1m}}{\sqrt{r_1^2 + \left(1/\omega C\right)^2}} = \frac{100}{\sqrt{173^2+100^2}} = 0.5\ A,$$

$$\psi_{i1} = \psi_u - arctg\frac{-1/\omega C}{r_1} = 30° + arctg\frac{100}{173} = 60°,$$

$$U_{Cm} = I_{1m}\cdot\frac{1}{\omega C} = 0{,}5\cdot 100 = 50\ V, \quad \psi_{uC} = \psi_{i1} - 90° = -30°;$$

$$u_C(t_-) = U_{Cm}\cdot sin(100t + \psi_{uC}) = 50\cdot sin(100t - 30°)\ V.$$

The independent initial condition with account of the second commutation law is as follows: $u_C(0_+) = u_C(0_-) = 50\cdot sin(-30°) = -25\ V$.

2. In accordance with the classical calculation method, we have

$$u_C(t) = u_{Cs}(t) + u_{Ct}(t),$$

$$i_1(t) = i_{1s}(t) + i_{1t}(t), \quad i_2(t) = i_{2s}(t) + i_{2t}(t), \quad i_3(t) = i_{3s}(t) + i_{3t}(t).$$

3. We calculate the steady-state components by the complex-notation method:

$\underline{U}_m = 100\cdot e^{j30°}\ V$,

$$\underline{Z} = r_1 + \frac{r_2\cdot\left(-j\frac{1}{\omega C}\right)}{r_2 - j\frac{1}{\omega C}} = 173 + \frac{100\cdot(-j100)}{100 - j100} = 228.7\cdot e^{-j12.6°}\ Ohm,$$

$$\underline{I}_{1sm} = \frac{\underline{U}_m}{\underline{Z}} = \frac{100\cdot e^{j30}}{228.7\cdot e^{-j12.6}} = 0.437\cdot e^{j42.6°}\ A,$$

$$\underline{I}_{2sm} = \underline{I}_{1sm}\cdot\frac{-j\frac{1}{\omega C}}{r_2 - j\frac{1}{\omega C}} = 0.437\cdot e^{j42.6°}\cdot\frac{-j100}{100 - j100} = 0.309\cdot e^{-j2.4°}\ A,$$

$$\underline{I}_{3sm} = \underline{I}_{1sm}\cdot\frac{r_2}{r_2 - j\frac{1}{\omega C}} = 0.437\cdot e^{j42.6°}\cdot\frac{100}{100 - j100} = 0.309\cdot e^{j87.6°}\ A,$$

$\underline{U}_{Csm} = \underline{I}_{2sm}\cdot r_2 = 0.309\cdot e^{-j2.4°}\cdot 100 = 30.9\cdot e^{-j2.4°}\ V.$

The instantaneous current values are: $i_{1s}(t) = 0.437\cdot sin(100t + 42.6°)\ A$,
$i_{2s}(t) = 0.309\cdot sin(100t - 2.4°)\ A$, $i_{3s}(t) = 0.309\cdot sin(100t + 87.6°)\ A$.

The instantaneous and initial values of the steady-state component of the voltage across the capacitor are:

$$u_{Cs}(t) = 30.9\cdot sin(100t - 2.4°)\ V, \quad u_{Cs}(0_+) = 30.9\cdot sin(-2.4°) = -1.29\ V.$$

The initial value of the transient component of the voltage across the capacitor is:

$$u_{Ct}(0_+) = u_C(0_+) - u_{Cs}(0_+) = -25 + 1.29 = -23.71\ V.$$

4. Let's generate and solve the characteristic equation:

$$Z(p) = \frac{1}{pC} + \frac{r_1 r_2}{r_1 + r_2} = 0, \quad p = -\frac{r_1 + r_2}{C r_1 r_2} = -\frac{127 + 100}{10^{-4}\cdot 173\cdot 100} = -157.8\ s^{-1}.$$

Current transient components are: $i_{1t} = A\cdot e^{pt}$, $i_{2t} = B\cdot e^{pt}$, $i_{3t} = D\cdot e^{pt}$.

5. The integration constants are determined under the equivalent scheme for the initial time moment only for the transient components (fig. 7.26,b):

$$A = i_{1t}(0_+) = -\frac{u_{Ct}(0)}{r_1} = -\frac{-23.71}{173} = 0.137;$$

$$B = i_{2t}(0_+) = \frac{u_{Ct}(0)}{r_2} = \frac{-23.71}{100} = -0.237;$$

$$D = i_{3t}(0_+) = i_{1t}(0_+) - i_{2t}(0_+) = 0.173 + 0.237 = 0.374.$$

6. Work out the final formulae for the currents:

$$i_1(t) = i_{1s}(t) + i_{1t}(t) = 0.437\cdot sin(100t + 42.6°) + 0.137\cdot e^{-157.8t}\ A,$$
$$i_2(t) = i_{2s}(t) + i_{2t}(t) = 0.309\cdot sin(100t - 2.4°) - 0.237\cdot e^{-157.8t}\ A,$$
$$i_3(t) = i_{3s}(t) + i_{3t}(t) = 0.309\cdot sin(100t + 87.6°) + 0.374\cdot e^{-157.8t}\ A.$$

7.1.2. Transient processes in the circuits with two storage elements

7-12 (7.23). In scheme fig. 7.27, compute the transient currents by the classical method. The circuit parameters are: $U = 100\ V$, $r = 100\ Ohm$, $L = 0.1\ H$, $C = 1.6\ \mu F$. Plot $i_r(t)$.

Solution. 1. This time, there are zero independent initial conditions:

$$u_C(0_+) = u_C(0_-) = 0; \quad i_L(0_+) = i_L(0_-) = 0.$$

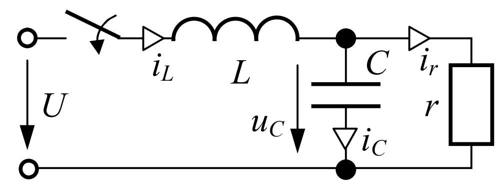


Fig. 7.27

2. The steady-state current components are:

$$i_{Cs} = 0, \quad i_{Ls} = i_{rs} = \frac{U}{r} = \frac{100}{100} = 1\ A.$$

3. The characteristic equation is: $\dfrac{1}{pC} + \dfrac{rpL}{r + pL} = 0$ or $LrC\cdot p^2 + L\cdot p + r = 0.$

The characteristic equation roots are:

$$p_{1,2} = \frac{-L \pm \sqrt{L^2 - 4r^2 LC}}{2LrC} = \frac{-0.1 \pm \sqrt{0.1^2 - 4\cdot 100^2 \cdot 0.1\cdot 1.6\cdot 10^{-6}}}{2\cdot 0.1\cdot 100\cdot 1.6\cdot 10^{-6}} = \frac{-0.1 \pm 0.06}{32\cdot 10^{-6}}\ s^{-1},$$

$$p_1 = -1250\ s^{-1}, \quad p_2 = -5000\ s^{-1}.$$

4. The transient components have the following view:

$$i_{Lt}(t) = A_1\cdot e^{p_1 t} + A_2\cdot e^{p_2 t}, \quad i_{rt}(t) = B_1\cdot e^{p_1 t} + B_2\cdot e^{p_2 t}, \quad i_{Ct}(t) = D_1\cdot e^{p_1 t} + D_2\cdot e^{p_2 t}.$$

The initial conditions for the transient components are:

$i_{Lt}(0_+) = A_1 + A_2$, $\quad i_{rt}(0_+) = B_1 + B_2$, $\quad i_{Ct}(0_+) = D_1 + D_2$,

$i_{Lt}'(0_+) = p_1\cdot A_1 + p_2\cdot A_2$; $\quad i_{rt}'(0_+) = p_1\cdot B_1 + p_2\cdot B_2$; $\quad i_{Ct}'(0_+) = p_1\cdot D_1 + p_2\cdot D_2$.

5. In order to determine the integration constants, we calculate the initial conditions. For that, we generate the equation system under Kirchhoff's laws for a zero time moment; moreover, we take into account the coupling equation between the current and voltage of the capacitor:

$$\begin{cases} i_L(0_+) - i_C(0_+) - i_r(0_+) = 0, \\ L\cdot i_L'(0_+) + u_C(0_+) = U, \\ u_C(0_+) - i_r(0_+)\cdot r = 0, \\ i_C(0_+) = C\cdot u_C'(0_+). \end{cases}$$

From here, the initial values of currents are:

$$i_r(0_+) = u_C(0_+)/r = 0; \quad i_C(0_+) = i_L(0_+) - i_r(0_+) = 0.$$

The initial values of the derivatives are: $u_C'(0_+) = i_C(0_+)/C = 0$;

$$i_r'(0_+) = u_C'(0_+)/r = 0; \quad i_L'(0_+) = (U - u_C(0_+))/L = 100/0.1 = 1000\ A/s;$$
$$i_C'(0_+) = i_L'(0_+) - i_r'(0_+) = 1000\ A/s.$$

The initial conditions for transient components are:

$i_{Lt}(0_+) = i_L(0_+) - i_{Ls}(0_+) = 0 - 1 = -1\ A$, $\quad i_{Lt}'(0_+) = i_L'(0_+) - i_{Ls}'(0_+) = 1000 - 0 = 1000\ A/s$,

$i_{rt}(0_+) = i_r(0_+) - i_{rs}(0_+) = 0 - 1 = -1\ A$, $\quad i_{rt}'(0_+) = i_r'(0_+) - i_{rs}'(0_+) = 0$,

$i_{Ct}(0_+) = i_C(0_+) - i_{Cs}(0_+) = 0,\qquad i_{Ct}'(0_+) = i_C'(0_+) - i_{Cs}'(0_+) = 1000 - 0 = 1000\ A/s.$

Thus, we obtain and solve the following three systems of equations:

$$\begin{cases} A_1 + A_2 = -1, \\ p_1 \cdot A_1 + p_2 \cdot A_2 = 1000; \end{cases} \qquad \begin{cases} A_2 = -1 - A_1 = -1 + 1.065 = 0.065, \\ A_1 = \dfrac{1000 + p_2}{p_1 - p_2} = \dfrac{1000 - 5000}{-1250 + 5000} = -1.065. \end{cases}$$

$$\begin{cases} B_1 + B_2 = -1, \\ p_1 \cdot B_1 + p_2 \cdot B_2 = 0; \end{cases} \qquad \begin{cases} B_2 = -1 - B_1 = -1 + 1.33 = 0.33, \\ B_1 = \dfrac{p_2}{p_1 - p_2} = \dfrac{-5000}{3750} = -1.33. \end{cases}$$

$$\begin{cases} D_1 + D_2 = 0, \\ p_1 \cdot D_1 + p_2 \cdot D_2 = 0; \end{cases} \qquad \begin{cases} D_2 = -D_1 = -0.266, \\ D_1 = \dfrac{1000}{p_1 - p_2} = \dfrac{1000}{3750} = 0.266. \end{cases}$$

6. Write down the final expressions for currents in accordance with the classical calculation method:

$i_L(t) = i_{Ls}(t) + i_{Lt}(t) = 1 - 1.065 \cdot e^{-1250t} + 0.065 \cdot e^{-5000t}\ A,$

$i_r(t) = i_{rs}(t) + i_{rt}(t) = 1 - 1.33 \cdot e^{-1250t} + 0.33 \cdot e^{-5000t}\ A,$

$i_C(t) = i_{Cs}(t) + i_{Ct}(t) = 0.266 \cdot e^{-1250t} - 0.266 \cdot e^{-5000t}\ A.$

7. Let's plot $i_r(t)$ (fig. 7.28). The transient time with account $|p_2| > |p_1|$ is $T_{tp} = \dfrac{4}{|p_1|} = \dfrac{4}{1250}\ s = 3.2\ ms.$

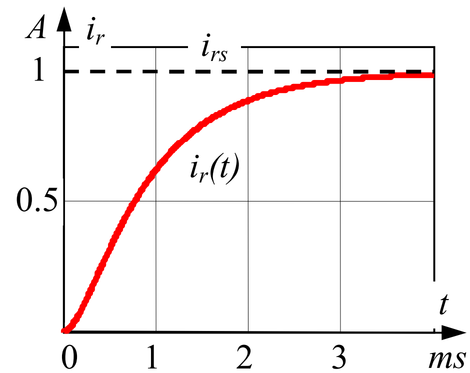


Fig. 7.28

7-13 (7.24). Calculate the transient currents in scheme fig. 7.29 using the classical method. The circuit parameters are: $U = 100\ V$, $r = 50\ Ohm$, $L = 0.2\ H$, $C = 40\ \mu F$. Plot $i_1(t)$.

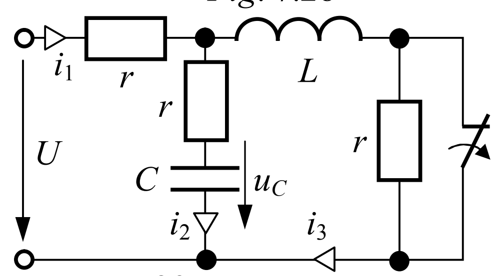


Fig. 7.29

Solution. 1. The independent initial conditions are:

$$u_C(0_+) = u_C(0_-) = 0;$$

$$i_3(0_+) = i_3(0_-) = \frac{U}{r} = \frac{100}{50} = 2\ A.$$

2. The steady-state current components are: $i_{2s} = 0,\ i_{1s} = i_{3s} = \dfrac{U}{2r} = \dfrac{100}{100} = 1\ A.$

3. The characteristic equation is:

$$\frac{1}{pC} + r + \frac{r(pL + r)}{r + pL + r} = 0 \quad \text{or} \quad 2LrC \cdot p^2 + (L + 3r^2C)p + 2r = 0,$$

or $2 \cdot 0.2 \cdot 50 \cdot 40 \cdot 10^{-6} \cdot p^2 + (0.2 + 3 \cdot 50^2 \cdot 40 \cdot 10^{-6})p + 2 \cdot 50 = 0,$

or $0.8 \cdot 10^{-3} \cdot p^2 + 0.5 \cdot p + 100 = 0.$

The characteristic equation roots are: $p_{1,2} = -312.5 \pm j165.4\ s^{-1}.$

4. The transient components have the following view:

$$i_{1t}(t) = A \cdot e^{-\alpha t} \cdot \sin(\omega t + \psi_1),\quad i_{2t}(t) = B \cdot e^{-\alpha t} \cdot \sin(\omega t + \psi_2),\quad i_{3t}(t) = D \cdot e^{-\alpha t} \cdot \sin(\omega t + \psi_3),$$

where the decay coefficient is $\alpha = |Re(p_1)| = 312.5\ s^{-1}$;

angular velocity of the free oscillations is $\omega = Im(p_1) = 165.4\ rad/s^{-1}$.

The initial conditions for the transient components are:

$$i_{1t}(0_+) = A \cdot \sin\psi_1, \qquad i_{1t}'(0_+) = -\alpha \cdot A \cdot \sin\psi_1 + \omega \cdot A \cdot \cos\psi_1;$$

$i_{2t}(0_+) = B\cdot sin\psi_2,\qquad i_{2t}'(0_+) = -\alpha\cdot B\cdot sin\psi_2 + \omega\cdot B\cdot cos\psi_2;$

$i_{3t}(0_+) = D\cdot sin\psi_3,\qquad i_{3t}'(0_+) = -\alpha\cdot D\cdot sin\psi_3 + \omega\cdot D\cdot cos\psi_3.$

5. In order to determine the integration constants, we calculate the initial conditions. Toward this end, let's generate the equation system under Kirchhoff's laws for a zero time moment; moreover, we take into account the coupling equation between the current and voltage of the capacitor:

$$\begin{cases} i_1(0_+) - i_2(0_+) - i_3(0_+) = 0, \\ r\cdot i_1(0_+) + r\cdot i_2(0_+) + u_C(0_+) = U, \\ r\cdot i_1(0_+) + L\cdot i_3'(0_+) + r\cdot i_3(0_+) = U, \\ i_2(0_+) = C\cdot u_C'(0_+). \end{cases}$$

From here, the initial values of currents are: $i_1(0_+) = i_2(0_+) + 2$,

$$i_2(0_+) = \frac{U - u_C(0) - 2r}{2r} = \frac{100 - 0 - 2\cdot 50}{2\cdot 50} = 0; \quad i_1(0_+) = 2\ A.$$

The initial values of the derivatives are: $u_C'(0_+) = i_2(0_+)/C = 0$;

$$i_3'(0_+) = \frac{U - r\cdot(i_1(0) + i_3(0))}{L} = \frac{100 - 50\cdot(2+2)}{0.2} = -500\ A/s;$$

$$i_1'(0_+) = -i_2'(0_+) = i_3'(0_+)/2 = -250\ A/s.$$

The initial conditions for the transient components are:

$i_{1t}(0_+) = i_1(0_+) - i_{1s}(0_+) = 2 - 1 = 1\ A,\qquad i_{1t}'(0_+) = i_1'(0_+) - i_{1s}'(0_+) = -250\ A/s,$

$i_{2t}(0_+) = i_2(0_+) - i_{2s}(0_+) = 0,\qquad i_{2t}'(0_+) = i_2'(0_+) - i_{2s}'(0_+) = 250\ A/s,$

$i_{3t}(0_+) = i_3(0_+) - i_{3t}(0_+) = 2 - 1 = 1\ A,\qquad i_{3t}'(0_+) = i_3'(0_+) - i_{3s}'(0_+) = -500\ A/s.$

Thus, we obtain and solve the following three equation systems:

$$\begin{cases} A\cdot sin\psi_1 = 1, \\ -\alpha\cdot A\cdot sin\psi_1 + \omega\cdot A\cdot cos\psi_1 = -250; \end{cases}$$

$$A\cdot cos\psi_1 = \frac{-250 + \alpha\cdot A sin\psi_1}{\omega} = \frac{-250 + 312.5}{165.4} = 0.3779,$$

$$A = \sqrt{(A sin\psi_1)^2 + (A cos\psi_1)^2} = \sqrt{1^2 + 0.3779^2} = 1.069,$$

$$tg\psi_1 = \frac{A sin\psi_1}{A cos\psi_1} = \frac{1}{0.3779} = 2.646, \qquad \psi_1 = 69.3°.$$

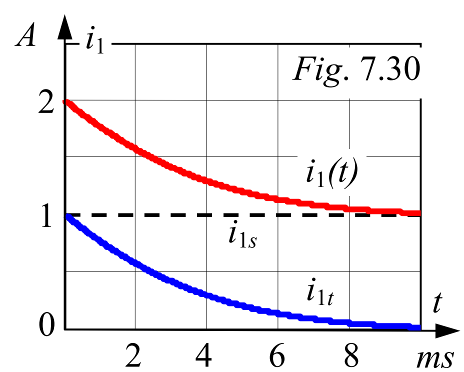


Fig. 7.30

$$\begin{cases} B\cdot sin\psi_2 = 0, \\ -\alpha\cdot B\cdot sin\psi_2 + \omega\cdot B\cdot cos\psi_2 = 250; \end{cases} \qquad B = 1.511, \quad \psi_2 = 0°.$$

$$\begin{cases} D\cdot sin\psi_3 = 1, \\ -\alpha\cdot D\cdot sin\psi_3 + \omega\cdot D\cdot cos\psi_3 = -500; \end{cases}$$

$$D\cdot cos\psi_3 = \frac{-500 + 312.5}{165.4} = -1.134,$$

$$D = \sqrt{1^2 + 1.134^2} = 1.512, \qquad tg\psi_3 = \frac{1}{-1.134} = -0.882,$$

but $cos\psi_3 < 0$, that's why $\psi_3 = 180° + arctg(-0.882) = 138.6°$.

6. Write down the final expressions for the currents in accordance with the classical calculation method:

$$i_1(t) = i_{1s}(t) + i_{1t}(t) = 1 + 1.069\cdot e^{-312.5t}\cdot sin(165.4t + 69.3°)\ A,$$

$$i_2(t) = i_{2s}(t) + i_{2t}(t) = 1.511\cdot e^{-312.5t}\cdot sin(165.4t)\ A,$$

$$i_3(t) = i_{3s}(t) + i_{3t}(t) = 1 + 1.512\cdot e^{-312.5t}\cdot sin(165.4t + 138.6°)\ A.$$

7. Let's plot $i_1(t)$ (fig. 7.30). The transient time is $T_{tp} = \frac{4}{\alpha} = \frac{4}{312.5}\ s = 12.8\ ms.$

The period of the free oscillations is

$$T_0 = \frac{2\pi}{\omega} = \frac{2\pi}{165.4} = 0.038\ s = 38\ ms.$$

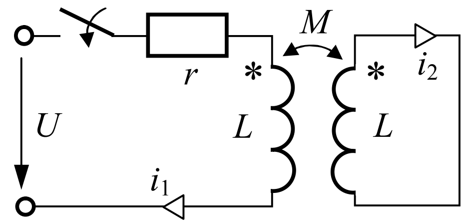


Fig. 7.31

7-14 (7.30). Calculate the transient currents in the scheme fig. 7.31. Numerical data are:

$U = 50\ V$, $r = 10\ Ohm$, $L = 0.1\ H$, $M = 0.05\ H$.

Solution. 1. The independent initial conditions are zero: $i_1(0_+) = i_2(0_+) = 0$.

2. The steady-state currents are: $i_{1s} = \frac{U}{r} = \frac{50}{10} = 5\ A$, $i_{2s} = 0$.

3. The equation system under Kirchhoff's laws for the circuit state after switching is:

$$\begin{cases} r\cdot i_1 + L\frac{di_1}{dt} - M\frac{di_2}{dt} = U, \\ -M\frac{di_1}{dt} + L\frac{di_2}{dt} = 0. \end{cases}$$

Having presented the equation system in the operational form and having equated its determinant to zero, we obtain the characteristic equation:

$$\Delta(p) = \begin{vmatrix} r + pL & -pM \\ -pM & pL \end{vmatrix} = rpL + p^2L^2 - p^2M^2 = 0.$$

The roots of the equation are: $p_1 = 0$, $p_2 = -\frac{rL}{L^2 - M^2} = -\frac{10\cdot 0.1}{0.1^2 - 0.05^2} = -133.3\ s^{-1}$.

4. The transient components of the currents have a view:

$i_{1t}(t) = A_1\cdot e^{p_1t} + A_2\cdot e^{p_2t} = A_1 + A_2\cdot e^{p_2t}$;

$i_{2t}(t) = B_1\cdot e^{p_1t} + B_2\cdot e^{p_2t} = B_1 + B_2\cdot e^{p_2t}$.

5. Let's calculate the dependent initial conditions (values of the current derivatives at the initial moment) with the aid of the equation system given in step 3 above for the time moment $t = 0_+$:

$$\begin{cases} Li_1'(0_+) - Mi_2'(0_+) = U - r\cdot i_1(0_+) = 50, \\ -Mi_1'(0_+) + Li_2'(0_+) = 0; \end{cases} \text{or} \begin{cases} 0.1i_1'(0_+) - 0.05i_2'(0_+) = 50, \\ -0.05i_2'(0_+) + 0.1i_2'(0_+) = 0. \end{cases}$$

The system solution is: $i_1'(0_+) = 667\ A/s$, $i_2'(0_+) = 333\ A/s$.

6. The initial values of the transient currents and their derivatives are:

- on the one hand, $i_{1t}(0_+) = A_1 + A_2$, $i_{1t}'(0_+) = A_2\cdot p_2$;
  $i_{2t}(0_+) = B_1 + B_2$, $i_{2t}'(0_+) = B_2\cdot p_2$;
- on the other hand, $i_{1t}(0_+) = i_1(0_+) - i_{1s}(0_+) = 0 - 5 = -5\ A$,
  $i_{1t}'(0_+) = i_1'(0_+) - i_{1s}'(0_+) = 667\ A/s$,
  $i_{2t}(0_+) = i_2(0_+) - i_{2s}(0_+) = 0$,
  $i_{2t}'(0_+) = i_2'(0_+) - i_{2s}'(0_+) = 333\ A/s$.

We obtain and solve the equation systems:

$A_1 + A_2 = -5$, $A_2\cdot p_2 = 667$; $A_2 = 667/(-133,3) = -5$, $A_1 = 0$;

$B_1 + B_2 = 0$, $B_2\cdot p_2 = 333$; $B_2 = 333/(-133.3) = -2.5$, $B_1 = -B_2 = 2.5$.

7. Finally, we have: $i_1(t) = i_{1s} + i_{1t} = 5 - 5\cdot e^{-133.3t}\ A$, $i_2(t) = 2.5 - 2.5\cdot e^{-133.3t}\ A$.

The fact $i_2(\infty) \neq 0$ is explained by zero secondary resistance $r_2 = 0$, which means the secondary circuit is made of the superconductor.

7-15 (7.31). Calculate the transient currents in scheme fig. 7.32. Numerical data are: $E = 10\ V$, $r = 1\ Ohm$, $L_1 = 0.1\ H$, $L_2 = 0.05\ H$, $M = 0.05\ H$.

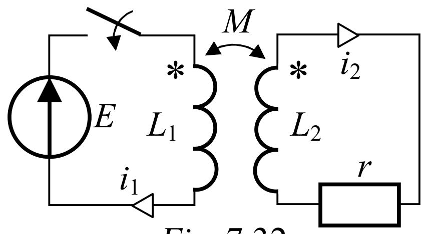


Fig. 7.32

Solution. 1. The independent initial conditions are zero: $i_1(0_+) = i_2(0_+) = 0$.

2. The steady-state currents are: $i_{1s} = E/0 = \infty$, $i_{2s} = 0$.

3. The equation system under Kirchhoff's laws for the circuit state after switching is:

$$\begin{cases} L_1\dfrac{di_1}{dt} - M\dfrac{di_2}{dt} = E, \\ -M\dfrac{di_1}{dt} + r\cdot i_2 + L_2\dfrac{di_2}{dt} = 0. \end{cases}$$

Having presented the equation system in the operational form and having equated its determinant to zero, we obtain the characteristic equation:

$$\Delta(p) = \begin{vmatrix} pL_1 & -pM \\ -pM & r + pL_2 \end{vmatrix} = p^2L_1L_2 + pL_1r - p^2M^2 = 0.$$

The roots of equation are: $p_1 = 0,\ p_2 = -\dfrac{rL_1}{L_1L_2 - M^2} = -\dfrac{1\cdot 0.1}{0.1\cdot 0.05 - 0.05^2} = -40\,s^{-1}$.

4. The transient components of the currents have a view:

$$i_{1t}(t) = A_1 + A_2\cdot e^{p_2t};\quad i_{2t}(t) = B_1 + B_2\cdot e^{p_2t}.$$

5. Let's calculate the dependent initial conditions (values of the current derivatives at the initial moment) with the aid of the equation system given in step 3 above for the time moment $t = 0_+$:

$$\begin{cases} L_1i_1'(0_+) - Mi_2'(0_+) = E = 10, \\ -Mi_1'(0_+) + L_2i_2'(0_+) = 0; \end{cases} \quad \text{or} \quad \begin{cases} 0{,}1i_1'(0_+) - 0{,}05i_2'(0_+) = 10, \\ -0{,}05i_2'(0_+) + 0{,}05i_2'(0_+) = 0. \end{cases}$$

The system solution is: $i_1'(0_+) = 200\ A/s$, $i_2'(0_+) = 200\ A/s$.

6. The initial values of the transient currents and their derivatives are:

- on the one hand, $\quad i_{1t}(0_+) = A_1 + A_2,\ i_{1t}'(0_+) = A_2\cdot p_2;$
  $i_{2t}(0_+) = B_1 + B_2,\ i_{2t}'(0_+) = B_2\cdot p_2;$
- on the other hand, $\quad i_{1t}(0_+) = i_1(0_+) - i_{1s}(0_+) = -\infty,$
  $i_{1t}'(0_+) = i_1'(0_+) - i_{1s}'(0_+) = 200\ A/c,$
  $i_{2t}(0_+) = i_2(0_+) - i_{2s}(0_+) = 0,$
  $i_{2t}'(0_+) = i_2'(0_+) - i_{2s}'(0_+) = 200\ A/c.$

The first equation system $\quad A_1 + A_2 = -\infty,\quad A_1\cdot p_1 + A_2\cdot p_2 = 200$
cannot be solved, that's why we solve the second equation system:

$$B_1 + B_2 = 0,\quad B_2\cdot p_2 = 200;\qquad B_2 = \frac{200}{-40} = -5,\quad B_1 = -B_2 = 5.$$

7. Thus, the secondary current is: $i_2(t) = 5 - 5\cdot e^{-40t}\ A$.

We obtain the primary current from the first equation of the system drawn under Kirchhoff's laws:

$$\frac{di_1}{dt}=\frac{E+M\dfrac{di_2}{dt}}{L_1}=\frac{10+0.05\cdot(-5)\cdot(-40)\cdot e^{-40t}}{0.1}=100+100\cdot e^{-40t}\ A/s;$$

$$i_1(t)=\int_0^t\frac{di_1}{dt}dt=\int_0^t(100+100e^{-40t})dt=100t-2.5\cdot e^{-40t}-2.5\ A.$$

Unusual expression for the primary current is explained by zero resistance of the primary loop $r_1 = 0$. Here $(100t - 2.5)$ is not a steady-state component but a sum of the steady-state component and one of two parts of the transient component. The constant component of the secondary current $5 = 5\cdot e^{-0t}$ is not a steady-state component but one of two parts of the transient component as well. The transient component of the secondary current is not died away because the primary current goes up to infinity. Certainly, such situation may be observed in practice but approximately and for a short time interval.

7-16 (7.32). Compute the transient currents in scheme fig. 7.33. Numerical data are as follows: $U = 100\ V$, $r_1 = r_2 = 50\ Ohm$, $L_1 = 0.1\ H$, $L_2 = 0.2\ H$, $M = 0.05\ H$.

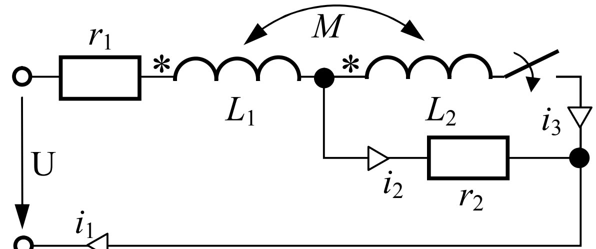


Fig. 7.33

Solution. 1. The independent initial conditions are:

$$i_1(0_+)=i_1(0_-)=\frac{U}{r_1+r_2}=\frac{100}{50+50}=1\ A,$$

$$i_3(0_+)=i_3(0_-)=0.$$

2. The steady-state current components are: $i_{2s}=0,\ i_{1s}=i_{3s}=\dfrac{U}{r_1}=\dfrac{100}{50}=2\,A.$

3. The equation system under Kirchhoff's laws for the circuit state after switching is:

$$\begin{cases} i_1 - i_2 - i_3 = 0,\\ r_1\cdot i_1 + L_1 i_1' + M i_3' + L_2 i_3' + M i_1' = U,\\ r_1\cdot i_1 + L_1 i_1' + M i_3' + r_2\cdot i_2 = U.\end{cases}$$

4. The characteristic equation obtained on the ground of the equations under Kirchhoff's laws is:

$$\Delta(p)=\begin{vmatrix} 1 & -1 & -1\\ r_1+p(L_1+M) & 0 & p(L_2+M)\\ r_1+pL_1 & r_2 & pM\end{vmatrix}=0,$$

$$p^2(L_1L_2-M^2)+p(r_1L_2+r_2L_1+r_2L_2+2r_2M)+r_1r_2=0,$$

$$p^2(0.1\cdot0.2-0.05^2)+p(50\cdot0.2+50\cdot0.1+50\cdot0.2+2\cdot50\cdot0.05)+50\cdot50=0,$$

$$p^2\cdot0.0175+p\cdot30+2500=0,$$

$$p_{1,2}=-657.1\pm769.3\ s^{-1};\quad p_1=-87.8\ s^{-1};\quad p_2=-1626.4\ c^{-1}.$$

5. The transient components of the currents are:

$$i_{1t}(t)=A_1\cdot e^{p_1t}+A_2\cdot e^{p_2t};\quad i_{2t}(t)=B_1\cdot e^{p_1t}+B_2\cdot e^{p_2t};\quad i_{3t}(t)=D_1\cdot e^{p_1t}+D_2\cdot e^{p_2t}.$$

The initial values of the transient components of currents and their derivatives are:

$i_{1t}(0_+) = A_1 + A_2$; $i_{2t}(0_+) = B_1 + B_2$; $i_{3t}(0_+) = D_1 + D_2$;

$i_{1t}'(0_+) = A_1p_1 + A_2p_2$; $i_{2t}'(0_+) = B_1p_1 + B_2p_2$; $i_{1t}'(0_+) = D_1p_1 + D_2p_2$.

6. We obtain the initial values from the previously generated equation system for zero time moment: $i_2(0_+) = i_1(0_+) - i_3(0_+) = 1\ A$,

$i_1'(0_+)\cdot(L_1+M) + i_3'(0_+)\cdot(L_2+M) = U - r_1\cdot i_1(0_+)$, $\quad 0.15i_1'(0_+) + 0.25i_3'(0_+) = 50$,

$L_1i_1'(0_+)+Mi_3'(0_+) = U - r_1\cdot i_1(0_+) - r_2\cdot i_2(0_+)$, $\quad 0.1i_1'(0_+) + 0.05i_3'(0_+) = 100 - 50 - 50 = 0$.

The equation system solution is:

$i_1'(0_+) = -142.9\ A/s$, $\quad i_3'(0_+) = 285.7\ A/s$, $\quad i_2'(0_+) = i_1'(0_+) - i_3'(0_+) = -428.6\ A/s$.

The initial values of the transient components and their derivatives are:

$i_{1t}(0_+) = i_1(0_+) - i_{1s}(0_+) = 1 - 2 = -1\ A$, $\quad i_{1t}'(0_+) = i_1'(0_+) = -142.9\ A/s$,

$i_{2t}(0_+) = i_2(0_+) - i_{2s}(0_+) = 1 - 0 = 1\ A$, $\quad i_{2t}'(0_+) = i_2'(0_+) = -428.6\ A/s$,

$i_{3t}(0_+) = i_3(0_+) - i_{3s}(0_+) = 0 - 2 = -2\ A$, $\quad i_{3t}'(0_+) = i_3'(0_+) = 285.7\ A/s$.

7. We obtain and solve the following equation systems:

$A_1 + A_2 = -1$, $\quad A_1p_1 + A_2p_2 = -142.9 \quad \Rightarrow \quad A_1 = -1.150$, $\quad A_2 = 0.150$;

$B_1 + B_2 = 1$, $\quad B_1p_1 + B_2p_2 = -428.6 \quad \Rightarrow \quad B_1 = 0.778$, $\quad B_2 = 0.222$;

$D_1 + D_2 = -2$, $\quad D_1p_1 + D_2p_2 = 285.7 \quad \Rightarrow \quad D_1 = -1.928$, $\quad D_2 = -0.072$;

8. Finally, we have: $i_1(t) = i_{1s}(t) + i_{1t}(t) = 2 - 1.150\cdot e^{-87.8t} + 0.150\cdot e^{-1626.4\cdot t}\ A$,

$i_2(t) = i_{2s}(t) + i_{2t}(t) = 0.778\cdot e^{-87.8t} + 0.222\cdot e^{-1626.4\cdot t}\ A$,

$i_3(t) = i_{3s}(t) + i_{3t}(t) = 2 - 1.928\cdot e^{-87.8t} - 0.072\cdot e^{-1626.4\cdot t}\ A$.

7.1.3. Transient processes at the instant change of the reactive parameters of the circuit parts (ill-conditioned commutation)

7-17 (7.35). Using the numerical data of problem 7.15, calculate the current i_2 in scheme fig. 7.34 when the switch is opening.

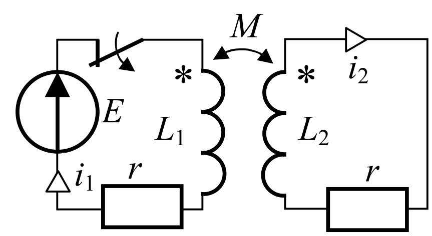


Fig. 7.34

Solution. The currents before the commutation are:

$$i_2(t_-) = 0, \quad i_1(t_-) = \frac{10}{1} = 10\ A.$$

The equation under Kirchhoff's voltage law for the secondary loop is

$$-M\frac{di_1}{dt} + r\cdot i_2 + L_2\frac{di_2}{dt} = 0.$$

Owing to the jump change of the primary current from 10 A down to 0 the derivatives $\frac{di_1}{dt}$ and $\frac{di_2}{dt}$ at the commutation moment reach the infinite meanings, so the quantity $r\cdot i_2$ can be neglected. Thus, we have an equality $Mdi_1 = L_2di_2$, which is integrated from $t = 0_-$ to $t = 0_+$ during the commutation period. During this period, currents change:

i_1 – from $i_1(0_-) = 10\ A$ to $i_1(0_+) = 0$,

i_2 – from $i_2(0_-) = 0$ to required $i_2(0_+)$.

$$L_2\int_0^{i_2(0_+)} di_2 = M\int_{i_1(0_-)}^{0} di_1 \quad \text{or} \quad L_2i_2(0_+) = -Mi_1(0_-) = -0.05\cdot 10 = -0.5\ Wb.$$

From here $i_2(0_+) = -\dfrac{0.5}{L_2} = -\dfrac{0.5}{0.05} = -10\ A$.

The circuit time constant is $\tau = \dfrac{L_2}{r} = \dfrac{0.05}{1} = 0.05\ s$.

The root of the characteristic equation is $p = -\tau^{-1} = -20\ s^{-1}$.

The required transient current is $i_2(t) = i_2(0)\cdot e^{pt} = -10\cdot e^{-20t}\ A$. "Minus" sign means the opposite direction of the current in comparison with that shown in the figure.

7-18 (7.36). Calculate the current of the first coil in scheme fig. 7.35 with the following numerical data: $U = 100\ V$, $r_1 = 60\ Ohm$, $r_2 = 40\ Ohm$, $L_1 = 0.1\ H$, $L_2 = 0.2\ H$, $M = 0.05\ H$. Plot the current of the first coil.

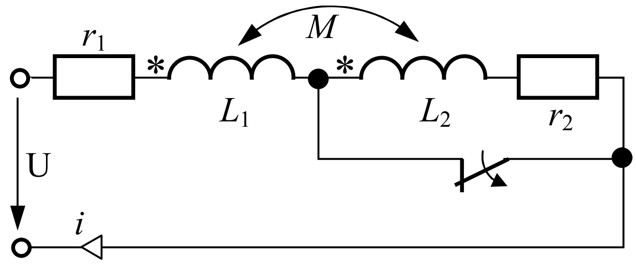


Fig. 7.35

Solution. 1. Before the commutation, currents in the coils are different: the current in the first coil is – $i_1(0_-) = \dfrac{U}{r_1} = \dfrac{100}{60} = 1.667\ A,$

but no current flows through the second coil – $i_2(0_-) = 0$.

Thus, before the commutation, the circuit flux linkage consists of the sum of the coils flux linkages:

$$\Psi(0_-) = \Psi_1(0_-) + \Psi_2(0_-) = L_1 i_1(0_-) + M i_2(0_-) + L_2 i_2(0_-) + M i_1(0_-) =$$
$$= 0.1\cdot 1.667 + 0 + 0 + 0.05\cdot 1.667 = 0.25\ Wb.$$

The independent initial condition in accordance with the first commutation law is: $\Psi(0_+) = \Psi(0_-) = 0.25\ Wb$. However, after the commutation, with account of the series connection of the coils with additive polarity, the circuit flux linkage is as follows $\Psi(0_+) = i(0_+)\cdot(L_1 + L_2 + 2M)$. From here

$$i(0_+) = \frac{\Psi(0_+)}{L_1 + L_2 + 2M} = \frac{0.25}{0.1 + 0.2 + 2\cdot 0.05} = 0.625\ A.$$

2. The steady-state current component is: $i_s = \dfrac{U}{r_1 + r_2} = \dfrac{100}{60 + 40} = 1\ A.$

3. The characteristic equation is: $p\cdot(L_1 + L_2 + 2M) + r_1 + r_2 = 0,$

$$p = -\frac{r_1 + r_2}{L_1 + L_2 + 2M} = -\frac{60 + 40}{0.1 + 0.2 + 2\cdot 0.05} = -\ 250\ s^{-1}.$$

4. The transient current component is $i_t = A\cdot e^{pt}$, where the integration constant is

$A = i_t(0) = i(0) - i_s = 0.625 - 1 = -\ 0.375.$

5. Finally, we have:

$i(t) = i_s(t) + i_t(t) = 1 - 0.375\cdot e^{-250t}\ A.$

6. The current curve is shown in fig. 7.36.

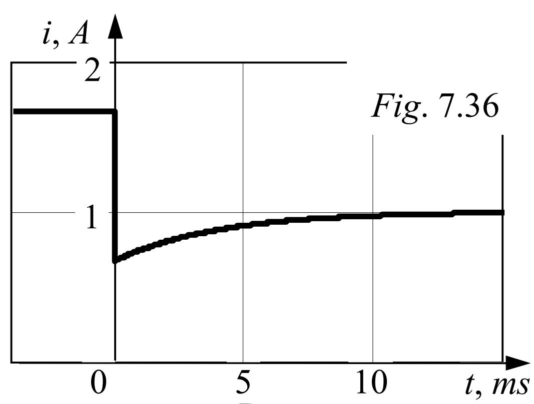


Fig. 7.36

7-19 (7.37). Calculate the current of the second coil in scheme fig. 7.37 with the following numerical data: $E = 11.7\ V$, $r_1 = 1\ Ohm$, $r_2 = 9\ Ohm$, $r = 3\ Ohm$, $L_1 = 10\ mH$, $L_2 = 20\ mH$, $M = 10\ mH$.

Solution. 1. Before the commutation:

$$R = r + \frac{r_1 r_2}{r_1 + r_2} = 3 + \frac{1\cdot 9}{1 + 9} = 3.9\ Ohm,$$

$$i_1(0_-) = \frac{E}{R}\cdot\frac{r_2}{r_1 + r_2} = \frac{11.7\cdot 9}{3.9\cdot 10} = 2.7\ A,$$

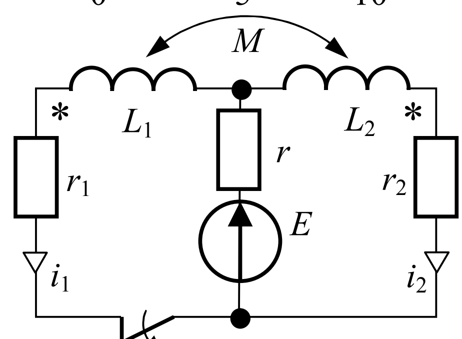


Fig. 7.37

$$i_2(0_-)=\frac{E}{R}\cdot\frac{r_1}{r_1+r_2}=\frac{11.7\cdot 1}{3.9\cdot 10}=0.3\ A.$$

2. According to Kirchhoff's voltage law the equation for the loop with the second coil for the commutation moment is:

$$r\cdot(i_1+i_2)+r_2\cdot i_2+L_2\frac{di_2}{dt}+M\frac{di_1}{dt}=E \text{ or } L_2di_2=-\,Mdi_1$$

$$\text{or } L_2\cdot(i_2(0_+)-i_2(0_-))=-\,M\cdot(i_1(0_+)-i_1(0_-)).$$

But $i_1(0_+)=0$, then $i_2(0_+)=\frac{M}{L_2}i_1(0_-)+i_2(0_-)=\frac{10}{20}\cdot 2.7+0.3=1.65\ A.$

3. The steady-state current component is $i_{2s}=\frac{E}{r+r_2}=\frac{11.7}{12}=0.975\ A.$

4. The root of the characteristic equation is $p=-\frac{r+r_2}{L_2}=-\frac{3+9}{0.02}=-\,600\ s^{-1}.$

5. The transient current component is $i_{2t}=A\cdot e^{pt}$,
where the integration constant is $A=i_{2t}(0_+)=i_2(0_+)-i_{2s}=1.65-0.975=0.675.$

6. Finally, we have: $i_2(t)=i_{2s}(t)+i_{2t}(t)=0.975+0.675\cdot e^{-600t}\ A.$

7-20 (7.38). Calculate the transient current $i_1(t)$ in scheme fig. 7.38 with the following numerical data: $E_1=36\ V$, $E_2=6\ V$, $r_1=300\ Ohm$, $r_2=r_3=600\ Ohm$, $C_1=300\ \mu F$, $C_2=200\ \mu F$.

Solution. Before the commutation, the voltages across the capacitors and their summary charge are:

$$u_{C1}(0_-)=\frac{E_1}{r_1+r_3}\cdot r_3=\frac{36\cdot 600}{300+600}=24\ V,$$

$u_{C2}(0_-)=-\,E_2=-\,6\ V.$

$q(0_-)=C_1u_{C1}(0_-)+C_2u_{C2}(0_-)=$

$=(300\cdot 24-200\cdot 6)\cdot 10^{-6}=60\cdot 10^{-4}\ C.$

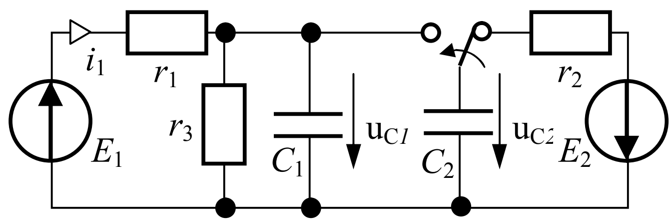


Fig. 7.38

After the commutation, the capacitors have the same voltage $u_C(0_+)=u_{C1}(0_+)=u_{C2}(0_+)$. That's why the capacitor charge after the commutation is $q(0_+)=(C_1+C_2)\cdot u_C(0_+)$. However, in accordance with the second commutation law $q(0_+)=q(0_-)$. From here

$$u_C(0_+)=\frac{q(0_+)}{C_1+C_2}=\frac{60\cdot 10^{-4}}{(300+200)\cdot 10^{-6}}=12\ V.$$

The initial value of the required current is

$$i_1(0)=\frac{E_1-u_C(0)}{r_1}=\frac{36-12}{300}=0.08\ A=80\ mA.$$

The steady-state current is $i_{1s}=\frac{E_1}{r_1+r_3}=\frac{36\cdot 10^{-3}}{300+600}=40\ mA.$

The transient component of the current is $i_{1t}=A\cdot e^{pt}$, where the root of the characteristic equation is $p=-\frac{r_1+r_3}{(C_1+C_2)r_1r_3}=-\frac{(300+600)\cdot 10^6}{(300+200)\cdot 300\cdot 600}=-\,10\ s^{-1},$

the integration constant is $A = i_{1t}(0) = i_1(0) - i_{1s} = 80 - 40 = 40$.

Finally, we have: $i_1(t) = i_{1s}(t) + i_{1t}(t) = 40 + 40 \cdot e^{-10t}$ mA.

7.1.4. State variable method

7-21 (7.40). Calculate the transient currents in scheme fig. 7.39 by the state variable method. The numerical data are: $E = 100$ V, $J = 5$ A, $r_1 = 20$ Ohm, $r_2 = 30$ Ohm, $r_3 = 10$ Ohm, $L = 0.09$ H, $C = 100$ μF.

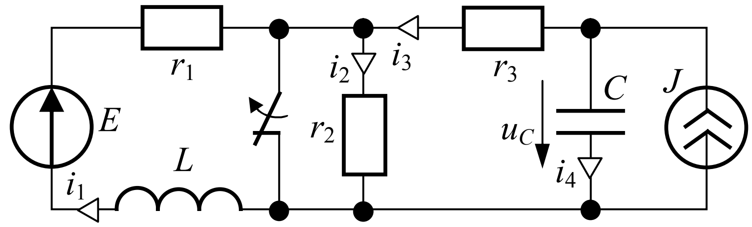


Fig. 7.39

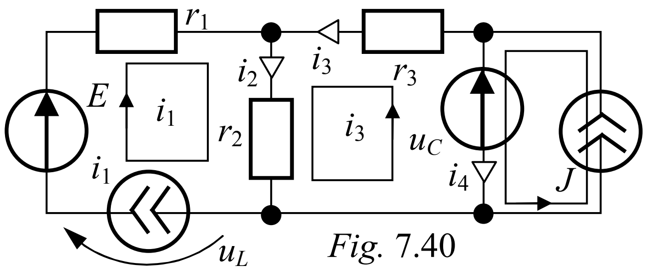


Fig. 7.40

Solution. 1. The current through inductance $i_1(t)$ and voltage across the capacitor $u_C(t)$ are set as the state variables. Output variables are the rest of the currents: $i_2(t)$, $i_3(t)$, $i_4(t)$.

2. The independent initial conditions are: $i_1(0_+) = i_1(0_-) = \dfrac{E}{r_1} = \dfrac{100}{20} = 5$ A, $u_C(0_+) = u_C(0_-) = J \cdot r_3 = 5 \cdot 10 = 50$ V.

3. For the state variables, we generate the system of the differential equations in Cauchy's form. The derivatives $i_1'(t)$ and $u_C'(t)$ are obtained, respectively, from $u_L = Li_1'(t)$ and $i_4 = Cu_C'(t)$. In order to find u_L and i_4 let's replace the inductance by the current source i_1 and the capacitance – by the voltage source u_C in the initial scheme . In a resistive scheme fig. 7.40, we determine u_L and i_4 as well as the output quantities $i_3(t)$ and $i_2(t)$. It may be done by any method of the composite DC-circuits analysis. Let's perform the calculation by the mesh current method. We have three independent loops with two known currents i_1 and J and one unknown current i_3. Let's generate an equation: $(r_2 + r_3) \cdot i_3 + r_2 \cdot i_1 = u_C$.

The equation solution is: $i_3 = \dfrac{u_C - r_2 i_1}{r_2 + r_3} = \dfrac{-r_2}{r_2 + r_3} \cdot i_1 + \dfrac{1}{r_2 + r_3} \cdot u_C$.

The rest of the necessary quantities are: $i_2 = i_1 + i_3 = \dfrac{r_3}{r_2 + r_3} \cdot i_1 + \dfrac{1}{r_2 + r_3} \cdot u_C$;

$$i_4 = C\frac{du_C}{dt} = J - i_3 = \frac{r_2}{r_2 + r_3} \cdot i_1 - \frac{1}{r_2 + r_3} \cdot u_C + J;$$

$$u_L = L\frac{di_1}{dt} = E - r_1 \cdot i_1 - r_2 \cdot i_2 = -\left(r_1 + \frac{r_2 r_3}{r_2 + r_3}\right) \cdot i_1 - \frac{r_2}{r_2 + r_3} \cdot u_C + E.$$

Two last equations are followed by the state equations in Cauchy's form:

$$\begin{cases} \dfrac{di_1}{dt} = -\dfrac{1}{L}\left(r_1 + \dfrac{r_2 r_3}{r_2 + r_3}\right) \cdot i_1 - \dfrac{r_2}{r_2 + r_3} \cdot \dfrac{1}{L} u_C + \dfrac{1}{L} E; \\ \dfrac{du_C}{dt} = \dfrac{r_2}{r_2 + r_3} \cdot \dfrac{1}{C} i_1 - \dfrac{1}{r_2 + r_3} \cdot \dfrac{1}{C} u_C + \dfrac{1}{C} J. \end{cases}$$

The rest of the equations are called the coupling ones. They are necessary to find the output variables.

4. We solve the equation system by the classical method:

$i_1(t) = i_{1s}(t) + i_{1t}(t); \quad u_C(t) = u_{Cs}(t) + u_{Ct}(t).$

There are direct-current sources in the circuit, that's why the steady-state components are direct-current too, their derivatives being equal to zero. In accordance with the superposition principle, the state equation system is true for both complete $i_1(t)$ and $u_C(t)$ and their components. The system of the state equations for the steady-state components is the following:

$$\begin{cases} -(r_1+\dfrac{r_2 r_3}{r_2+r_3})\cdot i_{1s} - \dfrac{r_2}{r_2+r_3}\cdot u_{Cs} = -E; \\ \dfrac{r_2}{r_2+r_3}\cdot i_{1s} - \dfrac{1}{r_2+r_3}\cdot u_{Cs} = -J. \end{cases} \quad \text{or} \quad \begin{cases} -27.5\cdot i_{1s} - 0.75\cdot u_{Cs} = -100; \\ 0.75\cdot i_{1s} - 0.025\cdot u_{Cs} = -5. \end{cases}$$

The system solution is: $i_{1s} = -1\ A, \quad u_{Cs} = 170\ V.$

5. Having presented the state equations in the operational form and having equated the system determinant to zero, we obtain the following characteristic equation:

$$\Delta(p) = \begin{vmatrix} \dfrac{1}{L}\left(r_1+\dfrac{r_2 r_3}{r_2+r_3}\right)+p & \dfrac{r_2}{L(r_2+r_3)} \\ \dfrac{-r_2}{C(r_2+r_3)} & \dfrac{1}{C(r_2+r_3)}+p \end{vmatrix} =$$

$$= p^2 + \left(\frac{1}{L}\left(r_1+\frac{r_2 r_3}{r_2+r_3}\right)+\frac{1}{C(r_2+r_3)}\right)\cdot p + \frac{r_1+r_2}{LC(r_2+r_3)} = p^2 + 555.6p + 138900 = 0.$$

The roots of the characteristic equation are: $p_{1,2} = -277.8 \pm j248.5\ s^{-1}$.

The transient components have a form:

$i_{1t}(t) = A\cdot e^{-\alpha\cdot t}\cdot sin(\omega t+\psi_1); \qquad u_{Ct}(t) = B\cdot e^{-\alpha\cdot t}\cdot sin(\omega t+\psi_u),$

Here $\alpha = |\mathrm{Re}(p_1)| = -277.8\ s^{-1}$ – decay coefficient,

$\omega = \mathrm{Im}(p_1) = 248.5\ s^{-1}$ – angular velocity of the free oscillations.

The initial values of the transient components and their derivatives are:

$i_{1t}(0_+) = A\cdot sin\psi_1; \qquad i_{1t}'(0_+) = -\alpha\cdot A\cdot sin\psi_1 + \omega\cdot A\cdot cos\psi_1;$

$u_{Ct}(0_+) = B\cdot sin\psi_u; \qquad u_{Ct}'(0_+) = -\alpha\cdot B\cdot sin\psi_u + \omega\cdot B\cdot cos\psi_u.$

6. The independent initial conditions are: $i_1(0_+) = 5\ A, \quad u_C(0_+) = 50\ V.$

We obtain the initial values of the derivatives from the state equations:

$$i_1'(0_+) = -\frac{1}{L}(r_1+\frac{r_2 r_3}{r_2+r_3})\cdot i_1(0_+) - \frac{r_2}{r_2+r_3}\cdot\frac{1}{L}u_C(0_+) + \frac{1}{L}E =$$

$$= -\frac{1}{0.09}\cdot 27.5\cdot 5 - 0.75\cdot\frac{1}{0.09}\cdot 50 + \frac{1}{0.09}\cdot 100 = -833.3\ A/s.$$

$$u_C'(0_+) = \frac{r_2}{r_2+r_3}\cdot\frac{1}{C}i_1(0_+) - \frac{1}{r_2+r_3}\cdot\frac{1}{C}u_C(0_+) + \frac{1}{C}J =$$

$$= 0.75\cdot 10^4\cdot 5 - 0.025\cdot 10^4\cdot 50 + 10^4\cdot 5 = 75000\ V/s.$$

The initial values of the transient components and their derivatives are:

$i_{1t}(0_+) = i_1(0_+) - i_{1s} = 5+1 = 6\ A, \qquad i_{1t}'(0_+) = i_1'(0_+) - i_{1s}' = -833.3\ A/s,$

$u_{Ct}(0_+) = u_C(0_+) - u_{Cs} = 50-170 = -120\ V, \qquad u_{Ct}'(0_+) = u_C'(0_+) - u_{Cs}' = 75000\ V/s.$

We obtain and solve the following equation systems:

$$\begin{cases} A\cdot sin\psi_1 = 6, \\ -\alpha\cdot A\cdot sin\psi_1 + \omega\cdot A\cdot cos\psi_1 = -833.3; \end{cases} \qquad \begin{cases} B\cdot sin\psi_u = -120, \\ -\alpha\cdot B\cdot sin\psi_u + \omega\cdot B\cdot cos\psi_u = 75000. \end{cases}$$

$A = 6.874$, $\psi_1 = 60.8°$, $B = 452.1$, $\psi_u = -15.4°$.

7. The final expressions for the state variables are:

$$i_1(t) = i_{1s}(t) + i_{1t}(t) = -1 + 6.874\cdot e^{-277.8t}\cdot sin(248.5t + 60.8°)\ A;$$

$$u_C(t) = u_{Cs}(t) + u_{Ct}(t) = 170 + 452.1\cdot e^{-277.8t}\cdot sin(248.5t - 15.4°)\ V.$$

8. The output quantities are:

$$i_2(t) = \frac{r_3}{r_2 + r_3}\cdot i_1 + \frac{1}{r_2 + r_3}\cdot u_C = 0.25\cdot i_1 + 0.025\cdot u_C = 4 + 11.83\cdot e^{-277.8t}\cdot sin(248.5t - 7.29°)\ A;$$

$$i_3(t) = \frac{-r_2}{r_2 + r_3}\cdot i_1 + \frac{1}{r_2 + r_3}\cdot u_C = -0.75\cdot i_1 + 0.025\cdot u_C = 5 + 11.25\cdot e^{-277.8t}\cdot sin(248.5t - 41.83°)\ A;$$

$$i_4(t) = \frac{r_2}{r_2 + r_3}\cdot i_1 - \frac{1}{r_2 + r_3}\cdot u_C + J = 0.75\cdot i_1 - 0.025\cdot u_C + 5 =$$

$$= 11.25\cdot e^{-277.8t}\cdot sin(248.5t + 138.17°)\ A.$$

7.2. COMPUTATION OF THE TRANSIENT PROCESSES BY OPERATIONAL METHOD

7.2.1. Transients in circuits with a single storage element

7-22 (7.45). The voltage impressed to circuit fig. 7.41,a obeys the law

$u(t) = 30t^2 + 18t + 10\ V.$

The circuit parameters are:

$r_1 = r_2 = 100\ Ohm$, $C = 10\ \mu F$.

Compute the capacitor current.

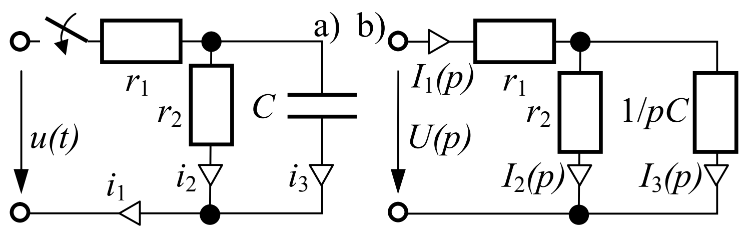


Fig. 7.41

Solution. Until there is an input voltage, the circuit is at rest. So, the problem under the question has zero independent initial conditions, its equivalent operational scheme being like in fig. 7.42,b.

The image of the impressed voltage may be determined with the aid of the table of Laplace transformations: $U(p) = \dfrac{30\cdot 2}{p^3} + \dfrac{18}{p^2} + \dfrac{10}{p}$.

The images of the 1st and 3rd currents are: $I_1(p) = \dfrac{U(p)}{r_1 + \dfrac{r_2\cdot 1/pC}{r_2 + 1/pC}}$;

$$I_3(p) = \frac{I_1(p)\cdot r_2}{r_2 + \dfrac{1}{pC}} = \frac{U(p)}{r_1 + \dfrac{r_2}{r_2 pC + 1}}\cdot\frac{r_2 pC}{r_2 pC + 1} = \frac{U(p) r_2 pC}{p r_1 r_2 C + r_1 + r_2} =$$

$$= \frac{60 r_2 C}{p^2(p r_1 r_2 C + r_1 + r_2)} + \frac{18 r_2 C}{p(p r_1 r_2 C + r_1 + r_2)} + \frac{10 r_2 C}{p r_1 r_2 C + r_1 + r_2}.$$

Let's resolve the latter expression into the vulgar fractions:

$$I_3(p) = \frac{A}{p} + \frac{B}{p^2} + \frac{D}{pr_1r_2C + r_1 + r_2}.$$

Having reduce the fractions in the expression for current $I_3(p)$ to the common denominator and having equated the numerators, we obtain the following equation:

$$Ap(pr_1r_2C + r_1 + r_2) + B(pr_1r_2C + r_1 + r_2) + Dp^2 = 60r_2C + p\cdot 18r_2C + p^2\cdot 10r_2C.$$

We equate the coefficients at the identical exponents of p and obtain the equation system:

coefficients at p^2: $ACr_1r_2 + D = 10r_2C$;

at p: $A(r_1 + r_2) + BCr_1r_2 = 18r_2C$;

at 1: $B(r_1 + r_2) = 60r_2C$.

From here $$B = \frac{60r_2C}{r_1 + r_2} = \frac{60\cdot 100\cdot 10^{-5}}{100 + 100} = 3\cdot 10^{-4};$$

$$A = \frac{18r_2C - BCr_1r_2}{r_1 + r_2} = \frac{18\cdot 10^{-3} - 3\cdot 10^{-5}}{200} = 9\cdot 10^{-5};$$

$$D = 10r_2C - ACr_1r_2 = 0.01 - 9\cdot 10^{-5}\cdot 10^{-5}\cdot 10^{4} = 0.01.$$

Finally, we have:

$$I_3(p) = \frac{9\cdot 10^{-5}}{p} + \frac{3\cdot 10^{-4}}{p^2} + \frac{0.01}{p\cdot 0.1 + 200} \doteq 9\cdot 10^{-5} + 3\cdot 10^{-4}\cdot t + 0.1\cdot e^{-2000t}\ A = i_3(t).$$

7-23 (7.46). Compute the voltage and current of the inductance in scheme fig. 7.42,a. Numerical data: $U = 24\ V$, $L = 0.25\ H$, $R_1 = 30\ Ohm$, $R_2 = 10\ Ohm$.

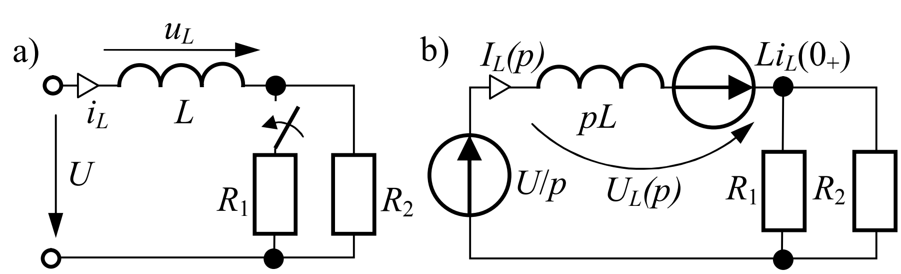


Fig. 7.42

Solution. 1. Let's calculate the independent initial condition and write down the inner operational EMF: $i_L(0_+) = i_L(0_-) = \frac{U_0}{R_2} = \frac{24}{10} = 2.4\ A$,

$$Li_L(0_+) = 0.25\cdot 2.4 = 0.6\ V\cdot s.$$

2. The equivalent operational scheme is presented in fig. 7.42,b.

3. As both operational EMF are in the same branch, we determine the current image $I_L(p)$ by Ohm's law:

$$I_L(p) = \frac{\frac{U}{p} + Li_L(0_+)}{pL + \frac{R_1R_2}{R_1 + R_2}} = \frac{U + pLi_L(0_+)}{p(pL + \frac{R_1R_2}{R_1 + R_2})} = \frac{\frac{U}{L} + pi_L(0_+)}{p(p + \frac{R_1R_2}{L(R_1 + R_2)})} = \frac{\frac{24}{0.25} + p\cdot 2.4}{p(p + \frac{30\cdot 10}{0.25\cdot 40})} =$$

$$= \frac{96 + p\cdot 2{,}4}{p(p + 30)} = \frac{F_1(p)}{pF_2(p)}.$$

On the ground of Kirchhoff's voltage law

$$U_L(p) = pL\cdot I_L(p) - Li_L(0_+) = pL\cdot\frac{\frac{U}{L} + pi_L(0_+)}{p\left(p + \frac{R_1R_2}{L(R_1+R_2)}\right)} - Li_L(0_+) =$$

$$= \frac{U + pL\cdot i_L(0_+) - Li_L(0_+)\left(p + \frac{R_1R_2}{L(R_1+R_2)}\right)}{p + \frac{R_1R_2}{L(R_1+R_2)}} = \frac{24 + p\cdot 0.6 - 0.6(p+30)}{p+30} = \frac{6}{p+30}.$$

4. We determine the current original $i_L(t)$ with the aid of the expansion theorem:

$$i_L(t) = \frac{F_1(0)}{F_2(0)} + \sum_{k=1}^{n}\frac{F_1(p_k)}{p_kF_2'(p_k)}e^{p_kt}.$$

The root of equation $F_2(p) = p + 30 = 0$ is $p = -30\ s^{-1}$,
derivative $F_2'(p) = 1$; $F_2(0) = 30$; $F_1(0) = 96$, $F_1(p) = 96 - 30\cdot 2.4 = 24$;

$$i_L(t) = \frac{96}{30} + \frac{24}{-30\cdot 1}e^{-30t} = 3.2 - 0.8e^{-30t}\ A.$$

The image of the voltage across the inductance $U_L(p) = \frac{6}{p+30}$ is a standard (reference) function. This is the image of an exponent, that's why $u_L(t) = 6e^{-30t}\ V$.

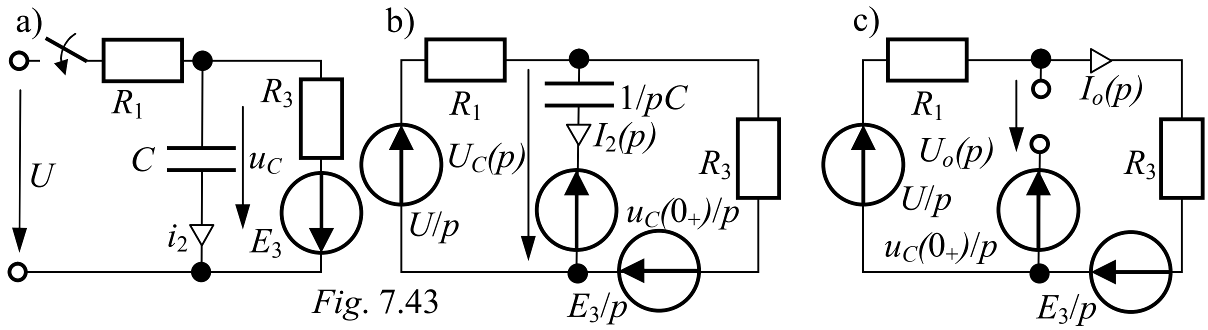


Fig. 7.43

7-24 (7.47). In scheme fig. 7.43,a, compute the current $i_2(t)$ and voltage across the capacitor $u_C(t)$, if $U = 240\ V$, $E_3 = 100\ V$, $R_1 = R_3 = 50\ Ohm$, $C = 1000\ \mu F$.

Solution. 1. The independent initial condition is: $u_C(0_+) = u_C(0_-) = -E_3 = -100\ V$.

The inner operational EMF is $\frac{u_C(0)}{p} = -\frac{100}{p}$.

2. The equivalent operational scheme is presented in fig. 7.43,b.

3. As in the given problem it is necessary to determine only current, we determine its image by the method of equivalent generator (fig. 7.43,c). According to Kirhhoff's voltage law:

$$U_o(p) + R_1\cdot I_o(p) = \frac{U}{p} - \frac{u_C(0_+)}{p} = \frac{240-(-100)}{p} = \frac{340}{p}.$$

Under Ohm's law $I_o(p) = \dfrac{\frac{U}{p} + \frac{E_3}{p}}{R_1 + R_3} = \dfrac{240+100}{p(50+50)} = \dfrac{3.4}{p}$.

Then $U_o(p) = \dfrac{340}{p} - 50\dfrac{3.4}{p} = \dfrac{170}{p}$.

The scheme input impedance concerning the separated terminals is

$$Z_{inp}(p) = \frac{R_1 R_3}{R_1 + R_3} = \frac{50 \cdot 50}{50 + 50} = 25\ Ohm.$$

The image of the required current $I_2(p)$ is:

$$I_2(p) = \frac{U_o(p)}{Z_{inp}(p) + \dfrac{1}{pC}} = \frac{170}{p(25 + \dfrac{1}{0.001p})} = \frac{6.8}{p + 40}.$$

On the ground of Kirchhoff's voltage law $U_C(p) - \dfrac{1}{pC} I_2(p) = \dfrac{u_C(0_+)}{p}$.

From here $U_C(p) = \dfrac{-100}{p} + \dfrac{1}{0.001p} \cdot \dfrac{6.8}{p+40} = \dfrac{-100p + 2800}{p(p+40)} = \dfrac{F_1(p)}{pF_2(p)}$.

4. The current original is found from the table of Laplace transforms:

$$i_2(t) = 6.8e^{-40t}\ A.$$

We find the voltage original with the aid of the expansion theorem:
the root of equation $F_2(p) = p + 40 = 0$ is $p = -40\ s^{-1}$,
derivative $F_2'(p) = 1$, $F_2(0) = 40$; $F_1(0) = 2800$, $F_1(p) = -100\cdot(-40) + 2800 = 6800$;

$$u_C(t) = \frac{2800}{40} + \frac{6800}{-40 \cdot 1} e^{-40t} = 70 - 170e^{-40t}\ V.$$

7.2.2. Transients in the circuits with two storage elements

7-25 (7.58). In scheme fig. 7.44,a, find the current $i_3(t)$ and voltage $u_C(t)$ by the operational method. The circuit parameters are: $E = 300\ V$, $r_1 = r_3 = 25\ Ohm$, $L = 0.02\ H$, $C = 100\ \mu F$.

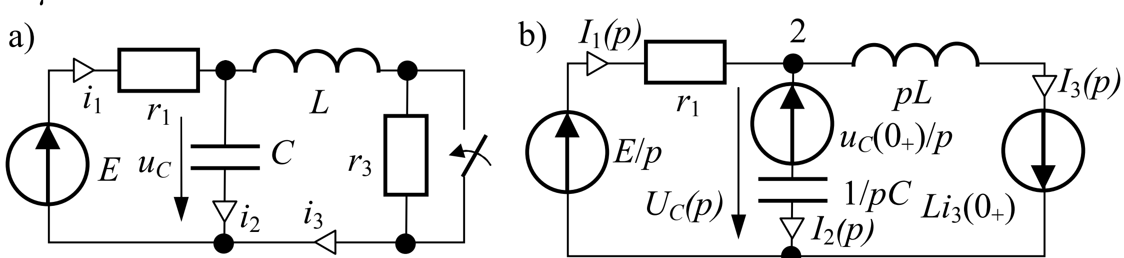


Fig. 7.44

Solution. 1. In order to construct the operational scheme, we determine the independent initial conditions: $i_3(0_+) = i_3(0_-) = \dfrac{E}{r_1 + r_2} = \dfrac{300}{25 + 25} = 6\ A$,

$$u_C(0_+) = u_C(0_-) = r_3 \cdot i_3(0_-) = 25 \cdot 6 = 150\ V.$$

2. The equivalent operational scheme is shown in fig. 7.44,b.

3. Perform the scheme calculation using the method of two nodes:

$$U_{23}(p) = U_C(p) = \frac{\dfrac{E}{p}\cdot\dfrac{1}{r_1} + \dfrac{u_C(0_+)}{p} pC - \dfrac{Li_3(0_+)}{pL}}{\dfrac{1}{r_1} + pC + \dfrac{1}{pL}} =$$

$$= \frac{EL + r_1 pLCu_C(0_+) - r_1 Li_3(0_+)}{r_1 LCp^2 + pL + r_1} = \frac{F_{1U}(p)}{F_2(p)};$$

$$I_3(p) = \frac{U_{23}(p) + Li_3(0_+)}{pL} = \frac{E + r_1 pCu_C(0_+) + i_3(0_+)pL(r_1 pC + 1)}{p(r_1 LCp^2 + pL + r_1)} = \frac{F_{1I}(p)}{F_2(p)}.$$

4. We find the originals under the expansion theorem.

Assume $F_2(p) = r_1 LCp^2 + pL + r_1 = 5\cdot 10^{-5}p^2 + 0.02p + 25 = 0$,

the roots of the equation are $p_{1,2} = -200 \pm j678 = -706.9\cdot e^{-j73.56°}\ s^{-1}$;

$F_2(0) = 25$; $F_2'(p) = 10^{-4}p + 1$; $F_2'(p_1) = j0.0678$.

The formula for the voltage across the capacitor is: $u_C(t) = 2\mathrm{Re}\left(\frac{F_{1U}(p_1)}{F_2'(p_1)}e^{p_1 t}\right)$;

$F_{1U}(p_1) = 300\cdot 0.02 + 25\cdot(-200 + j678)\cdot 0.02\cdot 10^{-4}\cdot 150 - 25\cdot 0.02\cdot 6 = 5.3\cdot e^{j73.56°}$;

$$u_C(t) = 2\mathrm{Re}\left(\frac{5.3e^{j73.56}}{j0.0678}e^{-200t}e^{j678t}\right) = 156.3e^{-200t}\cos(678t - 16.44°) =$$

$$= 156.3e^{-200t}\sin(678t + 73.56°)\ V.$$

Calculate the current $i_3(t) = \frac{F_{1I}(0)}{F_2(0)} + 2\mathrm{Re}\left(\frac{F_{1I}(p_1)}{F_2'(p_1)}e^{p_1 t}\right)$;

$F_{1I}(0) = 300$; $F_{1I}(p_1) = 300 + 25\cdot(-200 + j678)\cdot 10^{-4}\cdot 150 +$

$+ 6\cdot(-200 + j678)\cdot 0.02\cdot(25\cdot(-200 + j678)\cdot 10^{-4} + 1) = 265\cdot e^{j73.56°}$;

$$i_3(t) = \frac{300}{25} + 2\mathrm{Re}\left(\frac{265e^{j73.56}}{-706.9e^{-j73.56}\ j0.0678}e^{-200t}e^{j678t}\right) =$$

$$= 12 + 11.06e^{-200t}\sin(678t - 32.88°)\ A.$$

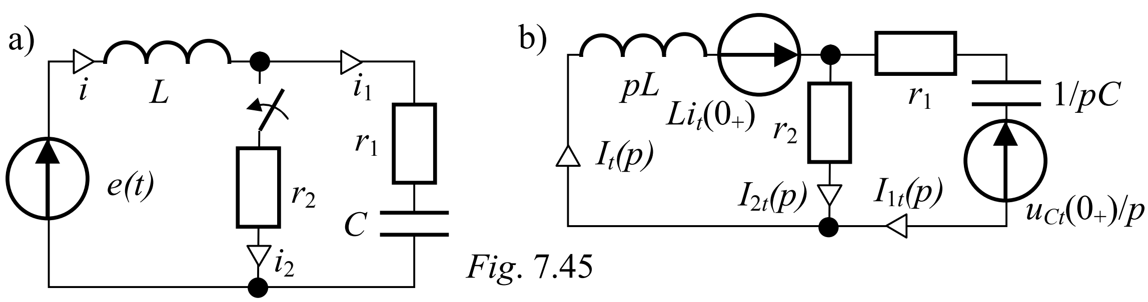


Fig. 7.45

7-26 (7.59). In scheme fig. 7.45, determine the current $i_2(t)$ at the following circuit parameters: $e(t) = E_m\sin(\omega_0 t + \psi)$; $E_m = 400\ V$, $\omega_0 = 100\ rad/s$, $\psi = -45°$; $L = 0.25\ H$, $C = 400\ \mu F$, $r_1 = 25\ Ohm$, $r_2 = 75\ Ohm$.

Solution. 1. Calculate the independent initial conditions.

$$\underline{I}_m = \frac{\underline{E}_m}{r_1 + j\omega_0 L + \frac{1}{j\omega_0 C}} = \frac{400e^{-j45}}{25 + j100\cdot 0.25 + \frac{10^6}{j100\cdot 400}} = 16\cdot e^{-j45°}\ A;$$

$$\underline{U}_{Cm} = \underline{I}_m\cdot\frac{1}{j\omega_0 C} = 16\cdot e^{-j45°}\cdot(-j25) = 400\cdot e^{-j135°}\ V;$$

$i(t_-) = 16\sin(\omega_0 t - 45°)\ A$; $u_C(t_-) = 400\sin(\omega_0 t - 135°)\ V$;

$i(0_+) = i(0_-) = 16sin(-45°) = -8\sqrt{2}\ A,\quad u_C(0_+) = u_C(0_-) = 400sin(-135°) = -200\sqrt{2}\ V.$

2. Calculate the steady-state components $i_s(t)$, $i_{2s}(t)$ and $u_{Cs}(t)$ by complex-notation method.

$$\underline{I}_{ms} = \frac{\underline{E}_m}{j\omega_0 L + \dfrac{r_2(r_1 - j\dfrac{1}{\omega_0 C})}{r_2 + r_1 - j\dfrac{1}{\omega_0 C}}} = \frac{400e^{-j45}}{j25 + \dfrac{75(25 - j25)}{75 + 25 - j25}} = 16\cdot e^{-j73.05°}\ A;$$

$$\underline{I}_{2ms} = \underline{I}_{ms}\cdot\frac{r_1 - j\dfrac{1}{\omega_0 C}}{r_2 + r_1 - j\dfrac{1}{\omega_0 C}} = 16\cdot e^{-j73.05°}\cdot\frac{25 - j25}{100 - j25} = 5.49\cdot e^{-j104°}\ A;$$

$$\underline{U}_{Cms} = \underline{I}_{1ms}\cdot\frac{1}{j\omega_0 C} = \underline{I}_{ms}\cdot\frac{r_2}{r_2 + r_1 - j\dfrac{1}{\omega_0 C}}\cdot\frac{1}{j\omega_0 C} = 16\cdot e^{-j73.05°}\cdot\frac{75}{100 - j25}\cdot(-j25) =$$

$$= 291.4\cdot e^{-j149.05°}\ V;$$

$i_{2s}(t) = 5.49sin(\omega_0 t - 104°)\ A,\quad i_s(t) = 16sin(\omega_0 t - 73.5°)\ A,$
$u_{Cs}(t) = 291.4sin(\omega_0 t - 149.05°)\ V.$

The initial values of the steady-state components are:

$i_s(0_+) = 16sin(-73.05°) = -15.3\ A,\quad u_{Cs}(0_+) = 291.4sin(-149.05°) = -150\ V.$

3. We apply the operational method to determine the transient component of the current $i_{2t}(t)$. For that we determine the initial values of the transient components of the inductance current and the capacitance voltage:

$i_t(0_+) = i(0_+) - i_s(0_+) = -8\sqrt{2} + 15.3 = 4\ A,$
$u_{Ct}(0_+) = u_C(0_+) - u_{Cs}(0_+) = -200\sqrt{2} + 150 = -133\ V.$

The operational equivalent scheme for transient components is presented in fig. 7.45,b.

4. Perform the calculations for the scheme obtained using the method of two nodes.

$$U_{12t}(p) = \frac{\dfrac{Li_t(0_+)}{pL} + \dfrac{u_{Ct}(0_+)}{p(r_1 + \dfrac{1}{pC})}}{\dfrac{1}{r_2} + \dfrac{1}{r_1 + \dfrac{1}{pC}} + \dfrac{1}{pL}} = \frac{[(r_1 pC + 1)Li_t(0_+) + u_{Ct}(0_+)pLC]r_2}{(r_1 + r_2)LCp^2 + (L + r_1 r_2 C)p + r_2};$$

$$I_{2t}(p) = \frac{U_{12t}(p)}{r_2} = \frac{(r_1 pC + 1)Li_t(0_+) + u_{Ct}(0_+)pLC}{(r_1 + r_2)LCp^2 + (L + r_1 r_2 C)p + r_2} = \frac{F_1(p)}{F_2(p)}.$$

5. We determine the current original $i_{2t}(t)$ with the aid of the expansion theorem.
The roots of the equation $F_2(p) = 0,.01p^2 + p + 75 = 0$ are $p_{1,2} = -50 \pm j50\sqrt{2}\ s^{-1}$;

$F_2'(p) = 0.02p + 1;\quad F_2'(p_1) = j\sqrt{2}.$

$F_1(p_1) = (10^{-2}\cdot(-50 + j50\sqrt{2}) + 1)\cdot 0.25\cdot 4 + 10^{-4}\cdot(-50 + j50\sqrt{2})\cdot(-133) = 1.188\cdot e^{-j11.3°};$

$$i_{2t}(t) = 2\mathrm{Re}\left(\frac{F_1(p_1)}{F_2'(p_1)}e^{p_1 t}\right) = 2\mathrm{Re}\left(\frac{1.188e^{-j11.3}}{j\sqrt{2}}e^{-50t}e^{j70.7t}\right) =$$

$$= 1.68e^{-50t}cos(70.7t - 101.3°) = 1.68e^{-50t}sin(70.7t - 11.3°)\ A.$$

Finally, we have:

$$i_2(t) = i_{2s}(t) + i_{2t}(t) = 5.49sin(100t - 104°) + 1.68e^{-50t}sin(70.7t - 11.3°)\ A.$$

7-27 (7.60). Calculate the transient currents in scheme fig. 7.46,a, using the operational method. The circuit parameters are: $U = 200\ V$, $r_1 = 40\ Ohm$, $r_2 = 60\ Ohm$, $L = 0.7\ H$, $C = 100\ \mu F$.

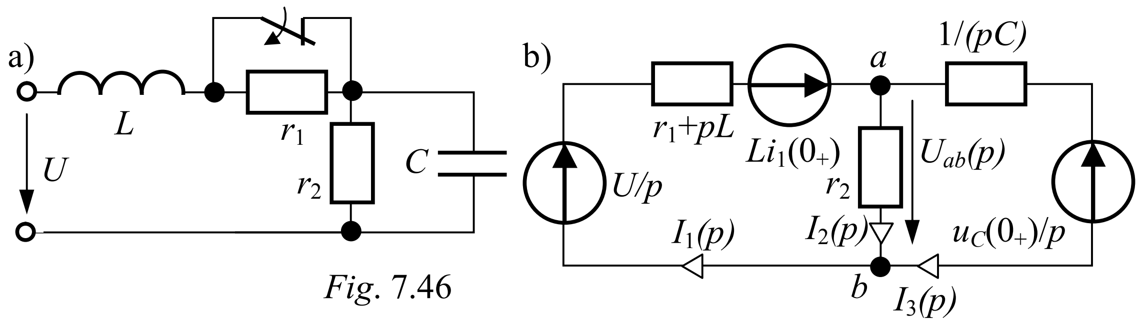


Fig. 7.46

Answers: the independent initial conditions are – $i_1(0_+) = 3.33\ A$, $u_C(0_+) = 200\ V$, the equivalent operational scheme is in fig. 7.47,b; images of the junction voltage and currents are:

$$U_{ab}(p) = \frac{p^2 LCr_2 u_C(0_+) + pr_2(Li_1(0_+) + u_C(0_+)Cr_1) + Ur_2}{p(p^2 LCr_2 + p(L + r_1 r_2 C) + r_1 + r_2)};$$

$$I_1(p) = \frac{p^2 Li_1(0_+)Cr_2 + pLi_1(0_+) + U}{p(p^2 LCr_2 + p(L + r_1 r_2 C) + r_1 + r_2)};$$

$$I_2(p) = \frac{p^2 LCu_C(0_+) + p(Li_1(0_+) + u_C(0_+)Cr_1) + U}{p(p^2 LCr_2 + p(L + r_1 r_2 C) + r_1 + r_2)};$$

$$I_3(p) = \frac{u_C(0_+)Cr_1}{p^2 LCr_2 + p(L + r_1 r_2 C) + r_1 + r_2};$$

the current originals are: $i_1(t) = 2 + 1.39 \cdot e^{-111.9t}sin(106.2t + 106.3°)\ A$;
$i_2(t) = 2 + 1.936 \cdot e^{-111.9t}sin(106.2t + 43.5°)\ A$; $i_3(t) = 1.794 \cdot e^{-111.9t}sin(106.2t)\ A$.

7-28 (7.61). Solve the problem 7.16 by the operational method.

Solution. 1. The independent initial conditions are:

$$i_1(0_+) = i_1(0_-) = \frac{U}{r_1 + r_2} = \frac{100}{50 + 50} = 1\ A, \quad i_3(0_+) = i_3(0_-) = 0.$$

2. The equivalent operational scheme is presented in fig. 7.47.

3. Perform the circuit calculation by the mesh current method. The equation system for the loop currents (the currents of the branches 1 and 3 are assumed to be the loop ones) has the view:

$$\begin{cases} I_1(p)(r_1 + pL_1 + r_2) + I_3(p)(pM - r_2) = \dfrac{U}{p} + L_1 i_1(0_+), \\ I_1(p)(pM - r_2) + I_3(p)(pL_2 + r_2) = Mi_1(0_+). \end{cases}$$

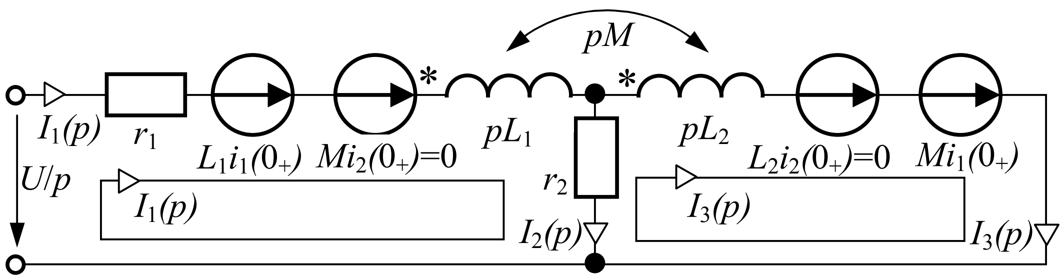


Fig. 7.47

Solve the system by Cramer's method:

$$\Delta(p)=\begin{vmatrix} pL_1+r_1+r_2 & pM-r_2 \\ pM-r_2 & pL_2+r_2 \end{vmatrix}=p^2(L_1L_2-M^2)+p((L_1+2M)r_2+L_2(r_1+r_2))+r_1r_2;$$

$$\Delta_1(p)=\begin{vmatrix} \frac{U}{p}+L_1i_1(0_+) & pM-r_2 \\ Mi_1(0_+) & pL_2+r_2 \end{vmatrix}=$$

$$=\frac{1}{p}(p^2(L_1L_2-M^2)i_1(0_+)+p(Mi_1(0_+)r_2+UL_2+L_1i_1(0_+)r_2)+Ur_2);$$

$$\Delta_3(p)=\begin{vmatrix} pL_1+r_1+r_2 & \frac{U}{p}+L_1i_1(0_+) \\ pM-r_2 & Mi_1(0_+) \end{vmatrix}=\frac{1}{p}\{p[i_1(0_+)(L_1r_2+M(r_1+r_2))-UM]+Ur_2\};$$

$$I_1(p)=\frac{\Delta_1(p)}{\Delta(p)}=\frac{p^2(L_1L_2-M^2)i_1(0_+)+p(Mi_1(0_+)r_2+UL_2+L_1i_1(0_+)r_2)+Ur_2}{p[p^2(L_1L_2-M^2)+p((L_1+2M+L_2)r_2+L_2r_1)+r_1r_2]}=$$

$$=\frac{p^2\cdot 0.0175+p\cdot 27.5+5000}{p(p^2\cdot 0.0175+p\cdot 30+2500)}=\frac{F_1(p)}{pF_2(p)};$$

$$I_3(p)=\frac{\Delta_3(p)}{\Delta(p)}=\frac{p[i_1(0_+)(L_1r_2+M(r_1+r_2))-UM]+Ur_2}{p[p^2(L_1L_2-M^2)+p((L_1+2M+L_2)r_2+L_2r_1)+r_1r_2]}=$$

$$=\frac{p\cdot 5+5000}{p(p^2\cdot 0.0175+p\cdot 30+2500)}=\frac{F_3(p)}{pF_2(p)}.$$

4. We determine the current originals with the aid of the expansion theorem.

The roots of equation $F_2(p)=p^2\cdot 0.0175+p\cdot 30+2500=0$:

$$p_{1,2}=-857.1\pm 769.3\ s^{-1};\quad p_1=-87.8\ s^{-1};\quad p_2=-1626.4\ s^{-1}.$$

$F_2(0)=2500$; $F_2'(p)=0.035\cdot p+30$; $F_2'(p_1)=26.93$; $F_2'(p_2)=-26.93$.

The calculation of the first current: $F_1(p)=p^2\cdot 0.0175+p\cdot 27.5+5000$;

$F_1(0)=5000$; $F_1(p_1)=2720$; $F_1(p_2)=6565$.

$$i_1(t)=\frac{F_1(0)}{F_2(0)}+\sum_{k=1}^{k=2}\frac{F_1(p_k)}{p_kF_2'(p_k)}e^{p_kt}=\frac{5000}{2500}+\frac{2720}{-87.5\cdot 26.93}\cdot e^{-87.5t}+$$

$$+\frac{6565}{-1626.4\cdot(-26.93)}\cdot e^{-1626.4t}=2-1.154\cdot e^{-87.5t}+0.150\cdot e^{-1626.4t}\ A.$$

The calculation of the third current: $F_3(p)=p\cdot 5+5000$;

$F_3(0)=5000$; $F_3(p_1)=4561$; $F_3(p_2)=-3132$.

$$i_3(t) = \frac{F_3(0)}{F_2(0)} + \sum_{k=1}^{k=2} \frac{F_3(p_k)}{p_k F_2'(p_k)} e^{p_k t} = \frac{5000}{2500} + \frac{4561}{-87.5 \cdot 26.93} \cdot e^{-87.5t} +$$

$$+ \frac{-3132}{-1626.4 \cdot (-26.93)} \cdot e^{-1626.4t} = 2 - 1.929 \cdot e^{-87.5t} - 0.072 \cdot e^{-1626.4t}\ A.$$

We determine the current of the second branch by Kirchhoff's current law:

$$i_2(t) = i_1(t) - i_3(t) = 2 - 1.154 \cdot e^{-87.5t} + 0.150 \cdot e^{-1626.4t} - 2 + 1.929 \cdot e^{-87.5t} +$$

$$+ 0.072 \cdot e^{-1626.4t} = 0.775 \cdot e^{-87.5t} + 0.222 \cdot e^{-1626.4t}\ A.$$

7.3. CALCULATION OF THE TRANSIENT PROCESSES WITH THE AID OF THE DUHAMEL INTEGRAL

7-29 (7.71). A circuit fig. 7.48,a is supplied with voltage $u_{inp}(t)$ (fig. 7.48,b). The circuit parameters are: $r_1 = r_2 = 10\ kOhm$, $C = 200\ \mu F$, $U_0 = 100\ V$, $t_1 = 1\ s$.

Compute the voltage across the capacitor with the aid of the Duhamel integral. Plot it.

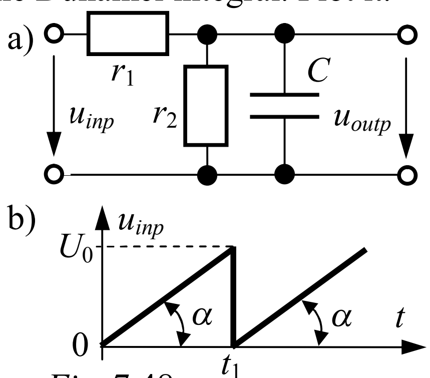


Fig. 7.48

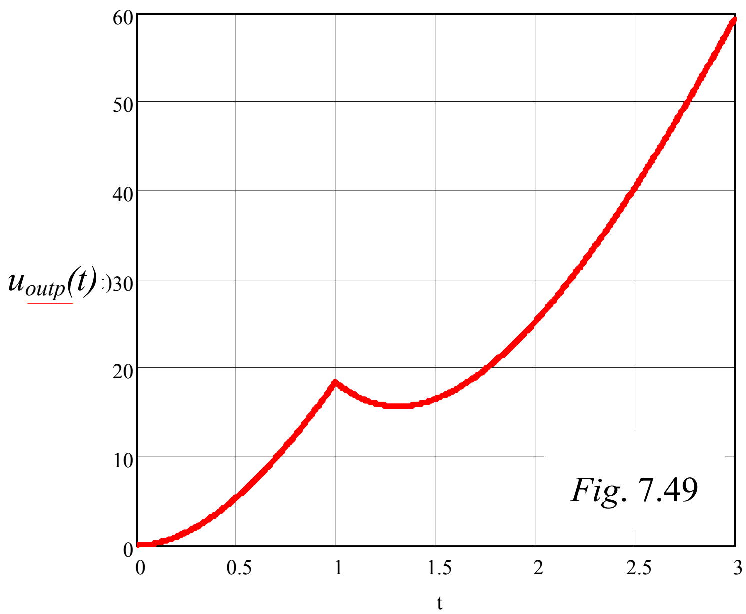


Fig. 7.49

Solution. Calculate the transient characteristic by the classical method. When the circuit fig. 7.48,a is connected to the unit voltage source, we have:

$$u_{outps} = \frac{1}{r_1 + r_2} r_2 = 10/(10+10) = 0.5\ V, \quad p = -\frac{1}{r_{equ} C} = -\frac{10^6}{5000 \cdot 200} = -1\ s^{-1},$$

$u_{outp}(0) = 0$; $\quad u_{outp}(t) = u_{outps} + (u_{outp}(0) - u_{outps})e^{pt} = 0.5 - 0.5e^{-t}\ V$,

finally, $g(t) = 0.5 - 0.5e^{-t}$.

Let's present the voltage $u_{inp}(t)$ analytically:

$$u_{inp}(t) = \begin{cases} u_{inp1}(t) = 100t\ V & if\ \ 0 \le t \le t_1; \\ u_{inp2}(t) = 100t - 100\ V & if\ \ t \ge t_1. \end{cases}$$

The voltage derivative is: $\quad u_{inp}'(t) = \begin{cases} u_{inp1}'(t) = 100\ V/s & if\ 0 \le t \le t_1; \\ u_{inp2}'(t) = 100\ V/s & if\ t \ge t_1. \end{cases}$

The output voltage in the interval $0 \le t \le t_1$:

$$u_{outp1}(t) = u_{inp1}(0) \cdot g(t) + \int_0^t u_{inp1}'(\tau) g(t - \tau) d\tau = 0 + \int_0^t 100(0.5 - 0.5e^{-(t-\tau)}) d\tau =$$

$$= 50t - 50 + 50e^{-t}\ V.$$

The output voltage in the interval $t \ge t_1$:

$$u_{outp2}(t) = u_{inp1}(0)\cdot g(t) + \int_0^{t_1} u'_{inp1}(\tau)g(t-\tau)d\tau +$$

$$+ (u_{inp2}(t_1) - u_{inp1}(t_1))g(t - t_1) + \int_{t_1}^{t} u'_{inp2}(\tau)g(t-\tau)d\tau =$$

$$= -100 + 50t + 50e^{-t} + 50e^{-(t-1)} = -100 + 50t + 68.4e^{-(t-1)}\ V.$$

Thus, $u_{outp}(t) = \begin{cases} u_{outp1}(t) = 50t - 50 + 50e^{-t}\quad V\quad if\ 0 \le t \le t_1; \\ u_{outp2}(t) = -100 + 50t + 68{,}4e^{-(t-1)}\ V\ if\ t \ge t_1 = 1\ s. \end{cases}$

In accordance with the latter expression, the graph $u_{outp}(t)$ is presented in fig. 7.49.

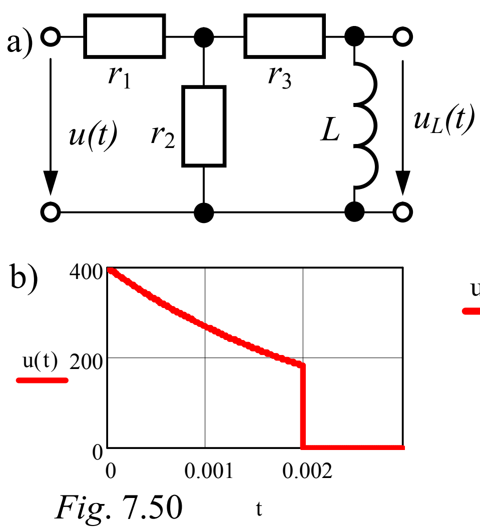


Fig. 7.50

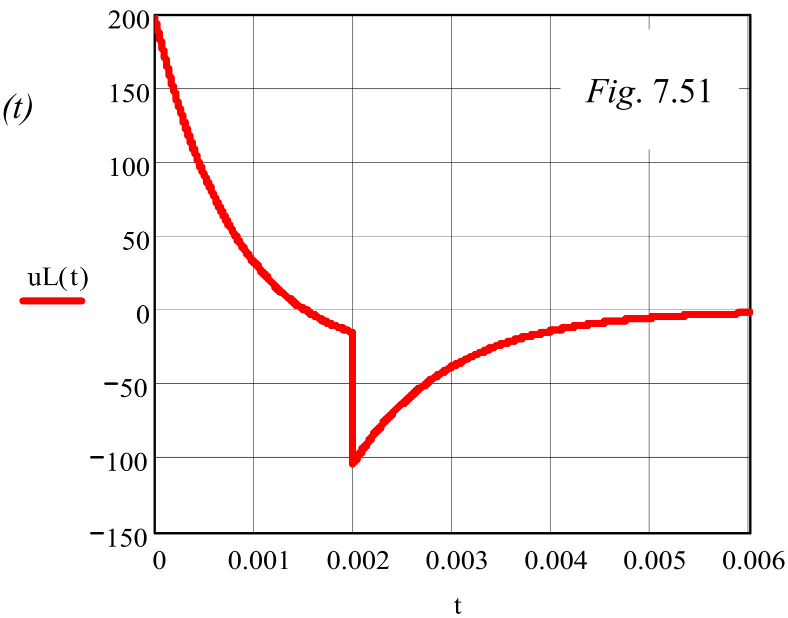


Fig. 7.51

7-30 (7.72). The circuit fig. 7.50,a is supplied with the voltage impulse $u(t) = 400e^{-400t}\ V$ of duration $t_1 = 2\ ms$ (fig. 7.50,b). The circuit parameters are $r_1 = r_2 = 100\ Ohm$, $r_3 = 50\ Ohm$, $L = 0.1\ H$. Calculate the voltage $u_L(t)$ and plot it with the aid of the Duhamel integral.

Answers: $g(t) = 0.5e^{-1000t}$, on the interval $0 \le t \le t_1$:

$$u_L(t) = u(0)\cdot g(t) + \int_0^{t} u'(\tau)g(t-\tau)d\tau = 333.3e^{-1000t} - 133.3e^{-400t}\ V,$$

on the interval $t \ge t_1$:

$$u_L(t) = u(0)\cdot g(t) + \int_0^{t_1} u'(\tau)g(t-\tau)d\tau + (-u(t_1))g(t-t_1) = -109.3e^{-1000t} - 89.87e^{-1000t+2}\ V.$$

The diagram $u_L(t)$ is shown in fig. 7.51.

7-31 (7.73). With the aid of the Duhamel integral, compute the current $i_2(t)$ in the circuit fig. 7.52,a when there is the disturbance because of the staircase voltage $u(t)$ (fig. 7.52,b), if $r_1 = r_2 = 20\ Ohm$, $r_3 = 10\ Ohm$, $C = 1000\ \mu F$, $t_1 = 20\ ms$.

Answers: $g(t) = 25 - 12.5e^{-50t}\ mS$, $u(t) = \begin{cases} u_1(t) = 1\ V\quad if\ 0 \le t \le t_1; \\ u_2(t) = 2\ V\ \ if\ \ t \ge t_1. \end{cases}$

on the interval $0 \le t \le t_1$: $i_2(t) = u_1(0) \cdot g(t) = 25 - 12.5e^{-50t}$ *mA*,
at $t \ge t_1$: $i_2(t) = u_1(0) \cdot g(t) + (u_2(t_1) - u_1(t_1))g(t-t_1) = 50 - 12.5e^{-50t} - 12.5e^{-50t+1}$ *mA*.
The diagram $i_2(t)$ is shown in fig. 7.53.

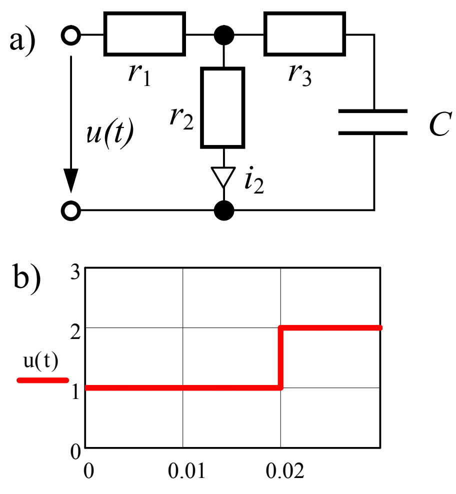


Fig. 7.52

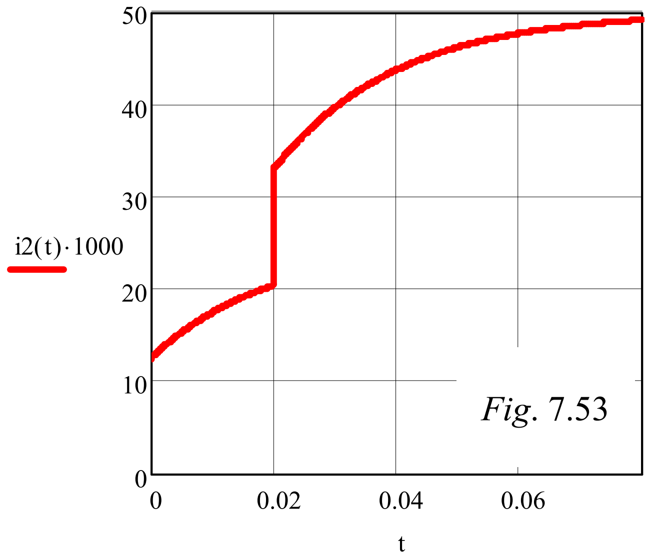


Fig. 7.53

7-32 (7.74). Compute the current through resistor r_1 in scheme fig. 7.54,a. The circuit parameters are: $r_1 = 50$ *Ohm*, $r_2 = r_3 = 100$ *Ohm*, $C = 200$ *μF*. The voltage of the source is set by the diagram fig. 7.54,b. Use the Duhamel integral.

Answers: $g(t) = 0.00667 + 0.00333 \cdot e^{-37.5t}$ *S*,

$$i_1(t) = \begin{cases} 133t + 1.78 - 1.78e^{-37.5t} \ A \ \ if \ 0 \le t \le 0.01s, \\ -133t + 0.889 + 2.334e^{-37.5(t-0.01)} \ A \ if \ 0.01 \le t \le 0.02s, \\ -0.174e^{-37.5(t-0.02)} \ A \ \ if \ t \ge 0.02s. \end{cases}$$

The current diagram is in fig. 7.54,c.

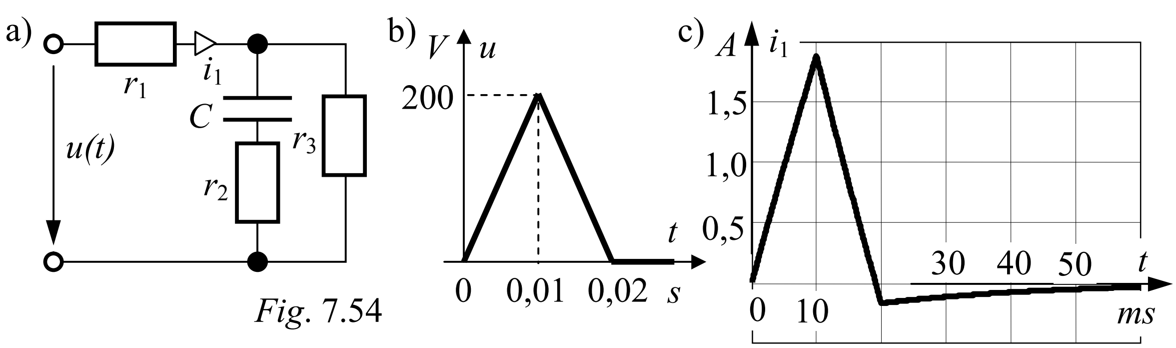


Fig. 7.54

7.4. APPLICATION OF THE COMPLEX TRANSFER FUNCTION WHEN CALCULATING THE TRANSIENT PROCESSES

7-33 (7.79). Calculate the transient voltage $u_2(t)$ (output quantity) when supplying the circuit in problem 5.13 with direct current (input quantity) $J = 0.05$ *A* by both the classical and operational methods. Formula for the complex transfer impedance obtained in the solution of the problem 5.13 is as follows:

$$\underline{Z}(j\omega) = \frac{b_1 j\omega + b_0}{j\omega + a_0} = \frac{83333}{j\omega + 66.67} \ Ohm.$$

Solution. 1. When classical method is applied to solve the problem, the required voltage $u_2(t)$ is found as the sum of the steady-state and transient components:

$$u_2(t) = u_{2s}(t) + u_{2t}(t).$$

As a D-C source is used, the steady-state component is constant too and may be determined through the complex transfer impedance at the frequency $\omega = 0$:

$$Z(0) = \frac{b_0}{a_0} = \frac{83333}{66.67} = 1250\ Ohm; \quad u_{2s} = Z(0)\cdot J = 1250\cdot 0.05 = 62.5\ V.$$

A complex transfer function is a so-called *system function*. Then, in order to obtain the characteristic equation it is sufficient to replace $j\omega$ by p and to equate the denominator of the fraction with zero. Doing in this way, we have:

$$p + a_0 = 0, \quad p = -a_0 = -66.67\ s^{-1}.$$

At the single root of the characteristic equation, the transient component has a view: $u_{2t}(t) = A\cdot e^{pt}$, where the integration constant A is found from the initial conditions:

$$A = u_{2t}(0) = u_2(0) - u_{2s}(0).$$

In order to obtain the initial value of voltage $u_2(0)$, we again use the complex transfer impedance, however, this time at the frequency equal to infinity:

$$Z(\infty) = \frac{b_1}{a_1} = \frac{0}{1} = 0.$$

Then $u_2(0) = Z(\infty)\cdot J = 0$, and $A = -u_{2s}(0) = -62.5$.

Finally, we have: $u_2(t) = 62.5 - 62.5\cdot e^{-66.67t}\ V$.

2. In order to compute the output voltage $u_2(t)$ by the operational method, we make use of the operational transfer impedance $Z(p)$, which is obtained from the complex transfer impedance through the replacement of $j\omega$ by p:

$$Z(p) = \frac{b_1 p + b_0}{p + a_0} = \frac{83333}{p + 66.67}.$$

The images of the source current and output voltage (with application of MathCAD) are: $J(s) := J \left| \begin{matrix} laplace, t \\ float, 4 \end{matrix} \right. \rightarrow \frac{.5000e-1}{s}$, i.e. $J(p) = \frac{0.05}{p}$;

$$U2(p) := Z(p)\cdot J(p) \qquad U_2(p) \left| \begin{matrix} simplify \\ float, 4 \end{matrix} \right. \rightarrow \frac{.1250e5}{(3.\cdot p + 200.)\cdot p}.$$

We find the original of the required voltage by means of the Laplace inversion:

$$u2(t) := U2(p) \left| \begin{matrix} invlaplace, p \\ float, 4 \end{matrix} \right. \rightarrow 62.5 - 62.5\cdot e^{(-66.67)\cdot t}.$$

7-34 (7.80). Solve the problem 7.33 using the spectral method on the condition that the source produces a single rectangular current impulse of the amplitude $J = 0.05\ A$ and duration $\tau = a_0^{-1} = 0.015\ s$.

Solution. The instantaneous value of the disturbance current may be presented in the following analytical way: $J(t) := \left| \begin{matrix} 0.05 & if\ 0 \le t \le \tau \\ 0 & otherwise \end{matrix} \right.$.

In order to perform Laplace transform using the program MathCAD let's present the instantaneous current by one formula with the help of the Heaviside function 1*(t)* which is presented in MathCAD as $\Phi(t)$: $J(t) := 0.05\cdot(\Phi(t) - \Phi(t-\tau))$.

The source current image is:

$$J(s) := J(t) \begin{vmatrix} laplace,t \\ float,4 \end{vmatrix} \rightarrow \frac{.5000e-1}{s} - .5000e\text{-}1 \cdot \frac{e^{(-.1500e-1)\cdot s}}{s},$$

i.e. $J(p) = \frac{0.05}{p}(1 - e^{-p\tau})$.

The spectral density of the source current may be obtained from the current image replacing p by $j\omega$: $J(\omega) := \frac{0.05}{j\cdot\omega}(1 - e^{-j\cdot\omega\cdot\tau})$.

The spectral density of the output voltage is found through the complex transfer impedance: $U2(\omega) := Z(\omega)\cdot J(\omega)$.

Eventually, with the aid of MathCAD we perform the calculation of the Fourier inversion integral: $u2(t) := \frac{1}{2\pi}\cdot\int_{-60000}^{+60000} U2(\omega)\cdot e^{j\cdot\omega\cdot t}\,d\omega$.

For any time moment, it is possible to obtain the answer. For instance,

$u2(0.005) = 17.717$ $u2(0.01) = 30.386$ $u2(0.05) = 2.113$.

Eventually, the voltage $u2(t)$ may be obtained in the graph form through the application of the function *invfourier*:

$$u2(t) := U2(\omega) \begin{vmatrix} invfourier,\omega \\ float,4 \end{vmatrix} \rightarrow 31.24\cdot\Phi(1.\cdot t) - 31.24\cdot\Phi[(-1).\cdot t] - 62.49\cdot e^{(-66.67)\cdot t}\cdot\ldots$$

The voltage curve $u2(t)$ is presented in fig. 7.55.

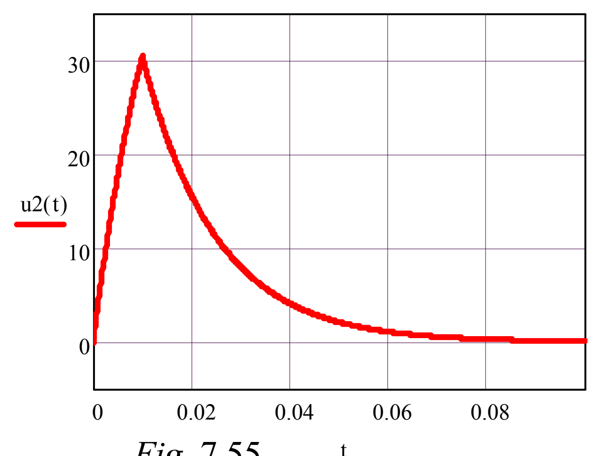


Fig. 7.55

8. ELECTRIC CIRCUITS WITH DISTRIBUTED PARAMETERS

8.1. ELECTRIC CIRCUITS WITH DISTRIBUTED PARAMETERS AT THE STEADY-STATE CONDITION

8.1.1. Calculation of the line parameters and its working modes

8-1 (8.1). In order to determine the parameters of the communication line 160 *km* length at the frequency 1000 *Hz*, the no-load and short-circuit tests were performed; the results of which were: $\underline{Z}_o = 887 \cdot e^{-j70°}$ *Ohm*, $\underline{Z}_S = 540 \cdot e^{j71°}$ *Ohm*. Determine the primary and secondary line parameters.

Solution. 1. Determine the secondary line parameters.

$$\underline{Z}_C = \sqrt{\underline{Z}_o \cdot \underline{Z}_S} = 692 \cdot e^{j0.5°}\ Ohm, \quad th\,\underline{\gamma} l = \sqrt{\frac{\underline{Z}_S}{\underline{Z}_o}} = 0.780 \cdot e^{j70.5°} = 0.26 + j0.735;$$

$$\frac{1 + th\underline{\gamma} l}{1 - th\underline{\gamma} l} = \frac{1 + (0.26 + j0.735)}{1 - (0.26 + j0.735)} = 1.399 \cdot e^{j75.11° = j1.311\ rad} = e^{2\alpha l} \cdot e^{j2\beta l}.$$

The latter equality splits into two ones: $e^{2\alpha l} = 1.399$

and $2\beta l = 1.311 + 2\pi k$.

From the first equality, we find the decay coefficient

$$2\alpha l = ln\ e^{2\alpha l} = ln\ 1.399 = 0.336, \qquad \alpha = 0.00105\ Np/km.$$

In order to estimate the number k of complete 2π radians along the whole line, let's determine approximately the total phase shift along the whole line at the speed $v = 150 \cdot 10^3$ *km/s*:

$$2\beta' l = 2l \cdot \frac{\omega}{v} = 2 \cdot 160 \cdot \frac{2\pi \cdot 1000}{150 \cdot 10^3} = 13.4\ rad > 4\pi, \quad k = 13.4/6.283 \approx 2.$$

Accordingly: $2\beta l = 1.311 + 2\pi \cdot 2$, from here $\beta = 0.04337$ *rad/km*.

Thus, $\underline{Z}_C = 692 \cdot e^{j0.5°}$ *Ohm*,

$$\underline{\gamma} = \alpha + j\beta = 0.00105 + j0.04337 = 0.04338 e^{j88.61°}\ 1/km.$$

2. Determine the primary line parameters:

$$\underline{Z}_0 = r_0 + j\omega L_0 = \underline{\gamma} \cdot \underline{Z}_C = 0.04338 e^{j88.61°} \cdot 692 e^{j0.5°} = 30.02 e^{j89.11°} = 0.46 + j30.02\ Ohm/km,$$

$$\underline{Y}_0 = g_0 + j\omega C_0 = \underline{\gamma}/\underline{Z}_C = 0.04338 e^{j88.61°}/(692 e^{j0.5°}) =$$

$$= 62.68 \cdot 10^{-6} \cdot e^{j88.11°} = (2.06 + j62.64) \cdot 10^{-6}\ S/km.$$

With account of the frequency $\omega = 2\pi f = 6283$ *rad/s*, we eventually have:

$r_0 = 0.46$ *Ohm/km*, $g_0 = 2.06 \cdot 10^{-6}$ *S/km*,

$L_0 = 30.02/6283$ *H/km* $= 4.79$ *mH/km*, $C_0 = 62.64 \cdot 10^{-6}/6283$ *F/km* $= 9.97$ *nF/km*.

8-2 (8.2). The no-load and short-circuit tests for the communication line $l = 120$ *km* long at the frequency 800 *Hz* give: $\underline{Z}_o = 182 e^{j3.55°}$ *Ohm*, $\underline{Z}_S = 209 e^{-j22,1°}$ *Ohm*. It is necessary to determine the secondary and primary line parameters as well as to calculate the line input impedance if it is loaded with impedance $\underline{Z}_l = 2\underline{Z}_C$.

Answers: $\underline{Z}_C = 195 e^{-j9.28°}$ *Ohm*, $\underline{\gamma} = 8.93 \cdot 10^{-3} + j18.40 \cdot 10^{-3}$ 1/*km*; $r_0 = 2.3$ *Ohm/km*, $g_0 = 30 \cdot 10^{-6}$ *S/km*, $L_0 = 0.65$ *mH/km*, $C_0 = 20$ *nF/km*, $\underline{Z}_{inp} = 191 e^{-j5°}$ *Ohm*.

8-3 (8.3). Some parameters of the communication line $l = 140$ *km* long working at the frequency $f = 1500$ *Hz* were found experimentally: $\underline{Z}_C = 710 \cdot e^{-j9°}$ *Ohm*, $\underline{Z}_0 = 19.2 \cdot e^{j70°}$ *Ohm/km*. It is necessary to calculate the primary and secondary parameters, to determine the input impedance at no-load and short-circuit conditions.

Solution. Angular velocity is $\omega = 2\pi f = 2\pi\cdot1500 = 9425$ *rad/s*.

Because $\underline{Z}_0 = r_0 + j\omega L_0 = 6.57 + j18.04$ *Ohm/km*, then the longitudinal primary parameters are $r_0 = \text{Re}(\underline{Z}_0) = 6.57$ *Ohm/km*, $L_0 = \frac{1}{\omega}\text{Im}(\underline{Z}_0) = \frac{18.04}{9425} = 1.91\cdot10^{-3}$ *H/km*.

Further, from $\underline{Z}_C = \sqrt{\underline{Z}_0/\underline{Y}_0}$ we find

$$\underline{Y}_0 = g_0 + j\omega C_0 = \frac{\underline{Z}_0}{\underline{Z}_C^2} = \frac{19.2\cdot e^{j70}}{\left(710\cdot e^{-j9}\right)^2} = 38.1\cdot10^{-6}\cdot e^{j88^\circ} = (1.33 + j38.08)\cdot10^{-6}\ S/km.$$

From here, the transversal primary parameters are

$$g_0 = \text{Re}(\underline{Y}_0) = 1.33\cdot10^{-6}\ S/km, \qquad C_0 = \frac{1}{\omega}\text{Im}(\underline{Y}_0) = \frac{38.08}{9425}\cdot10^{-6} = 4.04\cdot10^{-9}\ F/km.$$

The secondary parameters are:

- the characteristic impedance is $\underline{Z}_C = 710\cdot e^{-j9^\circ}$ *Ohm*,
- the propagation coefficient is

$$\underline{\gamma} = \sqrt{\underline{Z}_0\underline{Y}_0} = \sqrt{19.2e^{j70}\cdot38.1\cdot10^{-6}e^{j88}} = 27.1\cdot10^{-3}\cdot e^{j79^\circ} = (5.16 + j26.55)\cdot10^{-3}\ 1/km.$$

From here, the decay coefficient is – $\alpha = \text{Re}(\underline{\gamma}) = 5.16\cdot10^{-3}$ *Np/km*,

the phase constant is – $\beta = \text{Im}(\underline{\gamma}) = 26.55\cdot10^{-3}$ *rad/km*.

Let's calculate the hyperbolic functions: $\underline{\gamma}l = 0.722 + j3.717$;

$e^{\underline{\gamma}l} = 2.059\cdot e^{j213^\circ} = -1.727 - j1.121$; $\qquad e^{-\underline{\gamma}l} = 0.486\cdot e^{-j213^\circ} = -0.407 + j0.265$;

$sh\,\underline{\gamma}l = ½[e^{\underline{\gamma}l} - e^{-\underline{\gamma}l}] = -0.660 - j0.693 = 0.957\cdot e^{-j133.6^\circ}$;

$ch\,\underline{\gamma}l = ½[e^{\underline{\gamma}l} + e^{-\underline{\gamma}l}] = -1.067 - j0.428 = 1.150\cdot e^{-j158.1^\circ}$;

$th\,\underline{\gamma}l = sh\,\underline{\gamma}l/ch\,\underline{\gamma}l = 0.832\cdot e^{j24.5^\circ}$.

No-load and short-circuit impedances are:

$\underline{Z}_{1o} = \underline{Z}_C/th\,\underline{\gamma}l = 710\cdot e^{-j9^\circ}/(0.832\cdot e^{j24.5^\circ}) = 854\cdot e^{-j33.5^\circ}$ *Ohm*,

$\underline{Z}_{1S} = \underline{Z}_C\,th\,\underline{\gamma}l = 710\cdot e^{-j9^\circ}\cdot0.832\cdot e^{j24.5^\circ} = 591\cdot e^{j15.5^\circ}$ *Ohm*.

8-4 (8.4). The secondary parameters are known for a two-wire overhead communication line, at the frequency 50 *Hz*: $\underline{Z}_C = 440\cdot e^{-j10^\circ}$ *Ohm*, $\underline{\gamma} = (4 + j18)\cdot10^{-3}$ 1/*km*.

This line works as the D-C line and supplies the load $r_l = 400$ *Ohm*. The input voltage is $U_1 = 600$ *V*. Determine U_2 and I_1, if the line length is $l = 200$ *km*.

Solution. Determine the line primary parameters r_0 and g_0, which do not change in a D-C line:

$$r_0 = \text{Re}(\underline{\gamma}\cdot\underline{Z}_C) = \text{Re}(18.44\cdot10^{-3}\cdot e^{j77.5^\circ}\cdot440\cdot e^{-j10^\circ}) = 3.105\ Ohm/km,$$

$$g_0 = \text{Re}\left(\frac{\underline{\gamma}}{\underline{Z}_C}\right) = \text{Re}\left(\frac{18.44\cdot10^{-3}e^{j77.5}}{440e^{-j10}}\right) = 1.826\cdot10^{-6}\ S/km.$$

The secondary parameters in a D-C line are:

$$Z_C = \sqrt{\frac{r_0}{g_0}} = \sqrt{\frac{3.105}{1.826\cdot10^{-6}}} = 1303\ Ohm,$$

$$\gamma = \sqrt{r_0 g_0} = \sqrt{3.105\cdot1.826\cdot10^{-6}} = 2.382\cdot10^{-3}\ 1/km.$$

We find the output voltage and input current by means of the principal equations of the long line in hyperbolic functions with account of the correlation $I_2 = U_2/r_l$:

$$U_1 = U_2 \cdot ch\,\gamma l + I_2 Z_C \cdot sh\,\gamma l = U_2 \cdot ch\,\gamma l + \frac{U_2}{r_l} Z_C \cdot sh\,\gamma l = U_2 \cdot (ch\,\gamma l + \frac{Z_C}{r_l} sh\,\gamma l),$$

from here, the required voltage is

$$U_2 = \frac{U_1}{ch\gamma l + \dfrac{Z_C}{r_l} sh\gamma l} = \frac{600}{ch(2.382 \cdot 10^{-3} \cdot 200) + \dfrac{1303}{400} sh(0.4764)} = 220\ V.$$

The input current is

$$I_1 = \frac{U_2}{Z_C} \cdot sh\gamma l + I_2 \cdot ch\gamma l = \frac{U_2}{Z_C} \cdot sh\gamma l + \frac{U_2}{r_l} \cdot ch\gamma l = \frac{220}{1303} \cdot 0.4945 + \frac{220}{400} \cdot 1.116 = 0.697\ A.$$

8-5 (8.6). Determine the impedances of *T*- and *П*-equivalent schemes of the long line $l = 400\ km$ long with the parameters:

$$\underline{Z}_C = 391 e^{-j3,75°}\ Ohm \quad \text{and} \quad \underline{\gamma} = (0.187 + j1.058) \cdot 10^{-3}\ 1/km.$$

Answers. *T*-scheme: $\underline{Z}_{1T} = \underline{Z}_{2T} = 20.74 + j82.69\ Ohm$, $\underline{Z}_{0T} = 92.4 - j932.1\ Ohm$;
П-scheme: $\underline{Z}_{0П} = 37.1 + j158.9\ Ohm$, $\underline{Z}_{1П} = \underline{Z}_{2П} = 206 - j1781\ Ohm$.

8-6 (8.7). Three-phase steel-aluminum overhead transmission line 300 *km* long possesses the following parameters (per a phase):

$$r_0 = 0.08\ Ohm/km,\ g_0 = 3.75 \cdot 10^{-8}\ S/km,\ \omega L_0 = 0.42\ Ohm/km,\ \omega C_0 = 2.7\ \mu S/km.$$

Calculate the line secondary parameters, phase velocity and the wave length.

Find the phase voltage, current and power at the sending end of the line, its efficiency, if the voltage at the receiving end is 330 *kV*, power is 300 *MW* and the load power factor is equal to 0.92.

Calculate the voltage complexes of the incident and reflected waves at both ends of the line.

Solution. Assume, the line load is balanced and it is *Y*-connected. Accordingly, we perform a calculation for but one phase.

Calculate the line secondary parameters:

- the characteristic impedance is

$$\underline{Z}_C = \sqrt{\frac{r_0 + j\omega L_0}{g_0 + j\omega C_0}} = \sqrt{\frac{0.08 + j0.42}{3.75 \cdot 10^{-8} + j2.7 \cdot 10^{-6}}} = \sqrt{\frac{0.428 e^{j79.2}}{2.7 \cdot 10^{-6} e^{j89.2}}} = 398 \cdot e^{-j5°}\ Ohm,$$

- the propagation coefficient is

$$\underline{\gamma} = \sqrt{(r_0 + j\omega L_0)(g_0 + j\omega C_0)} = \sqrt{0.428 e^{j79.2} \cdot 2.7 \cdot 10^{-6} e^{j89.2}} =$$

$$= 1.07 \cdot 10^{-3} \cdot e^{j84.2°} = (0.108 + j1.069) \cdot 10^{-3}\ 1/km.$$

From here, the decay coefficient is – $\alpha = \text{Re}(\underline{\gamma}) = 0.108 \cdot 10^{-3}\ Np/km$,
the phase constant is – $\beta = \text{Im}(\underline{\gamma}) = 1.069 \cdot 10^{-3}\ rad/km$.

The phase velocity and the wave length are:

$$v = \frac{\omega}{\beta} = \frac{314}{1.069 \cdot 10^{-3}} = 294\,000\ km/s; \quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.069 \cdot 10^{-3}} = 5880\ km.$$

Let's calculate the hyperbolic functions: $\underline{\gamma} l = 0.033 + j0.321$;

$e^{\gamma l} = 1.033 \cdot e^{j18.4°} = 0.980 + j0.326;$ $e^{-\gamma l} = 0.968 \cdot e^{-j18.4°} = 0.919 - j0.305;$
$sh\ \gamma l = ½[e^{\gamma l} - e^{-\gamma l}] = 0.031 + j0.316 = 0.317 \cdot e^{j84.4°};$
$ch\ \gamma l = ½[e^{\gamma l} + e^{-\gamma l}] = 0.949 + j0.01 = 0.950 \cdot e^{j0.62°}.$

Determine the complexes of the phase voltage and current at the receiving end of the line: $\underline{U}_{2ph} = \frac{U_{2l}}{\sqrt{3}} = \frac{330}{\sqrt{3}} = 190.5\ kV;$ $cos\varphi_2 = 0.92$, then $\varphi_2 = 23.1°$;

$$I_{2ph} = \frac{P_2}{3U_{2ph}\,cos\varphi_2} = \frac{300}{3 \cdot 190.5 \cdot 0.92} = 0.571\ kA; \quad \psi_{i2} = \psi_{u2} - \varphi_2 = -\varphi_2;$$

$$\underline{I}_{2ph} = I_{2ph} \cdot e^{-j\varphi 2} = 0.571 \cdot e^{-j23.1°}\ kA.$$

Under the principal equations of the long line with hyperbolic functions, we calculate the complexes of the phase voltage and current at the sending end of the line:

$$\underline{U}_{1ph} = \underline{U}_{2ph} \cdot ch\ \gamma l + \underline{I}_{2ph}\underline{Z}_C \cdot sh\ \gamma l =$$
$$= 190.5 \cdot 0.95 \cdot e^{j0.62°} + 0.571 \cdot e^{-j23.1°} \cdot 398 \cdot e^{-j5°} \cdot 0.317 \cdot e^{j84.4°} = 229.3 \cdot e^{j15.6°}\ kV;$$

$$\underline{I}_{1ph} = \frac{\underline{U}_{2ph}}{\underline{Z}_C} \cdot sh\ \gamma l + \underline{I}_{2ph} \cdot ch\ \gamma l = \frac{190.5}{398e^{-j5}} \cdot 0.317 \cdot e^{j84.4°} + 0.571 \cdot e^{-j23.1°} \cdot 0.95 \cdot e^{j0.62°} =$$
$$= 0.505 \cdot e^{-j6.3°}\ kA.$$

The active power at the beginning of the line

$$P_1 = 3\text{Re}(\underline{U}_{1ph} \cdot \overset{*}{\underline{I}}_{1ph}) = 3 \cdot 229.3 \cdot 0.505 \cdot cos(15.6° + 6.3°) = 322\ MW.$$

The line efficiency is $\eta = P_2/P_1 = 300/322 = 0.93$.

Electromagnetic processes in a long line are considered as a result of the superposition of the incident and reflected waves –

$$\underline{U} = \underline{U}_{refl} + \underline{U}_{inc} = \underline{A}_1 e^{\gamma x} + \underline{A}_2 e^{-\gamma x},$$
$$\underline{I} = -\underline{I}_{refl} + \underline{I}_{inc} = -\frac{\underline{A}_1}{\underline{Z}_C} e^{\gamma x} + \frac{\underline{A}_2}{\underline{Z}_C} e^{-\gamma x},$$

where $\underline{A}_1$ and $\underline{A}_2$ – integration constants which may be determined through the voltage and current at the receiving end of the line:

$$\underline{A}_1 = \frac{\underline{U}_{2ph} - \underline{I}_{2ph}\underline{Z}_C}{2} e^{-\gamma l} = \frac{190.5 - 0.571e^{-j23.1} \cdot 398e^{-j5}}{2} \cdot 0.968 \cdot e^{-j18.4°} = 51.9 \cdot e^{j76.8°};$$

$$\underline{A}_2 = \frac{\underline{U}_{2ph} + \underline{I}_{2ph}\underline{Z}_C}{2} e^{\gamma l} = \frac{190.5 + 0.571e^{-j23.1} \cdot 398e^{-j5}}{2} \cdot 1.033 \cdot e^{j18.4°} = 209.3 \cdot e^{j3.1°}.$$

At the beginning of the line $x = 0$, that's why

$\underline{U}_{inc}(x = 0) = \underline{A}_2 = 209.3 \cdot e^{j3.1°}\ kV,$ $\underline{U}_{refl}(x = 0) = \underline{A}_1 = 51.9 \cdot e^{j76.8°}\ kV,$
$\underline{I}_{inc}(x = 0) = \underline{U}_{inc}(x = 0)/\underline{Z}_C = 0.526 \cdot e^{j8.1°}\ kA,$
$\underline{I}_{inc}(x = 0) = \underline{U}_{inc}(x = 0)/\underline{Z}_C = 0.13 \cdot e^{j81.8°}\ kA.$

At the end of the line $x = l$, that's why

$$\underline{U}_{inc}(x = l) = \underline{A}_2 \cdot e^{-\gamma l} = \frac{\underline{U}_{2ph} + \underline{I}_{2ph}\underline{Z}_C}{2} = 202.6 \cdot e^{-j15.3°}\ kV,$$

$$\underline{U}_{refl}(x = l) = \underline{A}_1 \cdot e^{\gamma l} = \frac{\underline{U}_{2ph} - \underline{I}_{2ph}\underline{Z}_C}{2} = 53.6 \cdot e^{j95.2°}\ kV,$$

$\underline{I}_{inc}(x = l) = \underline{U}_{inc}(x = l)/\underline{Z}_C = 0.509 \cdot e^{-j10.3°}\ kA,$
$\underline{I}_{refl}(x = l) = \underline{U}_{refl}(x = l)/\underline{Z}_C = 0.135 \cdot e^{j100.2°}\ kA.$

As one should expect, the incident wave decreases towards the end of the line because of the losses in the line, the reflected wave decaying in direction from the end of the line towards its beginning.

8-7 (8.8). At the end of the line described in problem 8.6, the following happens: a) load disconnection; b) three-phase short-circuit. For each case, determine the line voltages and currents at the line beginning and end, if the input phase voltage is the same as calculated in problem 8.6.

Determine additionally the values of voltage and current of the incident and reflected waves at the receiving end of the line.

Solution. To solve the problem, we use the principal equations of the long line with hyperbolic functions.

a) *at the load disconnection* (no-load condition), the current at the line end is $\underline{I}_{2ph} = 0$, that's why

$$\underline{U}_{2ph} = \frac{\underline{U}_{1ph}}{ch(\underline{\gamma}l)} = \frac{229.3e^{j15.6}}{0.95e^{j0.6}} = 241\cdot e^{j15°}\ kV, \qquad U_{2l} = \sqrt{3}\,U_{2ph} = \sqrt{3}\cdot 241 = 418\ kV;$$

$$\underline{I}_{1ph} = \frac{\underline{U}_{2ph}}{\underline{Z}_C} sh\ \underline{\gamma}l = \frac{241e^{j15}}{398e^{-j5}}\cdot 0.317\cdot e^{j84.4°} = 0.192\cdot e^{j104.4°}\ kA, \qquad I_{1l} = I_{1ph} = 0.192\ kA.$$

At no-load condition, the reflection coefficient is +1, it means the incident wave is reflected completely without the sign change. That's why $\underline{U}_{inc}(x = l) = \underline{U}_{refl}(x = l)$.

Taking this into account:

$$\underline{U}_{2ph} = \underline{U}_{inc}(x = l) + \underline{U}_{refl}(x = l) = 2\underline{U}_{inc}(x = l) = 241\cdot e^{j15°}\ kV,$$

from here $\underline{U}_{inc}(x = l) = \underline{U}_{refl}(x = l) = \underline{U}_{2ph}/2 = 120.5\cdot e^{j15°}\ kV,$

$$\underline{I}_{inc}(x = l) = \underline{I}_{refl}(x = l) = \underline{U}_{inc}(x = l)/\underline{Z}_C = 0.303\cdot e^{j10°}\ kA.$$

b) *at shot-circuit*, the output voltage is $\underline{U}_{2ph} = 0$, that's why

$$\underline{I}_{2ph} = \frac{\underline{U}_{1ph}}{\underline{Z}_C sh(\underline{\gamma}l)} = \frac{229.3e^{j15.6}}{398e^{-j5}\ 0.317e^{j84.4}} = 1.817\cdot e^{-j63.8°}\ kA, \qquad I_{2l} = I_{2ph} = 1.817\ kA;$$

$$\underline{I}_{1ph} = \underline{I}_{2ph}\cdot ch\ \underline{\gamma}l = 1.817\cdot e^{-j63.8°}\cdot 0.95\cdot e^{j0.6°} = 1.726\cdot e^{-j63.2°}\ kA, \qquad I_{1l} = I_{1ph} = 1.726\ kA.$$

At the short-circuit condition, the reflection coefficient is equal to –1. That's why $\underline{I}_{inc}(x = l) = -\underline{I}_{refl}(x = l)$. Taking this into account:

$$\underline{I}_{2ph} = \underline{I}_{inc}(x = l) - \underline{I}_{refl}(x = l) = 2\underline{I}_{inc}(x = l) = 1.817\cdot e^{-j63.8°}\ kA,$$

from here $\underline{I}_{inc}(x = l) = -\underline{I}_{refl}(x = l) = \underline{I}_{2ph}/2 = 0.909\cdot e^{-j63.8°}\ kA,$

$$\underline{U}_{inc}(x = l) = -\underline{U}_{refl}(x = l) = \underline{I}_{inc}(x = l)\cdot\underline{Z}_C = 362\cdot e^{-j58.8°}\ kV.$$

8-8 (8.9). Under a) no-load and b) short-circuit conditions, determine the line voltages and currents at the beginning and end of the line described in problem 8.6, if its length is 900 km, while the input phase voltage is $\underline{U}_{1ph} = 229.3\ kV$.

Answers: $ch\ \underline{\gamma}l = 0.574 + j0.080$, $sh\ \underline{\gamma}l = 0.056 + j0.824$;

a) $U_{2l} = 685\ kV$; $I_1 = 821\ A$; b) $I_2 = 697\ A$; $I_1 = 404\ A$.

Comparing the calculation results for two similar lines but of different length (in problem 8.6 the line length is 300 km, while in problem 8.8 it is 900 km), we see that in the second case the output voltage and the input current have sharply increased under no-load condition while the current has appreciably decreased under short-circuit condition. It can be concluded the no-load condition for high-voltage transmission line may be much dangerous than short-circuit condition.

8.1.2. Lines with matched load

8-9 (8.10). For a three-phase transmission line with the matched load, the complexes of the input phase voltage and the output phase current are known: $\underline{U}_1 = 100\ kV$, $\underline{I}_2 = 190e^{-j90°}\ A$. Determine the line efficiency, if its characteristic impedance is $\underline{Z}_C = 500e^{-j10°}\ Ohm$.

Solution. The load impedance is equal to a characteristic one for the line with matched load, then the output voltage is $\underline{U}_2 = \underline{I}_2 \cdot \underline{Z}_C = 0.19e^{-j90°} \cdot 500e^{-j10°} = 95e^{-j100°}\ kV$.

Because of the equality $U_2 = U_1 e^{-\alpha l}$ for the matched line, there is

$$e^{-\alpha l} = U_2/U_1 = 95/100 = 0.95.$$

The line efficiency is $\eta = e^{-2\alpha l} = 0.95^2 = 0.903$.

8-10 (8.11). Two-wire line 100 *km* in length is loaded with the impedance $\underline{Z}_C = 410e^{-j30°}\ Ohm$.

The input voltage is – $u_1(t) = 220\sqrt{2}\sin(314t + 120°)\ V$, the output one is – $u_2(t) = 188.7\sqrt{2}\sin(314t + 79.9°)\ V$. Determine the current and voltage at point *A* situated at a distance of 20 *km* from the line end, write down their instantaneous values.

Solution. As it is stated in the problem, the line is with the matched load, its principal equations being as follows:

$$\begin{cases} \underline{U}_1 = \underline{U}_2 \cdot e^{\gamma l} = \underline{U}_2 \cdot e^{\alpha l} \cdot e^{j\beta l}; \\ \underline{I}_1 = \underline{I}_2 \cdot e^{\gamma l} = \underline{I}_2 \cdot e^{\alpha l} \cdot e^{j\beta l}. \end{cases}$$

In order to determine the current and voltage at point *A* it is necessary to know the line parameters, namely: the decay coefficient α and phase constant β. We find them from the first equation of the line:

$$e^{\gamma l} = \underline{U}_1/\underline{U}_2 = 220e^{j120°}/188.7e^{j79.9°} = 1.166e^{j40.1° = +j0.7\ rad};$$

$$e^{\alpha l} = 1.166;\quad \alpha l = \ln 1.166 = 0.153;\quad \alpha = 0.00153\ Np/km;\quad \beta l = 0.7 + 2\pi k\ rad.$$

$$\beta_{or} = \frac{\omega}{v} = \frac{314}{3 \cdot 10^5} \approx 10^{-3}\ rad/km;\qquad \beta_{or} l \approx 0.1\ rad;$$

thus, $k = 0$ and $\beta = 0.007\ rad/km$.

Then the voltage at point *A* is:

$$\underline{U}_A = \underline{U}_2 \cdot e^{\gamma l} = 188.7e^{j79.9°} \cdot e^{0.00153 \cdot 20} \cdot e^{j0.007 \cdot 20} = 194.6e^{j79.9°} \cdot e^{j0.14\ rad = 8.02°} = 194.6e^{-j87.92°}\ V.$$

The ratio $\underline{Z}_{inp} = \underline{Z}_C$ is true for any cross-section of the line, that's why the current at point *A* may be calculated in the following way

$$\underline{I}_A = \underline{U}_A/\underline{Z}_C = 194.6e^{j87.92°}/(410e^{-j30°}) = 0.475e^{j117.92°}\ A.$$

The instantaneous voltage and current are:

$$u_A(t) = 194.6\sqrt{2}\sin(314t + 87.92°)\ V,\qquad i_A(t) = 0.475\sqrt{2}\sin(314t + 117.92°)\ A.$$

8-11 (8.12). A line 25 *km* long is supplied from the voltage source $e_1 = 141.4\sin 5000t\ V$ with inner resistance $r_{in} = 100\ Ohm$. The line parameters are: $\underline{Z}_C = 335.5 - j497.4\ Ohm$, $\underline{\gamma} = (3.48 + j19.70) \cdot 10^{-3}\ 1/km$, $\underline{Z}_l = \underline{Z}_C$. Determine the currents, voltages, powers at the line input and output, find the line efficiency.

Answers: $U_1 = 90.6\ V$, $I_1 = 0.151\ A$, $P_1 = 7.65\ W$;
$U_2 = 83.04\ V$, $I_2 = 0.138\ A$, $P_2 = 6.43\ W$; $\eta = 0.84$.

8-12 (8.13). Two-wire line 100 *km* long is loaded with impedance $\underline{Z}_C = 1410e^{-j30°}$ *Ohm*. At a distance of 20 *km* from the line end, the voltage is $u_a(t) = 141.4 sin(314t + 60°)$ *V*. At point "*b*" situated at a distance of 40 *km* from the line end, the current is known $i(t) = 0.164 sin(314t + 120°)$ *A*.

Determine instantaneous and effective values of the input voltage.

Answer: $u_1(t) = 715.6\sqrt{2}\,sin(314t + 180°)$ *V*; $U_1 = 715.6$ *V*.

8-13 (8.14). A line $l = 20$ *km* in length has the following secondary parameters: $\underline{Z}_C = 1350e^{-j24°}$ *Ohm* and $\underline{\gamma} = 0.0175 + j0.039$ 1/*km*, it being loaded with the impedance equal to the characteristic one. Determine the power P_2 delivered to the load as well as the power P_1 supplied into the line, if the input voltage is $U_1 = 10$ *V*.

Answers: $P_1 = 67.67$ *mW*, $P_2 = 33.6$ *mW*.

8-14 (8.15). An air communication line possesses the following parameters:

$$r_0 = 2.84\ Ohm/km,\quad g_0 = 0.7\ \mu S/km,\quad L_0 = 1.94\ mH/km,\quad C_0 = 6.25\ nF/km.$$

Determine: 1) at which consumer impedance there is no reflected wave in the line at the medium rated frequency 800 *Hz*; 2) the voltage, current and power of the source as well as the efficiency of the line 59 *km* long, the load voltage being 20 *V*, the load impedance being as in paragraph 1.

Solution. Let's calculate the line secondary parameters at the frequency 800 *Hz*:

$\omega = 2\pi f = 2\pi\cdot 800 = 5027$ *rad/s*,

$$\underline{Z}_C = \sqrt{\frac{r_0 + j\omega L_0}{g_0 + j\omega C_0}} = \sqrt{\frac{2.84 + j5027\cdot 1.94\cdot 10^{-3}}{0.7\cdot 10^{-6} + j5027\cdot 6.25\cdot 10^{-9}}} = \sqrt{\frac{10.15e^{j73.8}}{31.56\cdot 10^{-6}e^{j88.7}}} =$$

$$= 567\cdot e^{-j7.46°}\ Ohm,$$

$$\underline{\gamma} = \sqrt{(r_0 + j\omega L_0)(g_0 + j\omega C_0)} = \sqrt{10.15e^{j73.8}\ 31.56\cdot 10^{-6}e^{j88.7}} = 17.9\cdot 10^{-3}\cdot e^{j81.3°}\ 1/km.$$

From here, the decay coefficient is – $\alpha = \text{Re}(\underline{\gamma}) = 2.71\cdot 10^{-3}$ *Np/km*.

There is no reflected wave in the line with matched load, it means its impedance is $\underline{Z}_l = \underline{Z}_C = 567\cdot e^{-j7.46°}$ *Ohm*.

Then the output voltage, current and power are:

$$\underline{U}_2 = 20\ V,\quad \underline{I}_2 = \frac{\underline{U}_2}{\underline{Z}_C} = \frac{20}{567e^{-j7.46}} = 0.0353\cdot e^{j7.46°}\ A,$$

$$P_2 = \text{Re}(\underline{U}_2\cdot \overset{*}{\underline{I}}_2) = 20\cdot 0.0353\cdot cos(-7.46°) = 0.7\ W.$$

In case of the matched load, the line efficiency may be found under the formula

$$\eta = e^{-2\alpha l} = e^{-2\cdot 2.71\cdot 0.059} = 0.73.$$

The input power is $P_1 = P_2/\eta = 0.7/0.73 = 0.96$ *W*.

The input voltage and current are $U_1 = U_2\cdot e^{\alpha l} = 20\cdot e^{\alpha l} = 20e^{2.71\cdot 0.059} = 23.4$ *V*,

$$I_1 = I_2\cdot e^{\alpha l} = 0.0353\cdot e^{2.71\cdot 0.059} = 0.041\ A.$$

8-15 (8.16). A D-C generator with voltage 10 *kV* supplies the overhead and cable lines connected in series. The overhead line parameters are: $l_1 = 20$ *km*, $r_{01} = 4$ *Ohm/km*, $g_{01} = 10^{-6}$ *S/km*. The cable parameters are: $l_2 = 40$ *km*, $r_{02} = 0.5$ *S/km*, $g_{02} = 0.5\cdot 10^{-6}$ *S/km*. The cable is connected with the matched load. Determine the powers of the generator and consumer, find the efficiency of the overhead line, as well as that of the cable line and of the line as a whole.

Answers: $P_g = 94.3\ kW$, $P_l = 82.2\ kW$, $\eta_1 = 0.907$; $\eta_2 = 0.96$; $\eta_{total} = 0.872$.

8.1.3. Distortionless lines

8-16 (8.19). An overhead two-wire communication line 100 *km* in length possesses the parameters: $r_0 = 2.8\ Ohm/km$, $g_0 = 0.7\ \mu S/km$, $L_0 = 2\ mH/km$, it working under a matched condition at the frequency $\omega = 5000\ rad/s$. What additional inductance L_0' is to be added per each kilometer of the line to eliminate distortions and the output voltage would lag the input one by 100° in phase?

Solution. In accordance with the problem statement, the line phase shift is
$\beta l = 100° + 360°\cdot k = 1.745 + 2\pi k\ rad$, it means $\beta = (1.745 + 2\pi k)\cdot 10^{-2}\ rad/km$.

Let's estimate β. The phase velocity for the overhead line is practically equal to the light speed, that's why $\beta \approx \frac{\omega}{c} = \frac{5000}{300000} = 1.667\cdot 10^{-2}\ rad/km$.

Comparing two results obtained, we conclude that $k = 0$.

Finally, we have $\beta = 0.01745\ rad/km$.

For a distortionless line the following relationships are true

$$\beta = \omega\sqrt{(L_0 + L_0')C_0} \quad \text{and} \quad \frac{r_0}{L_0 + L_0'} = \frac{g_0}{C_0}$$

$$\text{or} \quad \sqrt{(L_0 + L_0')C_0} = \frac{\beta}{\omega} \quad \text{and} \quad \sqrt{\frac{L_0 + L_0'}{C_0}} = \sqrt{\frac{r_0}{g_0}}.$$

Multiplying two latter equalities, we find

$$(L_0 + L_0') = \frac{\beta}{\omega}\sqrt{\frac{r_0}{g_0}} = \frac{0.01745}{5000}\sqrt{\frac{2{,}8}{0.7\cdot 10^{-6}}} = 0.00698\ H/km.$$

Eventually, we have: $L_0' = 6.98 - 2 = 4.98\ mH/km$.

8-17 (8.20). A cable communication line is 150 *km* in length and the wave impedance is $Z_C = 60\ Ohm$. It takes signals 1 *ms* to pass along this line without any distortion, they decaying by 11.3 *dB*. Determine the line primary parameters.

Solution. The line decay is $\alpha l = 11.3\ dB = 11.3\cdot 0.115 = 1.3\ Np$.

From here, the decay coefficient is $\alpha = \frac{1.3}{l} = \frac{1.3}{150} = 8.66\cdot 10^{-3}\ Np/km$.

At the same time, a distortionless line obeys to the correlations:

$$\frac{L_0}{C_0} = \frac{r_0}{g_0}, \quad Z_C = \sqrt{\frac{L_0}{C_0}}, \quad \alpha = \sqrt{r_0 g_0}, \quad v = \frac{1}{\sqrt{L_0 C_0}}.$$

Furthermore, it may be assumed that $v = 150\ 000\ km/s$.

From these correlations, we find:

$r_0 = \alpha\cdot Z_C = 8.66\cdot 10^{-3}\cdot 60 = 0.52\ Ohm/km$,

$$g_0 = \frac{\alpha^2}{r_0} = \frac{8.66^2\cdot 10^{-6}}{0.52} = 1.44\cdot 10^{-4}\ S/km,$$

$$L_0 = \frac{Z_C}{v} = \frac{60}{1.5\cdot 10^5} = 0.4\cdot 10^{-3}\ H/km = 0.4\ mH/km,$$

$$C_0 = \frac{1}{Z_C v} = \frac{1}{60\cdot 1.5\cdot 10^5} = 0.111\cdot 10^{-6}\ F/km = 0.111\ \mu F/km.$$

8-18 (8.21). The primary parameters of the two-wire copper 4*mm* telephone line (at $f = 100\ kHz$) are: $r_0 = 14\ Ohm/km$, $L_0 = 2\cdot 10^{-3}\ H/km$, $g_0 = 5\cdot 10^{-6}\ S/km$, $C_0 = 6.36\ nF/km$. Calculate which inductance L_1 should be added per each kilometer to obtain the distortionless line. What are the line secondary parameters under these conditions?

Answers: $L_1 = \frac{r_0 C_0}{g_0} - L_0 = 15.8\cdot 10^{-3}\ H/km,\ Z_C = \sqrt{\frac{L_0 + L_1}{C_0}} = 1673\ Ohm,$

$$\underline{\gamma} = \alpha + j\beta = \sqrt{r_0 g_0} + j\omega\sqrt{L_0 C_0} = 8.37\cdot 10^{-3} + j6.685\ 1/km.$$

8.1.4. Zero-loss line

8-19 (8.23). A voltmeter at the receiving end of the zero-loss line reads 100 *V*. Determine the ammeter reading, it being mounted at distance of 2 *m* from the line's end, the generator frequency and the line wave impedance are $f = 3\cdot 10^7\ Hz$ and $Z_C = 1000\ Ohm$.

Solution. High impedance voltmeter is connected at the line end. Under this circumstance, the line works in no-load condition, and there is no current at the line end $I_2 = 0$, that's why one of the principal equations of the zero-loss line takes a view:

$$\underline{I}(y) = j\frac{\underline{U}_2}{Z_C} sin(\beta y).$$

Phase velocity of an overhead line is practically equal to speed of light $c = 3\cdot 10^8\ m/s$. Then the phase constant of the line is:

$$\beta = \frac{\omega}{v} = \frac{2\pi f}{c} = \frac{2\pi\cdot 3\cdot 10^7}{3\cdot 10^8} = 0.2\pi\ rad/m.$$

Eventually, the ammeter reading is

$$I_A = |\underline{I}(y{=}2)| = \frac{U_2}{Z_C} sin(\beta y) = \frac{100}{1000} sin(0.2\pi\cdot 2) = 0.0951\ A.$$

8-20 (8.24). A generator with wave length $\lambda = 20\ m$ supplies an ideal zero-loss line $l = 5\ m$ in length. At the line's end, there is an ammeter reading 0.17 *A*, furthermore, at the middle of the line, there is a voltmeter reading 120 *V*. Determine the wave impedance of the line. Take $r_V = \infty$, $r_A = 0$.

Solution. Zero-loss line's equation at the short-circuit condition has a view:

$$\underline{U}(y) = jZ_C\cdot \underline{I}_2\cdot sin(\beta y).$$

In the centre of the line $sin(\beta y) = sin(\frac{2\pi}{\lambda}\cdot\frac{2,5}{20}\lambda = 0.25\pi = 45°) = 0.7071.$

Then the required impedance is $Z_C = 120/(0.17\cdot 0.7071) = 998$ Ohm.

8-21 (8.25). Determine the parameters of the overhead zero-loss line 30 *m* in length having the wave impedance 600 *Ohm* and working at the frequency 15 *MHz*. Calculate

the power delivered into the line as well as its efficiency at the load impedance 300 *Ohm* and voltage $U_2 = 120$ *V*.
Answers: $\underline{\gamma} = j\beta = j0.314\ 1/m$, $L_0 = 2\ \mu H/m$, $C_0 = 5.55\ nF/m$, $P_1 = 48\ W$, $\eta = 1$.

8-22 (8.26). Determine the current of a generator supplying voltage $U_g = 120\ V$ to a zero-loss line loaded with resistance $r_l = 380\ Ohm$, the line possessing the following parameters: $l = 100\ m$, $\lambda = 60\ m$, $L_0 = 5.3\ \mu H/m$.
Answers: $Z_C = 1590\ Ohm$, $\beta = 0.1047\ rad/m$, $\underline{Z}_{inp} = 255.6e^{j59.69°}\ Ohm$, $\underline{I}_1 = 0.047e^{-j59.69°}\ A$.

8-23 (8.27). There is an inductive load $L = 17.3\ \mu H$ at the end of the overhead zero-loss line $l = 12\ m$ in length. Determine the distance between the line end and the nearest voltage and current anti-nodes, if the source frequency is $\omega = 10^8\ rad/s$ while the line parameters are $L_0 = 10\ \mu H/m$, $C_0 = 1.11\ pF/m$.
Solution. Inductive reactance ωL may be simulated by the segment of the short-circuited zero-loss line of length l_1. Let's calculate the segment length.

The inductive reactance in a complex form is presented as $j\omega L$, the input impedance of the short-circuited zero-loss line is $\underline{Z}_S = jZ_C\, tg(\beta l_1)$, it means $\omega L = Z_C\, tg(\beta l_1)$,

where the line secondary parameters are $Z_C = \sqrt{\dfrac{L_0}{C_0}} = \sqrt{\dfrac{10\cdot 10^{-6}}{1.11\cdot 10^{-12}}} = 3000\ Ohm$,

$$\beta = \omega\sqrt{L_0 C_0} = 10^8\sqrt{10^{-5}\cdot 1.11\cdot 10^{-12}} = 0.333\ rad/m.$$

We obtain: $l_1 = \dfrac{1}{\beta} arctg\left(\dfrac{\omega L}{Z_C}\right) = \dfrac{1}{0.333} arctg\left(\dfrac{10^8\cdot 17.3\cdot 10^{-6}}{3000}\right) = \dfrac{\pi}{2}\ m.$

The wave length is $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{0.333} = 6\pi\ m.$

In case of the short-circuited zero-loss line, the voltage anti-node nearest to the line end is situated at a distance equal to the quarter of the wave length:

$$\tfrac{1}{4}\lambda = 1.5\pi\ m.$$

For the line under consideration with account of the segment l_1:

$$1.5\pi - l_1 = 1.5\pi - 0.5\pi = \pi = 3.14\ m.$$

The current anti-node is shifted in comparison with the voltage anti-node by the quarter of the wave length. That's why the coordinate of the current anti-node is as follows: $\pi + 1.5\pi = 2.5\pi = 7.85\ m$.

8-24 (8.28). In order to match the line with the load at the frequency 100 *MHz*, it is necessary to add the inductive reactance 800 *Ohm*, which can be simulated by the segment of the zero-loss line. Determine the shortest possible segment of the short-circuited zero-loss line made of copper wires with radius $r = 2\ mm$, the distance between the wires being $d = 20\ cm$. Calculate the impedance of the segment at the open-circuited line end.

Direction. The primary parameters of zero-loss line may be calculated by the following formulae: $L_0 = \frac{\mu_0}{\pi} ln\frac{d}{r} = 4\cdot 10^{-7}\cdot ln\frac{d}{r}$ H/m; $C_0 = \frac{\pi\varepsilon_0}{ln\frac{d}{r}}$.

Answer: $y_{min} = 46.1$ cm, $\underline{Z}_o = -j381.9$ Ohm.

8-25 (8.29). A zero-loss line with parameters $l = 30$ m, $f = 15$ MHz, $Z_C = 600$ Ohm supplies a load: $U_2 = 120$ V, $r_l = 300$ Ohm. Determine the position of the voltage minimums and maximums along the line.

Solution. It is known that there are standing waves in a zero-loss line in no-load and short-circuit conditions. As this time the reflection coefficient is $n = \pm 1$, the incident and reverse waves are identical in value, their superposition results in the voltage nodes and anti-nodes distributed along the line in no-load (short-circuit) condition in accordance with the respective expressions:

$$y_{node(anti\text{-}node)} = (2k + 1)\cdot\lambda/4; \qquad y_{anti\text{-}node(node)} = k\lambda/2; \qquad k = 0, 1, 2, ...$$

At the arbitrary non-matched load the reflection coefficient is $n < 1$, and the incident and reflected waves are not equal to each other. As a result of the wave superposition in the line there are the voltage minimums and maximums. It is obvious that they are observed in the points where the direct and reverse waves are in phase (maximums) or out of phase (phase shift is 180°) (minimums).

1. Determine the reflection coefficient n from the load and estimate the direct and reverse wave voltages U_d, U_{rev}.

$$n = \frac{r_l - Z_C}{r_l + Z_C} = \frac{300 - 600}{300 + 600} = -0.333, \quad \text{it means} \quad U_{rev} = -0.333U_d.$$

At the line end the voltage is determined as

$$U_2 = U_d + U_{rev} = U_d - 0.333\cdot U_d = 0.667\cdot U_d = 120\ V.$$

As $n < 0$, then at the line end there is a voltage minimum $U_{min} = 120$ V, furthermore: $U_d = U_{min}/0.667 = 180$ V, $U_{rev} = -0.333U_d = -60$ V.

The voltage maximum is $U_{max} = 180 + 60 = 240$ V.

2. So, the first voltage minimum is at the line end. Further, the voltage minimums takes place every $\lambda/2$. Estimate the wave length:

$$\lambda = \frac{2\pi}{\beta} = \frac{v}{f} = 300\cdot 10^3/(15\cdot 10^6) = 20\ m.$$

The voltage maximums are shifted in comparison with minimums by a quarter of the wave length $\lambda/4 = 5$ m and further they also takes place every $\lambda/2$ along the line.

8.2. TRANSIENT PROCESSES IN THE LONG LINES

8.2.1. Calculation of waves arising

8-26 (8.30). A zero-loss line with parameters $Z_C = 250\,Ohm$, $l = 140\,km$, $v = 280\cdot 10^3\,km/s$ is connected to a D-C voltage source $E_0 = 120\,kV$ possessing the inner inductance $L_0 = 0.15\,H$. Receiving end of the line is open-circuited (fig. 8.1,a). It is required to plot the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for two time moments: $t_1 = 0.75l/v$ and $t_2 = 1.5l/v$.

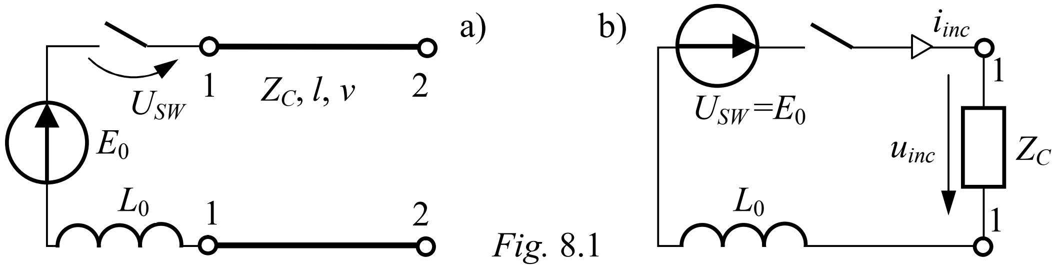


Fig. 8.1

Solution. 1. At the time moment $t_f = t_1 = 0.75l/v = 0.375\ mc$, there is but the incident wave in the line. We calculate its parameters based on the equivalent scheme for the section "1-1" (fig. 8.1,b): $\quad i_{inc}(t) = i_s(t) + Ae^{pt}$;

$$i_{inc}(0_+) = i_{inc\partial}(0_-) = 0,\quad i_s(t) = E_0/Z_C = 120\cdot 10^3/250 = 480\ A,$$

$$A = i_{inc}(0_+) - i_s(0_+) = 0 - 480 = -480,\qquad p = -Z_C/L_0 = -250/0.15 = -1667\ 1/s.$$

$$i_{inc}(t) = 480 - 480e^{-1667t}\ A;\quad u_{inc}(t) = Z_C\cdot i_{inc}(t) = 120 - 120e^{-1667t}\ kV.$$

In order to obtain the voltage and current dependencies on the coordinate according to which the diagram should be constructed, we pass to the argument $[t_f - \frac{x}{v}]$:

$$i_{inc}(t_f; x) = 480 - 480\,e^{-1667[0.375\cdot 10^{-3} - x/280\cdot 10^3]} = 480 - 480\,e^{-1.667[0.375 - x/280]}\ A;$$

$$u_{inc}(t_f; x) = 120 - 120\,e^{-1.667[0.375 - x/280]}\ kV.$$

At $t_f = 0.375\ ms$ the expressions are true for but the coordinate $x \le v\cdot t_f = 105\ km$.

The voltage distribution plot $u_{inc}(t_f; x)$ along the line for time moment t_1 is presented in fig. 8.2. The current curve $i_{inc}(t_f; x)$ is analogous, because $i_{inc} = u_{inc}/Z_C$.

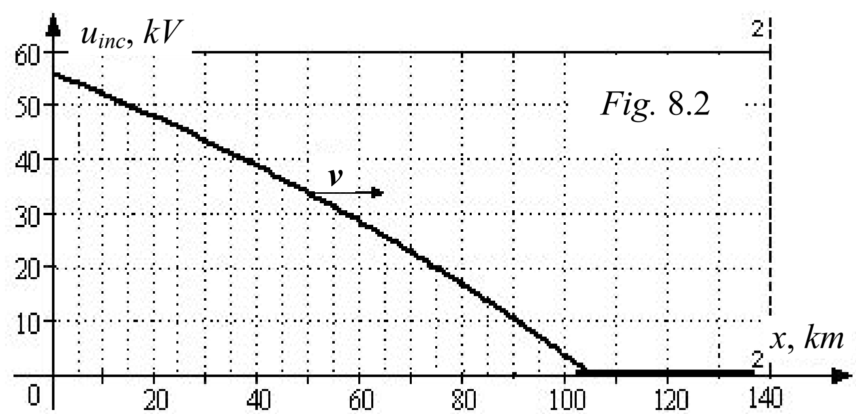


Fig. 8.2

2. At the time moment $t_f = t_2 = 1.5l/v = 0.75\ ms$, in the line there are both the incident and reflected waves. As the line end is open the wave is completely reflected without the sign change: $n_2 = 1$. Thus, the expressions to construct the plots are:

$$u_{inc}(t_f; x) = 120 - 120\,e^{-1.667[0.75 - x/280]}\ kV,\qquad 0 \le x \le 210\ km,$$

$$i_{inc}(t_f; x) = 480 - 480\,e^{-1.667[0.75 - x/280]}\ A;$$

$$u_{refl}(t_f; y) = 120 - 120\,e^{-1.667[0.25 - y/280]}\ kV,\qquad 0 \le y \le 70\ km,$$

$$i_{refl}(t_f; y) = 480 - 480\,e^{-1.667[0.25 - y/280]}\ A.$$

Notice: the incident wave lifetime is $t' = t_2 = 0.75\ ms$, the reflected wave lifetime is $t'' = t_2 - t_{tr} = 0.75 - 0.5 = 0.25\ ms$.

The calculated values of waves for separate line points are shown in the table 8.1

Table 8.1

x, km	u_{inc}, kV	i_{inc}, A	y, km	u_{refl}, kV	i_{refl}, A
210	0	0	35	22.57	90.3
105	55.78	223.1	17.5	32.21	128.9
70	67.86	271.4	0	40.9	163.6
0	85.63	342.5	–	–	–

The plots of the incident, reflected waves and summary values of the voltage u and current i are presented separately (fig. 8.3). Moreover: $u = u_{inc} + u_{refl}$; $i = i_{inc} - i_{refl}$.

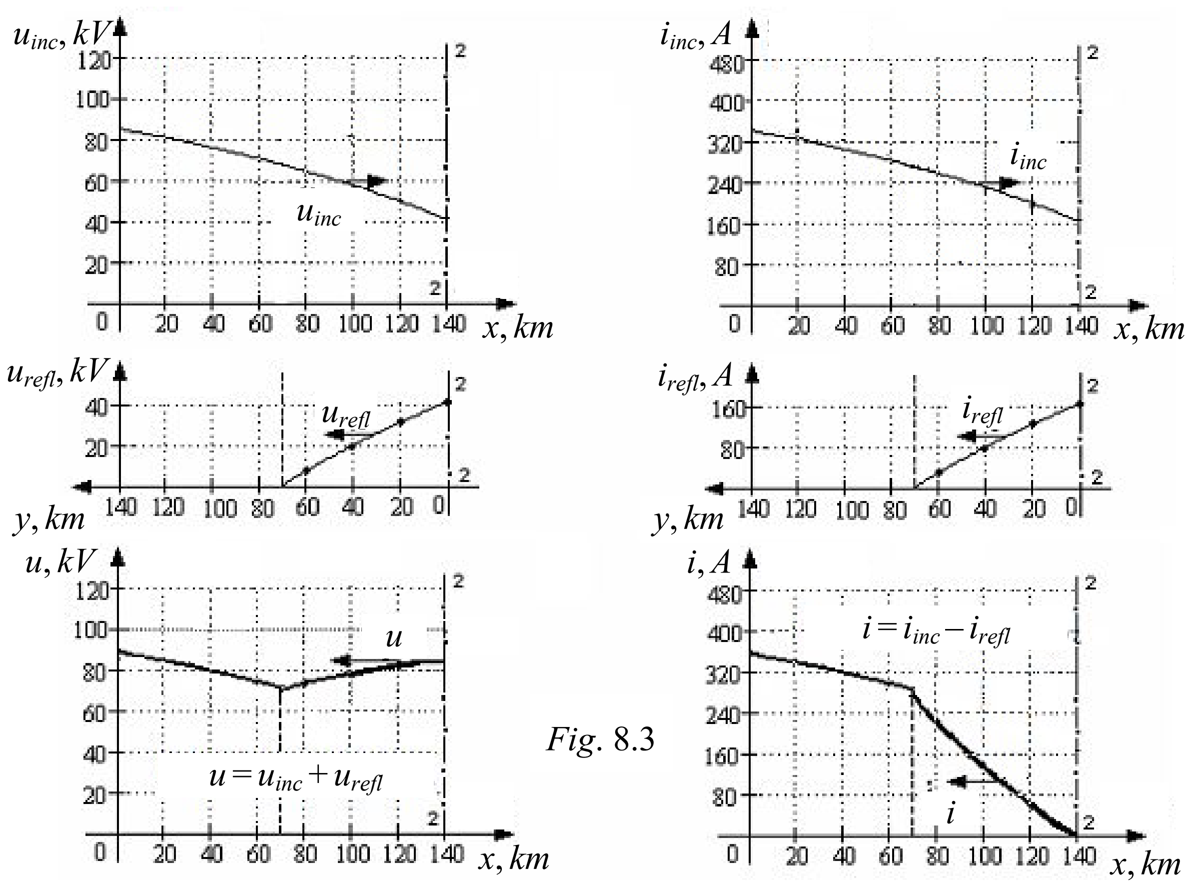


Fig. 8.3

8-27 (8.31). A zero-loss line initially working under no-load condition is suddenly loaded with rC-circuit (fig. 8.4). Determine the parameters of the reverse wave arising and plot the wave distribution along the line as the time interval $t_f = 150\ \mu s$ passes after the load commutation. Numerical data: $E_0 = 100\ V$, $Z_C = 250\ Ohm$, $l = 25\ km$, $v = 100{\cdot}10^3\ km/s$, $r_l = 150\ Ohm$, $C_l = 0.125\ \mu F$.

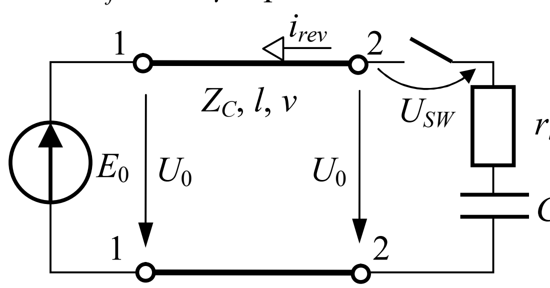


Fig. 8.4

Solution. Considering the line condition before the commutation, we determine the voltage across the switch:

$i(t_-) = 0$, $U_{SW} = u(t_-) = U_0 = E_0 = 100\ V$.

We draw out the equivalent scheme for the section 2-2 for the moment when the reverse wave arises (fig. 8.5): emf $E_{equ} = U_{SW}$ is directed with the polarity opposite to the polarity of the voltage across the switch, the line being presented by its wave impedance.

Using the equivalent scheme, we calculate the load current.

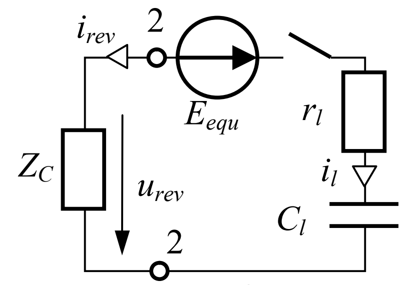


Fig. 8.5

$u_C(0_+) = u_C(0_-) = 0,\quad p = -1/[(r_l + Z_C)\cdot C_l] = -20\cdot 10^3$ 1/*s*,

$$i_l(t) = i_s + i_t = 0 + Ae^{pt} = \frac{E_{equ}}{r_l + Z_C} e^{-20000t} = 0.25e^{-20000t}\ A.$$

Current and voltage of the reverse wave are as follows:

$$i_{rev}(t, y = 0) = -i_l = -0.25e^{-20000t}\ A,$$

$$u_{rev}(t, y = 0) = Z_C\cdot i_{rev} = -62.5e^{-20000t}\ V.$$

In order to perform the voltage and current epures in the function of a coordinate, we pass to the argument $[t_f - y/v]$.

$$i_{rev}(y;\ t_f) = -0.25e^{-20\cdot[0.15 - y/100]}\ A,\qquad y \le v\cdot t_f = 15\ km;$$

$$u_{rev}(y;\ t_f) = -62.5e^{-20\cdot[0.15 - y/100]}\ V.$$

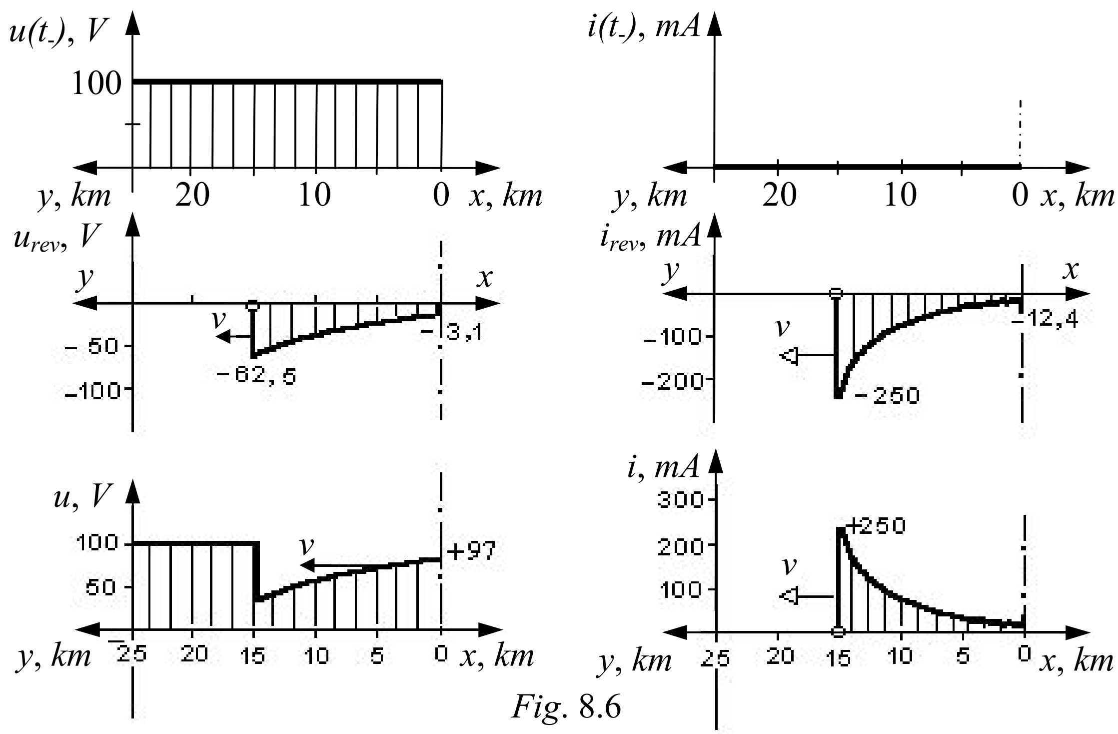


Fig. 8.6

Epures of the initial, reversed waves as well as the summary graphs ($u = u(t_-) + u_{rev}$ and $i = i(t_-) - i_{rev}$) are presented separately in fig. 8.6.

8-28 (8.32). An overhead line ($l = 70$ *km*, $Z_C = 400$ *Ohm*) is supllied from the generator with voltage $U_0 = 100$ *kV* ($r_0 = 0$), and the line has been working under no-load condition for a long time .

Plot the voltage and current distribution along the line for the time moment when 0.2 *ms* passes after a non-ramified active-inductive load $r = 200$ *Ohm*, $L = 100$ *mH* is connected to the line end.

Solution. Let's draw the initial scheme and work out a scheme to calculate the parameters of the arising reverse wave (fig. 8.7,a and b).

The voltage and current along the line before the commutation are:

$$u(t_-) = U_0 = 100\ kV, \quad i(t_-) = 0.$$

The voltage across the switch at the commutation moment is: $U_{SW} = U_0 = 100\ kV$.

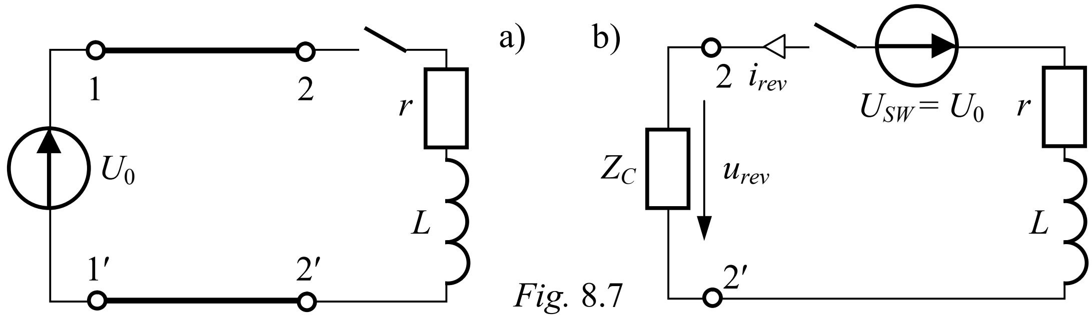


Fig. 8.7

The arising reverse wave is calculated using the scheme fig. 8.7,b at zero independent initial condition ($i_{rev}(0_+) = 0$):

$$i_{rev}(t) = \frac{-U_0}{r+Z_C}(1 - e^{-\frac{r+Z_C}{L}t}) = \frac{-100\cdot 10^3}{200+400}(1 - e^{-\frac{600}{0.1}t}) = -167 + 167\cdot e^{-6000t}\ A,$$

$$u_{rev}(t) = Z_C\cdot i_{rev} = 400\cdot(-167 + 167\cdot e^{-6000t})\cdot 10^{-3} = -66.7 + 66.7\cdot e^{-6000t}\ kV.$$

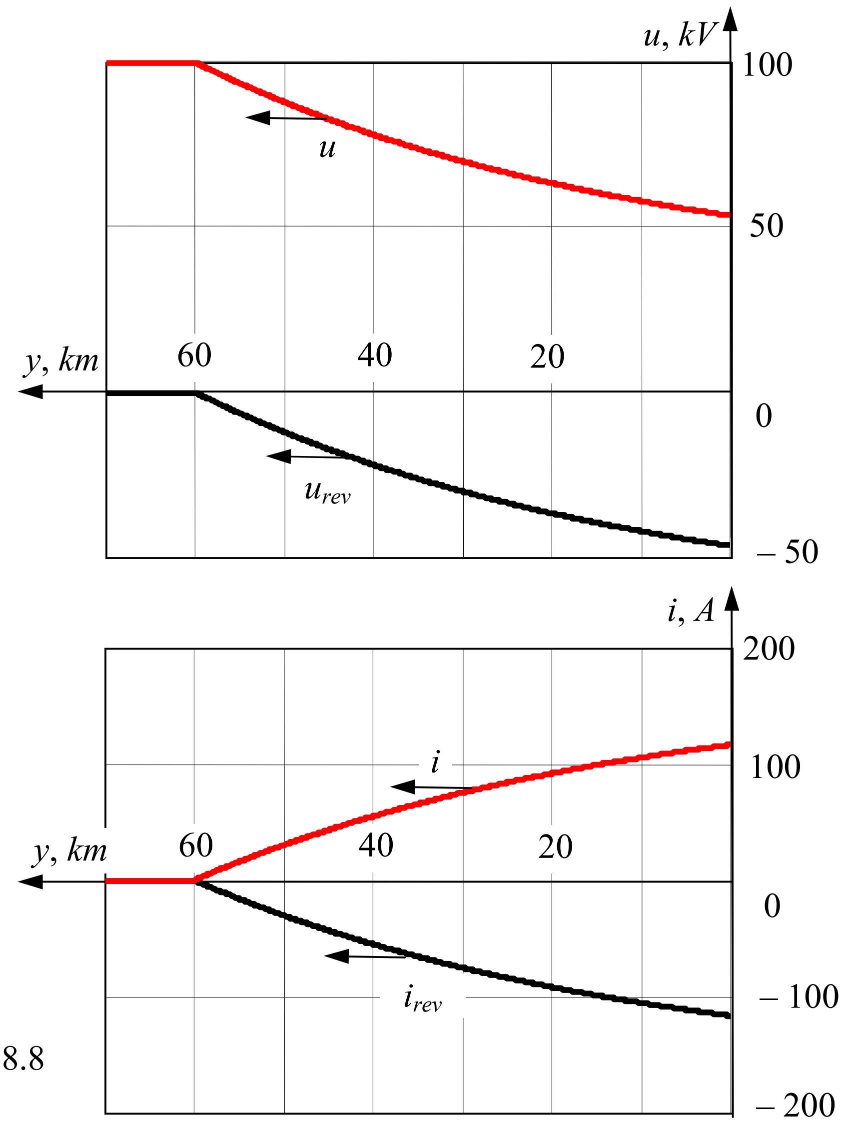


Fig. 8.8

Formulae $u_{rev}(t)$ and $i_{rev}(t)$ are relevant for coordinate $y = 0$. In order to pass from the time functions to the coordinate functions at $t_f = 0.2\ ms$, we perform a substitution $t \to (t_f - y/v)$:

$$i_{rev}(t_f, y) = -167 + 167\cdot exp[-6000(2\cdot 10^{-4} - \frac{y}{3\cdot 10^5})],$$

$$u_{rev}(t_f, y) = -66.7 + 66.7\cdot exp[-6000(2\cdot 10^{-4} - \frac{y}{3\cdot 10^5})].$$

During the given fixed time period t_f, the wave passes the distance: $y_f = v\cdot t_f = 3\cdot 10^5\cdot 2\cdot 10^{-4} = 60\ km$. That's why the formulae $u_{rev}(t_a, y)$ and $i_{rev}(t_f, y)$ are true for corrdinates $y \le y_f = 60\ km$. At $y > y_f$ u_{rev} and i_{rev} are equal to zero.

Total voltage and current values in the line are found by superposing the condition before the commutation and the reverse wave: $u = u(t_-) + u_{rev}$ $i = i(t_-) - i_{rev}$.

Eventually,

$$u(y) = \begin{cases} 33.3 + 66.7\, exp(-6000(2\cdot 10^{-4} - \frac{y}{3\cdot 10^5}))\ kV & if\ 0 \le y \le 60\, km, \\ 100\ kV \quad if\ 60\, km \le y \le 70\, km. \end{cases}$$

$$i(y) = \begin{cases} 167 - 167\, exp(-6000(2\cdot 10^{-4} - \frac{y}{3\cdot 10^5}))\ A & if\ 0 \le y \le 60\, km, \\ 0 \quad if\ 60\, km \le y \le 70\, km. \end{cases}$$

The voltage and current epures for the time moment $t_f = 2\ mc$ are presented in fig. 8.8.

8-29 (8.33). Inductive load $L = 0.015\ H$ is connected to the centre of the working line (fig. 8.9,a), it possessing the parameters $Z_C = 300\ Ohm$, $v = 240\cdot 10^3\ km/s$, $l = 120\ km$. The load resistance and the source emf are: $r = 600\ Ohm$, $E_0 = 1.2\ kV$. Determine the parameters of the arising reverse and direct waves, plot the voltage and current along the line after time $t_f = 200\ \mu s$ has elapsed after the commutation.

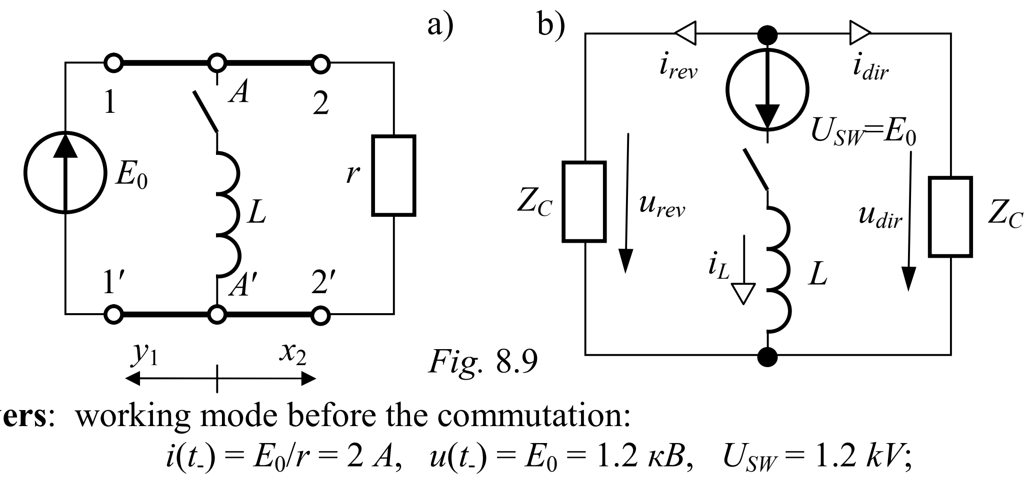


Fig. 8.9

Answers: working mode before the commutation:

$$i(t_-) = E_0/r = 2\ A,\quad u(t_-) = E_0 = 1.2\ кB,\quad U_{SW} = 1.2\ kV;$$

equivalent scheme to compute the direct and reverse waves is in fig. 8.9,b:

$$i_L(t) = 8 - 8e^{-10000t}\ A,\quad i_{dir}(t, x_2{=}0) = i_{rev}(t, y_1{=}0) = -0.5 i_L(t) = -4 + 4e^{-10000t}\ A;$$

$t_f = 0.2\ ms$, $i_{dir}(t_f, x_2) = -4 + 4exp[-10\cdot(0.2 - x_2/240]\ A$, $0 \le x_2 \le 48\ km$;

$u_{dir}(t_f, x_2) = -1.2 + 1.2exp[-10\cdot(0.2 - x_2/240]\ kV$, $0 \le x_2 \le 48\ km$;

$i_{rev}(t_f, y_1) = -4 + 4exp[-10\cdot(0.2 - y_1/240]\ A$, $0 \le y_1 \le 48\ km$;

$u_{rev}(t_f, y_1) = -1.2 + 1.2exp[-10\cdot(0.2 - y_1/240]\ kV$, $0 \le y_1 \le 48\ km$;

total voltage and current values in each of the line halves are determined in accordance with the following formulae: $i_1 = i(t_-) - i_{rev}$, $u_1 = u(t_-) + u_{rev}$,

$i_2 = i(t_-) + i_{dir}$, $u_2 = u(t_-) + u_{dir}$;

the voltage and current epures for the time moment t_f are presented in fig. 8.10.

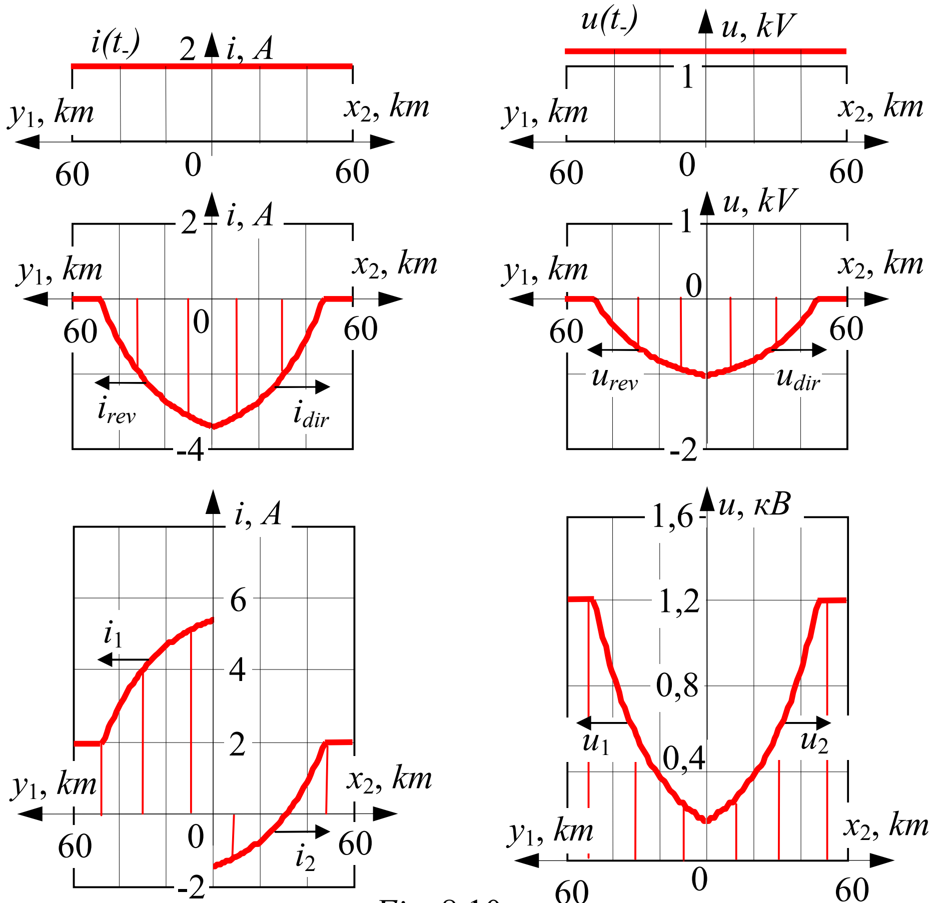


Fig. 8.10

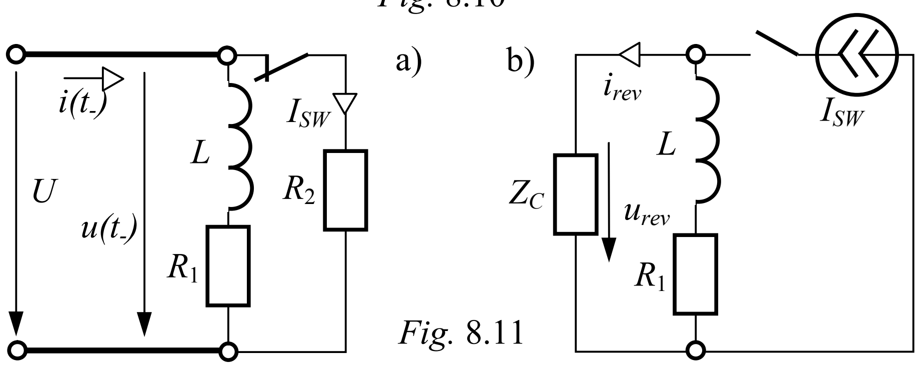


Fig. 8.11

8-30 (8.34). An antenna cable with parameters $l=20\,m$, $L_0=0.5\,\mu H/m$, $C_0=90\,pF/m$ is loaded in accordance with fig. 8.11,a: $R_1 = 150\,Ohm$, $R_2=50\,Ohm$, $L=0.01\,mH$, the input voltage being $U=1\,V$. Commutation (disconnection of the branch with R_2) causes the transient process in the cable. Plot the voltage u and current i distribution along the line for the time moment t_f, when the wave which arises reaches ¾ of the cable length.

Solution. The preliminary calculations are:

- wave phase velocity is $v = \dfrac{1}{\sqrt{L_0 C_0}} = \dfrac{1}{\sqrt{5\cdot 10^{-7}\cdot 9\cdot 10^{-11}}} = 1.49{\cdot}10^8$ *m/s*;

- characteristic impedance is $Z_C = \sqrt{\dfrac{L_0}{C_0}} = \sqrt{\dfrac{5\cdot 10^{-7}}{9\cdot 10^{-11}}} = 74.54$ *Ohm*;

- wave travel duration is $t_{tr} = \dfrac{l}{v} = \dfrac{20}{1.49\cdot 10^8} = 0.134{\cdot}10^{-6}\ s = 0.134\ \mu s$;

- specified fixed time moment is $t_f = ¾\, t_{tr} = 0.1\ \mu s$.

We analyze the circuit condition before the commutation:

$$u(t_-) = U = 1\ V, \quad R_l = \frac{R_1 R_2}{R_1 + R_2} = \frac{150\cdot 50}{150+50} = 37.5\ Ohm,$$

$$i(t_-) = u(t_-)/R_l = 1/37.5 = 0.0267\ A, \quad I_{SW} = u(t_-)/R_2 = 1/50 = 0.02\ A.$$

The voltage and current of the arising reverse wave are calculated using the equivalent scheme fig. 8.11,b at zero independent initial condition $i_L(0_+) = 0$. Note, the independent initial condition is considered to be zero despite the fact that there is a current flowing through the inductance before the commutation.

$$i_{rev}(t) = i_s + Ae^{pt}; \quad i_s = I_{SW}\cdot\frac{R_1}{R_1 + Z_C} = 0.02\cdot\frac{150}{150+74.54} = 0.01336\ A,$$

$$p = \frac{-R_1 - Z_C}{L} = \frac{-150-74.54}{10^{-5}} = -2.245{\cdot}10^7\ s^{-1}.$$

The circuit time constant and the transient process duration are:

$$\tau = |p|^{-1} = 4.45{\cdot}10^{-8}\ s, \qquad T_{tp} = 4\tau = 1.78{\cdot}10^{-7}\ s \approx 0.2\ \mu s.$$

The current initial value $i_{rev}(0)$ with account $i_L(0_+) = 0$ is as follows:

$$i_{rev}(0_+) = I_{SW} = 0.02\ A.$$

Then the integration constant is $A = i_{rev}(0_+) - i_s = 0.02 - 0{,}01336 = 6{,}639{\cdot}10^{-3}$.

Current and voltage of the reverse wave in function of time at zero corrdinate are:

$$i_{rev}(t) = 13.36 + 6.639\,e^{-2.245\cdot 10^7 t}\ A, \quad u_{rev}(t) = Z_C\cdot i_{rev} = 0.996 + 0.495\,e^{-2.245\cdot 10^7 t}\ V.$$

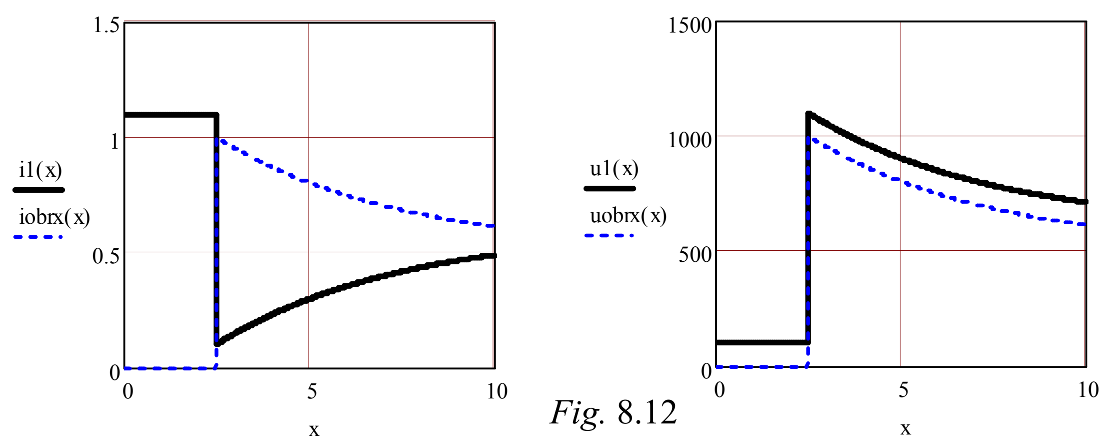


Fig. 8.12

Current and voltage of the reverse wave against time at the fixed time moment t_f:

$$i_{rev}(t_f, y) = 13.36 + 6.639 exp[-2.245{\cdot}10^7{\cdot}(10^{-7} - y/v)] =$$
$$= 13.36 + 6.639 exp[-2.245{\cdot}(1 - y/14.9)]\ A,$$

$$u_{rev}(t_f, y) = 0.996 + 0.495 exp[-2.245{\cdot}(1 - y/14.9)]\ V.$$

The latter formulae are true but for $y \le v \cdot t_f = 15\ m$. At $y \ge 15\ m$ there is no reverse wave.

We find the total current and voltage in the line in accordance with the formulae: $u = u(t_-) + u_{rev}$, $i = i(t_-) - i_{rev}$. The current and voltage epures are presented in fig. 8.12.

8-31 (8.36). In circuit fig. 8.13,a with parameters $E = 10\ kV$, $L = 0.09\ H$, $C = 0.4\ \mu F$, $r_0 = 100\ Ohm$, $R = 400\ Ohm$, $Z_C = 500\ Ohm$, $l = 10\ km$, $v = 100 \cdot 10^3\ km/s$, a commutation takes place. Plot the voltage $u(t_f, x)$ and current $i(t_f, x)$ distribution along the line for the time moment t_f, when the incident wave reaches the line centre.

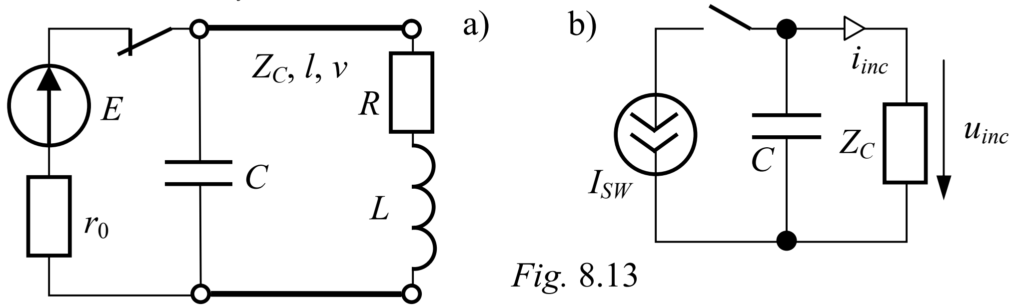


Fig. 8.13

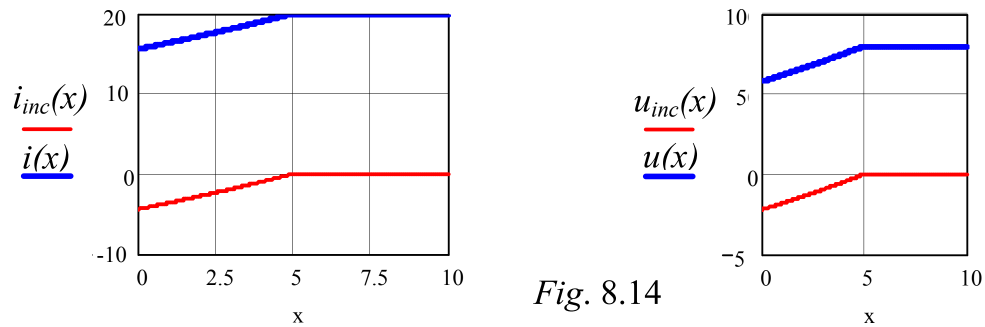


Fig. 8.14

Answers. $I_{SW} = i(t_-) = 20\ A$, $u(t_-) = 8\ kV$; scheme to calculate the incident wave is in fig. 8.13,b: $u_{inc}(t) = -10 + 10e^{-5000t}\ kV$, $i_{inc}(t) = -20 + 20e^{-5000t}\ A$;

$t_f = 0.05\ ms$,

$u_{inc}(t_f, x) = -10 + 10exp[-5\cdot(0.05 - x/100)]\ kV$, $\quad 0 \le x \le 5\ km$,

$i_{inc}(t_f, x) = -20 + 20exp[-5\cdot(0.05 - x/100)]\ A$;

$u(t_f, x) = u(t_-) + u_{inc}(t_f, x)$, $\quad i(t_f, x) = i(t_-) + i_{inc}(t_f, x)$;

the current and voltage epures are in fig. 8.14.

8.2.2. Computation of reflected waves

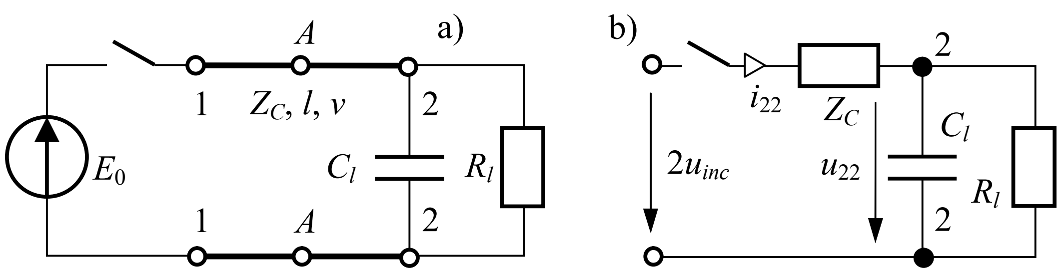


Fig. 8.15

8-32 (8.37). A loaded zero-loss line with parameters $v = 280 \cdot 10^3\ km/s$, $Z_C = 250\ Ohm$, $l = 140\ km$ is connected to an ideal source of the direct voltage $E_0 = 120\ kV$ (fig. 8.15,a). The load parameters are: $R_l = 750\ Ohm$, $C_l = 1.066\ \mu F$. It is required:

- to plot the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for the time moment when $t_f = 0.75$ *ms* passes after the line turning on;

- to plot the diagram of voltage $u_A(t)$ variation against time in section *AA* in the line centre for the time interval equal to 2.5 wave travel: $0 \le t \le 2.5t_{tr}$.

Solution. Part I. The wave travel time is $t_{tr} = l/v = 0.5$ *ms*. Accordingly, by the moment under consideration $t_f = 0.75$ *ms*, in the line there will be both the incident and reflected waves, furthermore, the reflected wave lifetime is $t'_f = 0.25$ *ms*. Let's calculate these waves.

1. The incident wave parameters: $u_{inc}(t;\, x{=}0) = E_0 = 120\ kV$,
$i_{inc}(t;\, x{=}0) = E_0/Z_C = 120\cdot10^3/250 = 480\ A$.

2. It takes the wave $t_{tr} = 0.5$ *ms* to reach the section "2-2" where it faces the heterogeneity. The wave partially passes into the load, and partially, it's reflected. The parameters of the reflected wave are calculated through the current or voltage i_{22}, u_{22} in cross-section "2-2" of the line. In the equivalent scheme for the cross-section "2-2" (fig. 8.15,b), it is easier to calculate the capacitor voltage, which presents the required voltage u_{22}:

$$u_{22}(t) = u_C(t) = u_s + Ae^{pt} = \frac{2u_{inc}}{Z_C + R_l}\cdot R_l + [u_C(0_+) - u_s(0_+)]\cdot e^{pt};$$

$$u_C(0_+) = u_C(0_-) = 0,\ \ u_s = \frac{2\cdot120\cdot10^3}{250+750}\cdot750 = 180\ kV,\quad A = 0 - 180 = -180\ kV,$$

$$Z_{inp}(p) = \frac{1}{pC} + \frac{Z_C\cdot R_l}{Z_C + R_l} = \frac{1}{1.066\cdot10^{-6}\cdot p} + \frac{250\cdot750}{250+750} = 0,\qquad p = -5000\ 1/s,$$

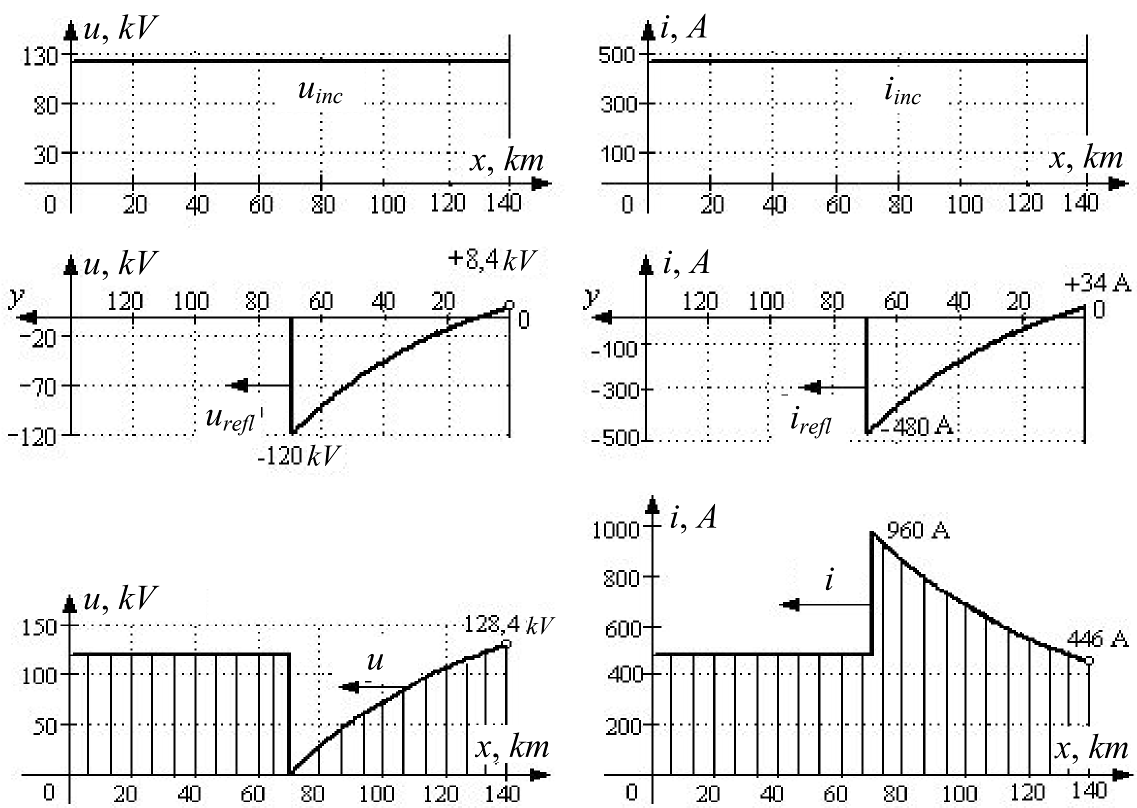


Fig. 8.16

Thus, $u_{22}(t) = 180 - 180e^{-5000t}$ kV.

From the correlation $u_{22}(t) = u_{inc} + u_{refl}$ and then under Ohm's law, we find:

$$u_{refl}(t) = u_{22}(t) - u_{inc}(t) = 180 - 180e^{-5000t} - 120 = 60 - 180e^{-5000t}\ kV,$$

$$i_{refl}(t) = \frac{u_{refl}}{Z_C} = \frac{(60-180e^{-5000t})\cdot 10^3}{250} = 240 - 720\,e^{-5000t}\ A.$$

3. In order to plot the voltage and current distribution along the line, we pass to the argument $[t_f - y/v]$:

$u_{inc}(t_f; x) = 120\ kV$, $\quad i_{inc}(t_f; x) = 480\ A$, $\quad 0 \le x \le 140\ km$;

$u_{refl}(t'_f; y) = 60 - 180exp[-5\cdot(0.25 - y/280)]\ kV$, $\quad t'_f = 0.25\ ms$,

$i_{refl}(t'_f; y) = 240 - 720\ exp[-5\cdot(0.25 - y/280)]\ A$, $\quad 0 \le y \le 70\ km$.

Under these expressions, the voltage and current distribution along the line is plotted in fig. 8.16. The total values of the voltage and current are

$$u = u_{inc} + u_{refl} \quad \text{and} \quad i = i_{inc} - i_{refl}.$$

Part II. In order to plot the voltage $u_A(t)$ variation in cross-section AA in the line centre for the time interval equal to 2.5 wave travels $0 \le t \le 2.5t_{tr}$, we use the results obtained in the first part of the solution.

Until the incident wave reaches the cross-section AA, that is during the time interval $t \le t_1 = 0.5t_{tr} = 0.25\ ms$, in the cross-section AA we have: $u_A(t) = 0$.

From the moment $t_1 = 0.25\ ms$ up to moment $t_2 = 0.75\ mc$, when the incident wave reaches the line end and then the reflected wave reaches the point A, the voltage $u_A(t)$ is equal to the voltage of the incident wave only $u_A(t) = u_{inc} = 120\ kV$.

At the moment t_2 the reflected wave reaches the point A, the incident and reflected waves are superimposed on one another:

$$u_A(t) = u_{inc} + u_{refl} = 120 + [60 - 180exp(-5000(t - 0.75\cdot 10^{-3})]\ kV.$$

This law of the voltage $u_A(t)$ variation lasts for the time interval t_{tr}, when the reflected wave reaches the beginning of the line (0.25 ms) and an incident wave which has arisen reaches the point A again (0.25 ms): $u_{inc2} = n_1\cdot u_{refl}$.

The process of the voltage variation in time in the point A of the line is presented in fig. 8.17:

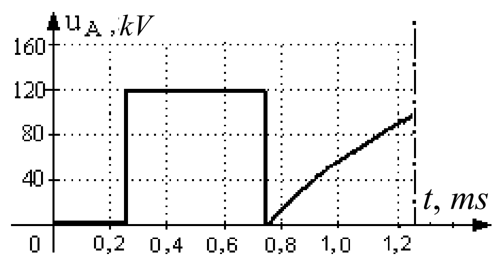


Fig. 8.17

$t = 0 - 0.25\ ms$ $\quad u_A(t) = 0$;

$t = 0.25 - 0.75\ ms$ $\quad u_A(t) = 120\ kV$;

$t = 0.75 - 1.25\ ms$ $\quad u_A(t) = 180 - 180exp(-5000(t - 0.75\cdot 10^{-3})\ kV$.

8-33 (8.38). The loaded zero-loss line ($R_l = 750\ Ohm$) with parameters $Z_C = 250\ Ohm$, $l = 140\ km$, $v = 280\cdot 10^3\ km/s$ is connected to a D-C voltage source $E_0 = 120\ kV$ with inner inductance $L_0 = 0.15\ H$ (fig. 8.18). Plot the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for the time moment $t_f = 1.5l/v$.

Answers: $i_{inc}(t_f; x) = 480 - 480e^{-1.667[0.75 - x/280]}\ A$; $\quad u_{inc}(t_f; x) = Z_C\cdot i_{inc}$;

$i_{refl}(t'_f; y) = -0.5i_{inc} = -240 + 240e^{-1.667\cdot[0.25 - y/280]}\ A$.

The diagrams are presented in fig. 8.19.

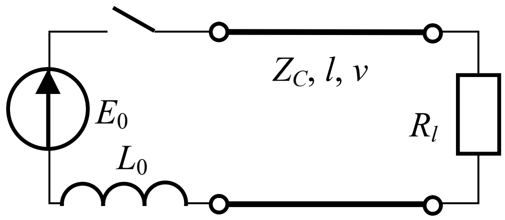


Fig. 8.18

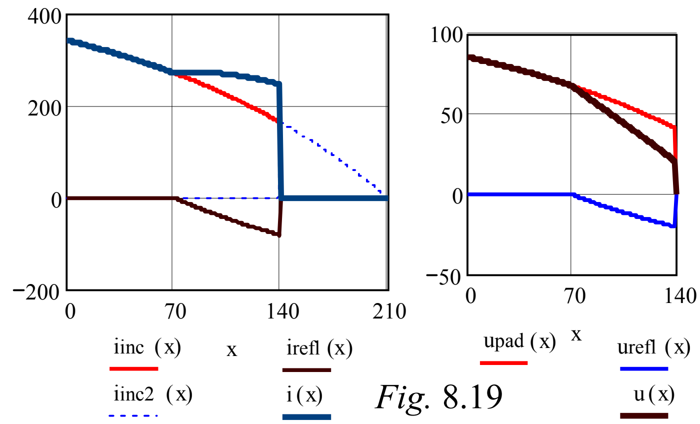


Fig. 8.19

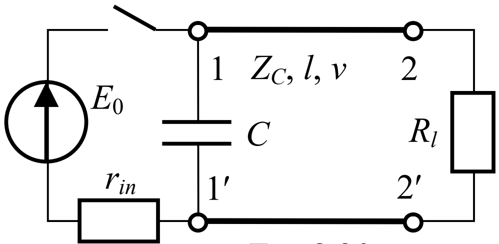


Fig. 8.20

8-34 (8.39). The loaded cable line ($R_l = 225$ *Ohm*) with parameters $Z_C = 75$ *Ohm*, $l = 50$ *km*, $v = 125\cdot10^3$ *km/s* is connected to a D-C voltage source ($E_0 = 1.2$ *kV*, $r_{in} = 5$ *Ohm*). At the input the line is protected from the disturbance with the aid of the capacitor $C = 53.33$ *μF* (fig. 8.20). Plot the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for the time moment $t_f = 1.5l/v$.

Answers: $i_{inc}(t_f; x) = 15 - 15e^{-4\cdot[0.6 - x/125]}$ *A*; $u_{inc}(t_f; x) = Z_C\cdot i_{inc}$;

$i_{refl}(t'_f; y) = -0.5 i_{inc} = -7.5 + 7.5e^{-4\cdot[0.2 - y/125]}$ *A*.

The diagrams are in fig. 8.21.

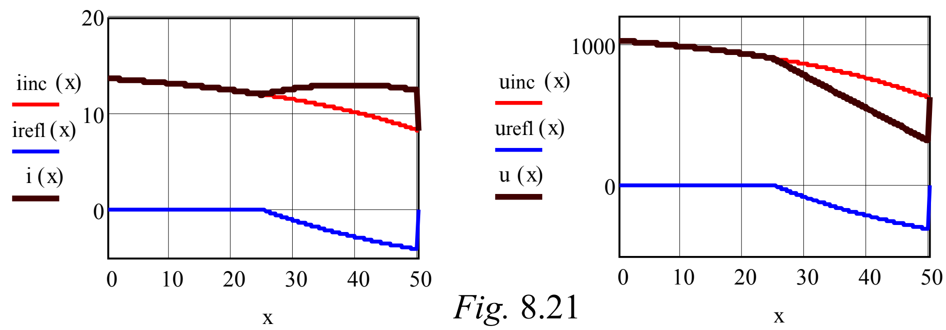


Fig. 8.21

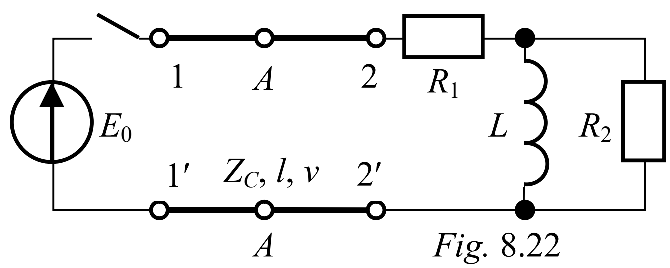


Fig. 8.22

8-35 (8.41). The loaded zero-loss line with parameters $Z_C = 250$ *Ohm*, $l = 140$ *km*, $v = 280\cdot10^3$ *km/s* is connected to an ideal D-C voltage source $E_0 = 120$ *kV* (fig. 8.22). The load parameters: $R_1 = 500$ *Ohm*, $L = 37.5$ *mH*, $R_2 = 250$ *Ohm*. It is required:

- to plot the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for the time moment when $t_f = 0.9$ *ms* passes after the line turning on;
- to plot the voltage $u_A(t)$ variation in section *AA* in the line centre for the time interval equal to two wave travels: $0 \le t \le 2t_{tr}$.

Answers: $i_{inc}(t) = 480$ *A*, $u_{inc}(t) = 120$ *kV*;

$u_{22}(t) = 160 + 20e^{-5000t}$ *kV*, $i_{22}(t) = 320 - 80e^{-5000t}$ *A*, $t'_f = 0.4$ *ms*,

$u_{refl}(t'_f; y) = 40 + 20e^{-5\cdot[0.4 - y/280]}$ *kV*, $i_{refl}(t'_f; y) = 160 + 80e^{-5\cdot[0.4 - y/280]}$ *A*.

The diagrams are in fig. 8.23.

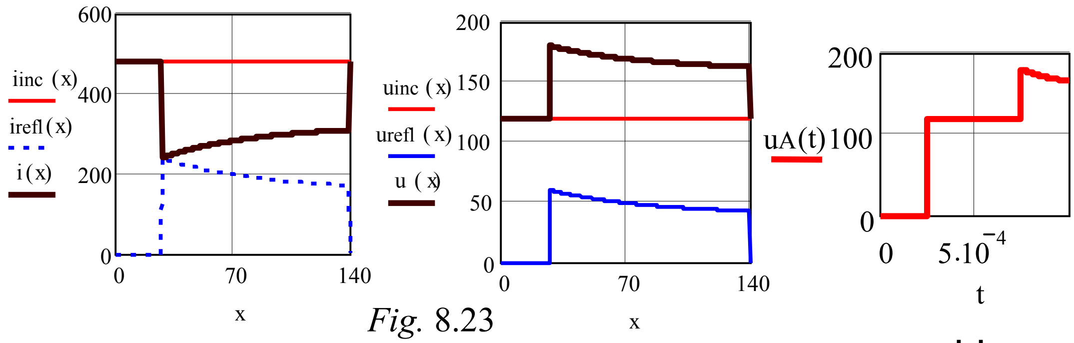


Fig. 8.23

8-36 (8.42). The loaded zero-loss line with parameters $v = 280\cdot10^3$ *km/s*, $Z_C = 300$ *Ohm*, $l = 140$ *km* is connected to an ideal D-C voltage source $E_0 = 120$ *kV* (fig. 8.24). The load parameters: $R_l = 950$ *Ohm*, $C_l = 0.1$ *μF*. Plot:

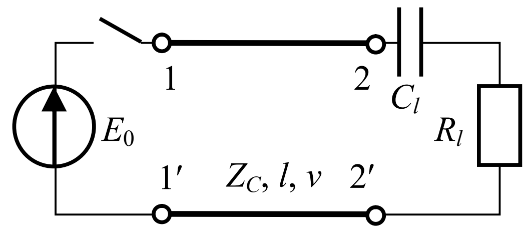


Fig. 8.24

- the current i_{22} diagram at the line's end in function of time;
- diagrams of the voltage $u(t_f, y)$ and current $i(t_f, y)$ distribution along the line for the time moment when $t_f = 0.86$ *ms* passes after the line turning on.

Answers: $i_{22}(t) = 192e^{-8000t}$ *A*; $u_{refl}(t'_f, y) = 120 - 57.6e^{-8\cdot[0.36 - y/280]}$ *kV*,
$i_{refl}(t'_f, y) = 400 - 192e^{-8\cdot[0.36 - y/280]}$ *A*.

The diagrams are in fig. 8.25.

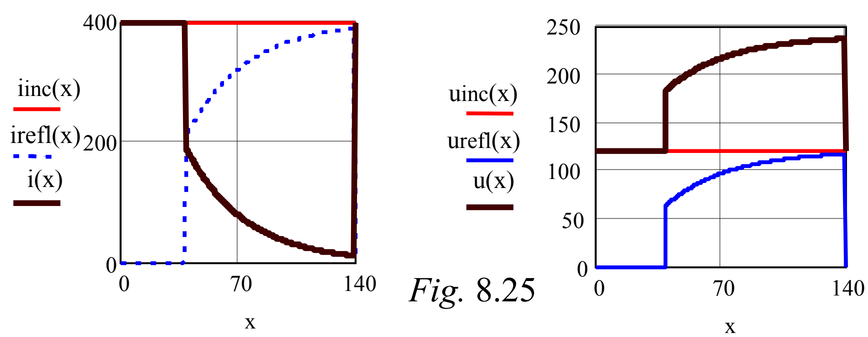


Fig. 8.25

8-37 (8.43). Using the conditions of problem 8.30, plot the voltage u and current i distribution along the cable for the time moment t_f, when the wave reflected from the line beginning passes ½ of the line length.

Solution. Let's use the calculation results of problem 8.30:

$v = 1.49\cdot10^8$ *m/s*; $Z_C = 74.54$ *Ohm*; $t_{tr} = 0.134$ *μs*; $t_f = (1+½)t_{tr} = 0.2$ *μs*.

$u(t_-) = U = 1$ *V*, $i(t_-) = 0.0267$ *A*,

$i_{rev}(t) = 13.36 + 6.639\,e^{-2.245\cdot10^7 t}$ *mA*, $u_{rev}(t) = 0.996 + 0.495\,e^{-2.245\cdot10^7 t}$ *V*.

The current and voltage of the reverse wave in function of the coordinates at the fixed time moment t_f are: $i_{rev}(t_f, y) = 13.36 + 6.639 exp[-2.245\cdot(2 - y/14.9)]$ *mA*,
$u_{rev}(t_f, y) = 0.996 + 0.495 exp[-2.245\cdot(2 - y/14.9)]$ *V*.

The inner resistance of the source is zero, that's why the reflection coefficient from the line beginning is $n_1 = \dfrac{0 - Z_C}{0 + Z_C} = -1$. Then the formulae of the voltage and current of the direct wave for zero coordinate in function of time are as follows (furthermore, time is measured from the moment of reflection):

$i_{dir}(t) = -13.36 - 6.639\,e^{-2.245\cdot10^7 t}$ mA, $u_{dir}(t) = -0.996 - 0.495\,e^{-2.245\cdot10^7 t}$ V.

These formulae but in function of coordinate with account of the direct wave lifetime $t_f - t_{tr} = 0.066\ \mu s$ are:

$i_{dir}(t_f, x) = -13.36 - 6.639exp[-2.245\cdot(0.66 - x/14.9)]\ A$,

$u_{dir}(t_f, y) = -0.996 - 0.495exp[-2.245\cdot(0.66 - x/14.9)]\ V$.

Formelae are true for $x \le v\cdot(t_f - t_{tr}) = 10\ m$. At $x \ge 10\ m$ there is no direct wave.

The total current and voltage in the line are found under the formulae: $u = u(t_-) + u_{rev} + u_{dir}$, $i = i(t_-) - i_{rev} + i_{dir}$. The current and voltage epures are shown in fig. 8.26.

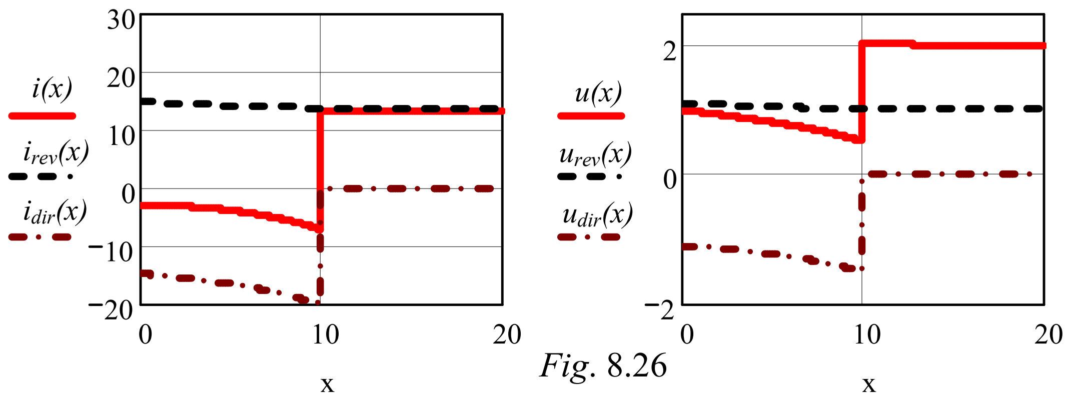


Fig. 8.26

8.2.3. Wave calculation when passing through *R*,*L*,*C*-elements

8-38 (8.45). The incident wave with a rectangular voltage front $u_{inc}(t) = 220\ kV$ travels along an overhead line with parameters $Z_{C1} = 220\ Ohm$, $l_1 = 150\ km$, $v_1 = 300\cdot10^3\ km/s$, it passes through the elements $R = 180\ Ohm$, $L = 30\ mH$ into the cable with parameters $Z_{C2} = 88\ Ohm$, $l_2 = 75\ km$, $v_2 = 150\cdot10^3\ km/s$, the cable end is opened (fig. 8.27).

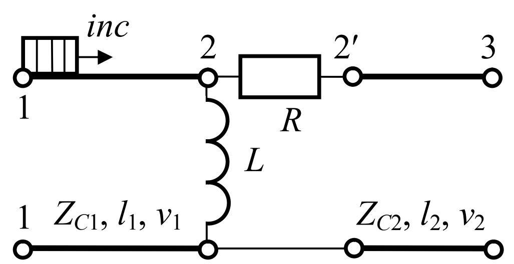


Fig. 8.27

It is necessary to plot current $i_{22}(t)$ and voltage $u_{22}(t)$ at the end of the fisrt line against time, as well as the total voltage and current distribution along both lines for the time moment $t_f = 0.5l_1/v_1$, which is measured from moment when the wave reaches the point of lines connection.

Solution. 1. Determine the current and voltage of the incident wave:

$u_{inc}(t, x_1{=}0) = 220\ kV$; $i_{inc}(t, x_1{=}0) = u_{inc}/Z_{C1} = 220\cdot10^3/220 = 1000\ A$.

2. After time $t_{tr} = l_1/v_1 = 0.5\ ms$ has elapsed, the wave reaches the section “2-2”, where it faces the heterogeneity. The wave is partially absorbed by inductance, partially it’s reflected, and partially in the form of the refracted wave passes into the second line. In order to determine the reflected and refracted waves it is necessary to calculate either current i_{22} or voltage u_{22} in cross-section “2-2”, further they may be used to obtain expressions u_{refl}, i_{refl}, u_{refr}, i_{refr}.

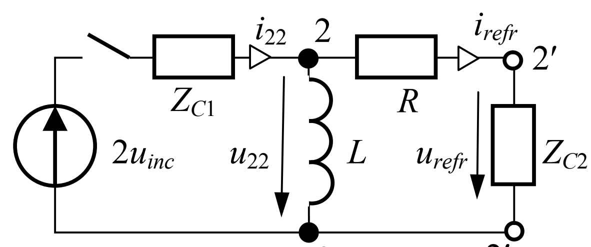


Fig. 8.28

The current $i_{22}(t)$ is computed by the equivalent scheme of the line for section “2-2” (fig. 8.28). We perform the calculation by the classical method:

$i_L(0_+) = i_L(0_-) = 0$, $i_{22}(t) = i_s + Ae^{pt}$;

$i_s(t) = 2u_{inc}/Z_{C1} = 2\cdot 220\cdot 10^3/220 = 2000\ A$,

$$i_{22}(0_+) = \frac{2u_{inc}}{Z_{C1} + R + Z_{C2}} = \frac{2\cdot 220\cdot 10^3}{220+180+88} = 902\ A,$$

$A = i_{22}(0_+) - i_s(0_+) = 902 - 2000 = -1098\ A$;

$$pL + \frac{Z_{C1}\cdot(R+Z_{C2})}{Z_{C1}+R+Z_{C2}} = 0, \qquad p = -4027\ 1/s, \qquad \tau = -1/p = 0.248\ s \approx 0.25\ ms.$$

Thus, $i_{22}(t) = 2000 - 1098e^{-4027t}\ A$.

The voltage $u_{22}(t)$ is found under Kirchhoff's voltage law:

$$u_{22}(t) = 2u_{inc} - Z_{C1}\cdot i_{22}(t) = 440 - 220\cdot(2 - 1.098e^{-4027t}) = 241.6e^{-4027t}\ kV.$$

The latter expressions are used to plot $u_{22}(t)$, $i_{22}(t)$ in fig. 8.29. These expressions are true for but time interval $t \le 2t_{tr} = 2\cdot 0.5 = 1\ ms$.

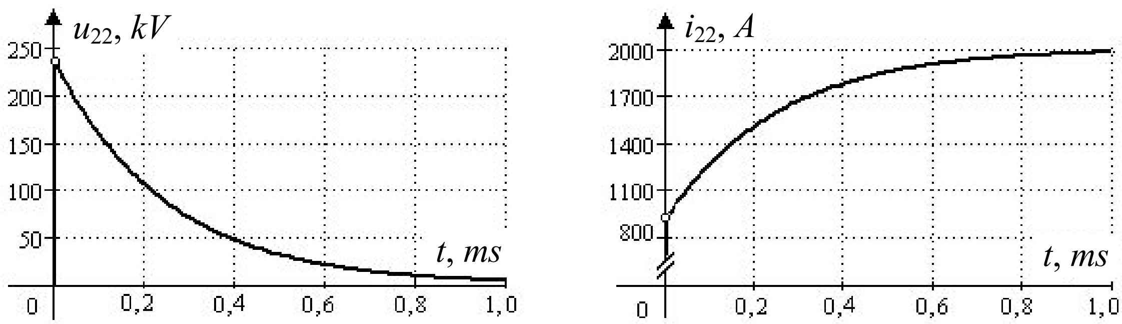


Fig. 8.29

3. Expressions for the reflected and refracted waves in function of time are found through the voltage $u_{22}(t)$.

Reflected wave: $u_{refl}(t, y_1{=}0) = u_{22}(t) - u_{inc} = -220 + 241.6e^{-4027t}\ kV$;

$i_{refl}(t, y_1{=}0) = u_{refl}/Z_{C1} = -1000 + 1098e^{-4027t}\ A$.

Refractred wave: $i_{refr}(t, x_2{=}0) = u_{22}/(R + Z_{C2}) = 901.5e^{-4027t}\ A$;

$u_{refr}(t, x_2{=}0) = Z_{C2}\cdot i_{refr} = 88\cdot 0.9015e^{-4027t} = 79.33e^{-4027t}\ kV$.

4. In order to plot the current and voltage distribution along both lines (fig. 8.30), we rewrite the expressions for the reflected and refracted waves in function of argument $[t_f - y_1/v_1]$ or $[t_f - x_2/v_2]$. In the first line there are the incident and reflected waves:

$u_{refl}[t_f, y_1] = -220 + 241.6\cdot exp[-4.027\cdot(0.25 - y_1/300)]\ kV$,

$i_{refr}[t_f, y_1] = -1000 + 1098\cdot exp[-4.027\cdot(0.25 - y_1/300)]\ A, \qquad y_1 \le v_1\cdot t_f = 75\ km$.

In the second line there is only a refracted wave:

$u_{refr}[t_f, x_2] = 79.33\cdot exp[-4.027\cdot(0.25 - x_2/150)]\ kV$,

$i_{refr}[t_f, x_2] = 901.5\cdot exp[-4.027\cdot(0.25 - x_2/150)]\ A, \qquad x_2 \le v_2\cdot t_f = 37.5\ km$.

Let's tabulate the results of the calculation of the current and voltage values for both lines. See table 8.2.

Table 8.2

y_1, *km*	u_{refl}, *kV*	i_{refl}, *A*	x_2, *km*	u_{refr}, *kV*	i_{refr}, *A*
75	+21.65	+98.4	37.5	79.33	901.5
60	–22.42	–101.9	30.0	64.86	737.1
45	–58.46	–265.7	22.5	53.03	602.7
30	–87.92	–399.6	15.0	43.36	492.8
15	–112.0	–509.1	0	28.99	329.4
0	–131.7	–598.6	---	---	---

Fig. 8.30
See «Album»

8-39 (8.46). The incident wave with a rectangular voltage front $u_{inc} = 220\ kV$ moves along an overhead line with parameters $Z_{C1} = 220\ Ohm$, $l_1 = 150\ km$, $v_1 = 300{\cdot}10^3\ km/s$. It passes through the elements $R = 180\ Ohm$, $L = 30\ mH$ into the cable with parameters $Z_{C2} = 88\ Ohm$, $l_2 = 75\ km$, $v_2 = 150{\cdot}10^3\ km/s$, the cable end is opened (fig. 8.31).

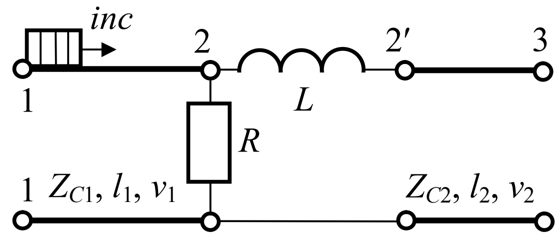


Fig. 8.31

It is required: to plot current $i_{22}(t)$ and voltage $u_{22}(t)$ at the end of the fisrt line against time, as well as the total volltage and current distribution along both lines for the time moment $t_f = 0.25\ ms$, which is measured from the moment when the wave reaches the point of lines connection.

Explanations. In order to differentiate the points of *R*,*L*-section belonging to both lines, they are marked at the scheme as 2-2 and 2′-2′.

Answers: $u_{22}(t) = 93.17 + 104.83e^{-8000t}\ kV$, $i_{22}(t) = 1576. - 476.e^{-8000t}\ A$;

$u_{refl}[t_f; y_1] = - 126.8 + 104.8exp[- 8{\cdot}(0.25 - y_1/300)]\ kV$,

$i_{refl}[t_f; y_1] = - 576.5 + 476.5exp[- 8{\cdot}(0.25 - y_1/300)]\ A$, $\quad y_1 \le v_1{\cdot}t_f = 75\ km$.

In the 2nd line there is but the refracted wave:

$u_{refr}[t_f; x_2] = 93.18 - 93.18exp[- 8{\cdot}(0.25 - x_2/150)]\ kV$,

$i_{refr}[t_f; x_2] = 1059 - 1059exp[- 8{\cdot}(0.25 - x_2/150)]\ A$, $\quad x_2 \le v_2{\cdot}t_f = 37.5\ km$.

The diagrams are in fig. 8.32.

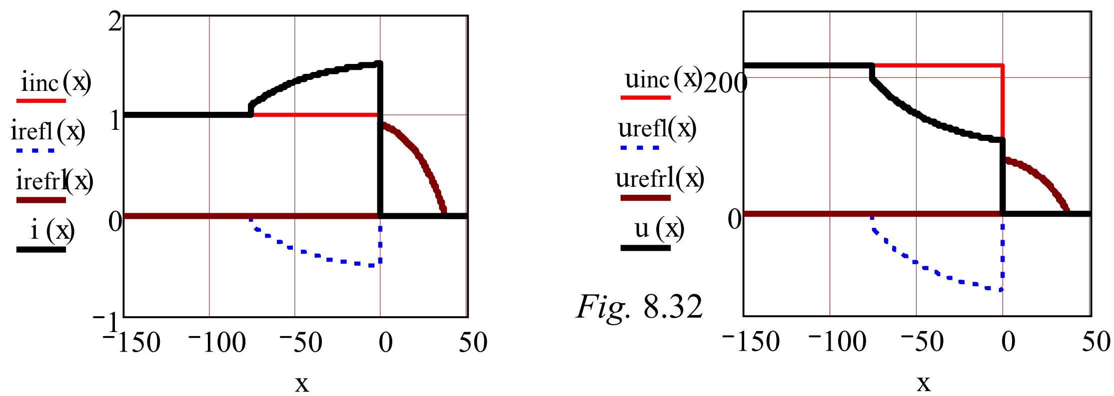


Fig. 8.32

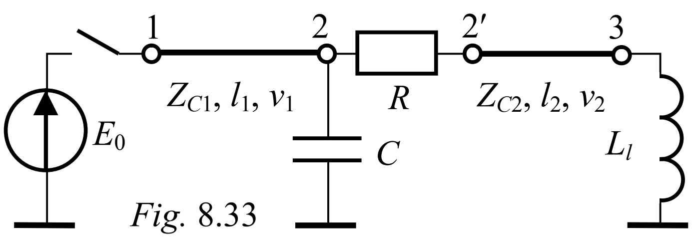


Fig. 8.33

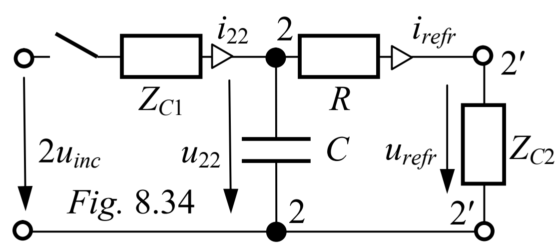


Fig. 8.34

8-40 (8.47). An overhead line with parameters $v_1 = 300{\cdot}10^3\ km/s$, $l_1 = 150\ km$, $Z_{C1} = 400\ Ohm$ is connected with another overhead line $v_2 = 300{\cdot}10^3\ km/s$, $l_2 = 150\ km$, $Z_{C2} = 250\ Ohm$ through the elements $R = 150\ Ohm$, $C = 2\ \mu F$, they are connected to an ideal voltage source $E_0 = 110\ kV$ (fig. 8.33). It is required to plot the voltage u and current i distribution along both lines for the time moment $t_f = 0.4\ ms$, which is measured from the moment when the wave reaches the point of lines connection. $L_l = 20\ mH$.

Solution. 1. Let's write down the incident wave parameters:

$$u_{inc}(t;\, x=0) = E_0 = 110\ kV, \quad i_{inc}(t; x=0) = u_{inc}/Z_{C1} = 110/0.4 = 275\ A.$$

2. After the time $t_{tr} = 0.5\ ms$ has elapsed the wave reaches the section 2-2, here it faces the heterogeneity and it is partially reflected and partially passes as a refracted wave into the second line. Construct the equivalent scheme for section 2-2; with the aid of this scheme, we compute the current i_{22} or the voltage u_{22} (fig. 8.34) by classical method. This time, it is easier to find the voltage $u_{22} = u_C$:

$$u_C(0) = 0, \quad p = -1/\tau;$$

$$\tau = \frac{Z_{C1}(R + Z_{C2})}{Z_{C1} + R + Z_{C2}} \cdot C = \frac{400 \cdot (150 + 250)}{400 + 150 + 250} \cdot 2 \cdot 10^{-6} = 0.4 \cdot 10^{-3}\ s; \quad p = -2500\ s^{-1}.$$

$$u_C(t) = u_{Cs} + A \cdot e^{pt}; \quad u_{Cs} = 2u_{inc} \cdot \frac{R + Z_{C2}}{Z_{C1} + R + Z_{C2}} = 220 \cdot \frac{150 + 250}{400 + 150 + 250} = 110\ kV,$$

$$A = u_C(0) - u_{Cs} = -110;$$

Finally, we have: $u_{22}(t;\, y=0) = u_C(t) = 110 - 110 \cdot e^{-2500t}\ kV,$

$$i_{22}(t;\, y=0) = \frac{2u_{inc} - u_{22}}{Z_{C1}} = \frac{220 - 110 + 110e^{-2500t}}{0.4} = 275 + 275 \cdot e^{-2500t}\ A.$$

The latter expressions are true over $2t_{tr}$, i.e. for the time interval until the waves reach the ends of their lines, then they are reflected and again reach the section 2-2.

3. Through the expressions of current and voltage at the end of the 1st line, we can work out the expressions for the reflected and refracted waves:

$$u_{refl}(t;\, y_1=0) = u_{22} - u_{inc} = -110 \cdot e^{-2500t}\ kV, \quad i_{refl}(t;\, y_1=0) = i_{inc} - i_{22} = -275 \cdot e^{-2500t}\ A.$$

$$i_{refr}(t;\, x_2=0) = \frac{u_{22}}{R + Z_{C2}} = \frac{110 - 110e^{-2500t}}{(150 + 250) \cdot 10^{-3}} = 275 - 275 \cdot e^{-2500t}\ A,$$

$$u_{refr}(t;\, x_2=0) = Z_{C2} \cdot i_{refr} = 68.75 - 68.75 \cdot e^{-2500t}\ kV.$$

4. In order to plot the current and voltage in fuction of coordinate, we rewrite the expressions obtained above as functions of $[t_f - y_1/v_1]$ and $[t_f - x_2/v_2]$:

$u_{refl}(y_1;\, t_f) = -110 \cdot exp[-2.5 \cdot (0.4 - y_1/300)]\ kV, \quad 0 < y_1 < v_1 \cdot t_f = 120\ km,$

$i_{refl}(y_1;\, t_f) = -275 \cdot exp[-2.5 \cdot (0.4 - y_1/300)]\ A;$

$u_{refr}(x_2;\, t_f) = 68.75 - 68.75 \cdot exp[-2.5 \cdot (0.4 - x_2/300)]\ kV, \quad 0 < x_2 < v_2 \cdot t_f = 120\ km,$

$i_{refr}(x_2;\, t_f) = 275 - 275 \cdot exp[-2.5 \cdot (0.4 - x_2/300)]\ A.$

Calculation results are tabulated in the table 8.3. The diagrams are performed in fig. 8.35.

Table 8.3

y_1, x_2, km	u_{refl}, kV	i_{refl}, A	u_{refr}, kV	i_{refr}, A
120	–110	–275	0	0
90	–85.67	–214.2	15.2	60.83
60	–66.72	–166.8	27.05	108.2
30	–51.96	–129.9	36.25	145.1
0	–40.47	–101.2	43.46	173.8

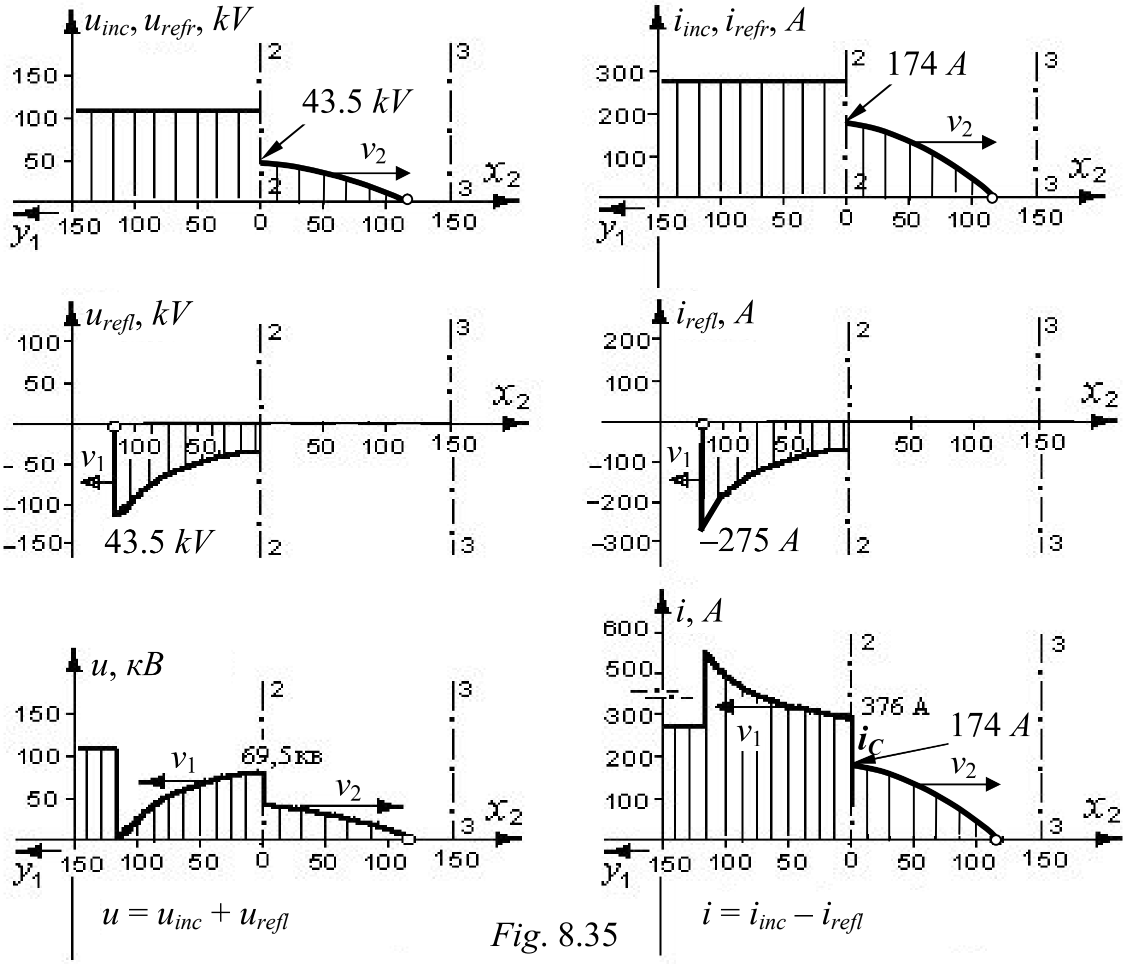


Fig. 8.35

8-41 (8.48). The incident wave $U_0 = 220\ kV$ with a rectangular front moves along the overhead line fig. 8.36,a with parameters $Z_C = 220\ Ohm$, $l = 200\ km$. It passes through the elements $r = 440\ Ohm$, $C = 0.455\ \mu F$ into the next identical overhead line.

It is required: 1) to plot the current and voltage at the end of the 1st line in function of time; 2) to plot the total voltage and current distribution along both lines for the time moment $t_f = 0.5\ ms$, which is measured from the moment when the wave reaches the point of lines connection.

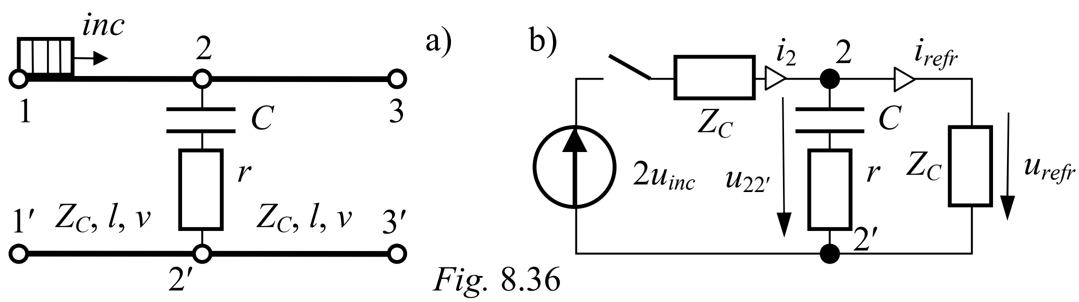


Fig. 8.36

Answers: 1) scheme to determine $i_2(t)$ and $u_{22'}(t)$ is presented in fig. 8.36,b,

$$i_2(t) = 1000 + 200\cdot e^{-4000t}\ A,\quad u_{22'}(t) = 220 - 44\cdot e^{-4000t}\ kV;$$

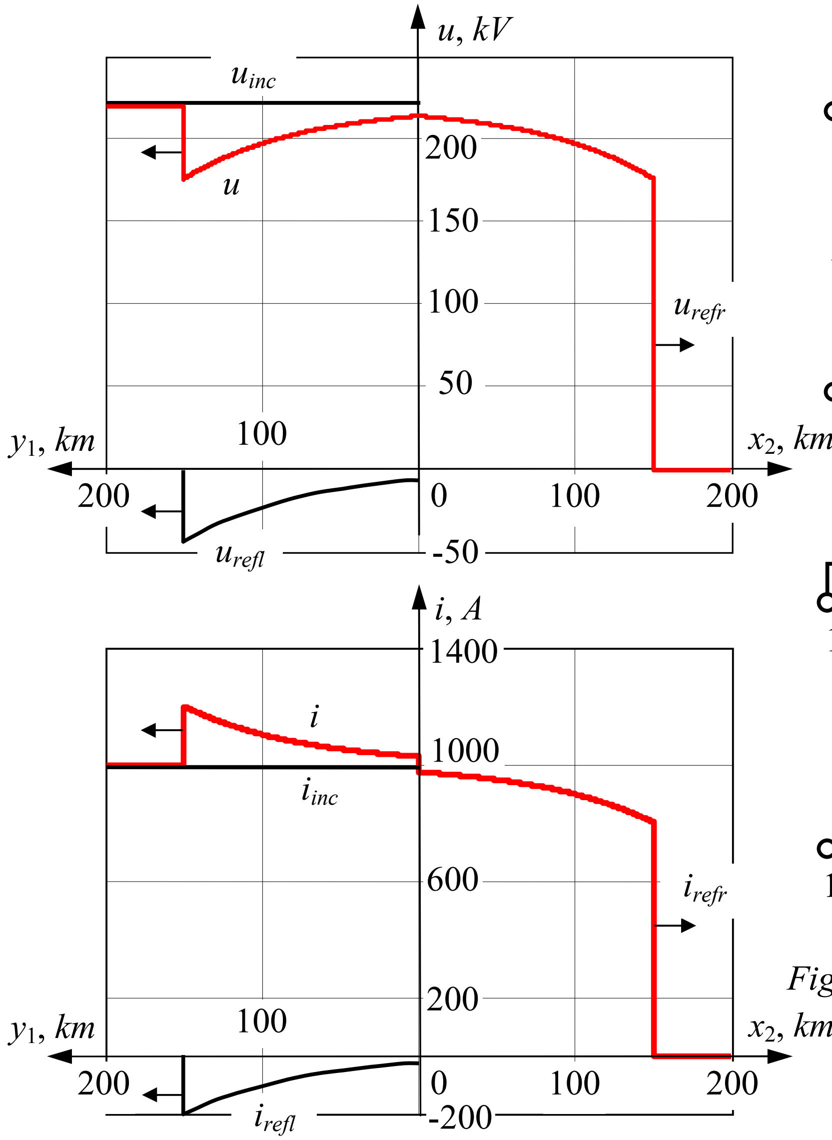
2) $u_{refl}(t_f, y_1) = -44\cdot exp[-4000\cdot(5\cdot 10^{-4} - y_1/(3\cdot 10^5))]\ kV,$

$i_{refl}(t_f, y_1) = -200\cdot exp[-4000\cdot(5\cdot 10^{-4} - y_1/(3\cdot 10^5))]\ A,$

$u_{refr}(t_f, x_2) = 220 - 44\cdot exp[-4000\cdot(5\cdot 10^{-4} - x_2/(3\cdot 10^5))]\ kV,$

$i_{refr}(t_f, x_2) = 1000 - 200\cdot exp[-4000\cdot(5\cdot 10^{-4} - x_2/(3\cdot 10^5))]\ A,$

the voltage and current epures are presented in fig. 8.37.



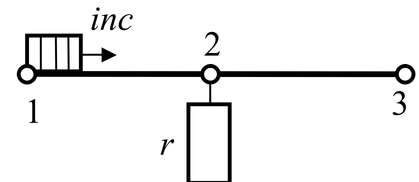


Fig. 8.38

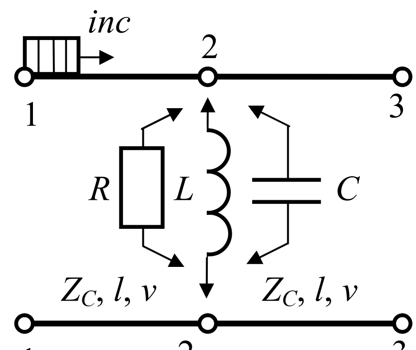


Fig. 8.39

Fig. 8.37

8-42 (8.49). The incident wave $u_{inc} = 220\ kV$ with a rectangular front moves along the overhead line fig. 8.38 with parameters $l = 150\ km$, $Z_C = 220\ Ohm$. It passes by elements $r = 50\ Ohm$, $L = 20\ mH$ into the next identical overhead line. It is required:

- to plot the current $i_{22}(t)$ and voltage $u_{22}(t)$ at the end of the 1st line;
- to plot the total voltage u and current i distribution along both lines for the time moment $t_f = 0.4\ ms$, which is measured from the moment when the wave reaches the point of lines connection.

Answers: $u_{22}(t) = 68.75 + 151.25e^{-8000t}\ kV$, $i_{22}(t) = 1688 - 688e^{-8000t}\ A$,

$u_{refl}(t_f, y_1) = -151.3 + 151.3exp[-8\cdot(0.4 - y_1/300)]\ kV$,

$i_{refl}(t_f, y_1) = -688 + 688exp[-8\cdot(0.4 - y_1/300)]\ A$, $y_1 \le v_1\cdot t_f = 120\ km$;

$u_{refr}(t_f, x_2) = 68.8 + 151.3exp[-8\cdot(0.4 - x_2/300)]\ kV$,

$i_{refr}(t_f, x_2) = 313 + 688exp[-8\cdot(0.4 - x_2/300)]\ A$, $x_2 \le v_2\cdot t_f = 120\ km$.

8-43 (8.50). The incident wave $u_{inc} = 1.2\ kV$ with a rectangular front moves along the cable line fig. 8.39 with parameters: $l = 60\ km$, $Z_C = 80\ Ohm$. It passes through the correction element into the next identical cable line. It is required to plot the total voltage u and current i distribution along both lines for the time moment $t_f = 0.3\ ms$, which is measured from the moment when the wave reaches the point of lines connection, the correction element being:

a) $R = 120\ Ohm$, b) $L = 5\ mH$, c) $C = 6.25\ \mu F$.

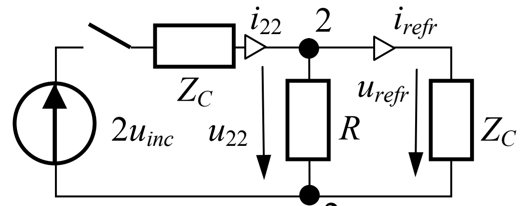


Fig. 8.40

Solution. Case **a)**: $R = 120\ Ohm$.

1. We calculate the incident wave's parameters

$$u_{inc}(t;\ x=0) = 1200\ V = const,$$

$$i_{inc}(t;\ x=0) = u_{inc}/Z_C = 1200/80 = 15\ A = const.$$

2. If the load is purely resistive, the reflected wave may be determined through the incident wave with the aid of the reflection coefficient n. For simplicity, let's construct the equivalent scheme for section "2-2" (fig. 8.40):

$$R_l = \frac{R \cdot Z_C}{R + Z_C} = \frac{120 \cdot 80}{120 + 80} = 48\ Ohm, \qquad n = \frac{R - Z_C}{R + Z_C} = \frac{48 - 80}{48 + 80} = -\,0.25.$$

3. Then the voltage and current of the reflected wave are:

$u_{refl}(t;\ y_1) = n \cdot u_{inc} = -\,0.25 \cdot 1200 = -\,300\ V = const,$

$i_{refl}(t;\ y_1) = n \cdot i_{inc} = -\,0.25 \cdot 15 = -\,3.75\ A = const, \qquad y_1 \le v_1 \cdot t_f = 45\ km.$

4. The voltage and current at the end of the 1st line are:

$$u_{22}(t) = u_{inc}(t) + u_{refl}(t) = 900\ V, \qquad i_{22}(t) = i_{inc}(t) - i_{refl}(t) = 18.75\ A.$$

5. The voltage and current of the refracted wave are (see the equivalent scheme):

$u_{refr}(t;\ x_2) = u_{22} = 900\ V = const,$

$i_{refr}(t;\ x_2) = u_{refr}/Z_C = 900/80 = 11.25\ A = const, \qquad x_2 \le v_2 \cdot t_f = 45\ km.$

The voltage and current epures along both lines for case a) are presented in fig. 8.41.

As there is a reactive element the calculation of the reflected wave in **cases b)** and **c)** is performed with the aid of the reflection coefficient in operational form.

Case **b)**: $I_{inc}(p) = \dfrac{15}{p}$; $Z(p) = \dfrac{pL \cdot Z_C}{pL + Z_C}$; $n(p) = \dfrac{Z(p) - Z_C}{Z(p) + Z_C} = \dfrac{-\,8000}{p + 8000}$;

$$I_{refl}(p) = n(p) \cdot I_{inc}(p) = \frac{-1.2 \cdot 10^5}{p(p + 8000)}; \quad i_{refl}(t) = -\,15 + 15e^{-8000t}\ A,$$

$i_{refl}(t;\ y_1) = -\,15 + 15exp[-\,8 \cdot (0.3 - y_1/150]\ A, \qquad y_1 \le v_1 \cdot t_f = 45\ km,$

$u_{refl}(t;\ y_1) = Z_C \cdot i_{refl}(t;\ y_1) = -\,1.2 + 1.2exp[-\,8 \cdot (0.3 - y_1/150]\ kV;$

$u_{22}(t) = u_{inc}(t) + u_{refl}(t) = 1.2e^{-8000t}\ kV, \qquad i_{22}(t) = i_{inc}(t) - i_{refl}(t) = 30 - 15e^{-8000t}\ A;$

$u_{refr}(t;\ x_2) = 1.2exp[-\,8 \cdot (0.3 - x_2/150]\ kV,$

$i_{refr}(t;\ x_2) = 15exp[-\,8 \cdot (0.3 - x_2/150]\ A, \qquad x_2 \le v_2 \cdot t_f = 45\ km.$

Case **c)**: $I_{inc}(p) = \dfrac{15}{p}$; $Z(p) = \dfrac{\frac{1}{pC} \cdot Z_C}{\frac{1}{pC} + Z_C}$; $n(p) = \dfrac{Z(p) - Z_C}{Z(p) + Z_C} = \dfrac{-\,p}{p + 4000}$;

$$I_{refl}(p) = n(p) \cdot I_{inc}(p) = \frac{-15}{p + 4000}; \quad i_{refl}(t) = -\,15e^{-4000t}\ A,$$

$i_{refl}(t;\ y_1) = -\,15exp[-\,4 \cdot (0.3 - y_1/150]\ A, \qquad y_1 \le v_1 \cdot t_f = 45\ km,$

$u_{refl}(t;\ y_1) = Z_C \cdot i_{refl}(t;\ y_1) = -\,1.2exp[-\,4 \cdot (0.3 - y_1/150]\ kV;$

$u_{22}(t) = u_{inc}(t) + u_{refl}(t) = 1.2 - 1.2e^{-4000t}\ кB, \qquad i_{22}(t) = i_{inc}(t) - i_{refl}(t) = 15 + 15e^{-4000t}\ A;$

$u_{refr}(t;\ x_2) = 1.2 - 1.2exp[-\,4 \cdot (0.3 - x_2/150]\ kV,$

$i_{refr}(t;\, x_2) = 15 - 15exp[-4\cdot(0.3 - x_2/150]\ A, \qquad\qquad x_2 \le v_2\cdot t_f = 45\ km.$

By the way, the problem for case b) or c) may be solved by the equivalent scheme as well similarly to problems 8.38 – 8.42.

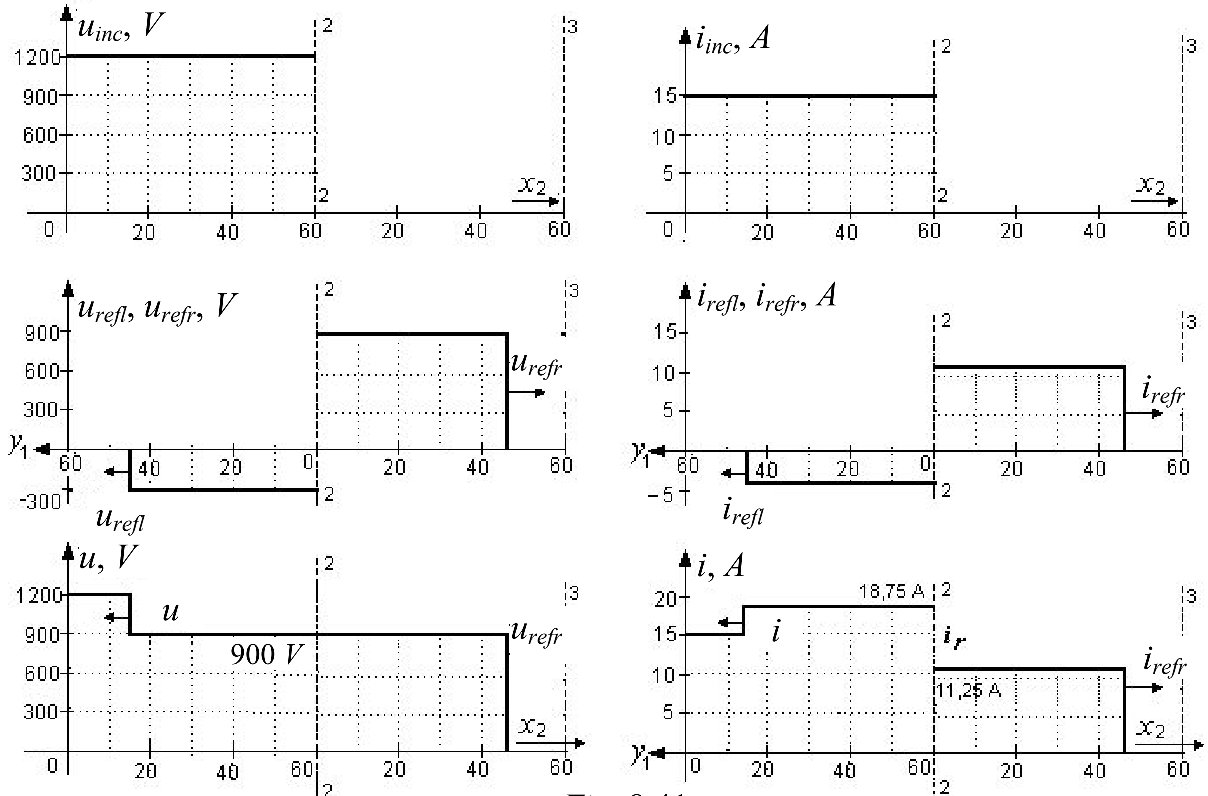


Fig. 8.41

8.2.4. Multiple wave reflections in lines

8-44 (8.52). An overhead zero-loss line (fig. 8.42) of length l and the wave impedance Z_C is connected to an ideal EMF source E_0. Calculate the transient process and plot the voltage and current u_{11}, i_{11} at the sending end and u_{22}, i_{22} – at the receiving end of the line for two cases: $R_l = 4Z_C$ and $R_l = Z_C/3$.

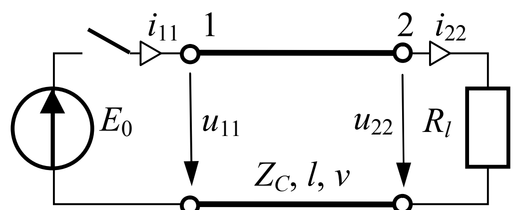


Fig. 8.42

Solution. 1. The load resistance is greater than the line wave impedance $R_l = 4Z_C$.

Perform some subsidiary calculations:

$$n_1 = \frac{0 - Z_C}{0 + Z_C} = -1, \qquad n_2 = \frac{4Z_C - Z_C}{4Z_C + Z_C} = +0.6.$$

$u_s = E_0$, $i_s = E_0/(4Z_C) = 0.25I_0$,

$u_{1inc} = E_0$, $i_{1inc} = E_0/Z_C = I_0$.

The subsequent calculations are tabulated in table 8.4, which is used for plotting (fig. 8.43).

From table 8.4 and graphs fig. 8.43, it follows the transient process has oscillatory character because the reflection coefficients at the line befinning and end are of different sign ($n_1 = -1$ and $n_2 = +0.6$). In this case the transient process lasts for 14-15 wave triavels, i.e. $t_{tp} = 15t_{tr}$.

Table 8.4

No	Time interval	u_{11}	i_{11}	Wave in line	u_{22}	i_{22}
1	$0 \div 1l/v$	E_0	I_0	$u_{1inc} = E_0 \rightarrow$	0	0
2	$1 \div 2l/v$	E_0	I_0	$\leftarrow u_{1refl} = 0.6E_0$	$1{,}6E_0$	$0{,}4I_0$
3	$2 \div 3l/v$	E_0	$-0.2I_0$	$u_{2inc} = -0.6E_0 \rightarrow$	$1{,}6E_0$	$0{,}4I_0$
4	$3 \div 4l/v$	E_0	$-0.2I_0$	$\leftarrow u_{2refl} = -0.36E_0$	$0{,}64E_0$	$0{,}16I_0$
5	$4 \div 5l/v$	E_0	$0.52I_0$	$u_{3inc} = +0.36E_0 \rightarrow$	$0{,}64E_0$	$0{,}16I_0$
6	$5 \div 6l/v$	E_0	$0.52I_0$	$\leftarrow u_{3refl} = 0.216E_0$	$1{,}22E_0$	$0{,}304I_0$
7	$6 \div 7l/v$	E_0	$0.09I_0$	$u_{4inc} = -0.216E_0 \rightarrow$	$1{,}22E_0$	$0{,}304I_0$
15th travel				$u_{8inc} = [-0.6]^7 \cdot u_{1inc} = -0.028E_0 = 2.8\% u_s$		
17th travel				$u_{9inc} = [-0.6]^8 \cdot u_{1inc} = +0.0168E_0 = 1.68\% u_s$		

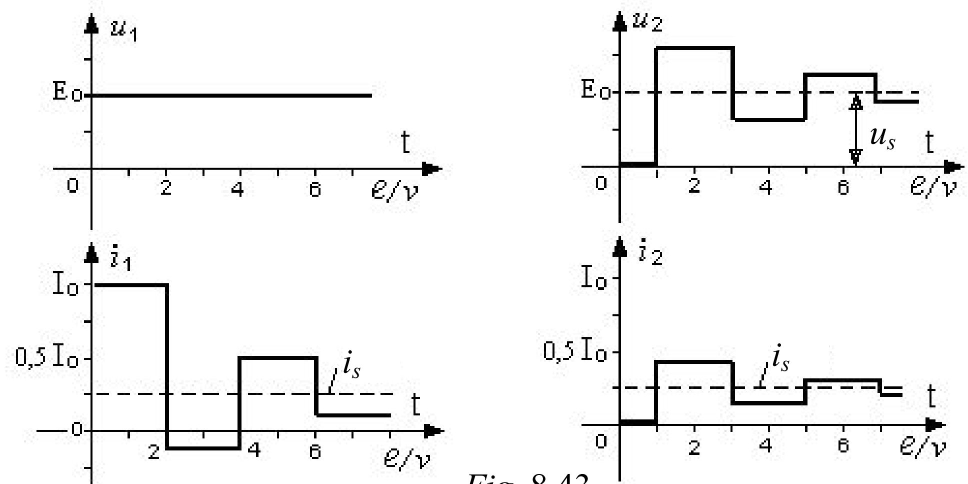


Fig. 8.43

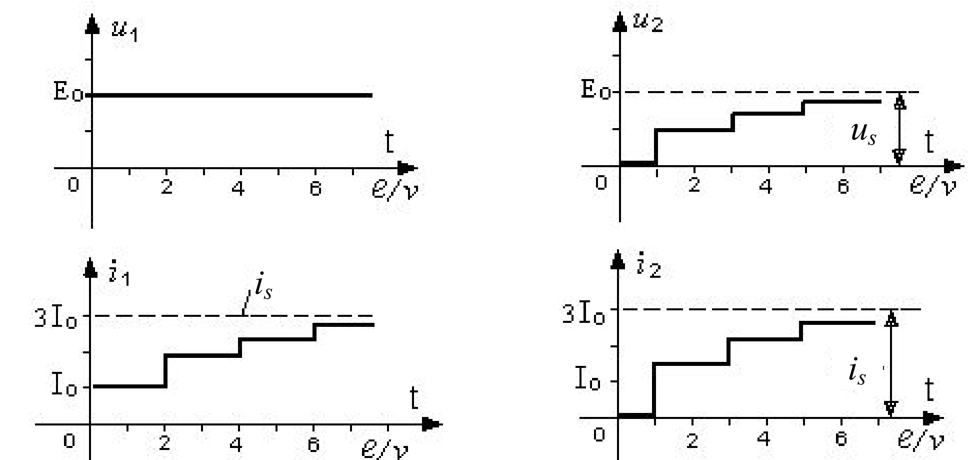


Fig. 8.44

2. The load resistance is less than the line wave impedance $R_l = Z_C/3$.
Perform analogous subsidiary calculations:

$$u_s = E_0, \quad i_s = E_0/(0.33Z_C) = 3I_0, \quad u_{1inc} = E_0, \quad i_{1inc} = E_0/Z_C = I_0,$$

$$n_1 = \frac{0 - Z_C}{0 + Z_C} = -1, \qquad n_2 = \frac{0.33Z_C - Z_C}{0.33Z_C + Z_C} = -0.5.$$

Subsequent calculations are tabulated in table 8.5, which is used for plotting (fig. 8.44).

Table 8.5

No	Time interval	u_{11}	i_{11}	Wave in line	u_{22}	i_{22}
1	$0 \div 1l/v$	E_0	I_0	$u_{1inc} = E_0 \rightarrow$	0	0
2	$1 \div 2l/v$	E_0	I_0	$\leftarrow u_{1refl} = -0.5E_0$	$0{,}5E_0$	$1{,}5I_0$
3	$2 \div 3l/v$	E_0	$2I_0$	$u_{2inc} = +0.5E_0 \rightarrow$	$0{,}5E_0$	$1{,}5I_0$
4	$3 \div 4l/v$	E_0	$2I_0$	$\leftarrow u_{2refl} = -0.25E_0$	$0{,}75E_0$	$2{,}25I_0$
5	$4 \div 5l/v$	E_0	$2{,}5I_0$	$u_{3inc} = +0.25E_0 \rightarrow$	$0{,}75E_0$	$2{,}25I_0$
6	$5 \div 6l/v$	E_0	$2{,}5I_0$	$\leftarrow u_{3refl} = -0.125E_0$	$0{,}875E_0$	$2{,}625I_0$
7	$6 \div 7l/v$	E_0	$2{,}75I_0$	$u_{4inc} = +0.125E_0 \rightarrow$	$0{,}875E_0$	$2{,}625I_0$
11^{th} travel				$u_{6inc} = [n_1 \cdot n_2]^5 \cdot u_{1inc} = 0.031E_0 = 3.13\% u_s$		
12^{th} travel				$u_{6refl} = n_2 \cdot [n_1 \cdot n_2]^8 \cdot u_{1inc} = 1.56\% u_s$		

If the reflection coefficients are of the same sign ($n_1 = -1$ and $n_2 = -0.5$) the transient process is of an aperiodic nature. In this case the transient process lasts for 11-12 wave travels, i.e. $t_{tp} = 12t_{tr}$.

8-45 (8.54). An overhead line ($l = 300\ km$, $Z_C = 200\ Ohm$) connected to a generator ($E = 100\ kV$, $r_0 = 50\ Ohm$) has been carrying the load $R_1 = 950\ Ohm$ for a long time. The load changes when the resistance $R_2 = 111.8\ Ohm$ is connected in parallel to the resistance R_1. Plot the time dependencies of the current and voltage in section *A-A* at the line centre.

Solution. For illustrative purposes, let's draw the initial scheme and work out a scheme to determine the parameters of the arising reverse wave (fig. 8.45,a and b).

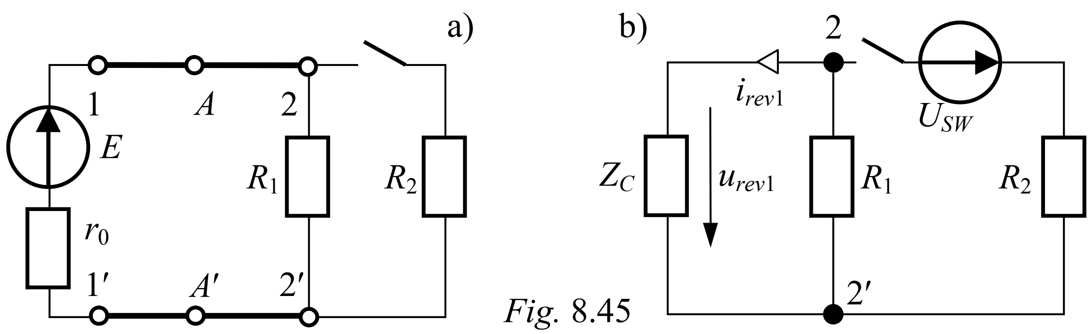


Fig. 8.45

The current and voltage along the line before the commutation as well as the voltage across the switch in the commutation moment are as follows:

$$I(t_-) = \frac{E}{r_0 + R_1} = \frac{100 \cdot 10^3}{50 + 950} = 100\ A, \qquad U(t_-) = U_{SW} = R_1 \cdot I(t_-) = 950 \cdot 0.1 = 95\ kV.$$

Perform some subsidiary calculations:

- the load resistance after commutation is $R_l = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{950 \cdot 111.8}{950 + 111.8} = 100\ Ohm$;

- the reflection coefficients at the line beginning and end are:

$$n_1 = \frac{r_0 - Z_C}{r_0 + Z_C} = \frac{50-200}{50+200} = -0.6; \quad n_2 = \frac{R_l - Z_C}{R_l + Z_C} = \frac{100-200}{100+200} = -0.333;$$

- the wave travel duration is $t_{tr} = \frac{l}{v} = \frac{300}{3 \cdot 10^5}\ s = 1\ ms$;

- steady-state condition in the line after the commutation is:

$$I_s = \frac{E}{r_0 + R_l} = \frac{100 \cdot 10^3}{50+100} = 666.7\ A, \quad U_s = R_l \cdot I_s = 100 \cdot 0.6667 = 66.67\ kV.$$

- 5% of the steady-state values of line voltage and current:

$$5\% I_s = 33.3\ A, \quad 5\% U_s = 3.33\ kV.$$

The 1st reverse wave is calculated based on the scheme fig. 8.45,b:

$$i_{rev1} = \frac{-U_{SW}}{R_2 + \frac{R_1 Z_C}{R_1 + Z_C}} \cdot \frac{R_1}{R_1 + Z_C} = \frac{-95 \cdot 10^3}{111.8 + \frac{950 \cdot 200}{950+200}} \cdot \frac{950}{950+200} = -283.3\ A,$$

$u_{rev1} = Z_C \cdot i_{rev1} = 200 \cdot (-0.2833) = -56.67\ kV$.

The voltage and current of the subsequent arising direct and reverse waves are calculated under the formulae: $i_{dir\,q} = n_1 \cdot i_{rev\,q}$; $u_{dir\,q} = n_1 \cdot u_{rev\,q}$;

$i_{rev\,q+1} = n_2 \cdot i_{dir\,q}$; $u_{rev\,q+1} = n_2 \cdot u_{dir\,q}$.

We keep on calculating until the voltage and current of the newly arising wave become less than 5% of the steady-state values. The total voltage and current values in section *A-A* are calculated by the formulae:

$$u = U(t_-) + \Sigma u_{rev} + \Sigma u_{dir}; \quad i = I(t_-) - \Sigma i_{rev} + \Sigma i_{dir}.$$

It is recommended to perform the calculations in the table form.

Table 8.6

№ of travel	Time interval, *ms*	Parameters of arising waves, *A* and *kV*	Current in section *A-A′* *i*, *A*	Voltage in section *A-A′* *u*, *kV*
1	0÷0.5	$i_{rev1} = -283.3$; $u_{rev1} = -56.67$	100	95
1-2	0.5÷1.5	$i_{dir1} = 170$; $u_{dir1} = 34$	383.3	38.33
2-3	1.5÷2.5	$i_{rev2} = -56.67$; $u_{rev2} = -11.33$	553.3	72.33
3-4	2.5÷3.5	$i_{dir2} = 34$; $u_{dir2} = 6.8$	610	61
4-5	3.5÷4.5	$i_{rev3} = -11.33$; $u_{rev3} = -2.27$	644	67.8
5-6	4.5÷5.5		655.3	65.53

By data of table 8.6, we plot *u(t)*, *i(t)* (fig. 8.46).

8-46 (8.55). Solve problem 8.45 under the condition that the switch opens using the following numerical data: $l = 3\ km$, $Z_C = 20\ Ohm$,

$v = 1.5 \cdot 10^5\ km/s$, $E = 6\ kV$, $r_0 = 2\ Ohm$, $R_1 = 40\ Ohm$, $R_2 = 10\ Ohm$.

Answers: $I(t_-) = 600\ A$, $U(t_-) = 4.8\ kV$,

$I_{SW} = 480\ A$; $I_s = 142.9\ A$, $U_s = 5716\ V$; $n_1 = -0.818$, $n_2 = 0.333$;

the scheme to calculate the 1st reverse wave is in fig. 8.47;

the current and voltage values of the arising reverse and direct waves are:

$i_{rev} = 320; -87.3; 23.8; -6.5\ A$; $u_{rev} = Z_C \cdot i_{rev} = 6400; -1746; 476; -130\ V$;

$i_{dir} = -262; 71.3; -19.5\ A$; $u_{dir} = Z_C \cdot i_{dir} = -5240; 1426; -390\ V$;

the sequence of the current and voltage values in section *A-A′* are:
$i_{A\text{-}A'}$ = 600; 280; 18; 105.3; 176.6; 152.8; 133.3; 139.8 *A*;
$u_{A\text{-}A'}$ = 4800; 11200; 5960; 4214; 5640; 6116; 5726; 5596 *V*.

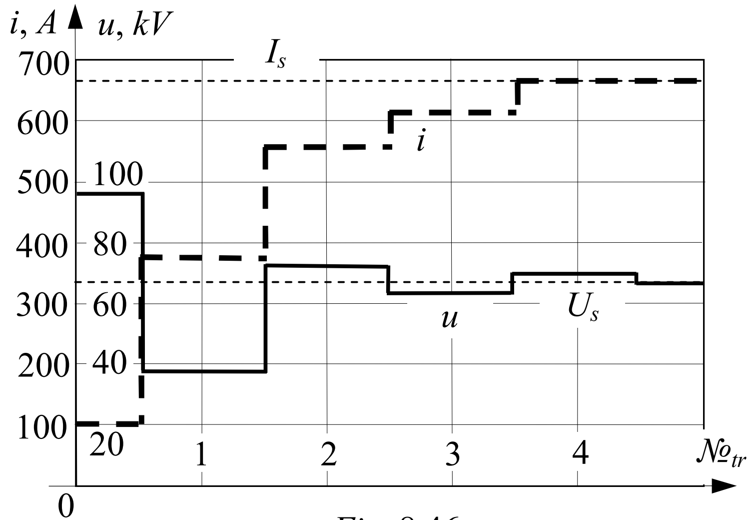


Fig. 8.46

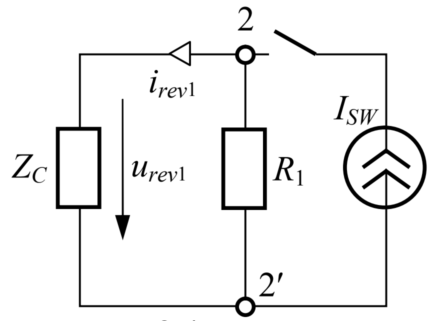


Fig. 8.47

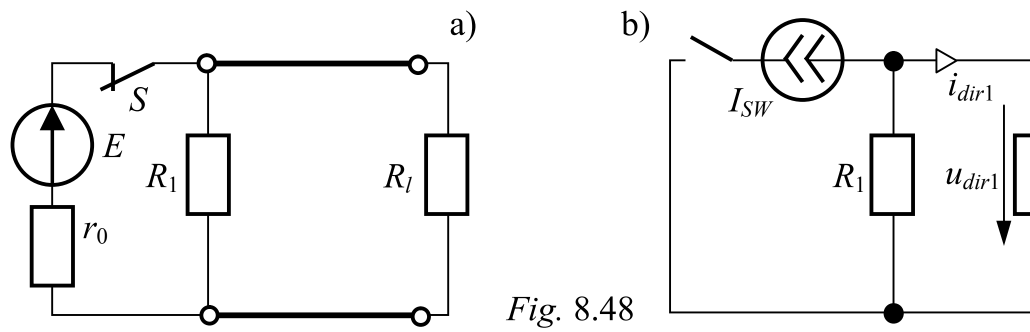


Fig. 8.48

8-47 (8.56). The connection scheme is presented in fig. 8.48,a. The commutation occurs by disconnecting the switch *S*. The numerical data: $l = 75$ *km*, $Z_C = 110$ *Ohm*, $v = 3\cdot10^5$ *km/s*, $E = 110$ *kV*, $r_0 = 6.111$ *Ohm*, $R_1 = 440$ *Ohm*, $R_l = 55$ *Ohm*. Plot the time dependencies of the current at the line beginning and voltage at the line end.

Answers: $I_{SW} = 2$ *кA*, $I(t_-) = 1.778$ *kA*, $U(t_-) = 97.78$ *kV*; $I_s = 0$, $U_s = 0$;
$n_1 = 0.6$, $n_2 = -0.333$; the scheme to calculate the 1st direct wave is in fig. 8.48,b;
the current and voltage values of the arising reverse and direct waves are:
$i_{dir} = -1.6$; 0.32; –0.064 *кA*;
$u_{dir} = -176$; 35.2; –7.04 *kV*;
$i_{rev} = 0.533$; –0.107 *кA*;
$u_{rev} = 58.7$; –11.73 *kV*;
the sequence of the total current and voltage values are:
$i = 0.178$; –0.035; 0.008 *kA*;
$u = 97.78$; –19.52; 3.95; –3.09 *kV*;
the diagrams are in fig. 8.49.

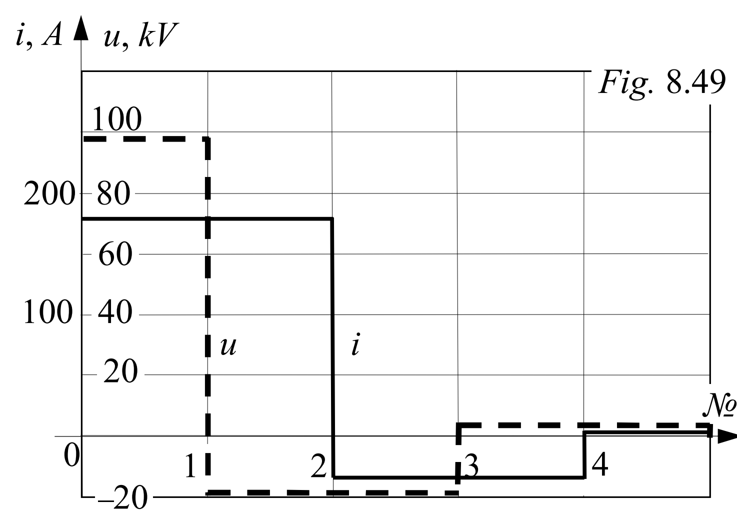


Fig. 8.49

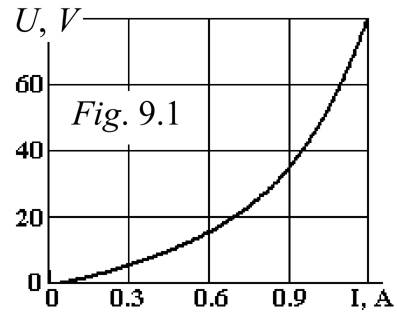
9. NON-LINEAR A-C CIRCUITS

9.1. CIRCUITS WITH INERTIAL NON-LINEAR IMPEDANCES

9-1 (9.34). An incandescent lamp with VAC *U(I)* (see fig. 9.1) is supplied with the sinusoidal voltage $u(t) = 56.6\sin(314t)$ *V*.

Determine the effective current and the lamp power.

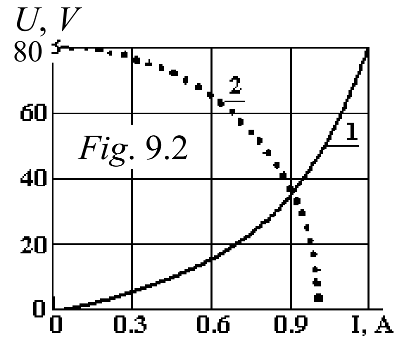
Answer: 0.95 *A*, 38 *W*.



9-2 (9.35). A circuit consisting of an incandescent lamp (VAC in fig. 9.1) connected in series with a capacitor $x_C = 80$ *Ohm* is supplied with sinusoidal voltage

$$u_0(t) = 113.1\sin(314t)\ V.$$

Determine the lamp current and voltage; verify the circuit power balance.



Solution. The applied voltage $U_0 = 80$ *V* is balanced by a phasor sum of voltages across the lamp *U* and capacitor $(I \cdot x_C)$. It means the solution of problem is reduced to graphical solution of the equation system:

$$\begin{cases} U = f(I), & (9.1) \\ U = \sqrt{U_0^2 - (I \cdot x_C)^2}, & (9.2) \end{cases}$$

where expression (9.1) is VAC of the incandescent lamp while (9.2) is the circuit equation under Kirchhoff's voltage law which is by essence the source VAC with EMF U_0 and inner impedance x_C.

The graphical solution of the system (9.1)-(9.2) is given in fig. 9.2. Line 1 is the lamp VAC, line 2 is the plot of the 2nd equation of the system. The intersection point sets the system solution as well as the condition for a non-linear element: $U = 35$ *V*, $I = 0.9$ *A*. The phase shift between the source current and voltage is: $\varphi = -arctg(x_C \cdot I/U) = -64^o$. The source active and reactive powers are: $P = U_0 \cdot I \cdot \cos\varphi = 31.5$ *W*, $Q = U_0 \cdot I \cdot \sin\varphi = -64.8$ *VAr*. The lamp power is: $P = UI = 31.5$ *W*, the capacitor power is: $Q = -I^2 \cdot x_C = -64.8$ *VAr*.

9-3 (9.36). In the statement of problem 9.2 the reactance is replaced by a resistance $r = 42$ *Ohm*. Determine the lamp current and voltage; verify the power balance.

Answer: 0.95 *A*; 40 *V*; 76 *W*.

9-4 (9.37). In the statement of problem 9.1, an ideal diod is connected in series with the lamp. Determine the effective values of the lamp voltage and current as well as its power.

Answer: 28.3 *V*; 0.8 *A*; 22.6 *W*.

9.2. CIRCUITS WITH INERTIALESS ELEMENTS

9.2.1. Graphical and analytical calculation methods

9-5 (9.38). Calculate the voltage $u(\omega t)$ across the non-linear resistor with symmetrical characteristic *u(i)* by means of the graphical method, if the circuit current in fig. 9.3 varies under the sinusoidal law $i = j(t) = I_m\sin(\omega t) = 5\sin(\omega t)$ *A*, while the resistor VAC is set by table 9.1.

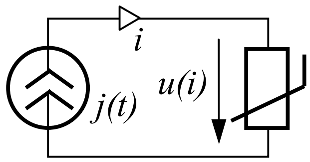


Fig. 9.3

Table 9.1

u, V	0	3.5	16	46.5	104	197.5	336
i, A	0	1	2	3	4	5	6

Expand the curve $u(\omega t)$ into the Fourie series, using only the 1st and 3rd harmonics. Determine the effective voltage across the resistance.

Solution. The graphical calculation of the circuit is given in fig. 9.4. The results of the voltage calculation for different moments ωt_p (with step 15° for the positive half-wave) are tabulated in table 9.2. To make the current and voltage comparison more convenient, the current curve $i(\omega t)$ is shown in fig. 9.4 as well.

Table 9.2

p	1	2	3	4	5	6	7	8	9	10	11	12
ωt_p, *deg*	15	30	45	60	75	90	105	120	135	150	165	180
u_p, *V*	5.8	28.4	73.6	130.4	178.7	197.5	178.7	130.4	73.6	28.4	5.8	0

The calculated curve $u(\omega t)$ meets two symmetry conditions, that's why it does not contain any direct component, even harmonics and cosines. Finally,

$$u(\omega t) = U_{1m}\sin\omega t + U_{3m}\sin3\omega t + U_{5m}\sin5\omega t + \dots ,$$

where $U_{km} = \frac{2}{n}\sum_{p=1}^{n} u_p \sin(k\cdot\omega t_p)$, and $n = 12$ parts for the half-period.

After substituting with the data from table 9.2, we obtain:

$$U_{1m} = 150.7\ V,\quad U_{3m} = -46.83\ V.$$

With account of the first two items of the Fourie series, we have:

$$u(\omega t) = 150.7\sin(\omega t) - 46.83\sin(3\omega t)\ V.$$

The effective voltage is

$$U_m = \sqrt{\frac{1}{n}\sum_{p=1}^{n} u_p^2} = \sqrt{\frac{1}{12}\sum_{p=1}^{n}(5.8^2 + 28.4^2 + \dots + 5.8^2 + 0)} = 111.6\ V.$$

9-6 (9.39). Compute the current of an ideal coil with a ferromagnetic core with the aid of the graphical method, the coil being connected to a sinusoidal voltage source $u = U_m\cos\omega t$ with an effective value $U = 76.4\ V$ and frequency $f = 50$ *cps*. WbAC of the coil for instantaneous values is set by table 9.3.

Table 9.3

Ψ, *Wb*	0	0.1	0.15	0.2	0.25	0.3	0.35	0.4
i, A	0	0.092	0.311	0.737	1.44	2.47	3.15	5.9

Expand the non-sinusoidal current $i(\omega t)$ into the Fourier series, using only the first two items. Calculate the effective current I.

Answer: $i(\omega t) = 2.81\sin\omega t - 0.937\sin3\omega t\ A$; $I = 2.10\ A$.

9-7 (9.41). Approximate the VAC of a non-linear resistor given in problem 9.5 (table 9.1) with a polynomial $u = a\cdot i + b\cdot i^3$. Analytically compute the voltage across the resistor $u(\omega t)$.

Answer: $u = 2i + 1.5i^3$, where $i[A]$, $u[V]$; $u = 150.6\sin\omega t - 46.9\sin3\omega t\ V$.

Fig. 9.4
See «Album»

9-8 (9.45). The core cross-section of an ideal coil is $S = 10\ cm^2$, the core mean length is $l = 50\ cm$, number of coil turns is $w = 100$. The magnetization curve of the core material is set by table 2.10. The core possesses an air gap l_a (fig. 9.5,a). Compute and plot WbAC of the coil $\Phi(i)$ for instantaneous values and for two lengths of the air gap: $l_a = 0.1$ and $0.25\ mm$.

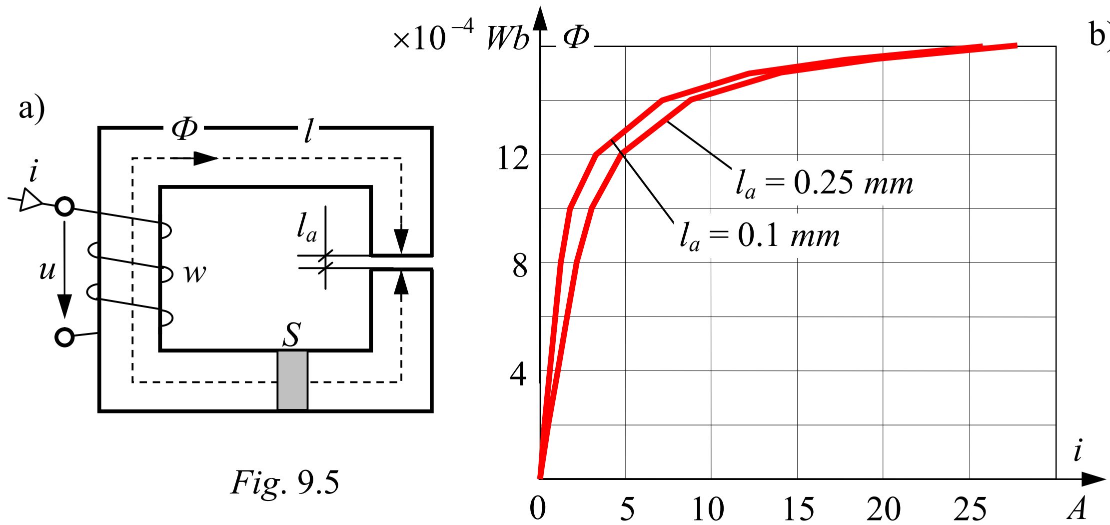


Fig. 9.5

Solution. Taking the different values of B, we compute the corresponding values $\Phi = BS$, which set the current instantaneous values

$$i = \frac{Hl + H_a l_a}{w}, \quad \text{where} \quad H_a = \frac{B}{\mu_0} = 8\cdot10^3 B\ A/cm, \quad \text{while} \quad B[T].$$

The calculation results of WbAC are tabulated in table 9.4, the curve for two lengths of the air gap ($l_{a1} = 0.1\ mm$ and $l_{a2} = 0.25\ mm$) is given in fig. 9.5,b.

Table 9.4

Φ, mWb	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.5	1.55	1.6
$i(l_{a1})$, A	0	0.29	0.58	0.88	1.22	1.8	3.29	7.12	12.2	17.7	25.8
$i(l_{a2})$, A	0	0.5	1.05	1.6	2.18	3	4.73	8.8	14	19.6	27.7

9-9 (9.46). The coil described in problem 9.8 is connected to a sinusoidal voltage source of effective value $U = 31\ V$ and frequency $f = 50\ cps$, the core having an air gap $l_a = 0.25\ mm$.

Compute the current instantaneous and effective values as well as the coil equivalent inductance using the analytical approximation of WbAC with expression $i = a\Phi^3$.

Solution. Assume $u = U_m cos\omega t$, then $\Phi = \Phi_m sin\omega t$,

where $\Phi_m = \dfrac{U_m}{\omega w} = \dfrac{31\sqrt{2}}{314\cdot100} = 14\cdot10^{-4}\ Wb.$

In accordance with the obtained flux value the current value is $i = I_m = 8.8\ A$; on the ground of these two values, we determine the approximation factor:

$$a = \frac{I_m}{\Phi_m^3} = \frac{8.8}{(14\cdot10^{-4})^3} = 32.07\cdot10^8\ A/Wb^3.$$

The current instantaneous value at the flux $\Phi = \Phi_m sin\omega t$ is

$$i = a\Phi_m^3 sin^3\omega t = \frac{3}{4}a\Phi_m^3 sin\omega t - \frac{1}{4}a\Phi_m^3 sin3\omega t = 6.6 sin\omega t - 2.2 sin3\omega t\ A.$$

The effective current is $I=\sqrt{0.5(I_{1m}^2+I_{3m}^2)}=\sqrt{0.5(6.6^2+2.2^2)}=4.92\ A$,

the coil equivalent reactance is $x_{equ}=\dfrac{U}{I}=\dfrac{31}{4.92}=6.3\ Ohm$,

equivalent inductance is $L_{equ}=\dfrac{x_{equ}}{\omega}=\dfrac{6.3}{314}=0.02\ H$.

9-10 (9.47). A reactor with a steel core and a linear capacitor of capacity $C=30\ \mu F$ are connected in series to a sinusoidal current source $i=I_m sin\omega t$ with frequency $f=50\ cps$.

Neglecting the steel loss, the reactor coil resistance and the leakage flux, find the current amplitude which guarantees the voltage resonance condition in the circuit; additionally, determine the effective circuit voltage as well as the voltages across a reactor and a capacitor in the resonance condition.

The relationship between the instantaneous values of the reactor flux linkage and its current is set by the equation $\Psi=ai+bi^3$, where $a=0.4\ Wb/A$, $b=-0.03\ Wb/A^3$, $i[A]$, $\Psi[Wb]$.

Solution. Firstly, determine the limiting value of the reactor current I_{max} for which the given approximation of WbAC is true. According to physics, there cannot be any negative-going part in WbAC $\Psi(i)$. That's why, we determine I_{max} from the condition $\dfrac{d\Psi}{dt}=0$: $a+3bI_{max}^2=0$, from here $I_{max}=\sqrt{\dfrac{-a}{3b}}=\sqrt{\dfrac{0,4}{3\cdot 0.03}}=2.11\ A$.

Thus, the required solution is to meet the condition $I_m<I_{max}$.

For a series resonance loop, in accordance with Kirchhoff's voltage law

$$u_L+u_C=u,\ \text{while}\ u_L=\frac{d\Psi}{dt},\ u_C=\frac{1}{C}\int idt\,.$$

In a steady-state periodical mode if the current is sinusoidal $i=I_m sin\omega t$, the voltage is $u_C=-\dfrac{I_m}{\omega C}cos\omega t$.

The flux linkage is $\Psi=ai+bi^3=(a+{}^3\!/\!_4 bI_m^3)sin\omega t-{}^1\!/\!_4 bI_m^3 sin3\omega t$,

then $u_L=\omega(a+{}^3\!/\!_4 bI_m^3)cos\omega t-{}^1\!/\!_4 b\cdot 3\omega I_m^3 cos3\omega t$.

Due to the voltage resonance $u_L+u_C=0$, which is possible only at mains frequency: $u_L^{(1)}+u_C^{(1)}=0$, from here, having omitted the multiplier $cos\omega t$, we obtain

$$\omega(a+{}^3\!/\!_4 bI_m^3)-\frac{I_m}{\omega C}=0.$$

After substituting with numerical data, we obtain a cubic equation

$$7.065I_m^3+106.2I_m-125.6=0,$$

the canonical form of which is $y^3+3py+2q=0$: $I_m^3+15.03I_m-17.78=0$, here $p=5.01$, $q=-8.89$.

The discriminant $D=q^2+p^3=204.8>0$, $\sqrt{D}=14.31$.

If the discriminant is positive, the canonic equation has one real root, which is found using Cardan's formulae $y_1=u+v$, where $u=\sqrt[3]{-q+\sqrt{D}}$, $v=\sqrt[3]{-q-\sqrt{D}}$.

In our case $u=\sqrt[3]{8.89+14.31}=2.852$, $v=\sqrt[3]{8.89-14.31}=-1.757$.

The required value is $I_m=u+v=1.095\ A$.

The instantaneous voltages are

across the capacitor at resonance $u_C = -\dfrac{I_m}{\omega C}\cos\omega t = -116.3\cos\omega t\ V,$

across the reactor $u_L = 116.3\cos\omega t + 9.28\cos 3\omega t\ V,$

at the scheme input $u = u_L + u_C = 9.28\cos 3\omega t\ V.$

The effective values are: $U_C = 82.25\ V,\ \ U_L = 82.5\ V,\ \ U = 6.56\ V.$

9-11 (9.50). A non-linear resistor is connected in series with a linear inductance $L = 0.1\ H$. The resistor's VAC is approximated by a polynomial

$$u = ai + bi^3 = 2i + 1.5i^3, \text{ where } u[V],\ i[A].$$

The obtained circuit is supplied with the sinusoidal voltage $U = 220\ V$ of frequency $f = 50\ cps$.

Calculate the first current harmonic by the describing function method.

Solution. Under Kirchhoff's voltage law for a series circuit, we have

$$u(i) + L\frac{di}{dt} = U_m \sin\omega t.$$

The required first current harmonic is $i = I_m\sin(\omega t + \psi)$. After substitution, the equation above takes a form:

$$aI_m\sin(\omega t + \psi) + bI_m^3\sin^3(\omega t + \psi) + \omega LI_m\cos(\omega t + \psi) = U_m\sin\omega t. \qquad (9.3)$$

Furthermore, there is an identity $\sin^3\alpha = ¾\sin\alpha - ¼\sin 3\alpha,$

let's omit the item $-¼bI_m^3\sin 3(\omega t + \psi)$, considering it as a negligible calculation inaccuracy;

$$\sin(\omega t + \psi) = \cos\psi\cdot\sin\omega t + \sin\psi\cdot\cos\omega t,\ \ \cos(\omega t + \psi) = \cos\psi\cdot\cos\omega t - \sin\psi\cdot\sin\omega t.$$

Let's equate the coefficients at $\sin\omega t$ and $\cos\omega t$ in the left and right sides of the equation (9.3) to each other. We get the equation system

$$\begin{cases}(aI_m + ¾bI_m^3)\cos\psi - \omega LI_m\sin\psi = U_m,\\(aI_m + ¾bI_m^3)\sin\psi + \omega LI_m\cos\psi = 0.\end{cases}$$

The system with numerical data is

$$\begin{cases}(2I_m + 1.125I_m^3)\cos\psi - 31.4I_m\sin\psi = 311,\\(2I_m + 1.125I_m^3)\sin\psi + 31.4I_m\cos\psi = 0,\end{cases}$$

it being solved with the aid of a computer: $Im := 1 \quad \psi := -1$

Given $(2\cdot Im + 1.125\cdot Im^3)\cdot\cos(\psi) - 31.4\cdot Im\cdot\sin(\psi) = 311$

$(2\cdot Im + 1.125\cdot Im^3)\cdot\sin(\psi) + 31.4\cdot Im\cdot\cos(\psi) = 0$

$Qt := MinErr(Im, \psi) \qquad Qt = \begin{pmatrix}5.948\\-0.644\end{pmatrix}$

Answer: $i^{(1)} = 5.948\sin(314t - 0.644)\ A.$

9.2.2. Piecewise-linear approximation method

9-12 (9.55). A half-wave rectifier is applied to charge a low-power accumulator, its scheme being presented in fig. 9.6,a. It is known that $E = 5\ V$; $u(t) = 10\cdot\sin(314\cdot t)\ V$; $r = 2\ Ohm$, the diod is ideal.

Plot the current; determine its average and effective values as well as the maximum voltage across the diode.

Answer: 0.545 A, 1.04 A, 15 V; the current plot is in fig. 9.6,b.

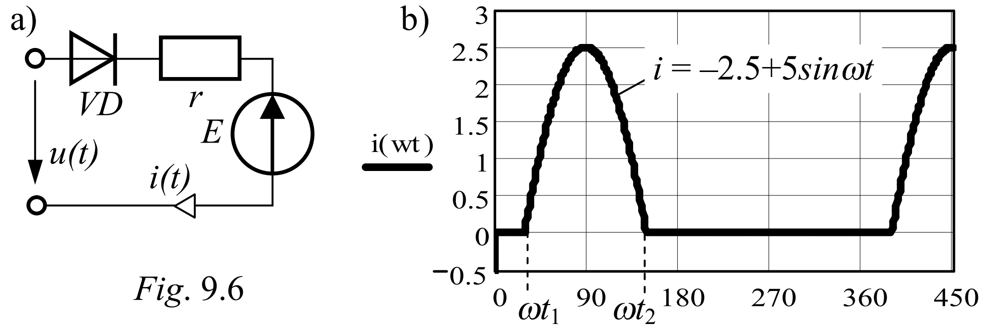


Fig. 9.6

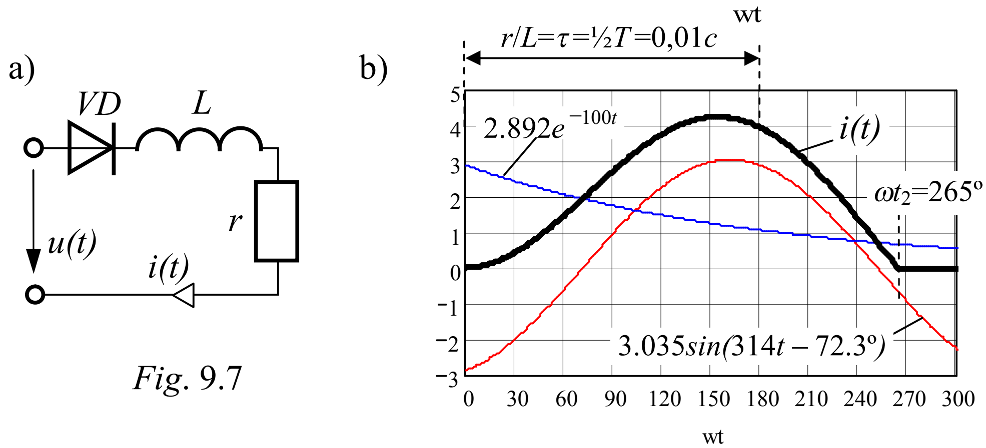


Fig. 9.7

9-13 (9.56). A single-phase half-wave rectifier (fig. 9.7,a) carries the $r–L$ load. Known are: $r = 10$ *Ohm*, $L = 0.1$ *H*, the source voltage is $u(t) = 100\cdot sin(314\cdot t)$ *V*.

Plot the current; determine its average and effective values.

Answer: 1.73 *A*, 2.37 *A*; the current plot is in fig. 9.7,b.

Note. The moment when the diode is blocking $\omega t_2 = 265^o$ is determined through the solution of the equation $3.035 sin(314\cdot t_2 - 1.262) + 2.892\, e^{-100\cdot t_2} = 0$.

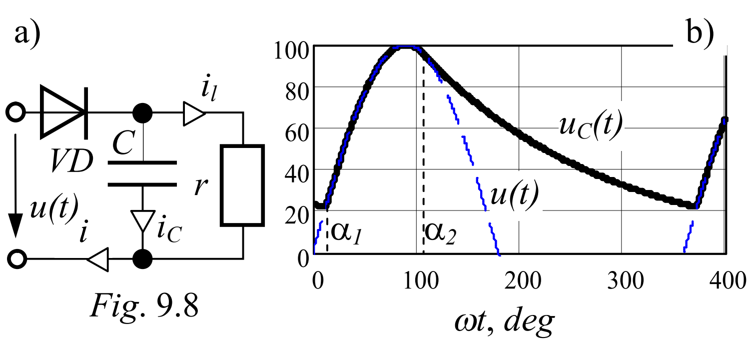


Fig. 9.8

9-14 (9.57). A single-phase half-wave rectifier with a smoothing capacitor $C = 100$ *μF* is loaded with resistance $r = 100$ *Ohm* (fig. 9.8,a), the source voltage being $u(t) = 100\cdot sin(314\cdot t)$ *V*.

Plot the load voltage $u_C(t)$; determine its average and effective values.

Answer: 57.1 *V*, 62.2 *V*; the voltage plot is in fig. 9.8,b.

Note. $\alpha_2 = \pi - arctg(\omega rC) = 1.88\ rad = 107.7^o$. Moment $\alpha_1 = 12.6^o = 0.22\ rad$ (or t_1) is found from the condition of the diode conducting, when the source voltage and voltage across the capacitor become equal: $U_m sin(\alpha_1) = U_m sin(\alpha_2)\cdot exp\left(-\frac{\alpha_1 - \alpha_2 + 2\pi}{\omega rC}\right)$.

This transcendental equation can be solved graphically or by a numerical method (e.g. step-by-step method).

9-15 (9.58). In circuit fig. 9.9,a there are D-C and A-C sources: $u(t) = 10\cdot sin(\omega t)$ *V*, $E = 5$ *V*. Resistances are: $r_1 = 5$ *Ohm*, $r_2 = 4$ *Ohm*, $r_3 = 2$ *Ohm*. Assume the diode *VD* to be ideal; compute the currents.

Solution. An ideal diode may be in one of two states – conducting or blocking, each one is presented by the corresponding equivalent scheme, respectively, fig. 9.9,b and 9.9,c.

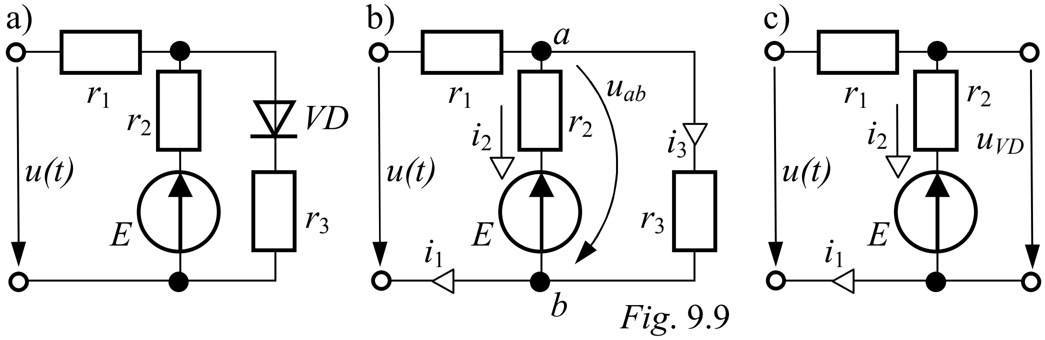


Fig. 9.9

Compute the circuit fig. 9.9,b by the nodal-pair method. The junction voltage is

$$u_{ab}=\frac{\frac{u}{r_1}+\frac{E}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}=\frac{\frac{10\,sin(\omega t)}{5}+\frac{5}{4}}{5^{-1}+4^{-1}+2^{-1}}=2.105sin(\omega t)+1.316\ V.$$

The branch currents are:

$$i_1=\frac{u-u_{ab}}{r_1}=\frac{10\,sin\,\omega t-2.105\,sin\,\omega t-1.316}{5}=1.579sin(\omega t)-0.263\ A,$$

$$i_3=\frac{u_{ab}}{r_3}=\frac{2.105\,sin\,\omega t+1.316}{2}=1.053sin(\omega t)+0.658\ A,$$

$$i_2=i_1-i_3=1.579sin(\omega t)-0.263-1.053sin(\omega t)-0.658=0.526sin(\omega t)-0.921\ A.$$

The angular range of the diode conducting state is determined from the condition the diode's current is always positive $i_3 > 0$:

$$1.053sin(\omega t)+0.658>0;\quad sin(\omega t)>-0.625;$$

$$-38.7° < \omega t < 218.7°.$$

Perform the computation of circuit fig. 9.9,c, corresponding to the blocking state of the diode:

$$i_3=0;\quad i_1=i_2=\frac{u-E}{r_1+r_2}=\frac{10\,sin\,\omega t-5}{5+4}=1.111sin(\omega t)-0.556\ A,$$

$$u_{VD}=u-r_1\cdot i_1=10sin(\omega t)-5.555sin(\omega t)+2.778=4.445sin(\omega t)+2.778\ V.$$

Condition of the diode blocking state – $u_{VD} < 0$ – gives an inequality:

$$4.445sin(\omega t)+2.778<0;\quad sin(\omega t)<-0.625;\qquad 218.7° < \omega t < 321.3°.$$

Finally, we have:

$$i_1(\omega t)=\begin{cases}1.579\,sin\,\omega t-0.263\ A\ if\ 0\le\omega t\le 218.7\ ,\\ 1.111\,sin\,\omega t-0.556\ A\ if\ 218.7\ \le\omega t\le 321.3\ ,\\ 1.579\,sin\,\omega t-0.263\ A\ if\ 321.3\ \le\omega t\le 360\ .\end{cases}$$

$$i_2(\omega t)=\begin{cases}0.526\,sin\,\omega t-0.921\ A\ if\ 0\le\omega t\le 218.7\ ,\\ 1.111\,sin\,\omega t-0.556\ A\ if\ 218.7\ \le\omega t\le 321.3\ ,\\ 0.526\,sin\,\omega t-0.921\ A\ if\ 321.3\ \le\omega t\le 360\ .\end{cases}$$

$$i_3(\omega t)=\begin{cases}1.053\sin\omega t+0.658\ A\ if\ 0\le\omega t\le 218.7\ ,\\ 0\quad if\ 218.7\ \le\omega t\le 321.3\ ,\\ 1.053\sin\omega t+0.658\ A\ if\ 321.3\ \le\omega t\le 360\ .\end{cases}$$

9-16 (9.59). Compute the current of the A-C source in the scheme fig. 9.10,a with the parameters – $u(t)=100\cdot sin(\omega t)\ V$, $E=50\ V$, $r=20\ Ohm$, $U_S=10\ V$. Calculate its effective value and plot it. Assume the stabilitron is ideal.

Solution. The problem is solved by the method of piecewise-linear approximation only for one period of the alternating voltage. Stabilitron VAC at the piecewise-linear approximation (so-called VAC of an ideal stabilitron) is presented in fig. 9.10,b.

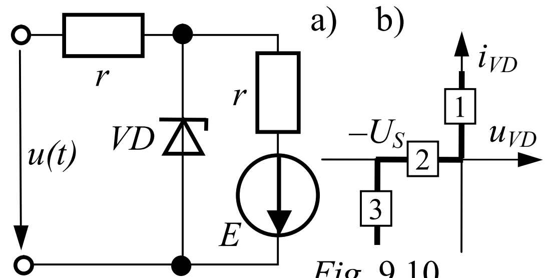


Fig. 9.10

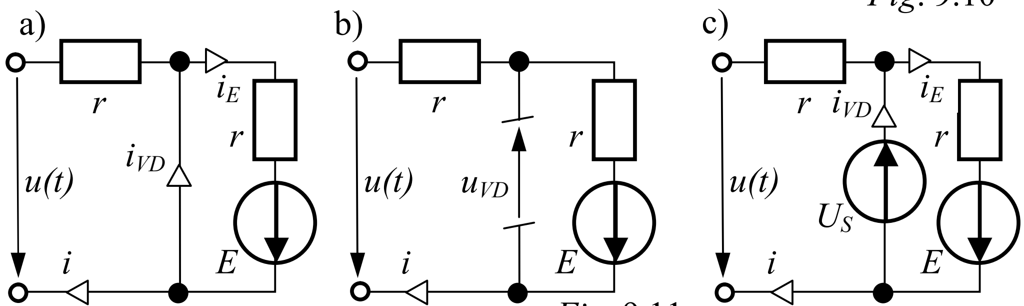


Fig. 9.11

Within the interval $0\le\omega t\le\omega t_1$, the stabilitron is conducting owing to the action of D-C source *E*, which means it works in the segment 1 of VAC. The circuit equivalent scheme for this interval has the view fig. 9.11,a. The circuit currents calculated by this scheme are as follows:

$$i=\frac{u}{r}=\frac{100\sin\omega t}{20}=5sin(\omega t)\ A,\qquad i_E=\frac{E}{r}=\frac{50}{20}=2.5\ A,\qquad i_{VD}=i_E-i=2.5-5sin(\omega t)\ A.$$

We determine the interval boundary from the condition the stabilitron is blocking ($i_{VD}(\omega t_1)=0$): $2.5-5sin(\omega t_1)=0$; $sin(\omega t_1)=0.5$; $\omega t_1=30°=\pi/6\ rad$.

In the interval $\omega t_1\le\omega t\le\omega t_2$, stabilitron is blocking (segment 2 of VAC), scheme takes a view fig. 9.11,b. The required current and voltage across stabilitron are:

$$i=\frac{u+E}{2r}=\frac{100\sin\omega t+50}{40}=2.5sin(\omega t)+1.25\ A,$$

$$u_{VD}=-u+ri=-100\cdot sin(\omega t)+50sin(\omega t)+25=-50sin(\omega t)+25\ V.$$

In order to find the interval boundary ωt_2, we solve the following equations

$u_{VD}(\omega t_2)=0$ – condition when the stabilitron is blocking,

$u_{VD}(\omega t_2)=-U_S$ – condition when the stabilitron becomes stabilized,

$-50sin(\omega t_2)+25=0$; $sin(\omega t_2)=0.5$; $\omega t_2=30°$ or $150°$;

$-50sin(\omega t_2)+25=-10$; $sin(\omega t_2)=0.7$; $\omega t_2=44.4°$ or $135.6°$.

Thus, there are different meanings of ωt_2, and it is necessary to choose the minimal one. However, the meaning $\omega t_2=30°$ does not satisfy the condition $\omega t_2>\omega t_1$, that's why the meaning $\omega t_2=44.4°=0.775\ rad$ has to be chosen.

In the interval $\omega t_2\le\omega t\le\omega t_3$, the stabilitron is stabilized (segment 3 of VAC). Work out the calculation scheme fig. 9.11,c and calculate the current:

$i = \dfrac{u - U_S}{r} = \dfrac{100\sin\omega t - 10}{20} = 5sin(\omega t) - 0.5\ A,\qquad i_E = \dfrac{U_S + E}{r} = \dfrac{10 + 50}{20} = 3\ A,$

$i_{VD} = i_E - i = 3.5 - 5sin(\omega t)\ A.$

We determine the interval boundary ωt_3 from condition the stabilitron is blocking ($i_{VD}(\omega t_3) = 0$): $3.5 - 5sin(\omega t_3) = 0$; $sin(\omega t_3) = 0.7$; $\omega t_3 = 44.4°$ or $135.6°$. However, the meaning $\omega t = 44.4°$ has already been passed, that's why we have to take $\omega t_3 = 135.6° = 2{,}366\ rad$.

In the interval $\omega t_3 \le \omega t \le \omega t_4$, the stabilitron is blocking again (scheme fig. 9.11,b). The required current is $i = 2.5sin(\omega t) + 1.25\ A$.

The condition of passing onto the segment 1 of VAC: $u_{VD}(\omega t_4) = 0$; $\omega t_4 = 30°$ or $150°$. As $\omega t_4 > 135.6°$, the interval boundary is $\omega t_4 = 150° = 5\pi/6\ rad$.

Starting from the moment $\omega t = \omega t_4$ and up to the period end $\omega t_5 = 360°$, the scheme fig. 9.11,a works again. Here $i(\omega t) = 5sin(\omega t)\ A$.

$$\text{Thus, } i(\omega t) = \begin{cases} 5\sin\omega t\ A & if\quad 0 \le \omega t \le \omega t_1 = 30 \quad = \dfrac{\pi}{6} rad, \\ 2.5\sin\omega t + 1.25\ A & if\ \omega t_1 \le \omega t \le \omega t_2 = 44.4 \quad = 0.775\, rad, \\ 5\sin\omega t - 0.5\ A & if\ \omega t_2 \le \omega t \le \omega t_3 = 135.6 \quad = 2.366\, rad, \\ 2.5\sin\omega t + 1.25\ A & if\ \omega t_3 \le \omega t \le \omega t_4 = 150 \quad = \dfrac{5\pi}{6} rad, \\ 5\sin\omega t\ A & if\quad \omega t_4 \le \omega t \le 360 \quad = 2\pi\, rad. \end{cases}$$

Current $i(\omega t)$ is plotted in fig. 9.12.

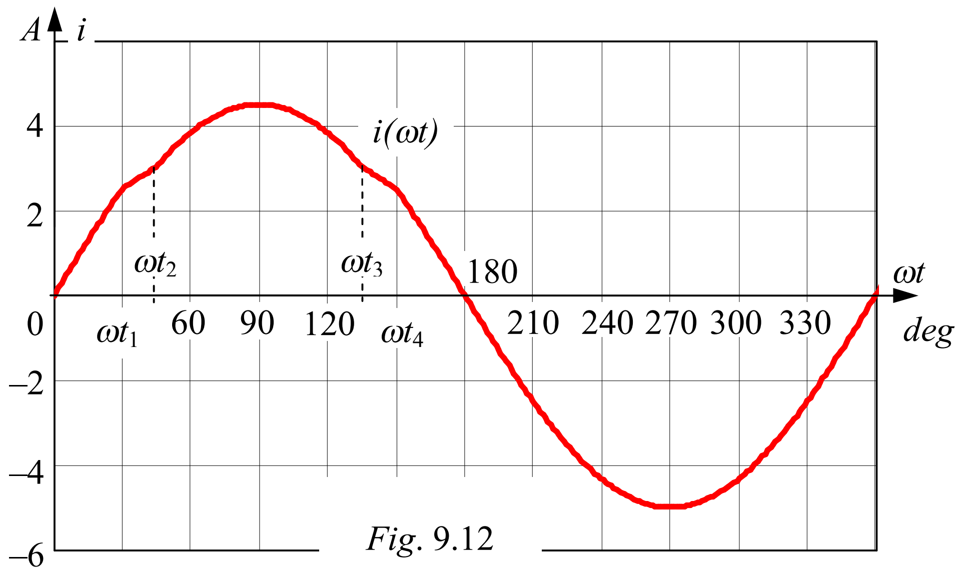


Fig. 9.12

The current effective value may be calculated under the formula

$$I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i(\omega t)^2\, d\omega t}\ .$$

As the diagram $i(\omega t)$ is symmetrical concerning the vertical axes drawn at $\omega t = ½\pi$ and $\omega t = \dfrac{3\pi}{2}$, the integration may be performed for but the half-period:

$$I = \sqrt{\frac{1}{\pi}\int_{\pi/2}^{3\pi/2} i(\omega t)^2 \, d\omega t} = 3.36\ A.$$

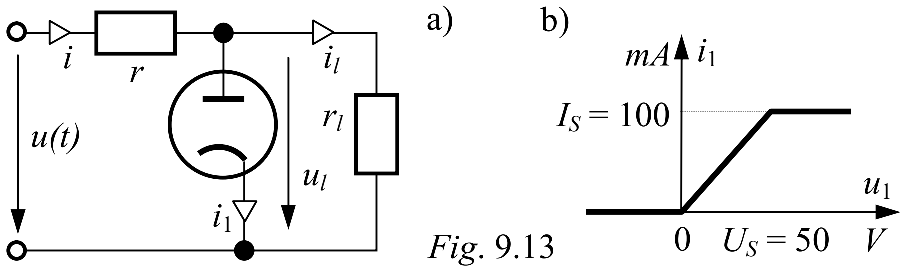


Fig. 9.13

9-17 (9.60). Plot the current through a load $r_l = 5\ kOhm$ (fig. 9.13,a) in a circuit with the diode tube, its VAC being approximated by the segments fig. 9.13,b. Circuit is supplied with the voltage $u = 320 \sin\omega t\ V$; $r = 1\ kOhm$.

Solution. Let's consider the interval $\omega t(0 \ldots \omega t_1)$, here $u > 0$ and the diode works in the slopping segment of VAC in the current range $i_1(0 \ldots I_S)$. While working within this range, the diode may be presented by linear resistance $r_{dif} = \frac{U_S}{I_S} = \frac{50}{0.1} = 500\ Ohm$ (scheme fig. 9.14,a).

According to the scheme fig. 9.14,a $\quad i_l = \frac{U_m \sin\omega t}{r + \frac{r_{dif} r_l}{r_{dif} + r_l}} \cdot \frac{r_{dif}}{r_{dif} + r_l} = 20 \sin\omega t\ mA,$

the voltage across diode and load is $\quad u_l = i_l \cdot r_l = 100 \sin\omega t\ V.$

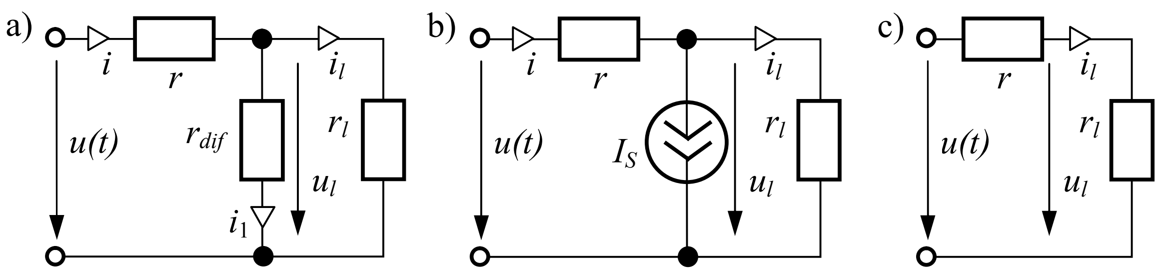


Fig. 9.14

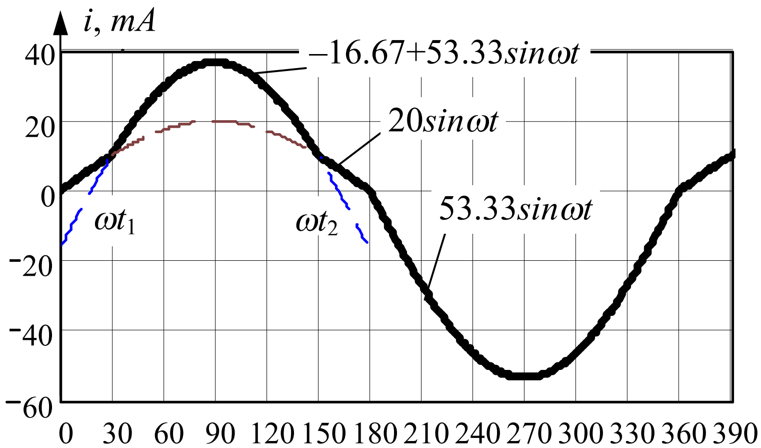


Fig. 9.15 $\qquad \omega t$, *deg*

The diode becomes saturated at the time moment t_1, when the current reaches the value I_S, while the voltage – value U_S:

$$U_S = u_l(\omega t_1) = 100 \sin\omega t_1,$$

from here

$$\omega t_1 = \arcsin\frac{U_S}{U_{lm}} = \arcsin\frac{50}{100} = 30^{\circ}.$$

The diode is out of the saturated state at the moment

$$\omega t_2 = 180^{\circ} - \omega t_1 = 150^{\circ}.$$

While the diode is in the saturation state, the calculation scheme takes a view fig. 9.14,b, the load current in interval $\omega t(\omega t_1 \ldots \omega t_2)$ is

$$i_l = -I_S\frac{r}{r+r_l} + \frac{U_m \sin\omega t}{r+r_l} = -16.67 + 53.33\sin\omega t\ mA.$$

In the interval $\omega t(\pi \ldots 2\pi)$ the diode is blocking, the corresponding scheme is in fig. 9.14,c; here $i_l = \dfrac{U_m \sin\omega t}{r+r_l} = 53.33\sin\omega t\ mA.$

The load current is plotted in fig. 9.15.

9.3. CALCULATION BY CHARACTERISTICS FOR EFFECTIVE VALUES

9-18 (9.65). Two tests are carried out in order to determine the steel-core coil parameters: 1) with a core: $U=100\ V$, $f=50\ cps$, $I=1.25\ A$, $\cos\varphi=0.2$; 2) without a core: $U=32\ V$, $I=4\ A$, $P=51.2\ W$. Construct an equivalent scheme, determine its parameters as well as the steel loss.

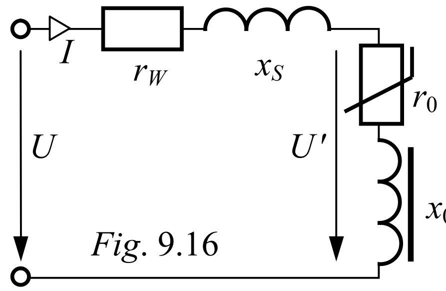


Fig. 9.16

Solution. A series equivalent scheme of the real coil with a ferromagnetic core is given in fig. 9.16.

The results of the test without a core are:

$$x_0 = 0,\ r_0 = 0,\ r_W = \frac{P}{I^2} = 3.2\ Ohm,\ \ Z = \frac{U}{I} = 8\ Ohm,$$

$$x_S = \sqrt{Z^2 - r_W^2} = 7.33\ Ohm.$$

The results of the test with a core are:

$$Z = U/I = 80\ Ohm,\quad R = Z\cdot\cos\varphi = 16\ Ohm,\quad r_0 = R - r_W = 12.8\ Ohm,$$

$$x_0 = \sqrt{Z^2 - R^2} - x_S = 71.05\ Ohm,$$

$$P_{st} = r_0\cdot I^2 = 20\ W \quad \text{or} \quad P_{st} = UI\cos\varphi - r_W I^2 = 20\ W.$$

9-19 (9.66). A steel-core coil is supplied from the A-C network, $f=50\ cps$. The instrument readings are: $U=120\ V$, $I=4\ A$, $P=280\ W$, $r_W=2.5\ Ohm$. Determine the self-induction emf, $\cos\varphi$, the steel loss. The leakage flux is negligible ($x_S=0$).

Solution. $R = \dfrac{P}{I^2} = 17.5\ Ohm,\quad r_0 = R - r_W = 15\ Ohm,\quad Z = \dfrac{U}{I} = 30\ Ohm,$

$$x_0 = \sqrt{Z^2 - R^2} = 24.4\ Ohm,\quad \cos\varphi = \frac{R}{Z} = 0.583,\quad P_{st} = r_0\cdot I^2 = 240\ W,$$

$$E = U' = \sqrt{r_0^2 + x_0^2}\cdot I = 115\ V.$$

9-20 (9.67). The choke current at $U=200\ V$, $f=50\ cps$ is equal to $5\ A$, furthermore, $P=300\ W$, $w=600$, $r_W=6\ Ohm$, $\Phi_m=1.2\ mWb$. Determine the equivalent scheme parameters as well as the steel loss and copper loss.

Solution. $Z = \dfrac{U}{I} = 40\ Ohm,\quad R = \dfrac{P}{I^2} = 12\ Ohm,\ r_0 = R - r_W = 6\ Ohm,$

$$P_W = r_W\cdot I^2 = 150\ Wm,\quad P_{st} = r_0\cdot I^2 = 150\ W,\ U' = \Phi_m\cdot 4.44 fW = 160\ V,$$

$$Z_0 = \frac{U'}{I} = 32\ Ohm,\quad x_0 = \sqrt{Z_0^2 - r_0^2} = 31.4\ Ohm,$$

$$x = \sqrt{Z^2 - R^2} = 38.2\ Ohm,\quad x_S = x - x_0 = 6.76\ Ohm.$$

9-21 (9.68). The choke magnetic core is made of steel Э42, $w = 410$, $r_W = 15\ Ohm$, $x_S = 10\ Ohm$. Determine the supply voltage of a choke ($f = 50\ cps$) to obtain the magnetic induction $B_m = 1.1\ T$ in value inside the core, moreover, $S = 20\ cm^2$, $\gamma = 7.8\cdot10^{-3}\ kg/cm^3$. At $B_m = 1.1\ T$ $H_m = 300\ A/m$, $P_0 = 1.46\ W/kg$, $Q_0 = 6.5\ VAr/kg$, the core mass is $G = 15.6\ kg$, $\xi = 1.05$.

Solution. $P_{st} = P_0\cdot G = 22.8\ W$, $Q_{st} = Q_0\cdot G = 101.4\ VAr$.

The core length is $l = \dfrac{G}{\gamma S} = 1\ m$.

Maximum current is $I_m = \dfrac{H_m l}{w} = 0.732\ A$, effective current is $I = \dfrac{I_m}{\sqrt{2}\xi} = 0.493\ A$.

The parameters of the series equivalent scheme are

$$r_0 = \frac{P_{st}}{I^2} = 93.9\ Ohm, \quad x_0 = \frac{Q_{st}}{I^2} = 417.6\ Ohm,$$

$$Z = \sqrt{(r_W + r_0)^2 + (x_S + x_0)^2} = 441\ Ohm.$$

The voltage across the choke is $U = Z\cdot I = 218\ V$.

9-22 (9.69). Two tests are carried out with a steel-core coil: with D-C source – $U_- = 20\ V$, $I_- = 10\ A$; with A-C source – $f = 50\ cps$, $U_\sim = 100\ V$, $I_\sim = 5\ A$, $P = 100\ W$.

Determine the steel loss and copper loss, the equivalent scheme parameters, power factor, loss angle. Neglect the leakage flux ($\Phi_S = 0$).

Solution. $r_W = U_-/I_- = 2\ Ohm$, $Z = \dfrac{U_\sim}{I_\sim} = 20\ Ohm$, $R = \dfrac{P}{I_\sim^2} = 4\ Ohm$, $r_0 = R - r_W = 2\ Ohm$,

$$x_0 = \sqrt{Z^2 - R^2} = 19.60\ Ом. \quad P_{st} = r_0\cdot I_\sim^2 = 50\ W, \quad P_W = r_W\cdot I_\sim^2 = 50\ W,$$

$$cos\varphi = \frac{R}{Z} = 0.2, \quad \text{loss angle } \delta = arctg\frac{g_0}{b_0} = arctg\frac{r_0}{x_0} = arctg\frac{2}{19.6} = 5.83°.$$

9-23 (9.70). The primary winding of a transformer (fig. 9.17) is supplied with the voltage $U_1 = 220\ V$ of frequency $f = 150\ cps$. The transformer core is made of the insulated sheets of steel 1512 with thickness $\Delta = 0.5\ mm$, the thickness of an insulation layer between sheets is $\delta = 0.01\ mm$. The core sizes are as follows: $a = 500\ mm$, $b = 50\ mm$, $c = 60\ mm$, $h = 600\ mm$. The number of winding turns is $w_1 = 75$, $w_2 = 35$. The air gap is $l_a = 0.5\ mm$.

It is required to calculate the voltage across the open secondary winding as well as the primary current under the condition the leakage flux is 5% of the main magnetic flux, resistance of the primary winding being $r_W = 0.461\ Ohm$.

Determine the parameters of both series and series-parallel equivalent schemes of the coil.

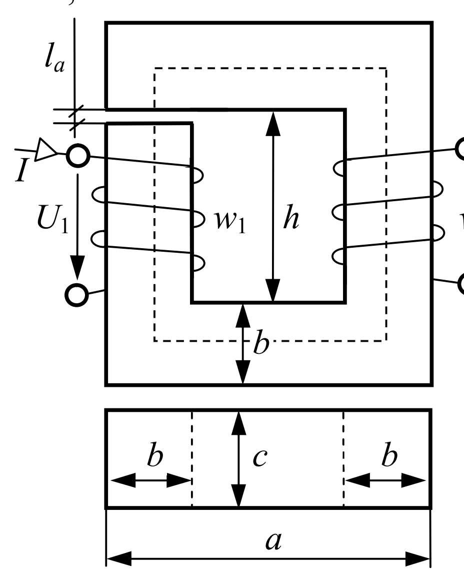


Fig. 9.17

Solution. Calculate the mean path length of the core l as well as the steel cross-section S.

$l = 2\cdot(a + h) = 2\cdot(500 + 600) = 2200\ mm = 2.2\ m$,

$S = b \cdot c \cdot k_f$, where the filling factor is $k_f = \frac{\Delta}{\Delta + \delta} = \frac{0.5}{0.5 + 0.01} = 0.98$;

$$S = 50 \cdot 60 \cdot 0.98 = 2941\ mm^2 = 2.941 \cdot 10^{-3}\ m^2.$$

The secondary winding is open and current does not flow through it, so it has no influence upon the primary current which is calculated by the step-by-step method.

Assume the component of voltage U_1 compensating EMF of the main magnetic flux is equal to $U' = 0.96U_1 = 211.2\ V$.

Then the amplitudes of the main magnetic flux and magnetic induction are as follows

$$\Phi_m = \frac{U'}{4.44 f w_1} = \frac{211.2}{4.44 \cdot 150 \cdot 75} = 4.23 \cdot 10^{-3}\ Wb, \quad B_m = \frac{\Phi_m}{S} = \frac{4.23 \cdot 10^{-3}}{2.94 \cdot 10^{-3}} = 1.44\ T.$$

The magnetization curve of steel 1512 (table 2.10) gives the maximum value of the magnetic intensity for $B_m = 1.44\ T$:

$$H_m = 14.26\ A/cm = 1426\ A/m.$$

Kirchhoff's voltage law for the magnetic circuit gives $H_m \cdot l + H_{am} \cdot l_a = I_{rm} \cdot w_1$, where $H_{am} = B_m / \mu_0$ – maximal value of the magnetic intensity in the air gap.

The maximum value of the current reactive component is as follows

$$I_{rm} = \frac{H_m \cdot l + H_{am} \cdot l_a}{w_1}.$$

In order to find its effective value we use the factor ξ which takes into account the unsinusoidality of the current curve resulting from the steel saturation. The factor ξ dependes on the magnetic induction amplitude and if $B_m = 1.44\ T$ it is equal to $\xi = 1.39$.

Then $$I_r = \frac{\frac{H_m \cdot l}{\xi} + \frac{B_m}{\mu_0} \cdot l_a}{\sqrt{2} \cdot w_1} = \frac{\frac{1426 \cdot 2.2}{1.39} + \frac{1.44 \cdot 5 \cdot 10^{-4}}{4\pi \cdot 10^{-7}}}{\sqrt{2} \cdot w_1} = 26.68\ A.$$

Active current component is determined through the steel loss, it being calculated under the formula $$P_{st} = p_{1/50} \cdot B_m^2 \left(\frac{f}{50}\right)^{1.3} M,$$

where $p_{1/50}$ – specific loss (per 1 *kg*) at the magnetic induction amplitude $B_m = 1\ T$ and frequency $f = 50\ cps$; in case of steel 1512 at the sheet thickness $\Delta = 0.5\ mm$ it is $3.3\ W/kg$; M – the core mass.

$M = g \cdot S \cdot l$, where g – specific gravity of steel, which is $7.8\ g/cm^3$.

$$M = 7.8 \cdot 29.41 \cdot 220 = 50470\ g = 50.47\ kg; \quad P_{st} = 3.3 \cdot 1.44^2 \cdot \left(\frac{150}{50}\right)^{1.3} \cdot 50.47 = 1436\ W.$$

The active current component is $I_a = \frac{P_{st}}{U'} = \frac{1436}{211.2} = 6.80\ A.$

The total current is $I = \sqrt{I_a^2 + I_r^2} = \sqrt{6.80^2 + 26.68^2} = 27.53\ A.$

The loss angle is $\alpha = arctg \frac{I_a}{I_r} = arctg \frac{6.80}{26.68} = 14.3°.$

Let's check correctness of U'. For this in accordance with fig. 9.18, we find the calculated value of the input voltage

$$U_{calc} = |\underline{I}(r_W + jx_S) + \underline{U}'|, \quad \text{here we assume } \underline{U}' = U' = 211.2\ V.$$

Then $\underline{I} = I\cdot e^{-j(90° - \alpha)} = 27.53\cdot e^{-j75.7°}\ A.$

Let's calculate x_S. In accordance with the problem task, $\Phi_S = 0.05\Phi$, as such

$$L_S = \frac{w_1\cdot\Phi_S}{I} = \frac{0.05\cdot w_1\cdot\Phi_m}{\sqrt{2}\cdot I} = \frac{0.05\cdot 75\cdot 4.23\cdot 10^{-3}}{\sqrt{2}\cdot 27.53} = 4.07\cdot 10^{-4}\ H.$$

The leakage reactance is $x_S = \omega\cdot L_S = 942\cdot 4.07\cdot 10^{-4} = 0.384\ Ohm.$

Finally, we have

$$U_{calc} = |\underline{I}(r_W + jx_S) + \underline{U}'| = |27.53\cdot e^{-j75.7°}\cdot(0.461 + j0.384) + 211.2| = 224.8\ V.$$

The calculation error is $\dfrac{U - U_{calc}}{U}\cdot 100\% = \dfrac{220 - 224.8}{220}\cdot 100\% = -2.18\ \%.$

It allows considering the current calculation to be correct.

Determine the equivalent scheme parameters. For a series scheme (fig. 9.16)

$$r_0 = \frac{P_{st}}{I^2} = \frac{1436}{27.53^2} = 1.89\ Ohm, \quad z_0 = \frac{U'}{I} = \frac{211.2}{27.53} = 7.67\ Ohm,$$

$$x_0 = \sqrt{z_0^2 - r_0^2} = \sqrt{7.67^2 - 1.89^2} = 7.43\ Ohm.$$

For a series-parallel scheme (fig. 9.18):

$$g_0 = \frac{I_a}{U'} = \frac{6.80}{211.2} = 0.032\ S, \quad b_0 = \frac{I_r}{U'} = \frac{26.68}{211.2} = 0.126\ S,$$

$$Y_0 = \frac{I}{U'} = \frac{27.53}{211.2} = 0.13\ S.$$

The voltage across the secondary winding is

$$U_2 = w_2\ \frac{d\Phi}{dt}, \text{ however } U' = w_1\ \frac{d\Phi}{dt}.$$

From here, we have

$$U_2 = \frac{w_2}{w_1}\cdot U' = \frac{35}{75}\cdot 211.2 = 98.6\ V.$$

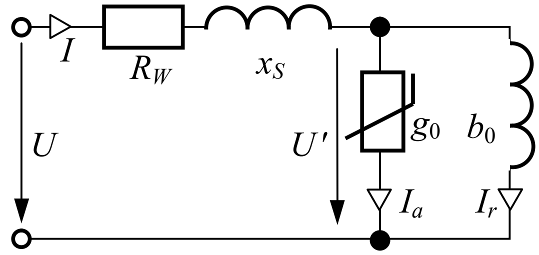


Fig. 9.18

9-24 (9.71). The volt-ampere characteristic of a steel-core choke at frequency $f = 50$ *cps* for effective values is approximated in definite range by the following expression

$$U_L = 200I - 15I^3, \quad \text{where} \quad U_L[V],\ I[A],$$

the choke being connected in series with linear capacitor of capacity 20 μF.

Having neglected the steel loss, the copper loss and the capacitor loss, determine the current value for the following cases: a) the voltage resonance condition, b) the input voltage reaches the maximum value while the whole circuit is inductive.

Determine in addition: 1) the minimum capacitance which does not allow reaching the resonance by varying the source voltage or current; 2) the maximum capacitance which does not allow using the given approximation to calculate the resonance current.

Answer: a) $U_L - U_C = 200I - 15I^3 - \dfrac{I}{2\pi fC} = 0, \quad I = 1.65\ A;$

b) $\dfrac{d(U_L - U_C)}{dI} = 40.8 - 45I^2 = 0, \quad I = 0.952\ A;$

1) $x_L(0) = \dfrac{dU_L}{dI}(0) = 200 - 45\cdot I^2 = 200\ Ohm = x_{Cmax}, \quad C_{min} = \dfrac{1}{2\pi f x_{C\,max}} = 15.9\ \mu F;$

2) $\dfrac{dU_L}{dI} = 200 - 45\cdot I^2 = 0,\ \ I = 2.11\ A,\ \ x_{Cmin} = x_L(2.11) = \dfrac{U_L}{I}(2.11) = 133.3\ Ohm,$

$$C_{max} = \frac{1}{2\pi f x_{C\,min}} = 23.89\ \mu F.$$

10. TRANSIENT PROCESSES IN NONLINEAR ELECTRIC CIRCUITS

10-1 (10.1). In scheme fig. 10.1, there are a source of sinusoidal current $j(t) = 0.5sin(500t+\psi)\ A$, two identical resistances $r_0 = r = 10\ kOhm$ and a varicap (voltage variable capacitor) $C(u)$. The transient process happens when closing the circuit breaker. The coulomb-volt characteristic of the varicap is presented in table 10.1.

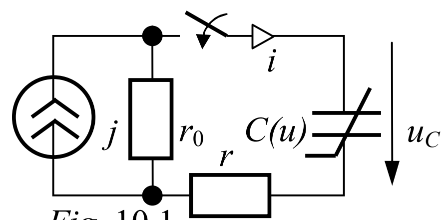


Fig. 10.1

Table 10.1

q, mC	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1
u_C, V	0	3	7	11	16	20	25	32	45.5	76	143	250

It is required to determine using the linearization method how many times the voltage across the varicap during the hardest transient process exceeds its amplitude in steady-state condition.

Solution. The circuit r, $C(u)$ is at rest before the commutation, that's why we deal with zero independent initial conditions $q(0_+) = 0,\ \ u_C(0_+) = 0$.

In a steady-state condition, the circuit state obeys to the equation under Kirchhoff's voltage law

$$i_s(r + r_0) + u_{Cs} = jr_0.$$

Assume the following inequality is true $i_s(r + r_0) \gg u_{Cs}$ (we will prove it later). Then

$$i_s = \frac{r_0}{r + r_0} j = 0.25sin(500t + \psi)\ A.$$

The steady-state value of the varicap charge is

$$q_s = \int i_s dt = \frac{-0.25}{500}\cdot cos(500t + \psi) = -0.5\cdot 10^{-3}\cdot cos(500t + \psi)\ C.$$

By the data from table 10.1, we plot the varicap CVC (fig. 10.2) and determine the point A of the steady-state condition by the amplitude value of the charge $q_{sm} = 0.5\ mC$. The amplitude value of the varicap voltage corresponding to the obtained point is $U_{Cm} = 20\ V$.

Let's replace the CVC of the varicap with the direct line in accordance with the linearization method, the direct line passing hrough point A, its equation is

$$q = Cu, \tag{10.1}$$

where $C = \dfrac{q_{sm}}{U_{Cm}} = \dfrac{0.5\cdot 10^{-3}}{20} = 25\cdot 10^{-6}\ F = 25\ \mu F$ – varicap capacity in point A.

The capacitor reactance is $\quad x_C = \dfrac{1}{\omega C} = \dfrac{1}{500\cdot 25\cdot 10^{-6}} = 80\ Ohm,$

i.e. relationship $i_s(r + r_0) \gg u_{Cs}$ is really true.

The circuit transient process is described by the equation

$$i(r + r_0) + u_C = jr_0, \tag{10.2}$$

moreover $i = \dfrac{dq}{dt}$, while from (10.1) $u_C = \dfrac{q}{C}$. (10.3)

Substituting (10.3) into (10.2), we obtain $C(r + r_0)\dfrac{dq}{dt} + q = jCr_0$. (10.4)

The solution of (10.4) is $q = q_s + q_t = -0.5 \cdot 10^{-3} \cdot cos(500t + \psi) + B \cdot e^{-\frac{t}{\tau}}$, (10.5)

where $\tau = C(r + r_0)$ – the circuit time constant;

B – the integration constant, which is determined from the condition $q(0) = 0$.

From (10.5), we obtain $B = 0.5 10^{-3} cos(\psi)$. Then finally we have

$$q = q_s + q_t = -0.5 \cdot 10^{-3} \cdot cos(500t + \psi) + 0.5 \cdot 10^{-3} \cdot cos(\psi) \cdot e^{-\frac{t}{C(r + r_0)}}. \quad (10.6)$$

Analyzing (10.6), we conclude that the hardest transient process is observed at $\psi = 0$ while the maximum value of charge q_{max} during the transient process occurs after the half-period has passed. From (10.6), we obtain

$$q_{max} = -0.5 \cdot 10^{-3} \cdot cos(\pi) + 0.5 \cdot 10^{-3} \cdot e^{-\frac{\pi}{\omega C(r + r_0)}} =$$

$$= 0.5 \cdot 10^{-3} \cdot (1 + e^{-\frac{\pi}{500 \cdot 25 \cdot 10^{-6} \cdot (10 + 10) \cdot 10^3}}) = 0.994 10^{-3}\ C = 0.994\ mC.$$

Using the varicap's CVC, we find the maximum voltage across the varicap during transients corresponding to value q_{max}: $u_{Cmax} = 250\ V$.

So, $\dfrac{u_{C\,max}}{U_{Cm}} = \dfrac{250}{20} = 12.5.$

Note that in a linear circuit the maximum overvoltage cannot exceed 2.

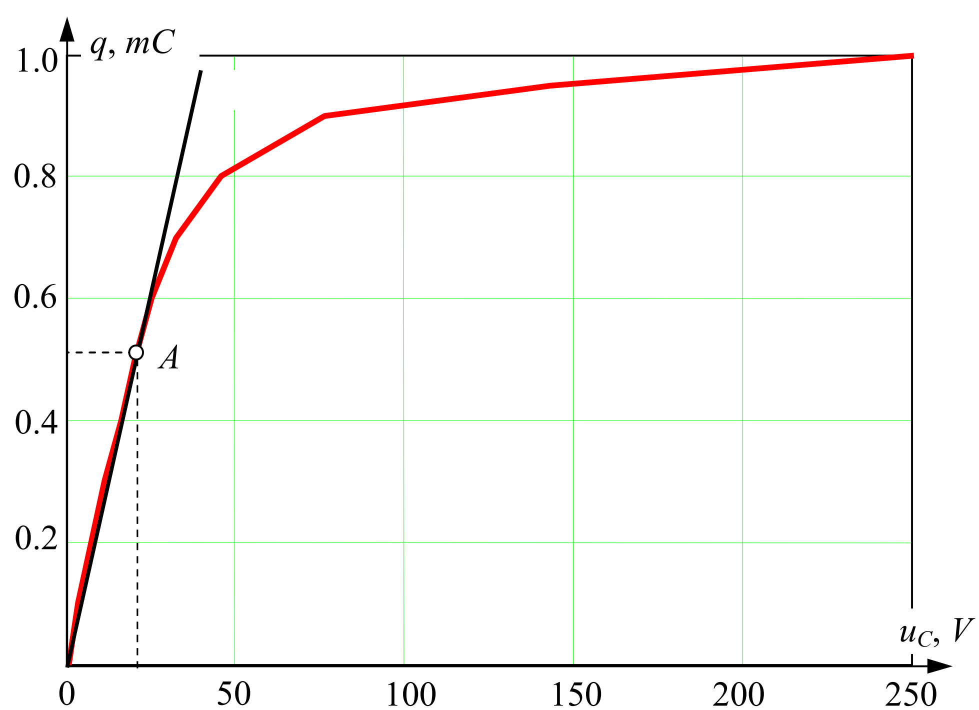


Fig. 10.2

10-2 (10.2). In the scheme fig. 10.3 containing a non-linear resistance (NE), the transient process happens when switching off the 3[rd] branch. NE characteristic is set by table 10.2.

Table 10.2

u_1, *V*	0	9	15	17.5	19	20.5	21.8	23	24	25	26	27	28
i_1, *A*	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.4	2	2.6

Having approximated the NE characteristic with two straight line segments, determine the instantaneous current through NE as well as the voltage across capacitor *C*. By the calculation results, plot the time dependences of the required quantities, if $U = 100$ *V*, $r_2 = 5$ *Ohm*, $r_3 = 52.9$ *Ohm*, $C = 200$ μF.

Solution. By the data of table 10.2, we plot $u_1(i_1)$ (NE VAC), it being presented in fig. 10.4.

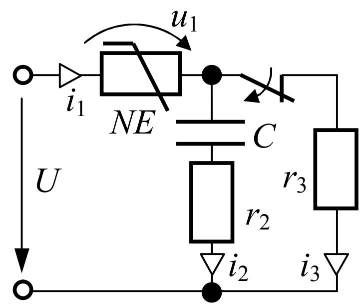


Fig. 10.3

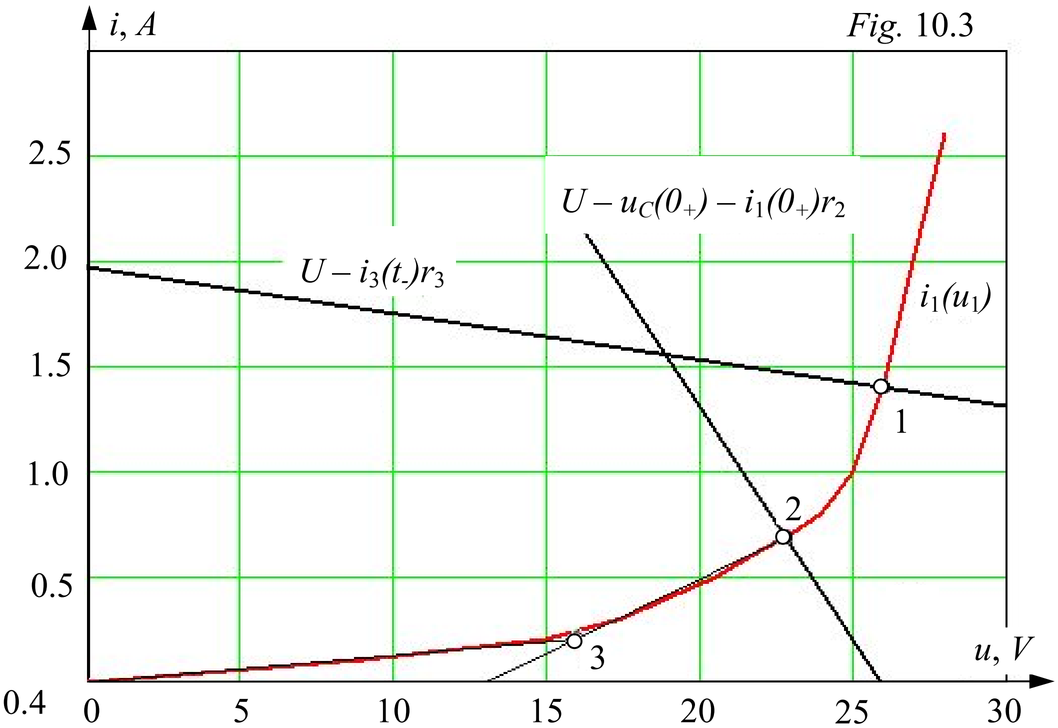


Fig. 10.4

Before the commutation $i_2(t_-) = 0$ (capacitor does not allow the direct current flowing), correspondingly, $i_1(t_-) = i_3(t_-)$, that's why the point (1) of the steady-state condition before the commutation is determined graphically in accordance with equation $u_1(t_-) + r_3 i_3(t_-) = U$.

From fig. 10.4, we obtain $i_1(t_-) = i_3(t_-) = 1.4$ *A*, $u_C(t_-) = r_3 i_3(t_-) = 52.9 \cdot 1.4 = 74$ *V*.

At the commutation moment, u_C keeps its value in accordance with the second commutation law, i.e. $u_C(0_+) = 74$ *V*.

On completing the transients, there are no currents in the circuit because of the capacitor; it means the point of the steady-state condition on NE VAC is in the coordinate origin, while the steady-state value of the voltage across *C* is $u_{Cs} = U = 100$ *V*.

At the commutation moment, the operating point (point 2) on NE VAC shifts in accordance with the expression

$$u_1(0_+) + u_C(0) + i_1(0_+) r_2 = U \quad \text{or} \quad u_1(0_+) + i_1(0_+) r_2 = 26.$$

From fig. 10.4, we obtain: $i_1(0_+) = i_1^{(2)} = 0.67\ A$; $u_1(0_+) = u_1^{(2)} = 22.7\ V$.

The operating part of NE VAC is approximated with two straight line segments: 2-3 and 3-0.

The coordinates of point 3 are: $i_1^{(3)} = 0.18\ A$, $u_1^{(3)} = 15.8\ V$.

The analytic expressions for segments 2-3 and 3-0 are

$$u_1 = 13.3 + i_1 r_{dif1}, \quad u_1 = i_1 r_{dif2},$$

where the differential resistances r_{dif1} and r_{dif2}

$$r_{dif1} = \frac{u_1^{(2)} - u_1^{(3)}}{i_1^{(2)} - i_1^{(3)}} = \frac{22.7 - 15.8}{0.67 - 0.18} = 14.08\ Ohm; \qquad r_{dif2} = \frac{u_1^{(3)}}{i_1^{(3)}} = \frac{15.8}{0.18} = 87.8\ Ohm.$$

In order to compute the transients, let's generate the circuit differential equations for post-commutation condition

$$u_1 + u_C + i_1 r_2 = U; \qquad i_1 = C\frac{du_C}{dt} \quad \text{or} \quad u_1 + u_C + Cr_2 \cdot \frac{du_C}{dt} = U. \qquad (10.7)$$

When NE operated in segment 2-3 of VAC, the equation (10.7) takes a view

$$13.3 + i_1 r_{dif1} + u_C + i_1 r_2 = U; \quad \text{or} \quad u_C + C(r_2 + r_{dif1}) \cdot \frac{du_C}{dt} = U - 13.3. \qquad (10.8)$$

The solution of the equation (10.8) is as follows $u_C = U - 13.3 + A_1 e^{p_1 t}$, where the root of the characteristic equation is

$$p_1 = -\frac{1}{(r_2 + r_{dif1}) \cdot C} = -\frac{1}{(5 + 14.08) \cdot 200 \cdot 10^{-6}} = -262\ s^{-1}.$$

We determine the integration constant A_1 from the condition that at $t = 0$ voltage $u_C(0_+) = 74\ V$, i.e. $A_1 = u_C(0) - U + 13.3 = 74 - 86.7 = -12.7\ V$.

The final solutions for voltage u_C and the circuit current i when NE operates in segment 2-3 are $u_C = 86.7 - 12.7 \cdot e^{-262t}\ V$,

$$i = \frac{du_C}{dt} = 200 \cdot 10^{-6} \cdot (-12.7) \cdot (-262) \cdot e^{-262t} = 0.67 \cdot e^{-262t}\ A. \qquad (10.9)$$

When NE operates in segment 3-0, the equation (10.7) takes a form

$$i_1 r_{dif2} + u_C + i_1 r_2 = U; \quad \text{or} \quad u_C + C(r_2 + r_{dif2}) \cdot \frac{du_C}{dt} = U. \qquad (10.10)$$

The solution of the equation (10.10) has a view $u_C = u_{Cs} + A_2 e^{p_2(t - t_1)}$, (10.11)
where the root of the characteristic equation is

$$p_2 = -\frac{1}{(r_2 + r_{dif2}) \cdot C} = -\frac{1}{(5 + 87.8) \cdot 200 \cdot 10^{-6}} = -53.9\ s^{-1},$$

and t_1 – time moment, when the point passes from segment 2-3 onto segment 3-0.

Determine t_1 from a condition that at $t = t_1$ the equation (10.9) has to give the result $i^{(3)} = 0.18\ A$, i.e. $0.18 = 0.67 \cdot e^{-262 t_1}$,

from here $t_1 = \dfrac{ln\dfrac{0.67}{0.18}}{262} = 5.0 \cdot 10^{-3}\ s = 5.0\ ms$.

We determine the integration constant A_2 from the condition that at $t = t_1$ the equations (10.9) and (10.11) for u_C have to give the same result, i.e.

$$86.7 - 12.7\cdot e^{-262\cdot 5\cdot 10^{-3}} = 100 + A_2,$$

from here $\qquad A_2 = 86.7 - 12.7\cdot e^{-262\cdot 5\cdot 10^{-3}} - 100 = -16.7\ V.$

The final answers for u_C and the circuit current i when NE operates in segment 3-0 are

$$u_C = 100 - 16.7\cdot e^{-53.9(t-t_1)}\ V,$$

$$i = C\frac{du_C}{dt} = 200\cdot 10^{-6}\cdot(-16.7)\cdot(-53.9)\cdot e^{-53.9(t-t_1)} = 0.18\cdot e^{-53.9(t-t_1)}\ A. \qquad (10.12)$$

The required plots are constructed under (10.9) and (10.12) with application of program MathCAD, they being presented in fig. 10.5.

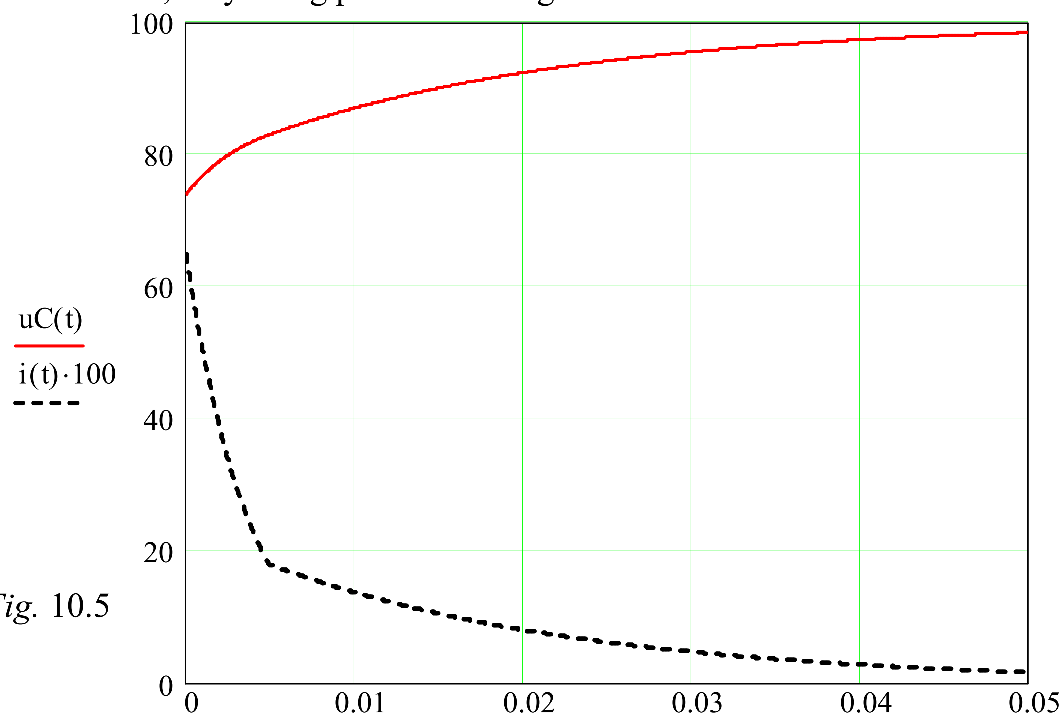


Fig. 10.5

10-3 (10.3). Determine the time dependence of the diode current in scheme fig. 10.6 during the transient process, if $J = 10\ mA$, $C = 5\ \mu F$ and the diode VAC is set by table 10.3. Additionally, find the time t_1 from the condition $u(t_1) = 0.8\ V$.

Table 10.3

i, mA	0	0.2	0.6	1	2.5	3.4	6	10
u, V	0	0.1	0.2	0.3	0.5	0.6	0.8	1

Solution. Let's apply the analytical approximation method; that is we express the diode characteristic by means of parabola $i = ku^2$. By scheme, we have $u_C = u$. In accordance with the second commutation law, $u_C(0_+) = u_C(0_-) = 0$. Consequently, the initial operating point of the diode is $u(0_+) = 0$, $i(0_+) = 0$ – the coordinate origin. Final operating point is $i_s = J = 10\ mA$, $u_s = 1\ V$. Approximate parabola has to pass through the final point, i.e.

$$i_s = J = ku_s^2, \qquad k = \frac{J}{u_s^2} = \frac{10\cdot 10^{-3}}{1^2} = 10^{-2}\ A/V^2.$$

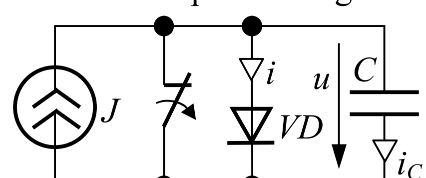


Fig. 10.6

After commutation,

$$i_C = C\frac{du}{dt}; \quad J = i_C + i = C\frac{du}{dt} + ku^2;$$

$$\frac{J-ku^2}{C}=\frac{du}{dt}; \quad dt=\frac{Cdu}{J-ku^2}=\frac{C}{J}\cdot\frac{du}{1-\frac{k}{J}u^2}=\frac{C}{J}\cdot\frac{du}{1-\left(\frac{u}{u_s}\right)^2}=\frac{Cu_s}{J}\cdot\frac{du/u_s}{1-\left(\frac{u}{u_s}\right)^2}.$$

Let's mark $\tau_{equ}=\frac{Cu_s}{J}=\frac{5\cdot10^{-6}\cdot1}{10\cdot10^{-3}}=0.5\cdot10^{-3}\ s.$

Taking into account that $\int\frac{dx}{1-x^2}=Arth(x)$, we obtain

$$t=\tau_{equ}\cdot Arth\left(\frac{u}{u_s}\right); \quad u=u_s\cdot th\left(\frac{t}{\tau_{equ}}\right); \quad i=ku^2=i_s\cdot th^2\left(\frac{t}{\tau_{equ}}\right).$$

We find t_1 from condition $u(t_1)=0.8\ V$: $\quad t_1=\tau_{equ}\cdot Arth(0.8)=0.549\cdot10^{-3}\ s.$

10-4 (10.4). Solve problem 10.3 using the step-by-step method. Compare the newly obtained result with the solution of problem 10.3.

Solution. The original equation for computation is

$$C\frac{du}{dt}+i=J \qquad \text{or} \qquad \frac{du}{dt}=\frac{1}{C}(J-i).$$

Replacing dt with Δt, we obtain the expression for the voltage increment at k-th interval $\quad \Delta u_k=\frac{\Delta t}{C}(J-i_{k-1}).$

We assume the current at the beginning of k-th interval to be equal to the current at the end of $(k-1)$-th interval. It allows passing from one interval to the subsequent one.

Assume Δt. It is recommended to take it in the range $(0.1\div0.2)\tau_{equ}$. Assume $\Delta t=0.1\,\tau_{equ}=5\cdot10^{-5}\ s=50\ \mu s$.

The circuit state in the commutation moment is: $u(0_+)=0$, $i(0_+)=0$.

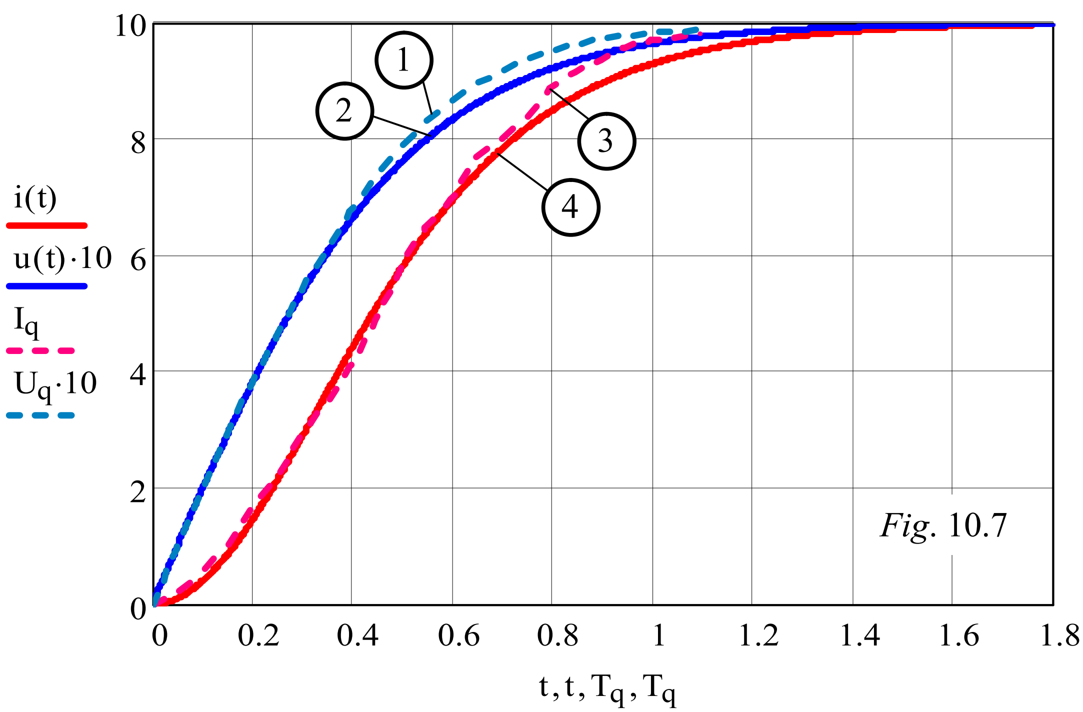


Fig. 10.7

The calcultion results of the transient process under the obtained formulae are given in table 10.4.

Based on the data in table 10.4 in fig. 10.7 the curves *u(t)* (curve 1) and *i(t)* (curve 3) are drawn with the application of the program MathCAD. The same dependencies obtained in problem 10.3 (curves 2 and 4, respectively) are also given there.

Table 10.4

k	t, *ms*	$i_{k\text{-}1}$, *mA*	Δu_k, *V*	$u_k = u_{k\text{-}1} + \Delta u_k$, *V*	i_k, *mA*
0	0	-	-	0	0
1	0.05	0	0.1	0.1	0.2
2	0.1	0.2	0.098	0.198	0.58
3	0.15	0.58	0.094	0.292	0.98
4	0.2	0.98	0.09	0.382	1.7
5	0.25	1.7	0.083	0.465	2.2
6	0.3	2.2	0.078	0.543	2.95
7	0.35	2.95	0.071	0.614	3.5
8	0.4	3.5	0.065	0.679	4.1
9	0.45	4.1	0.059	0.738	4.9
10	0.5	4.9	0.051	0.789	5.9
11	0.55	5.9	0.041	0.83	6.5
12	0.6	6.5	0.035	0.865	7
13	0.65	7	0.03	0.895	7.7
14	0.7	7.7	0.023	0.918	8
15	0.75	8	0.02	0.938	8.4
16	0.8	8.4	0.016	0.954	8.9
17	0.85	8.9	0.011	0.965	9.2
18	0.9	9.2	0.008	0.973	9.4
19	0.95	9.4	0.006	0.979	9.6
20	1.0	9.6	0.004	0.983	9.7
21	1.05	9.7	0.002	0.985	9.75
22	1.1	9.75	0.0015	0.9865	9.8

10-5 (10.6). A capacitor with capacity C is charged up to voltage U, then it is connected to a semiconductor diode (fig. 10.8). The diode VAC is approximately described by equation $i = \alpha u^2$.

1. Find the analytical dependence of the voltage across the diode (or the voltage across the capacitor) on time.

2. Compare it with the equation of the capacitor discharge through the constant resistance equal to the

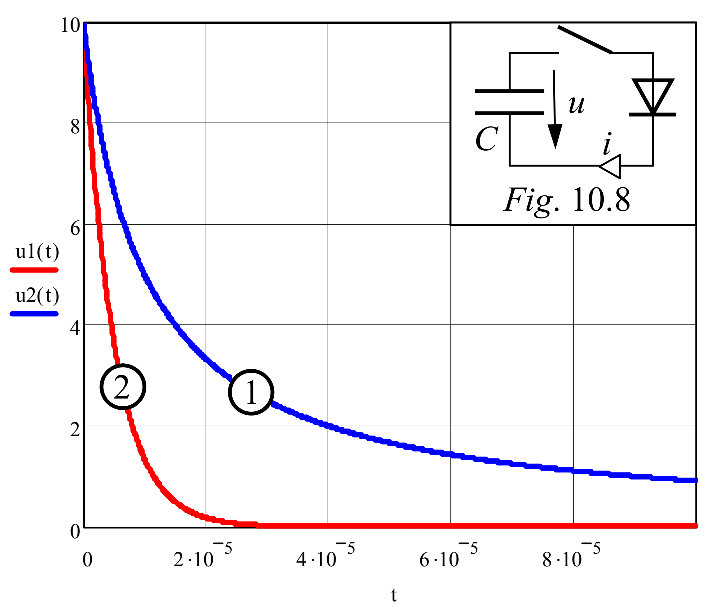


Fig. 10.8

differential resistance of the diode at the beginning of the discharge

$$r_{dif0} = du/di = 1/(2\alpha U).$$

Answers and comments:

$i = -Cdu/dt = \alpha u^2$; or $-\dfrac{C}{\alpha}\dfrac{du}{u^2} = dt$;

$$t = \frac{C}{\alpha u} + A = \frac{C}{\alpha u} - \frac{C}{\alpha U}.\ \text{So, in the first case}\ \ u1(t) = \frac{U}{1 + U\dfrac{\alpha t}{C}}.$$

In the second case $u2(t) = U\,e^{-\frac{t}{\tau}} = U\,e^{-\frac{t}{Cr_{dif0}}} = U\,e^{-\frac{2\alpha U}{C}t}$, the capacitor discharges quicker because in the first case the diode resistance increases with the voltage decreasing.

For illustrative purposes, graphs $u1(t)$ and $u2(t)$ are plotted in fig. 10.8 for the following values of the quantities: $U = 10\ V$, $\alpha = 0.01\ A/V^2$, $C = 10^{-6} F$.

10-6 (10.7). Initially, the switch shunts the current source $J = 200\ mA$, then it opens and the source becomes connected in series with the semiconductor *p-n* junction (fig. 10.9,a), the equivalent scheme is approximately presented by capacitance $C = 200\ pF$, connected in parallel with non-linear resistance VAC of which is given in fig. 10.9,b ($U_{dir} = 0.5\ V$; $r_{dif} = m_r \cdot tg\beta = 12\ Ohm$). Find out the law of voltage variation across *p-n* junction when it is in the conducting direction.

Answers and comments: $u(0_+) = 0$; $u_\infty = U_{dir} + r_{dif} \cdot J = 2.9\ V$.

The transient process is calculated by the piecewise-linear approximation method. In the interval $0 \div t_1$

$$r_{dif} = \infty,\quad u < U_{dir},\quad u(t) = \frac{1}{C}\int_0^t J dt = 10^9 t\ V;\quad u(t_1) = U_{dir},\quad t_1 = \frac{U_{dir}}{10^9} = 0.5\cdot 10^{-9}\ s.$$

In the interval $t > t_1$ $u(t) = u_\infty + A\,e^{p(t-t_1)}$, $p = \dfrac{-1}{r_{dif}C} = -417\cdot 10^6\ s^{-1}$,

$A = u(t_1) - u_\infty = -2.4$; $u(t) = 2.9 - 2.4\,e^{-417\cdot 10^6 (t - 0.5\cdot 10^{-9})}\ V.$

Finally we have: $u(t) = \begin{cases} 10^9 t\ V & \text{if}\quad 0 \le t \le t_1 = 0.5\cdot 10^{-9}\ s, \\ 2.9 - 2.4 e^{-417\cdot 10^6 (t - t_1)}\ V & \text{if}\quad t \ge t_1. \end{cases}$

The voltage plot $u(t)$ is presented in fig. 10.9,c.

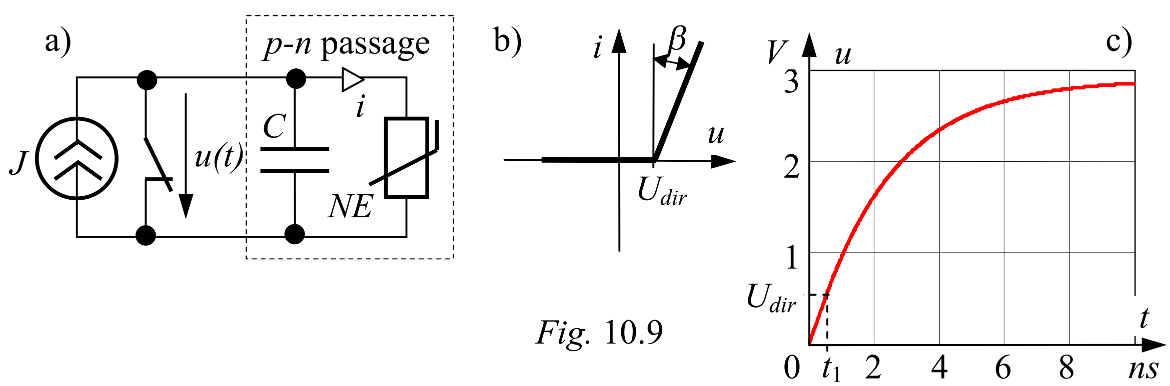


Fig. 10.9

10-7 (10.8). In the circuit fig. 10.10,a $U = 60\ V$, $r_1 = 30\ Ohm$, $r_2 = 20\ Ohm$, $L = 0.2\ H$, VAC of NE is set by table 10.5.

Compute the currents of the circuit transient process.

Answers and comments: the problem is solved by the linearization method. Steady-state condition before and after the commutation is computed by the method of an equivalent generator in a graphical manner. The circuit linear section is replaced with an equivalent generator which has the following parameters before the commutation:

$$U_o(t_-) = U = 60\ V,\quad r_{equ}(t_-) = r_1 = 30\ Ohm,\quad I_S(t_-) = U_o(t_-)/r_{equ}(t_-) = 2\ A.$$

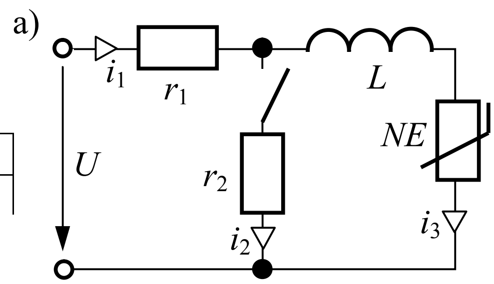
For point 1 (fig. 10.10,b): $i_3(t_-) = 1\ A$, $u_3(t_-) = 30\ V$.

In the condition after commutation

$$U_o(t_+) = U\frac{r_2}{r_1 + r_2} = 24\ V,\quad r_{equ}(t_+) = \frac{r_1 r_2}{r_1 + r_2} = 12\ Ohm,\qquad I_S(t_+) = 2\ A.$$

Table 10.5

U_3, V	0	18	30
I_3, A	0	0.5	1



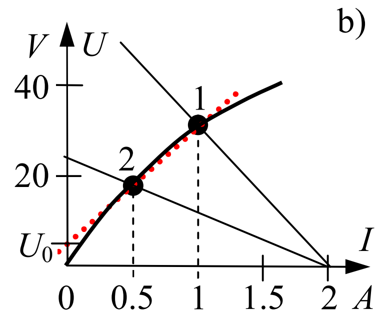


Fig. 10.10

For point 2: $i_{3\infty} = 0.5\ A$, $u_{3\infty} = 18\ V$.

Linear VAC through points 1 and 2: $r_{3dif} = \Delta u/\Delta i = 24\ Ohm$.

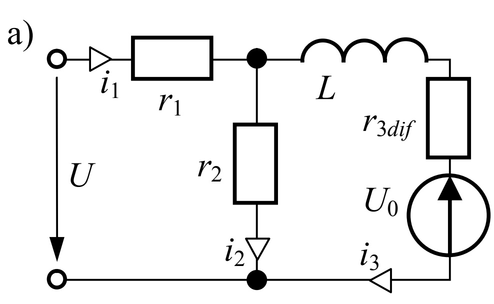
For the calculation scheme fig. 10.11,a: $i_3(t) = i_{3\infty} + Ae^{pt}$.

$$p = -\frac{r_{3dif} + \dfrac{r_1 r_2}{r_1 + r_2}}{L} = -\ 180\ s^{-1},\quad i_3(0_+) = 1\ A,\qquad A = i_3(0_+) - i_{3\infty} = 0.5.$$

$$\text{Thus,}\quad i_3(t) = 0.5 + 0.5\cdot e^{-180t}\ A,\quad u_L(t) = L\frac{di_3}{dt} = -\ 18e^{-180t}\ V.$$

The other currents and voltages are as follows: $u_{NE}(t)$ is determined graphically with the aid of the plot $i_3(t)$ and NE VAC; $u_2(t) = u_{NE} + u_L$; $i_2(t) = u_2/r_2$; $i_1(t) = i_2 + i_3$.

All the diagrams are presented in fig. 10.11,b.



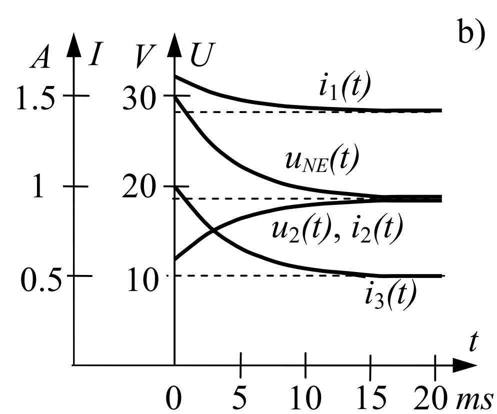


Fig. 10.11