MINISTRY OF EDUCATION AND SCIENCE, YOUTH AND SPORTS OF UKRAINE

DONETSK NATIONAL TECHNICAL UNIVERSITY

"Electromechanics and TFEE" Department
"English language" Department

Tutorial
on solving the problems on theoretical fundamentals of electrical engineering for students of specialty Electric Systems and Networks

Рассмотрено на заседании
кафедры «Электромеханика и ТОЭ»
28 января 2013 г., протокол № 5

Утверждено на заседании Совета ИМС
ДонНТУ.
Протокол № 2 от 26.02.2013 г.

Рекомендовано Ученым советом
ДонНТУ как учебное пособие.
Протокол № 2 от 22.03.2013 г.

Рег. номер 22, Объём – 19 п.л.

Donetsk – DonNTU – 2013

УДК 621.3.01. (075.3)
Т33

Tutorial on solving the problems on theoretical fundamentals of electrical engineering for students of specialty Electric Systems and Networks – Donetsk: DonNTU, 2013. – 304p.

This textbook is intended for the foreign students of English Technical Faculty.

The tutorial comprises the problems on all parts of the course TFEE which is studied by the students of specialty Electric Systems and Networks. Most of the problems are given with their solution. In essence, this tutorial is the authorized translation from Ukrainian to English of selected problems from the textbook "Теоретичні основи електротехніки. Збірник задач: навчальний посібник / О.В. Корощенко, В.Ф. Денник, О.А. Журавель та ін.; за заг. ред. О.В. Корощенка. – Донецьк: ДВНЗ «ДонНТУ», 2012. – 673 с.: іл." For every problem in the tutorial, the number of the corresponding problem from the original textbook is given in brackets.

Составители: А.В. Корощенко, доц., к.т.н.
О.И. Куксина, ст. преп.

Рецензенты: В.Е. Михайлов, доц., к.т.н.
О.Г. Каверина, проф., д.п.н.

CONTENTS

1. LINEAR D-C CIRCUITS

1.1. CIRCUIT CALCULATION UNDER OHM'S AND KIRCHHOFF'S LAWS

1-1 (2.1). In circuit fig. 1.1,a, determine currents for two positions of switch *S*, moreover $R_1 = 4\ Ohm$, $R_2 = 1\ Ohm$, $R_3 = 2\ Ohm$, $R_4 = 6\ Ohm$, $R_5 = 6\ Ohm$, $U = 36\ V$.

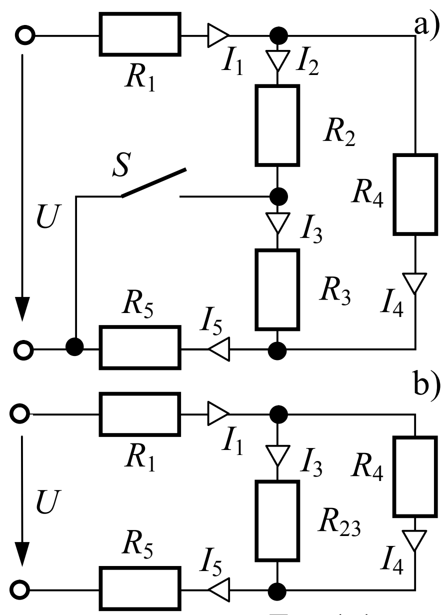


Fig. 1.1

Solution. 1. Assume the positive directions of currents (fig. 1.1,a). Perform the circuit calculation at opened switch *S*. Resistors R_2 and R_3 are connected in series; let's substitute them by an equivalent resistor R_{23}:

$$R_{23} = R_2 + R_3 = 1 + 2 = 3\ Ohm.$$

Equivalent scheme is in fig. 1.1,b. Resistors R_{23} and R_4 are connected in parallel, let's substitute them by an equivalent resistor R_{234}:

$$R_{234} = R_{23}{\cdot}R_4/(R_{23} + R_4) = 3{\cdot}6/(3{+}6) = 2\ Ohm.$$

We have a circuit with series connection of resistors R_1-R_{234}-R_5. Input resistance of the obtained circuit is:

$$R_{inp} = R_1 + R_{234} + R_5 = 4 + 2 + 6 = 12\ Ohm.$$

Input current under Ohm's law:

$$I_1 = I_5 = U/R_{inp} = 36/12 = 3\ A.$$

Other currents are determined under the rule of dispersion of currents into parallel branches:

$$I_2 = I_3 = I_1{\cdot}R_4/(R_{23} + R_4) = 3{\cdot}6/9 = 2\ A;$$

$$I_4 = I_1{\cdot}R_{23}/(R_{23} + R_4) = 3{\cdot}3/9 = 1\ A.$$

2. Perform the circuit calculation at closed switch *S* (fig. 1.2,a).

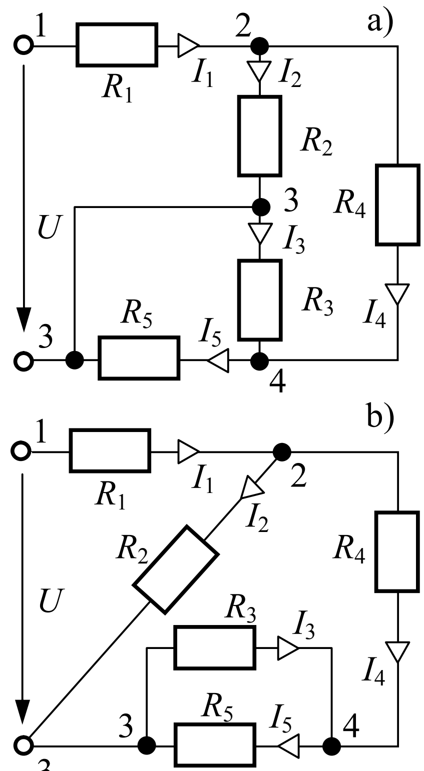


Fig. 1.2

Let's enumerate the points with different potentials (see fig. 1.2,a), construct the scheme in a convenient form (fig. 1.2,b). Two nodes are marked with figure «3», because they are connected through jump lead and they have the same potential. Resistors R_3 and R_5 are connected in parallel, that's why

$$R_{35} = R_3{\cdot}R_5/(R_3 + R_5) = 2{\cdot}6/(2 + 6) = 1.5\ Ohm.$$

New resistance R_{35} is connected in series with R_4:

$$R_{354} = R_{35} + R_4 = 1.5 + 6 = 7.5\ Ohm.$$

Then we adopt the parallel connection $R_{354}||R_2$:

$$R_{3542} = R_{354}R_2/(R_{354} + R_2) = 7.5{\cdot}1/(7.5 + 1) = 0.882\ Ohm.$$

Circuit input resistance:

$$R_{inp} = R_1 + R_{3542} = 4 + 0.882 = 4.882\ Ohm.$$

Let's calculate the currents: $I_1 = U/R_{inp} = 36/4.882 = 7.374\ A$;

$I_2 = I_1{\cdot}R_{354}/(R_{354} + R_2) = 7.374{\cdot}7.5/8.5 = 6.506\ A$;

$I_4 = I_1{\cdot}R_2/(R_{354} + R_2) = 7.374{\cdot}1/8.5 = 0.868\ A$;

$I_5 = I_4{\cdot}R_3/(R_3 + R_5) = 0.868{\cdot}2/(2 + 6) = 0.217\ A$; $I_3 = I_5 - I_4 = 0.217 - 0.868 = -\,0.651\ A$.

1-2 (2.2). Node potentials of a circuit part fig. 1.3 are measured with a voltmeter *V* and are equal to: $\varphi_1 = -\,15\ V$, $\varphi_2 = 52\ V$, $\varphi_3 = 64\ V$.

With the aid of Ohm's law and Kirchhoff's current law, determine all the currents shown in fig. 1.3 if $R_1 = 5$ *Ohm*, $R_2 = 10$ *Ohm*, $R_3 = 12$ *Ohm*, $E_1 = 80$ *V*, $E_3 = 70$ *V*.

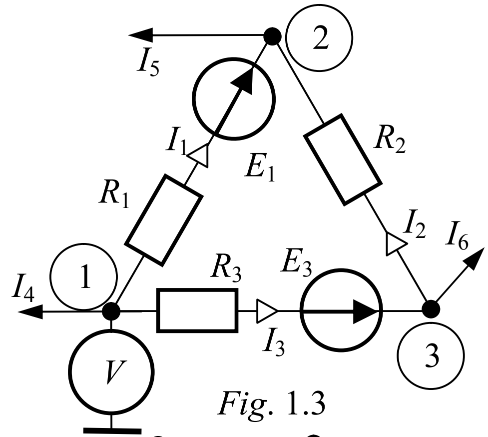


Fig. 1.3

Solution. In accordance with Ohm's law in generalized form, let's calculate:

$I_1 = (\varphi_1 - \varphi_2 + E_1)/R_1 = (-15 - 52 + 80)/5 = 2.6\ A$;

$I_2 = (\varphi_3 - \varphi_2)/R_2 = (64 - 52)/10 = 1.2\ A$;

$I_3 = (\varphi_1 - \varphi_3 + E_3)/R_3 = (-15 - 64 + 70)/12 = -0.75\ A$.

Under Kirchhoff's current law we determine the other currents:

$I_4 = -(I_1 + I_3) = -(2.6 - 0.75) = -1.85\ A$;

$I_5 = I_1 + I_2 = 2.6 + 1.2 = 3.8\ A$;

$I_6 = I_3 - I_2 = -0.75 - 1.2 = -1.95\ A$.

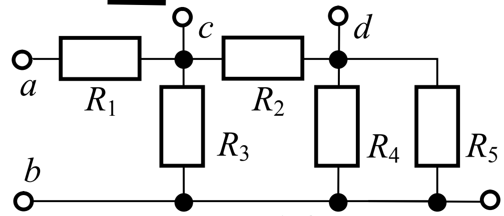


Fig. 1.4

1-3 (2.3). For a scheme fig. 1.4, determine equivalent resistances between terminals *a* and *b*, *c* and *d*, *d* and *f*, if $R_1 = 6$ *Ohm*, $R_2 = 5$ *Ohm*, $R_3 = 15$ *Ohm*, $R_4 = 30$ *Ohm*, $R_5 = 6$ *Ohm*.

Answers: $R_{df} = \left(R_4^{-1} + R_5^{-1} + (R_2 + R_3)^{-1}\right)^{-1} = 4\ Ohm$,

$$R_{ab} = R_1 + \frac{R_3 \cdot \left(R_2 + \dfrac{R_4 R_5}{R_4 + R_5}\right)}{R_3 + R_2 + \dfrac{R_4 R_5}{R_4 + R_5}} = 12\ Ohm, \qquad R_{cd} = \frac{R_2 \cdot \left(R_3 + \dfrac{R_4 R_5}{R_4 + R_5}\right)}{R_2 + R_3 + \dfrac{R_4 R_5}{R_4 + R_5}} = 4\ Ohm.$$

1-4 (2.4). Determine the circuit resistance between the points *a* and *b* at opened and at closed switch *S* (fig. 1.5) if $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 10$ *Ohm*.

Answer: at opened switch – 12.1 *Ohm*,
at closed one – 8.33 *Ohm*.

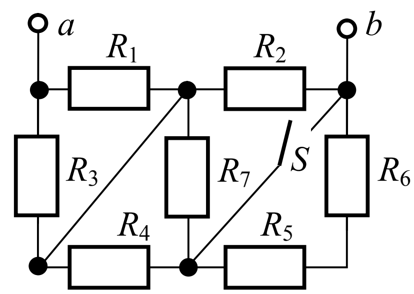


Fig. 1.5

1-5 (2.5). Determine the resistance of each presented circuit fig. 1.6 between terminals 1-1′ at no-load

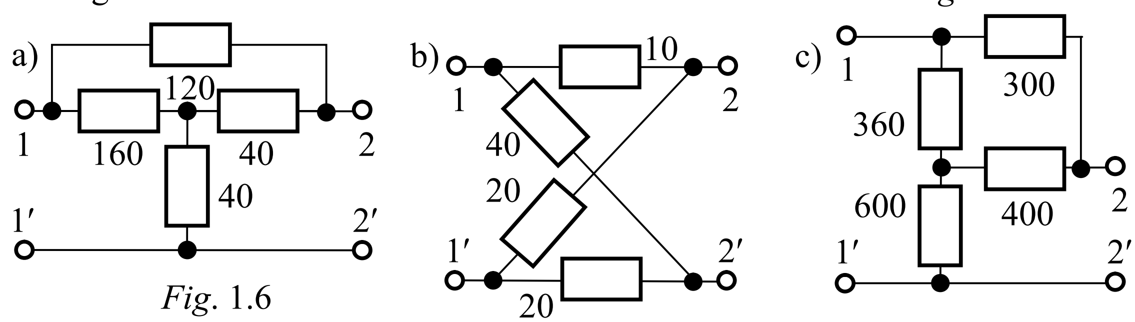


Fig. 1.6

condition (points 2 and 2′ are insulated) and short-circuit condition (points 2 and 2′ are joined). Resistances in *Ohms* are given in the scheme.

Answers: a) $R_{1O} = 120$ *Ohm*, $R_{1S} = 72$ *Ohm*; b) $R_{1O} = 20$ *Ohm*, $R_{1S} = 18$ *Ohm*; c) $R_{1O} = 838$ *Ohm*, $R_{1S} = 200$ *Ohm*.

1-6 (2,6). Loading characteristic of D-C generator is taken experimentally according to the scheme fig. 1.7,a, it being shown in fig. 1.7,b. A startup part of the characteristic is described with sufficient accuracy by the direct line equation $U = 110 - 0.25 \cdot I$, where $U[V]$, $I[A]$.

The nominal current of generator is $I_{nom} = 40\ A$, the maximum current protection is adjusted to $I_{max} = 60\ A$, the real short-circuit current is $I_S = 200\ A$.

Perform the equivalent schemes of a generator and determine its parameters.

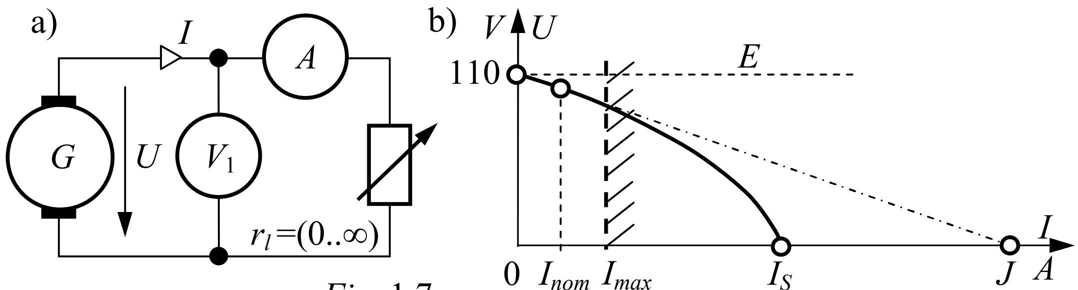


Fig. 1.7

Solution. Generator emf $E = U_O = 110\ V$, its inner resistance $r_i = 0.25\ Ohm$, calculated short-circuit current for a working part of the loading characteristic is $J = \dfrac{E}{r_i} = \dfrac{110}{0.25} = 440\ A$. An equivalent scheme of generator with series connection of emf source $E = 110\ V$ and resistance $r_i = 0.25\ Ohm$ is presented in fig. 1.8,a; a scheme with parallel connection of the current source and resistance is given in fig. 1.8,b.

In order to obtain the equivalent schemes we work out the equation in accordance with Kirchhoff's voltage law $I \cdot r_l + I \cdot r_i = E$.

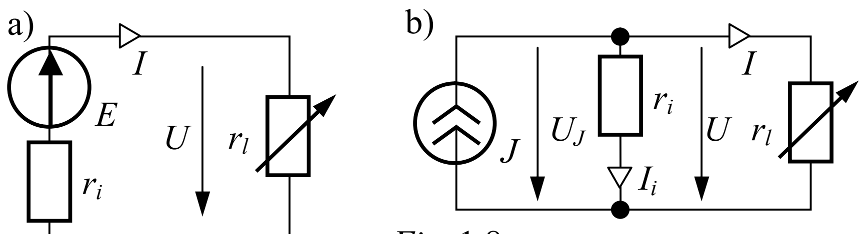


Fig. 1.8

Let's multiply this expression by the circuit current I and obtain

$$I^2 \cdot r_l + I^2 \cdot r_i = E \cdot I. \qquad (1.1)$$

In accordance with Joule's law

$I^2 \cdot r_l = P_l$ – power consumed by load,

$I^2 \cdot r_i = \Delta P_i$ – power dissipated in heat form in inner resistance of a generator,

$E \cdot I = P_G$ – power produced by generator (emf source).

Expression (1.1) reflects one of the principal features of an electric circuit: summary power generated by the energy sources is equal to summary power consumed by the circuit load. This feature may be formulated in another way: the *power balance* is true for any electric circuit.

For a parallel equivalent scheme in accordance with fig. 1.8,b on the ground of Kirchhoff's current law, we obtain $I + I_i = J.$ (1.2)

In this scheme, the voltage U_J across terminals of the current source and the voltage U across resistances are identical. Let's multiply the obtained expression (1.2) by $U_J = U$ and find out: $U \cdot I + U \cdot I_i = U_J \cdot J$.

However, Ohm's law gives $U = I \cdot r_l$, $U = I_i \cdot r_i$ and we come to the power balance expression for scheme with the current source: $I^2 \cdot r_l + I_i^2 \cdot r_i = P_G = U_J \cdot J.$ (1.3)

1-7 (2.7). For electric circuit fig. 1.9, the resistances $r_1 = 100$ *Ohm*, $r_2 = 150$ *Ohm*, $r_3 = 50$ *Ohm* and voltage $U = 150$ *V* are given. Calculate the currents at opened switch *S*. How much do currents change if the switch is closed?

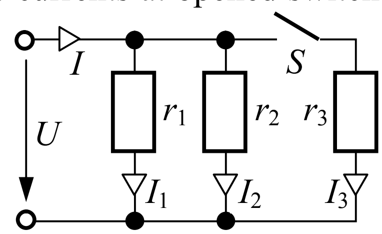


Fig. 1.9

Solution. At opened switch *S* the currents are determined under Ohm's law:

$I_3 = 0$, because the switch is opened;

$$I_1 = \frac{U}{r_1} = \frac{150}{100} = 1.5\ A; \qquad I_2 = \frac{U}{r_2} = \frac{150}{150} = 1.0\ A;$$

input current of the parallel connection is found in accordance with Kirchhoff's current law $I = I_1 + I_2 + I_3 = 1{,}5 + 1 + 0 = 2.5\ A$.

At closed switch, the currents are $I_1 = \frac{U}{r_1} = \frac{150}{100} = 1.5\ A$ – former value,

$$I_2 = \frac{U}{r_2} = \frac{150}{150} = 1.0\ A \text{ – doesn't change,}$$

$$I_3 = \frac{U}{r_3} = \frac{150}{50} = 3\ A.$$

Current of the common circuit part changes: $I = I_1 + I_2 + I_3 = 1.5 + 1 + 3 = 5.5\ A$.

1-8 (2.8). For circuit fig. 1.10, the instruments' readings are known: voltmeter *V* reads 120 *V*, wattmeter *W* reads 240 *W*. Resistances are given $r_1 = 16$ *Ohm*, $r_2 = 40$ *Ohm*. Determine the currents, resistance r_3, voltage *U*. Check a power balance.

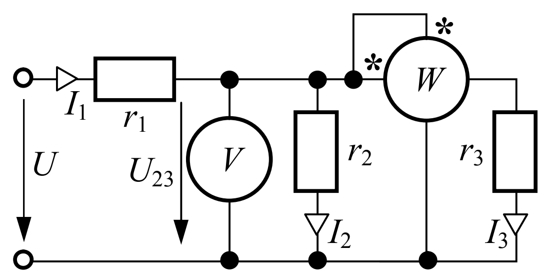


Fig. 1.10

Solution. Voltmeter *V* measures voltage $U_{23} = 120\ V$ across the section with parallel connection of resistances r_2 and r_3. Under Ohm's law, the current is

$$I_2 = \frac{U_{23}}{r_2} = \frac{120}{40} = 3\ A.$$

Wattmeter measures the power consumed by resistance r_3

$$P_3 = I_3^2 \cdot r_3 = U_{23} \cdot I_3,$$

that's why $I_3 = \frac{P}{U_{23}} = \frac{240}{120} = 2\ A$, and under Ohm's law $r_3 = \frac{U_{23}}{I_3} = \frac{120}{2} = 60$ *Ohm*.

The current through resistance r_1 in accordance with Kirchhoff's current law is

$$I_1 = I_2 + I_3 = 3 + 2 = 5\ A.$$

In accordance with Kirchhoff's voltage law for a loop r_1-r_2-U, the input circuit voltage in fig. 1.10 is $U = I_1 \cdot r_1 + U_{23} = 5 \cdot 16 + 120 = 200\ V$.

The generator power is $P_G = U \cdot I_1 = 200 \cdot 5 = 1000\ W$.

Summary consumed power is

$$\Sigma P_C = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 + I_3^2 \cdot r_3 = 5^2 \cdot 16 + 3^2 \cdot 40 + 2^2 \cdot 60 = 400 + 360 + 240 = 1000\ W.$$

As power balance $P_G = \Sigma P_C$ is true the problem is solved perfectly.

1-9 (2.9). In scheme fig. 1.11 a current $I_4 = 8\,A$ is measured by an ammeter of D'Arsonval system. Some circuit parameters are known: $E_1 = 120\ V$, $E_4 = 80\ V$, $E_5 = 6\ V$, $r_2 = r_4 = 6\ Ohm$, $r_3 = r_5 = 2\ Ohm$, $r_6 = 3\ Ohm$.

Determine the other currents, find the resistance r_1. Construct PC for outer loop.

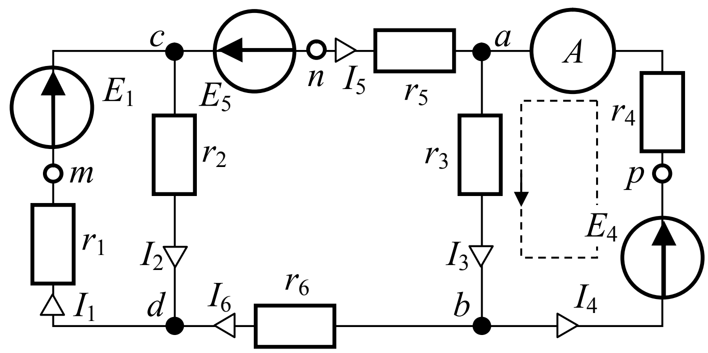


Fig. 1.11

Solution. In accordance with Kirchhoff's voltage law for the right loop, there is

$$I_4 \cdot r_4 + I_3 \cdot r_3 = E_4,\ \text{from where}\ \ I_3 = \frac{E_4 - I_4 \cdot r_4}{r_3} = \frac{80 - 8 \cdot 6}{2} = 16\ A.$$

Under Kirchhoff's current law for node a, we obtain $I_3 - I_5 - I_4 = 0$,

for node b $\quad I_6 + I_4 - I_3 = 0$,

from where $\quad I_5 = I_6 = I_3 - I_4 = 16 - 8 = 8\ A.$

Under Kirchhoff's voltage law for the middle loop, there is

$$I_6 \cdot r_6 - I_2 \cdot r_2 + I_5 \cdot r_5 + I_3 \cdot r_3 = -E_5,\quad \text{from where}$$

$$I_2 = \frac{I_6 r_6 + I_5 r_5 + I_3 r_3 + E_5}{r_2} = \frac{8 \cdot 3 + 8 \cdot 2 + 16 \cdot 2 + 6}{6} = 13\ A.$$

Under Kirchhoff's current law for node d, there is

$$I_1 = I_2 + I_6 = 13 + 8 = 21\ A.$$

Under Kirchhoff's voltage law for the left loop, there is

$$I_2 \cdot r_2 + I_1 \cdot r_1 = E_1,\ \text{from where}\ \ r_1 = \frac{E_1 - I_2 r_2}{I_1} = \frac{120 - 13 \cdot 6}{21} = 2\ Ohm.$$

A potential circle is a plot of the loop point potentials against the resistances of the loop. In an outer loop let's mark additional points n, m, p so that any element is defined with two points. Assume potential of a loop point to be equal to zero, for instance $\varphi_d = 0$. Assume the positive direction of loop, for instance in a clock-wise direction, and calculate potentials of the points next to the point d:

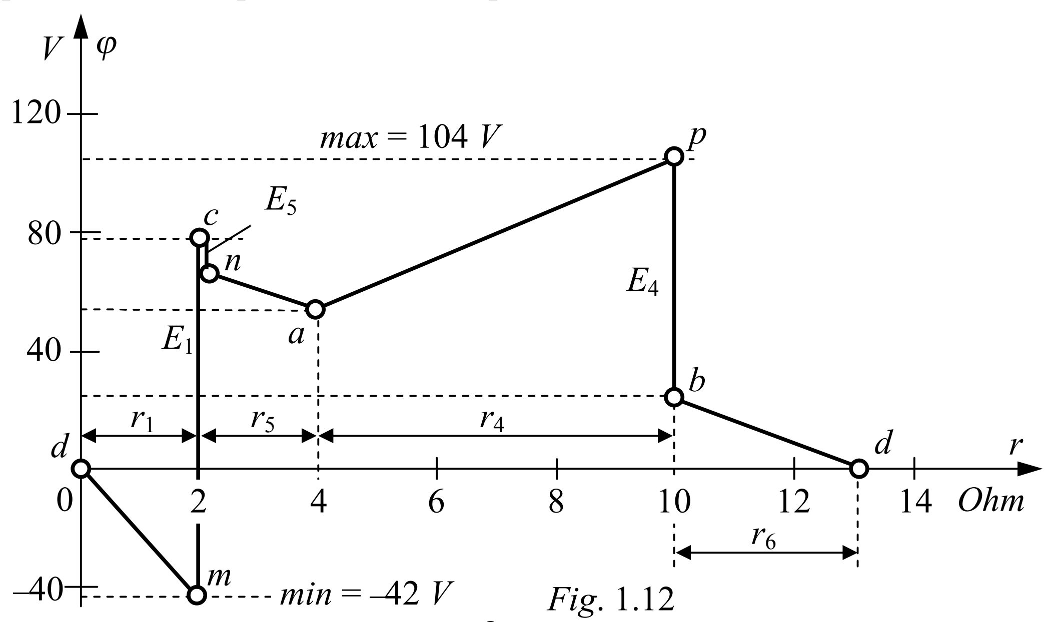


Fig. 1.12

$\varphi_m = \varphi_d - I_1 \cdot r_1 = 0 - 21 \cdot 2 = -42\ V$; $\varphi_c = \varphi_m + E_1 = -42 + 120 = 78\ V$;
$\varphi_n = \varphi_c - E_5 = 78 - 6 = 72\ V$; $\varphi_a = \varphi_n - I_5 \cdot r_5 = 72 - 8 \cdot 2 = 56\ V$;
$\varphi_p = \varphi_a + I_4 \cdot r_4 = 56 + 8 \cdot 6 = 104\ V$; $\varphi_b = \varphi_p - E_4 = 104 - 80 = 24\ V$;
$\varphi_d = \varphi_b - I_6 \cdot r_6 = 24 - 8 \cdot 3 = 0$.

The maximum potential is $max = 104\ V$, minimum potential is $min = -42\ V$, total resistance of the loop under consideration is

$$\Sigma r = r_1 + r_5 + r_4 + r_6 = 2 + 2 + 6 + 3 = 13\ Ohm.$$

Taking this into account, let's choose the axes' scales. A potential circle is presented in fig. 1.12.

1.2. METHOD OF DIRECT APPLICATION OF KIRCHHOFF'S LAWS

1-10 (2.13). Applying Kirchhoff's laws, calculate currents in scheme fig. 1.13 with parameters: $J = 3\ A$, $E = 30\ V$, $r_1 = 10\ Ohm$, $r_2 = 5\ Ohm$.

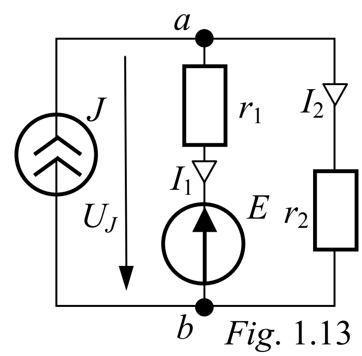


Fig. 1.13

Solution. Arbitrarily assumed positive directions of currents I_1 and I_2 as well as the voltage across terminals of the current source U_J are shown in scheme.

1) Circuit analysis: number of nodes – $Y = 2$; branches – $B = 3$; branches with unknown current – $B_T = 1$.

Number of equations by Kirchhoff's current law:
$N_I = Y - 1 = 1$;
number of equations by Kirchhoff's voltage law: $N_{II} = B - N_I - B_T = 1$.

2) the equation system is as follows: $\begin{cases} I_1 + I_2 - J = 0, \\ I_2 \cdot r_2 - I_1 \cdot r_1 = E. \end{cases}$

3) after simple transformations we obtain: $\begin{cases} I_1 + I_2 = 3, \\ -10 \cdot I_1 + 5 \cdot I_2 = 30. \end{cases}$

Solution of the equation system: $I_2 = 4\ A$, $I_1 = 3 - I_2 = 3 - 4 = -1\ A$.

4) Under Kirchhoff's voltage law for loop including U_J and r_2, we obtain:

$$U_J - I_2 \cdot r_2 = 0; \quad U_J = I_2 \cdot r_2 = 4 \cdot 5 = 20\ V.$$

5) Draw the power balance

$$U_J \cdot J - E \cdot I_1 = I_1^2 \cdot r_1 + I_2^2 \cdot r_2;$$

$$20 \cdot 3 - 30 \cdot (-1) = 1^2 \cdot 10 + 4^2 \cdot 5 \text{ or } 90\ (W) = 10 + 80\ (W).$$

Relative error is 0%, because all calculations are strict without round-ups.

1-11 (2.14). Applying Kirchhoff's laws, calculate currents in scheme fig. 1.14,a with parameters: $J_1 = 10\ A$, $E_2 = 100\ V$, $E_6 = 300\ V$, $r_3 = r_4 = r_5 = r_6 = 20\ Ohm$.

Solution. Arbitrarily assumed positive directions of U_J, I_2, I_3, I_4, I_5, I_6 are shown in scheme. For this scheme: $B = 5$, $Y = 4$.

In accordance with Kirchhoff's current law the equations for nodes «1», «2», «3», respectively, are as follows:

$$J_1 - I_2 + I_6 = 0; \qquad I_3 + I_5 - J_1 = 0; \qquad -I_3 - I_4 - I_6 = 0. \qquad (1.4)$$

To generate equations under Kirchhoff's voltage law we use the directed graph of the electric circuit fig. 1.14,b. Here the following feature of the electric circuit is taken into account: there is a branch with current source J_1, the inner resistance of which is $r_i = \infty$ and emf is $E_i = \infty$. That's why it is impossible to draw an equation by Kirchhoff's voltage law for a loop with a current source in the form $\Sigma\, I{\cdot}r = \Sigma\, E$, because it starts becoming indefinite: $I_5{\cdot}r_5 + J_1{\cdot}r_i = E_2 + E_i$ or $I_5{\cdot}r_5 + \infty = E_2 + \infty$.

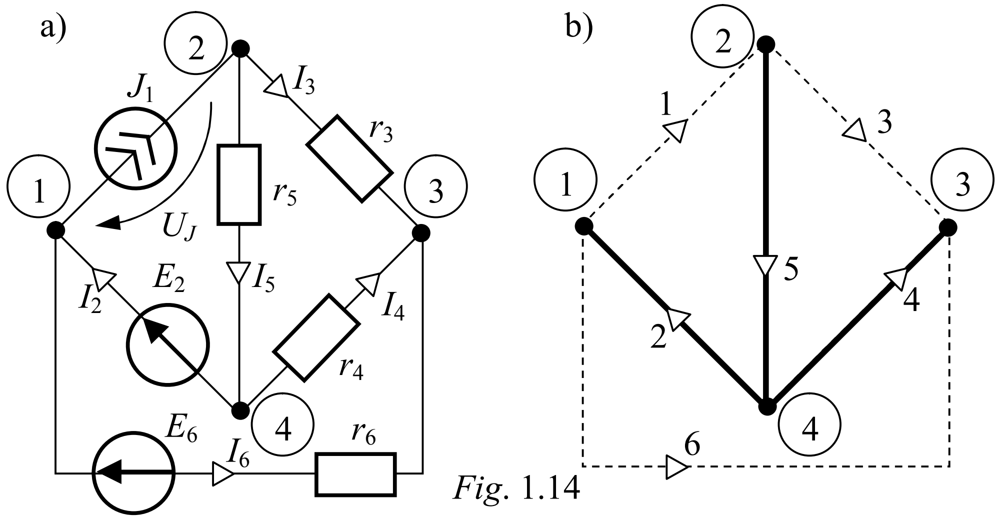


Fig. 1.14

Keep in mind the infinities in different parts of the equation may be joined in the left part of the equation and their difference gives the finite quantity U_J, which follows from the equation drawn under Kirchhoff's voltage law for a loop including voltage $U_J = -E_i + J_1{\cdot}r_i$ instead of the inner circuit of the current source: $I_5{\cdot}r_5 + U_J = E_2$.

Thus, mentioned above loop is used to determine the voltage $U_J = E_2 - I_5{\cdot}r_5$ across terminals of the current source. Then, to draw the equation system, we use the other independent loops.

For the loop with branches 3, 4, 5 we obtain $\quad I_3{\cdot}r_3 - I_4{\cdot}r_4 - I_5{\cdot}r_5 = 0, \quad (1.5)$

for the loop with branches 6, 4, 2 $\quad I_6{\cdot}r_6 - I_4{\cdot}r_4 = -E_6 + E_2. \quad (1.6)$

The equation system (1.4), (1.5), (1.6) includes 5 equations. Apply a method of substitution. From equations (1.4): $I_2 = J_1 + I_6$; $\quad I_5 = J_1 - I_3$; $\quad I_4 = -I_3 - I_6$.

Insert them into (1.5) and (1.6): $\begin{cases} I_3{\cdot}(r_3 + r_4 + r_5) + I_6{\cdot}r_4 - J_1{\cdot}r_5 = 0, \\ I_6{\cdot}(r_6 + r_4) + I_3{\cdot}r_4 = -E_6 + E_2. \end{cases}$

$$\begin{cases} 60{\cdot}I_3 + 20{\cdot}I_6 = 200, \\ 20{\cdot}I_6 + 40{\cdot}I_3 = -200. \end{cases}$$

Solve the system by Cramer's method: $\Delta = \begin{vmatrix} 60 & 20 \\ 20 & 40 \end{vmatrix} = 60{\cdot}40 - 20{\cdot}20 = 2000.$

$$\Delta_3 = \begin{vmatrix} 200 & 20 \\ -200 & 40 \end{vmatrix} = 200{\cdot}\begin{vmatrix} 1 & 20 \\ -1 & 40 \end{vmatrix} = 200{\cdot}(40 + 20) = 12000,$$

$$\Delta_6 = \begin{vmatrix} 60 & 200 \\ 20 & -200 \end{vmatrix} = 200{\cdot}\begin{vmatrix} 60 & 1 \\ 20 & -1 \end{vmatrix} = 200{\cdot}(-60 - 20) = -16000.$$

Currents are $\quad I_3 = \dfrac{\Delta_3}{\Delta} = \dfrac{12000}{2000} = 6\ A, \qquad I_6 = \dfrac{\Delta_6}{\Delta} = \dfrac{-16000}{2000} = -8\ A.$

$$I_2 = J_1 + I_6 = 10 - 8 = 2\ A,$$
$$I_4 = -I_3 - I_6 = -6 - (-8) = 2\ A,$$
$$I_5 = J_1 - I_3 = 10 - 6 = 4\ A.$$

The voltage across the terminals of the current source is

$$U_J = -E_2 + I_5{\cdot}r_5 = -100 + 4{\cdot}20 = -20\ V.$$

Power balance is $U_J \cdot J_1 + E_2 \cdot I_2 - E_6 \cdot I_6 = I_3^2 \cdot r_3 + I_4^2 \cdot r_4 + I_5^2 \cdot r_5 + I_6^2 \cdot r_6$.

$$-20 \cdot 10 + 100 \cdot 2 - 300 \cdot (-8) = 6^2 \cdot 20 + 2^2 \cdot 20 + 4^2 \cdot 20 + 8^2 \cdot 20,$$

$$2400\ W = 20 \cdot (36 + 4\ 16 + 64) = 20 \cdot 120 = 2400\ W.$$

Notion. Substitution method to reduce the number of equations is used to prove the mesh current method.

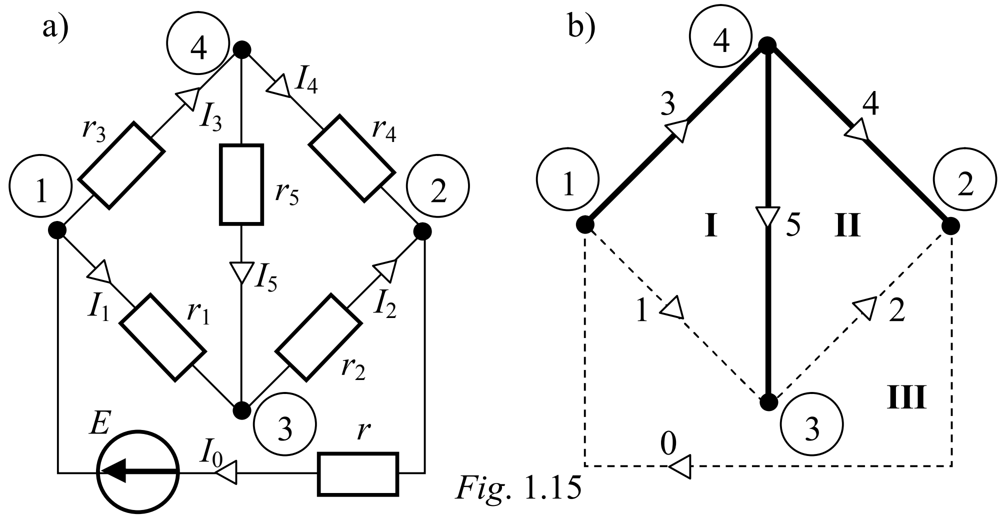


Fig. 1.15

1-12 (2.15). Bridge scheme fig. 1.15,a is supplied with energy from a real source having $E = 400\ V$, inner resistance $r = 10\ Ohm$. Resistances of the bridge branches are $r_1 = 20\ Ohm$, $r_2 = 40\ Ohm$, $r_3 = 60\ Ohm$, $r_4 = 30\ Ohm$. Bridge is loaded with resistance $r_5 = 30\ Ohm$.

Calculate currents applying Kirchhoff's laws.

Solution. Let's choose positive directions of currents and construct the circuit graph (fig. 1.15,b). Here branches 3, 4, 5 are the tree ones and branches 1, 2, 0 are the coupling ones, loops 1-5-3, 2-4-5, 0-3-4 are the main ones.

Number of unknown currents $B = 6$, number of nodes $Y = 4$, number of independent loops $K = 3$.

The Kirchhoff's equation system

Node 1:	$I_3 + I_1 - I_0 = 0;$	(1.7)
2:	$I_0 - I_2 - I_4 = 0;$	(1.8)
3:	$I_2 - I_5 - I_1 = 0;$	(1.9)
Loop I:	$I_1 \cdot r_1 - I_5 \cdot r_5 - I_3 \cdot r_3 = 0;$	(1.10)
II:	$I_2 \cdot r_2 - I_4 \cdot r_4 + I_5 \cdot r_5 = 0;$	(1.11)
III:	$I_0 \cdot r_0 + I_3 \cdot r_3 + I_4 \cdot r_4 = E.$	(1.12)

In order to reduce the number of equations we use the substitution method: from (1.7)-(1.9) we express the tree branch currents through the coupling branch currents and insert them into (1.10)-(1.12). We obtain a system of three equations:

$$\begin{cases} I_1 \cdot (r_1 + r_5 + r_3) - I_2 \cdot r_5 - I_0 \cdot r_3 = 0, \\ I_2 \cdot (r_2 + r_4 + r_5) - I_1 \cdot r_5 - I_0 \cdot r_4 = 0, \\ I_0 \cdot (r + r_3 + r_4) - I_1 \cdot r_3 - I_2 \cdot r_4 = E. \end{cases} \quad \text{or} \quad \begin{cases} 110 \cdot I_1 - 30 \cdot I_2 - 60 \cdot I_0 = 0, \\ -30 \cdot I_1 + 100 \cdot I_2 - 30 \cdot I_0 = 0, \\ -60 \cdot I_1 - 30 \cdot I_2 + 100 \cdot I_0 = 400. \end{cases}$$

By Cramer's method:

$$\Delta = \begin{vmatrix} 110 & -30 & -60 \\ -30 & 100 & -30 \\ -60 & -30 & 100 \end{vmatrix} = 10^3 \cdot (11 \cdot 10 \cdot 10 - 3 \cdot 3 \cdot 6 \cdot 2 - 6 \cdot 10 \cdot 6 - 3 \cdot 3 \cdot 11 - 3 \cdot 3 \cdot 10) = 443 \cdot 10^3;$$

$$\Delta_1 = \begin{vmatrix} 0 & -30 & -60 \\ 0 & 100 & -30 \\ 400 & -30 & 100 \end{vmatrix} = 400\cdot(30\cdot30 + 60\cdot100) = 276\cdot10^4;$$

$$\Delta_2 = \begin{vmatrix} 110 & 0 & -60 \\ -30 & 0 & -30 \\ -60 & 400 & 100 \end{vmatrix} = -400\cdot(-30\cdot110 - 30\cdot60) = 204\cdot10^4;$$

$$\Delta_0 = \begin{vmatrix} 110 & -30 & 0 \\ -30 & 100 & 0 \\ -60 & -30 & 400 \end{vmatrix} = 400\cdot(110\cdot100 - 30\cdot30) = 404\cdot10^4.$$

The currents of coupling branches are $I_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{276\cdot10^4}{443\cdot10^3} = 6.23\ A;$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{204\cdot10^4}{443\cdot10^3} = 4.61\ A; \quad I_0 = \frac{\Delta_0}{\Delta} = \frac{404\cdot10^4}{443\cdot10^3} = 9.12\ A.$$

The currents of the tree branches are $I_3 = I_0 - I_1 = 9.12 - 6{,}23 = 2.89\ A;$
$I_4 = I_0 - I_2 = 9.12 - 4.605 = 4.52\ A;$
$I_5 = I_2 - I_1 = 4.605 - 6.23 = -1.63\ A.$

The power balance is $E\cdot I_0 = \sum_{k=0}^{5} I_k^2 \cdot r_k$.

$400\cdot9.12 = 9.12^2\cdot10 + 6.23^2\cdot20 + 4.61^2\cdot40 + 2.89^2\cdot60 + 4.52^2\cdot30 + 1.63^2\cdot30,$

$\Sigma P_G = 3648\ W; \quad \Sigma P_C = 3648\ W.$

The power balance is true. The problem is solved correctly.

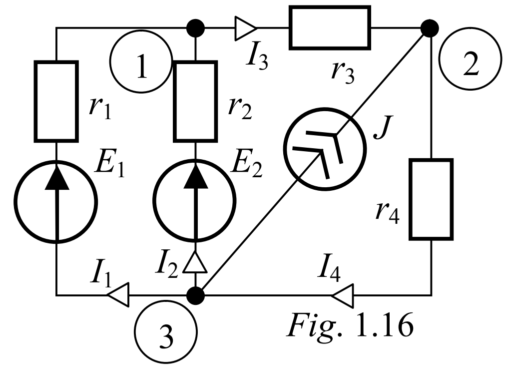


Fig. 1.16

1-13 (2.18). Determine currents under Kirchhoff's laws in the branches of a scheme fig. 1.16 and verify the power balance, if: $E_1 = 120\ V$, $E_2 = 60\ V$, $J = 4\ A$, $r_1 = r_2 = 20\ Ohm$, $r_3 = 5\ Ohm$, $r_4 = 15\ Ohm$.
Answers: $I_1 = 2\ A$, $I_2 = -1\ A$, $I_3 = 1\ A$, $I_4 = 5\ A$, $P = 480\ W$.

1.3. MESH CURRENT METHOD (LOOP CURRENT METHOD)

1-14 (2.20). For scheme fig. 1.11, it is known: $E_1 = 120\ V$, $E_4 = 80\ V$, $E_5 = 6\ V$, $r_1 = r_3 = r_5 = 2\ Ohm$, $r_2 = r_4 = 6\ Ohm$, $r_6 = 3\ Ohm$, $I_4 = 8\ A$. Find the other currents by MCM and check up the power balance.

Solution. Let's present the circuit graph (fig. 1.17), the branch 4 with known current being taken as the coupling branch. Then the loop current III starts becoming known:

$$I_{III} = I_4 = 8\ A.$$

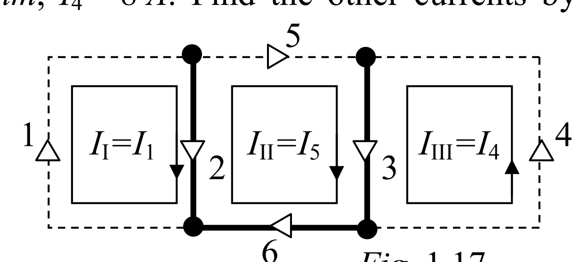


Fig. 1.17

Unknown loop currents are $I_I = I_1$, $I_{II} = I_5$.

Loop equations:
$$\begin{cases} I_1\cdot(r_1 + r_2) - I_5\cdot r_2 = E_1, \\ I_5\cdot(r_5 + r_3 + r_6 + r_2) - I_1\cdot r_2 + I_4\cdot r_3 = -E_5. \end{cases}$$

With taking into consideration $I_4 \cdot r_3 = 8 \cdot 2 = 16\ V$, we obtain:

$$\begin{cases} 8 \cdot I_1 - 6 \cdot I_5 = 120, \\ -6 \cdot I_1 + 13 \cdot I_5 = -22, \end{cases}$$

from where $I_5 = 8\ A$; $I_1 = 21\ A$.

$$I_2 = I_1 - I_5 = 21 - 8 = 13\ A; \quad I_6 = I_5 = 8\ A; \quad I_3 = I_5 + I_4 = 8 + 8 = 16\ A.$$

Equation of the power balance is

$$E_1 \cdot I_1 - E_5 \cdot I_5 + E_4 \cdot I_4 = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 + I_3^2 \cdot r_3 + I_4^2 \cdot r_4 + I_5^2 \cdot r_5 + I_6^2 \cdot r_6,$$

$$120 \cdot 21 - 6 \cdot 8 + 80 \cdot 8 = 21^2 \cdot 2 + 13^2 \cdot 6 + 16^2 \cdot 2 + 8^2 \cdot 6 + 8^2 \cdot 2 + 8^2 \cdot 3,$$

$$3112\ W = 3112\ W.$$

Power balance is true. The problem is solved correctly.

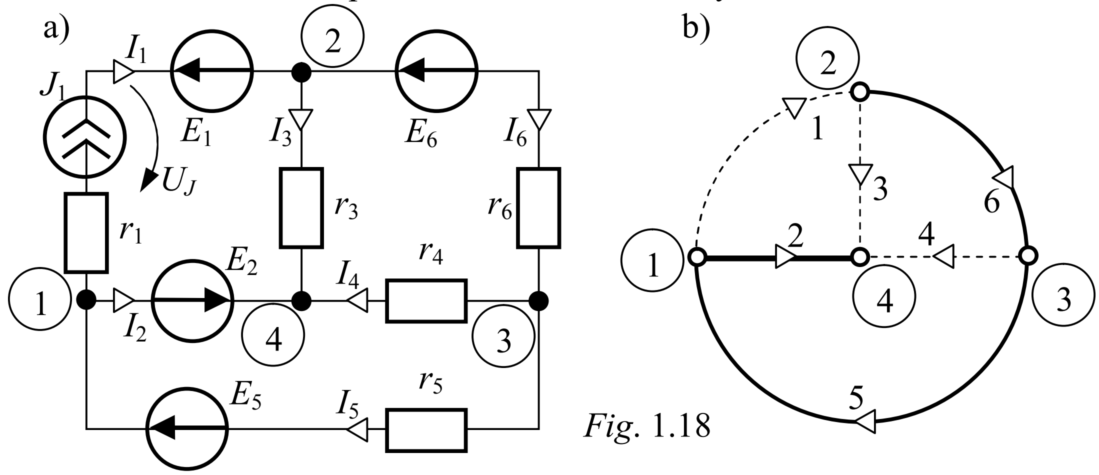


Fig. 1.18

1-15 (2.21). The following parameters are given for the scheme in fig. 1.18,a:

$$J_1 = 4\ A, \quad E_1 = 160\ V, \quad E_2 = 100\ V, \quad E_5 = 120\ V, \quad E_6 = 60\ V,$$

$$r_1 = 50\ Ohm, \quad r_3 = 40\ Ohm, \quad r_4 = 60\ Ohm, \quad r_5 = 30\ Ohm, \quad r_6 = 20\ Ohm.$$

Calculate currents.

Solution. Assumed current directions are shown in fig. 1.18,a. Keep in mind: there are two special branches.

The first one contains the current source J_1. To work, it demands a single condition: presence of a path for the current to flow at any circuit transformations or changes as a result of commutations.

The first branch resistance is $r_{b1} = r_1 + r_J = 50 + \infty = \infty$, where $r_J = \infty$ – theoretical value of inner resistance of idealized energy source termed the current source.

Conductance of this branch is $g_{b1} = r_{b1}^{-1} = \infty^{-1} = 0$.

The second scheme branch contains the idealized emf source only with zero inner resistance $r_E = 0$, that's why the resistance of the second branch is $r_{b2} = 0$, and conductance is $g_{b2} = r_{b2}^{-1} = \infty$.

Earlier it was said the branch with the current source has to be the coupling branch, which is taken into account in the scheme graph fig. 1.18,b.

When applying MCM in the loop equations the items $\pm I \cdot r$ appear, r being resistance of a common branch. So, it is worth adding the branches with ideal emf sources to the tree branches. Then the product is $I \cdot r = 0$ because $r = 0$. This is also taken into account in the circuit graph.

Thus, there are three loop currents, one flows along branches 1-6-5 and equals the current of source $I_{\text{I}} = J_1 = 4\ A$.

The second loop current $I_{II} = I_3$ flows along branches 3-2-5-6 and is unknown. The third loop current $I_{III} = I_4$ flows along branches 4-2-5 and is unknown too. Loop equations for unknown currents (moreover $r_2 = 0$) are:

$$\begin{cases} I_{II}\cdot(r_3 + r_5 + r_6) + I_{III}\cdot r_5 - J_1\cdot(r_6 + r_5) = -E_2 - E_5 + E_6, \\ I_{III}\cdot(r_4 + r_5) + I_{II}\cdot r_5 - J_1\cdot r_5 = -E_2 - E_5. \end{cases}$$

With numerals:

$$\begin{cases} 90\cdot I_{II} + 30\cdot I_{III} = 40, \\ 30\cdot I_{II} + 90\cdot I_{III} = -100, \end{cases}$$ from here $240\cdot I_{II} = 220$, $I_{II} = 0.917\ A$, $I_{III} = -1.417\ A$.

Branch currents are as follows

$I_3 = I_{II} = 0.917\ A$, $I_4 = I_{III} = -1.417\ A$, $I_2 = -I_{II} - I_{III} = -0.917 + 1.417 = 0.5\ A$,
$I_5 = -I_{II} - I_{III} + I_I = -0.917 + 1.417 + 4 = 4.5\ A$,
$I_6 = I_I - I_{III} = 4 - 0.917 = 3.083\ A$.

The voltage across the terminals of the current source is determined by Kirchhoff's voltage law for loop 1-4-2: $U_J - I_3\cdot r_3 - I_1\cdot r_1 = E_1 + E_2$,
from here $U_J = E_1 + E_2 + I_1\cdot r_1 + I_3\cdot r_3 = 160 + 100 + 4\cdot 50 + 0.917\cdot 40 = 496.7\ V$.

Power balance equation is:

$$U_J\cdot J_1 - E_1\cdot I_1 + E_2\cdot I_2 + E_5\cdot I_5 - E_6\cdot I_6 = I_1^2\cdot r_1 + I_3^2\cdot r_3 + I_4^2\cdot r_4 + I_5^2\cdot r_5 + I_6^2\cdot r_6.$$

Summary power of sources is:

$$\Sigma P_G = 496.7\cdot 4 - 160\cdot 4 + 100\cdot 0.5 + 120\cdot 4.5 - 60\cdot 3.083 = 1752\ W.$$

Summary power of consumers is:

$$\Sigma P_C = 4^2\cdot 50 + 0.917^2\cdot 40 + 1.417^2\cdot 60 + 4.5^2\cdot 30 + 3.083^2\cdot 20 = 1798\ W.$$

Average value of powers is:

$$\Sigma P_{av} = \frac{\Sigma P_G + \Sigma P_C}{2} = \frac{1752 + 1798}{2} = 1755\ W.$$

Divergence (absolute error) is:

$$\Delta P = |\Sigma P_G - \Sigma P_{av}| = |\Sigma P_C - \Sigma P_{av}| = 1798 - 1755 = 23\ W.$$

Relative error of calculations is: $\varepsilon\% = \dfrac{\Delta P}{\Sigma P_{CP}}\cdot 100 = \dfrac{23\cdot 100}{1755} = 1.31\%$,

it is appreciably less than allowed 3%. Hence, problem is solved correctly.

1.4. NODE POTENTIAL METHOD (NPM)

1-16 (2.27). Determine currents in problem 1.10 by NPM.

Solution. In scheme fig. 1.13 there are only 2 nodes. Assume $\varphi_b = 0$. Junction voltage for node a takes a form: $\varphi_a\cdot\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right) = J + \dfrac{E}{r_1}$.

With numerals, we obtain $\varphi_a\cdot\left(\dfrac{1}{10} + \dfrac{1}{5}\right) = 3 + \dfrac{30}{10}$ or $\varphi_a\cdot 3 = 60$,

from here $\varphi_a = 20\ V$.

Currents: $I_1 = \dfrac{\varphi_a - \varphi_b - E}{r_1} = \dfrac{20 - 0 - 30}{10} = -1\ A$,

$$I_2 = \frac{\varphi_a - \varphi_b}{r_2} = \frac{20 - 0}{5} = 4\ A.$$

Verification of the solution is realized by Kirchhoff's current law for node a:
$J = I_1 + I_2$: 3 = – 1 + 4. It's true. Problem is solved correctly.

1-17 (2.28). Calculate currents of the bridge scheme of problem 1.12 by NPM.
Solution. Consider node «2» as reference one, assume $\varphi_2 = 0$.

For nodes with unknown potentials φ_1, φ_3, φ_4 let's draw a system of the junction equations:

$$\begin{cases} \varphi_1\cdot\left(\frac{1}{r}+\frac{1}{r_1}+\frac{1}{r_3}\right)-\varphi_4\cdot\frac{1}{r_1}-\varphi_3\cdot\frac{1}{r}=\frac{E}{r}, \\ \varphi_3\cdot\left(\frac{1}{r}+\frac{1}{r_4}+\frac{1}{r_2}\right)-\varphi_1\cdot\frac{1}{r}-\varphi_4\cdot\frac{1}{r_2}=-\frac{E}{r}, \\ \varphi_4\cdot\left(\frac{1}{r_5}+\frac{1}{r_1}+\frac{1}{r_2}\right)-\varphi_1\cdot\frac{1}{r_1}-\varphi_3\cdot\frac{1}{r_2}=0. \end{cases}$$

With numerals

$$\begin{cases} \varphi_1\cdot\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{60}\right)-\varphi_4\cdot\frac{1}{20}-\varphi_3\cdot\frac{1}{10}=\frac{400}{10}, \\ \varphi_3\cdot\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{40}\right)-\varphi_1\cdot\frac{1}{10}-\varphi_4\cdot\frac{1}{40}=-\frac{400}{10}, \\ \varphi_4\cdot\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{40}\right)-\varphi_1\cdot\frac{1}{20}-\varphi_3\cdot\frac{1}{40}=0. \end{cases}$$

Or
$$\begin{cases} \varphi_1\cdot 10-\varphi_3\cdot 6-\varphi_4\cdot 3=2400, \\ -\varphi_1\cdot 12+\varphi_3\cdot 19-\varphi_4\cdot 3=-4800, \\ -\varphi_1\cdot 6-\varphi_3\cdot 3+\varphi_4\cdot 13=0. \end{cases}$$

Determinants: $\Delta=\begin{vmatrix} 10 & -6 & -3 \\ -12 & 19 & -3 \\ -6 & -3 & 13 \end{vmatrix}=886;\quad \Delta_1=\begin{vmatrix} 2400 & -6 & -3 \\ -4800 & 19 & -3 \\ 0 & -3 & 13 \end{vmatrix}=153600;$

$$\Delta_3=\begin{vmatrix} 10 & 2400 & -3 \\ -12 & -4800 & -3 \\ -6 & 0 & 13 \end{vmatrix}=-120000;\quad \Delta_4=\begin{vmatrix} 10 & -6 & 2400 \\ -12 & 19 & -4800 \\ -6 & -3 & 0 \end{vmatrix}=43200.$$

Node potentials: $\varphi_1=\frac{\Delta_1}{\Delta}=\frac{153600}{886}=173.4\ V,$

$$\varphi_3=\frac{\Delta_3}{\Delta}=\frac{-120000}{886}=-135.4\ V,\qquad \varphi_4=\frac{\Delta_4}{\Delta}=\frac{43200}{886}=48.8\ V.$$

Calculate the currents under Ohm's law in branches of the bridge scheme:

$$I_0=\frac{\varphi_3-\varphi_1+E}{r}=\frac{-135.4-173.4+400}{10}=9.12\ A,\qquad I_3=\frac{\varphi_1}{r_3}=\frac{173.4}{60}=2.89\ A,$$

$$I_1=\frac{\varphi_1-\varphi_4}{r_1}=\frac{173.4-48.8}{20}=6.23\ A,\qquad I_4=\frac{-\varphi_3}{r_4}=\frac{135.4}{30}=4.52\ A,$$

$$I_2=\frac{\varphi_4-\varphi_3}{r_2}=\frac{48.8-135.4}{40}=4.61\ A,\qquad I_5=\frac{-\varphi_4}{r_5}=\frac{-48.8}{30}=-1.63\ A.$$

The current values being obtained, they coincide with earlier calculated values by application of Kirchhoff's laws.

1-18 (2.29). Calculate currents of problem 1.15 by NPM.
Solution. In the given problem there is a branch №2 the resistance of which is $r_2 = 0$ and conductance $g_2 = \infty$. So, for nodes «1» and «4», which are the end points of branch №2, junction equations are confluent ones, from them it following $\varphi_4 - \varphi_1 = E_2$.

Assume for node «1» $\varphi_1 = 0$, then $\varphi_4 = E_2 = 100\ V$.

For nodes with unknown potentials, let's draw a system of the junction equations:

$$\begin{cases} \varphi_2 \cdot \left(\dfrac{1}{r_3} + \dfrac{1}{r_6} \right) - \varphi_3 \cdot \dfrac{1}{r_6} - \varphi_4 \cdot \dfrac{1}{r_3} = J_1 + \dfrac{E_6}{r_6}, \\ \varphi_3 \cdot \left(\dfrac{1}{r_6} + \dfrac{1}{r_4} + \dfrac{1}{r_5} \right) - \varphi_2 \cdot \dfrac{1}{r_6} - \varphi_4 \cdot \dfrac{1}{r_4} = -\dfrac{E_6}{r_6} - \dfrac{E_5}{r_5}. \end{cases}$$

With numerals and after simplifications, we obtain:

$$\begin{cases} \varphi_2 \cdot \left(\dfrac{1}{40} + \dfrac{1}{20} \right) - \varphi_3 \cdot \dfrac{1}{20} = 4 + \dfrac{60}{20} + \dfrac{100}{40}, \\ \varphi_3 \cdot \left(\dfrac{1}{20} + \dfrac{1}{60} + \dfrac{1}{30} \right) - \varphi_2 \cdot \dfrac{1}{20} = -\dfrac{60}{20} - \dfrac{120}{30} + \dfrac{100}{60}, \end{cases} \quad \text{or} \quad \begin{cases} \varphi_2 \cdot 3 - \varphi_3 \cdot 2 = 380, \\ -\varphi_2 \cdot 3 + \varphi_3 \cdot 6 = -320, \end{cases}$$

from here $\varphi_3 = 15\ V$, $\varphi_2 = 136.7\ V$.

The branch currents of scheme fig. 1.18 are calculated by Ohm's law:

$$I_3 = \frac{\varphi_2 - \varphi_4}{r_3} = \frac{136.7 - 100}{40} = 0.917\ A, \qquad I_4 = \frac{\varphi_3 - \varphi_4}{r_4} = \frac{15 - 100}{60} = -1.417\ A,$$

$$I_5 = \frac{\varphi_3 - \varphi_1 + E_5}{r_5} = \frac{15 + 120}{30} = 4.5\ A, \qquad I_6 = \frac{\varphi_2 - \varphi_3 - E_6}{r_6} = \frac{136.7 - 15 - 60}{20} = 3.085\ A,$$

$I_2 = \dfrac{\varphi_1 - \varphi_4 + E_2}{r_2} = \dfrac{0 - 100 + 100}{0} = \dfrac{0}{0}$ – uncertainty which is removed with the aid of Kirchhoff's current law for node «1», for instance: $I_2 = I_5 - I_1 = 4.5 - 4 = 0.5\ A$.

The current values being obtained, they coincide with earlier calculated values by MCM. That's why the power balance is needless.

1-19 (2.30). Calculate currents in scheme fig. 1.19 by NPM if $E_1 = 120\ V$, $E_2 = 80\ V$, $J_5 = 8\ A$, $r_1 = 20\ Ohm$, $r_2 = 40\ Ohm$, $r_3 = 25\ Ohm$, $r_4 = 15\ Ohm$, $r_5 = 80\ Ohm$. Check up the power balance.

Solution. Assume the positive directions of the branch currents I_1, I_2, I_3, I_{ab}, I_{dc}, voltage across terminals of the current source U_J and voltage U_{ad}. It is easy to see, in the scheme under study the wire resistances of parts *ab* and *cd* are equal to zero – $r_{ab} = r_{cd} = 0$, that's why at any currents in jump leads *ab*, *dc* theare are not any voltages across them – $U_{ab} = I_{ab} \cdot r_{ab} = U_{dc} = I_{dc} \cdot r_{cd} = 0$,

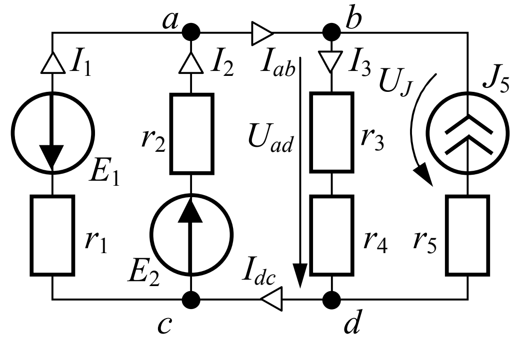


Рис. 1.19

consequently, $\varphi_a = \varphi_b$, $\varphi_d = \varphi_c$ and the scheme under study has but two different potentials whose difference is termed *junction voltage* $U_{ad} = \varphi_a - \varphi_d$, it being calculated as particular case of the junction equation for node with unknown potential φ_a at $\varphi_d = 0$:

$$U_{ad} = \frac{\sum\limits_a Eg + \sum\limits_a J}{\sum g} = \frac{-E_1 \cdot \frac{1}{r_1} + E_2 \cdot \frac{1}{r_2} + J_5}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3 + r_4}} = \frac{-\frac{120}{20} + \frac{80}{40} + 8}{\frac{1}{20} + \frac{1}{40} + \frac{1}{25+15}} = 40\ V.$$

The branch currents are calculated under Ohm's law:

$$I_1 = \frac{-U_{ad} - E_1}{r_1} = \frac{-40 - 120}{20} = -8\ A,$$

$$I_2 = \frac{E_2 - U_{ad}}{r_2} = \frac{80 - 40}{40} = 1\ A, \qquad I_3 = \frac{U_{ad}}{r_3 + r_4} = \frac{40}{25 + 15} = 1\ A.$$

Verification is fulfilled using Kirchhoff's current law

$I_1 + I_2 + J_5 = I_3$: $\quad -8 + 1 + 8 = 1$ – it is true.

Currents in the jump leads are found under Kirchhoff's current law:

$I_{ab} = I_1 + I_2 = -8 + 1 = -7\ A$, $\qquad I_{dc} = I_3 - J_5 = 1 - 8 = -7\ A$.

To draw the power balance, we determine voltage U_J across the terminals of the current source under Kirchhoff's voltage law:

$U_J + I_3 \cdot (r_3 + r_4) + J_5 \cdot r_5 = 0$, from here $U_J = -1 \cdot (25+15) - 8 \cdot 80 = -680\ V$.

The power balance equation is:

$$-E_1 \cdot I_1 + E_2 \cdot I_2 - U_k J J_5 = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 + I_3^2 \cdot (r_3 + r_4) + J_5^2 \cdot r_5 \quad \text{or}$$

$$-120 \cdot (-8) + 80 \cdot 1 - (-680) \cdot 8 = 8^2 \cdot 20 + 1^2 \cdot 40 + 1^2 \cdot (25+15) + 8^2 \cdot 80$$

$$6480\ W = 1280 + 40 + 40 + 5120 = 6480\ W.$$

1.5. EQUIVALENT SIMPLIFICATIONS IN ELECTRIC CIRCUITS

1-20 (2.35). Calculate the currents of the bridge scheme of problem 1.11 (fig. 1.15) by means of equivalent simplifications.

Solution. Let's check up the bridge equilibrium condition:

$r_2 \cdot r_3 = 40 \cdot 60 = 2400$; $\quad r_1 \cdot r_4 = 20 \cdot 30 = 600$.

Because of $r_1 \cdot r_4 \neq r_2 \cdot r_3$ the bridge is not balanced, all its currents differing from zero.

Let's substitute Δ-connection of resistances r_2-r_4-r_5 by equivalent Y-connection and obtain the scheme fig. 1.20:

$$r_a = \frac{r_4 \cdot r_5}{r_4 + r_5 + r_2} = \frac{30 \cdot 30}{30 + 30 + 40} = 9\ Ohm,$$

$$r_b = \frac{r_4 \cdot r_2}{r_4 + r_5 + r_2} = \frac{30 \cdot 40}{100} = 12\ Ohm,$$

$$r_c = \frac{r_2 \cdot r_5}{r_4 + r_5 + r_2} = \frac{40 \cdot 30}{30 + 30 + 40} = 12\ Ohm.$$

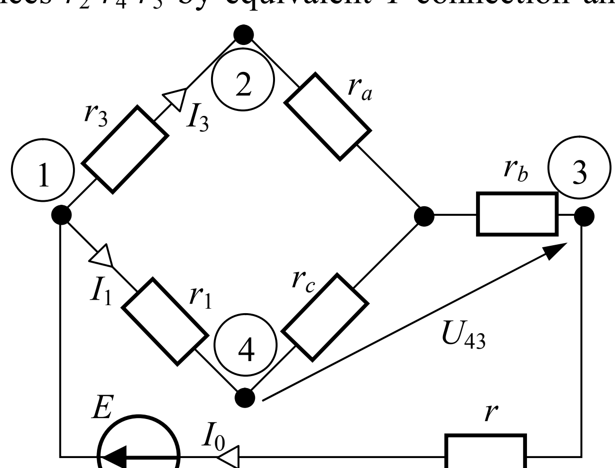


Fig. 1.20

Input circuit resistance as regard to the terminals of the emf source is

$$r_{inp} = r + \frac{(r_3 + r_a)(r_1 + r_c)}{r_3 + r_a + r_1 + r_c} + r_b =$$

$$= 10 + \frac{(60+9)(20+12)}{60+9+20+12} + 12 = 43.86\ Ohm.$$

The input current of the bridge scheme is $I_0 = \frac{E}{r_{inp}} = \frac{400}{43.86} = 9.12\ A.$

Currents of the parallel branches of scheme fig. 1.20 are

$$I_1 = I_0 \cdot \frac{r_3 + r_a}{r_3 + r_a + r_1 + r_c} = 9.12 \cdot \frac{69}{101} = 6.23\ A,$$

$$I_2 = I_0 \cdot \frac{r_1 + r_c}{r_3 + r_a + r_1 + r_c} = 9.12 \cdot \frac{32}{101} = 2.89\ A.$$

Voltage $U_{43} = I_1 \cdot r_c + I_0 \cdot r_b = 6.23 \cdot 12 + 9.12 \cdot 12 = 184.2\ V.$

Let's come back to the initial scheme and calculate the currents of the Δ-resistances:

$$I_2 = \frac{U_{34}}{r_2} = \frac{184.2}{40} = 4.61\ A,$$

$$I_4 = I_0 - I_2 = 9.12 - 4.61 = 4.51\ A, \qquad I_5 = I_2 - I_1 = 4.61 - 6.23 = -1.62\ A.$$

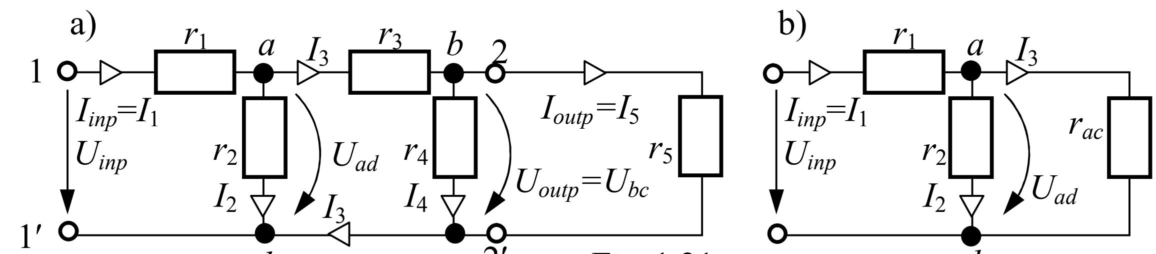


Fig. 1.21

1-21 (2.36). Determine currents in scheme fig. 1.21,a with the aid of equivalent simplifications if the input circuit voltage is $U = 400\ V$, resistances are $r_1 = 10\ Ohm$, $r_2 = 60\ Ohm$, $r_3 = 20\ Ohm$, $r_4 = 100\ Ohm$, load resistance at the output of the scheme is $r_5 = 50\ Ohm$.

Calculate the voltage transfer factor k_U and the current transfer factor k_I.

Solution. Variant 1. Let's substitute the series-parallel connection of resistances r_3, r_4, r_5 by equivalent one (fig. 1.21,b) r_{ac}:

$$r_{ac} = r_3 + \frac{r_4 \cdot r_5}{r_4 + r_5} = 20 + \frac{100 \cdot 50}{100 + 50} = 53,.33\ Ohm.$$

The input scheme resistance is:

$$r_{inp} = r_1 + \frac{r_2 \cdot r_{ac}}{r_2 + r_{ac}} = 10 + \frac{60 \cdot 53.33}{60 + 53.33} = 38.24\ Ohm.$$

The input scheme current is: $I_{inp} = I_1 = \frac{U_{inp}}{r_{inp}} = \frac{400}{38.24} = 10.46\ A.$

The voltage across the scheme branching fig. 1.21,b is:

$$U_{ad} = I_1 \cdot \frac{r_2 \cdot r_{ac}}{r_2 + r_{ac}} = 10.46 \cdot \frac{60 \cdot 53.33}{113.33} = 295.4\ V,$$

the currents are $I_2 = \frac{U_{ad}}{r_2} = \frac{295.4}{60} = 4.92\ A, \quad I_3 = \frac{U_{ad}}{r_{ac}} = \frac{295.4}{53.33} = 5.54\ A.$

The voltage across the right part scheme branching in fig. 1.21,a is

$$U_{bc} = U_{outp} = I_3 \cdot \frac{r_4 \cdot r_5}{r_4 + r_5} = 5.54 \cdot \frac{100 \cdot 50}{150} = 184.6\ V,$$

the currents of parallel branches are $I_4 = \frac{U_{bc}}{r_4} = \frac{184.6}{100} = 1.85\ A,$

$$I_5 = I_{outp} = \frac{U_{bc}}{r_5} = \frac{184.6}{50} = 3.69\ A.$$

The voltage transfer factor is $k_U = \frac{U_{outp}}{U_{inp}} = \frac{184.6}{400} = 0.462.$

The current transfer factor is $k_I = \frac{I_{outp}}{I_{inp}} = \frac{3.69}{10{,}46} = 0.353.$

Solution. Variant 2. It is convenient to compute a scheme with a single energy source by method of *proportional quantities*. First, assume the arbitrary value of current or voltage of the most remote part from the energy source; in our case we assume a current $I_5 = 10\ A$. Then with the aid of Kirchhoff's laws we calculate the input voltage (so called *influence*) which creates at the output the current I_5 (so called *circuit reaction*) which is equal to assumed value:

$$U_5 = I_5 \cdot r_5 = 10 \cdot 50 = 500\ V,$$

$$I_4 = \frac{U_5}{r_4} = \frac{500}{100} = 5\ A, \quad I_3 = I_5 + I_4 = 10 + 5 = 15\ A,$$

$$U_{ad} = I_3 \cdot r_3 + I_5 \cdot r_5 = 15 \cdot 20 + 500 = 800\ V,$$

$$I_2 = \frac{U_{ad}}{r_2} = \frac{800}{60} = 13.33\ A, \quad I_1 = I_2 + I_3 = 13.33 + 15 = 28.33\ A,$$

$$U_{inp}' = I_1 \cdot r_1 + U_{ad} = 28.33 \cdot 10 + 800 = 1083\ V.$$

Proportionality coefficient is determined $k = \frac{U_{inp}}{U_{inp}'} = \frac{400}{1083} = 0.369,$ all above-found expressions being multiplied by it to obtain sought quantities at the given input voltage $U_{inp} = 400\ V$. Then $I_1 = I_1 \cdot k = 28.33 \cdot 0.369 = 10.46\ A,$

$I_2 = I_2 \cdot k = 13.33 \cdot 0.369 = 4.92\ A,$ $I_3 = I_3 \cdot k = 15 \cdot 0.369 = 5.54\ A,$

$I_4 = I_4 \cdot k = 5 \cdot 0.369 = 1.85\ A,$ $I_5 = I_5 \cdot k = 10 \cdot 0.369 = 3.69\ A,$

$U_{ad} = U_{ad} \cdot k = 800 \cdot 0.369 = 295.4\ V,$ $U_5 = U_{outp} = U_5 \cdot k = 500 \cdot 0.369 = 185\ V,$

and this coincides with answer in variant 1.

1-22 (2.38). Determine currents in the circuit branches (fig. 1.22) having substituted the resistance triangle r_{ab}-r_{bc}-r_{ca} by an equivalent star. Numerical data: $E_A = 50\ V$, $E_B = 30\ V$, $E_C = 100\ V$,

$r_A = 3.5\ Ohm$, $r_B = 2\ Ohm$, $r_C = 7\ Ohm$,

$r_{ab} = 6\ Ohm$, $r_{bc} = 12\ Ohm$, $r_{ca} = 6\ Ohm$.

Answers: $I_A = -0.4\ A$, $I_B = -4.4\ A$,

$I_C = 4.8\ A$, $I_{ab} = 2.1\ A$, $I_{bc} = -2.3\ A$, $I_{ca} = 2.5\ A$.

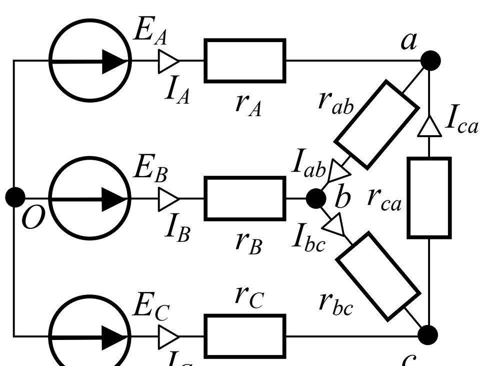


Fig. 1.22

1-23 (2.39). Calculate currents in the scheme fig. 1.23 by the simplification method, verify the power balance, if: $r_1 = r_2 = 6\ Ohm$, $r_3 = 3\ Ohm$, $r_4 = 12\ Ohm$, $r_5 = 4\ Ohm$, $J = 6\ A$.

Answers: $I_1 = 1\ A$, $I_2 = 1\ A$, $I_3 = 2\ A$, $I_4 = 1\ A$, $I_5 = 3\ A$.

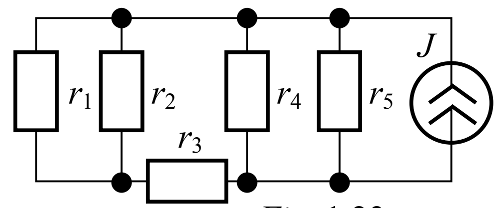


Fig. 1.23

A group of below-presented problems 1-24 – 1-27 is typical in courses "Electromagnetic transient processes in electric networks", "Electric systems and networks", "Relaying" at the computation of the short-circuit currents. However, these problems are solved by the methods of TFEE. Frequently, these calculations are performed in relative units with subsequent recalculations of all the quantities with the aid of their basic values.

1-24 (2.42). In an electric circuit fig. 1.24, it is known: $E_3 = 1.1$; $E_4 = 1.08$; $r_3 = 0.1$; $r_4 = 0.2$; $r_5 = 1$.

Do the following:

1. Determine the short-circuit current in point $K1$ (switches $S1$ and $S2$ are open) as well as the other circuit currents for the following cases, namely:

a) $E_1 = E_2 = 1$; $r_1 = r_2 = 0{,}3$;
b) $E_1 = E_2 = 1$; $r_1 = 0{,}3$; $r_2 = 0{,}4$;
c) $E_1 = 1$; $E_2 = 1{,}05$; $r_1 = 0{,}3$; $r_2 = 0{,}4$.

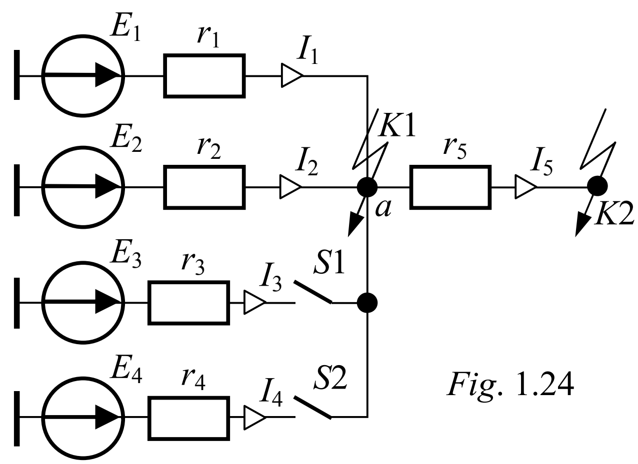


Fig. 1.24

2. Determine the short-circuit current in point $K2$ (switches $S1$ and $S2$ are closed) as well as the other circuit currents, if $E_1 = 1$; $E_2 = 1{,}05$; $r_1 = 0{,}3$; $r_2 = 0{,}4$.

Solution. 1. In the first case (short-circuit in point $K1$), the resistor r_5 has no influence and is not accounted for during calculation.

a) Perform an equivalent simplification – substitution of the branches 1-2 by single one (fig. 1.25). Because of $E_1 = E_2$ and $r_1 = r_2$, there occur $E_{12} = E_1 = 1$ and $r_{12} = ½r_1 = 0.15$.

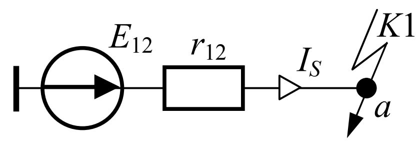


Fig. 1.25

The required short-circuit current is found under Ohm's law:

$$I_S = E_{12}/r_{12} = 1/0.15 = 6.667.$$

Currents of the branches 1 and 2:

$$I_1 = I_2 = ½I_S = 3.333.$$

b) As this time the branch resistances are different while the emf's are identical, then

$$E_{12} = E_1 = 1 \text{ and } r_{12} = \frac{r_1 r_2}{r_1 + r_2} = \frac{0.3 \cdot 0.4}{0.3 + 0.4} = 0.1714.$$

Short-circuit current and the branch currents are:

$$I_S = E_{12}/r_{12} = 1/0.1714 = 5.833,$$
$$I_1 = E_1/r_1 = 1/0.3 = 3.333, \quad I_2 = E_2/r_2 = 1/0.4 = 2.5.$$

c) In the last case under consideration, a simplification is performed on the ground of the formulae of two nodes method:

$$E_{12}=\frac{E_1/r_1+E_2/r_2}{r_1^{-1}+r_2^{-1}}=\frac{1/0.3+1.05/0.4}{0.3^{-1}+0.4^{-1}}=1.021;\quad r_{12}=\frac{r_1r_2}{r_1+r_2}=\frac{0.3\cdot 0.4}{0.3+0.4}=0.1714.$$

Short-circuit current and the branch currents are:

$$I_S=E_{12}/r_{12}=1.021/0.1714=5.958,$$
$$I_1=E_1/r_1=1/0.3=3.333,\quad I_2=E_2/r_2=1.05/0.4=2.625.$$

2. When all the sources of scheme fig. 1.24 are active and there is a short-circuit in point $K2$, we again perform a preliminary equivalent simplification on the ground of two nodes method and obtain a scheme fig. 1.26.

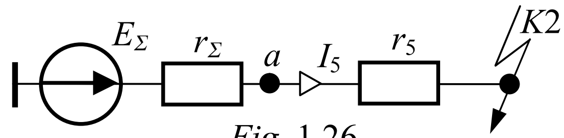


Fig. 1.26

$$E_\Sigma=\frac{E_1\cdot r_1^{-1}+E_2\cdot r_2^{-1}+E_3\cdot r_3^{-1}+E_4\cdot r_4^{-1}}{r_1^{-1}+r_2^{-1}+r_3^{-1}+r_4^{-1}}=\frac{1/0.3+1.05/0.4+1.1/0.1+1.08/0{,}2}{0.3^{-1}+0.4^{-1}+0.1^{-1}+0.2^{-1}}=$$
$$=1.073;$$

$$r_\Sigma=\left(r_1^{-1}+r_2^{-1}+r_3^{-1}+r_4^{-1}\right)^{-1}=\left(0.3^{-1}+0.4^{-1}+0.1^{-1}+0.2^{-1}\right)^{-1}=0.048.$$

The short-circuit current is: $I_S=I_5=E_\Sigma/(r_\Sigma+r_5)=1.073/(0.048+1)=1.024.$

Let's calculate a potential of point a: $\varphi_a=I_5r_5=1.024\cdot 1=1.024.$

The other circuit currents are determined by generalized Ohm's law:

$$I_1=(E_1-\varphi_a)/r_1=(1-1.024)/0.3=-0.08,$$
$$I_2=(E_2-\varphi_a)/r_2=(1.05-1.024)/0.4=0.065,$$
$$I_3=(E_3-\varphi_a)/r_3=(1.1-1.024)/0.1=0.76,$$
$$I_4=(E_4-\varphi_a)/r_4=(1.08-1.024)/0.2=0.28.$$

Verification: $I_1+I_2+I_3+I_4=-0.08+0.065+0.76+0.28=1.025\approx I_S.$

1-25 (2.43). In a scheme fig. 1.27 it is known: $E_1=1.1$; $E_2=1.05$; $r_1=0.1$; $r_2=0.2$; $r_3=0.8$; $r_4=0.5$; $r_5=0.6$; $r_6=1$.

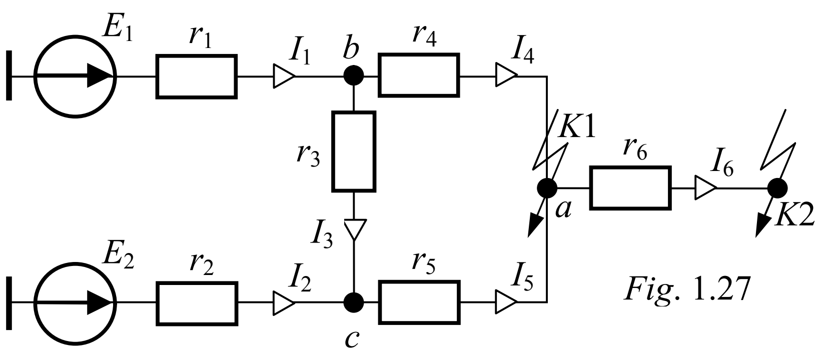


Fig. 1.27

Do the following:

1. Determine the short-circuit current in point $K1$ as well as the other circuit currents.
2. Determine the short-circuit current in point $K2$ as well as the other circuit currents.

Solution. Substitute the resistance triangle r_3-r_4-r_5 by the equivalent star r_a-r_b-r_c:

$$r_a=\frac{r_4\cdot r_5}{r_3+r_4+r_5}=\frac{0.5\cdot 0.6}{0.8+0.5+0.6}=0.1579,$$
$$r_b=\frac{r_3\cdot r_4}{r_3+r_4+r_5}=\frac{0.8\cdot 0.5}{0.8+0.5+0.6}=0.2105,$$

$$r_c = \frac{r_3 \cdot r_5}{r_3 + r_4 + r_5} = \frac{0.8 \cdot 0.6}{0.8 + 0.5 + 0.6} = 0.2526.$$

We have obtained the scheme fig. 1.28, it being analogous to scheme fig. 1.24 in problem 1-24. Its computation is developed under the same principles.

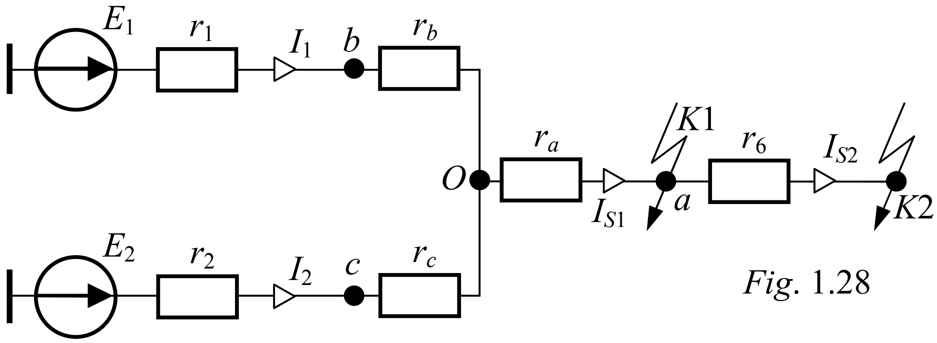


Fig. 1.28

$$E_\Sigma = \frac{E_1 \cdot (r_1 + r_b)^{-1} + E_2 \cdot (r_2 + r_c)^{-1}}{(r_1 + r_b)^{-1} + (r_2 + r_c)^{-1}} = \frac{1.1/(0.1 + 0.2105) + 1.05/(0.2 + 0.2526)}{(0.1 + 0.2105)^{-1} + (0.2 + 0.2526)^{-1}} =$$

$$= 1.080;$$

$$r_\Sigma = \left((r_1 + r_b)^{-1} + (r_2 + r_c)^{-1}\right)^{-1} = \left((0.1 + 0.2105)^{-1} + (0.2 + 0.2526)^{-1}\right)^{-1} = 0.1842.$$

1. A short-circuit current in point $K1$ and the branch currents are:

$$I_{S1} = E_\Sigma/(r_\Sigma + r_a) = 1.080/(0.1842 + 0.1579) = 3.157,$$

$$I_1 = (E_1 - I_{S1} \cdot r_a)/(r_1 + r_b) = (1.1 - 3.157 \cdot 0.1579)/(0.1 + 0.2105) = 1.937,$$

$$I_2 = (E_2 - I_{S1} \cdot r_a)/(r_2 + r_c) = (1.05 - 3.157 \cdot 0.1579)/(0.2 + 0.2526) = 1.218.$$

Let's calculate the potentials of points b and c:

$$\varphi_b = E_1 - I_1 \cdot r_1 = 1.1 - 1.937 \cdot 0.1 = 0.9063,$$

$$\varphi_c = E_2 - I_2 \cdot r_2 = 1.05 - 1.218 \cdot 0.2 = 0.8064.$$

The other currents of scheme fig. 1.27 are determined under Ohm's law and Kirchhoff's current law: $I_3 = (\varphi_b - \varphi_c)/r_3 = (0.9063 - 0.8064)/0.8 = 0.1249,$

$$I_4 = I_1 - I_3 = 1.937 - 0.1249 = 1.812,$$

$$I_5 = I_2 + I_3 = 1.218 + 0.1249 = 1.343.$$

2. A short-circuit current in point $K2$ and the branch currents are:

$$I_{S2} = E_\Sigma/(r_\Sigma + r_a + r_6) = 1.080/(0.1842 + 0.1579 + 1) = 0.8047,$$

$$I_1 = (E_1 - S_{S2} \cdot (r_a + r_6))/(r_1 + r_b) = (1.1 - 0.8047 \cdot (0.1579 + 1))/(0.1 + 0.2105) = 0.5418,$$

$$I_2 = (E_2 - I_{S2} \cdot (r_a + r_6))/(r_2 + r_c) = (1.05 - 0.8047 \cdot (0.1579 + 1))/(0.2 + 0.2526) = 0.2612.$$

Let's calculate the potentials of points b and c:

$$\varphi_b = E_1 - I_1 \cdot r_1 = 1.1 - 0.5418 \cdot 0.1 = 1.046,$$

$$\varphi_c = E_2 - I_2 \cdot r_2 = 1.05 - 0.2612 \cdot 0.2 = 0.9978.$$

The other currents in scheme fig. 1.27 are determined under Ohm's law and Kirchhoff's current law: $I_3 = (\varphi_b - \varphi_c)/r_3 = (1.046 - 0.9978)/0.8 = 0.0603,$

$$I_4 = I_1 - I_3 = 0.5418 - 0.0603 = 0.4815,$$

$$I_5 = I_2 + I_3 = 0.2612 + 0.0603 = 0.3215.$$

1-26 (2.44). In a scheme fig. 1.29 it is known: $E_1 = 1.1$; $E_2 = 1.05$; $r_1 = 0.1$; $r_2 = 0.2$; $r_3 = 0.8$; $r_4 = 1$. Determine a short-circuit current in point a as well as the other circuit currents.

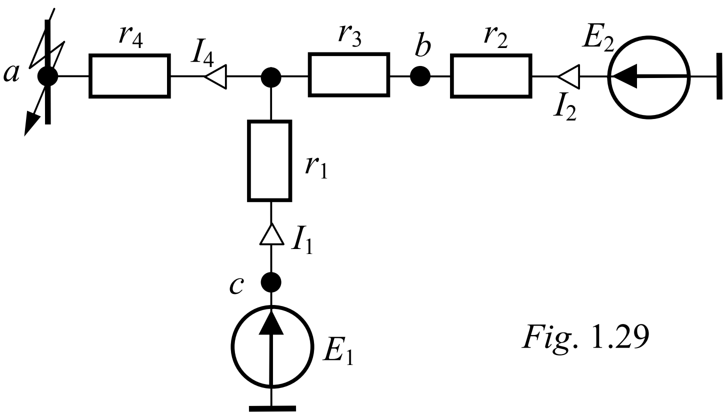


Fig. 1.29

Solution. Substitute a resistance star r_1- r_3-r_4 by an equivalent triangle r_{ab}-r_{bc}-r_{ca}:

$$r_{ab} = r_3 + r_4 + \frac{r_3 \cdot r_4}{r_1} = 0.8 + 1 + \frac{0.8 \cdot 1}{0.1} = 9.8,$$

$$r_{bc} = r_1 + r_3 + \frac{r_1 \cdot r_3}{r_4} = 0.1 + 0.8 + \frac{0.1 \cdot 0.8}{1} = 0.98,$$

$$r_{ca} = r_1 + r_4 + \frac{r_1 \cdot r_4}{r_3} = 0.1 + 1 + \frac{0.1 \cdot 1}{0.8} = 1.225.$$

It is possible to imagine two sources instead of a single ideal source E_1. As a result, we have an equivalent scheme fig. 1.30. Now, substitute the branches 2 and bc by single one. We obtain fig. 1.31:

$$E_{12} = \frac{E_1 \cdot r_{bc}^{-1} + E_2 \cdot r_2^{-1}}{r_{bc}^{-1} + r_2^{-1}} = \frac{1.1/0.98 + 1.05/0{,}2}{0.98^{-1} + 0.2^{-1}} = 1.058;$$

$$r_{2bc} = \left(r_{bc}^{-1} + r_2^{-1}\right)^{-1} = \left(0.98^{-1} + 0.2^{-1}\right)^{-1} = 0.1661.$$

The currents are calculated by Ohm's law:

$$I_{ab} = E_{12}/(r_{2bc} + r_{ab}) = 1.058/(0.1661 + 9.8) = 0.1062,$$

$$I_{11} = E_1/r_{ca} = 1.1/1.225 = 0.8980.$$

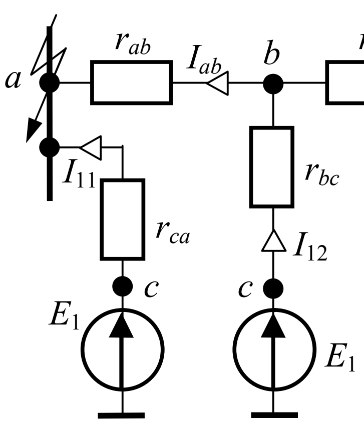


Fig. 1.30

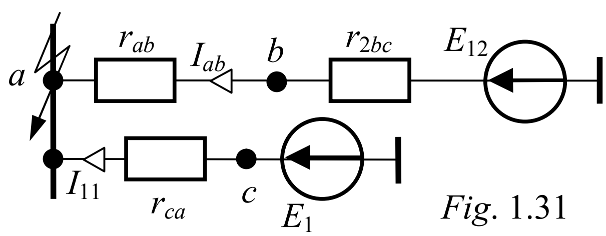


Fig. 1.31

A short-circuit current is: $I_S = I_4 = I_{ab} + I_{11} = 0.1062 + 0.8980 = 1.0042$.

Determine the currents of schemes fig. 1.30 and 1.29:

$$I_2 = (E_2 - I_{ab} \cdot r_{ab})/r_2 = (1.05 - 0.1062 \cdot 9.8)/0.2 = 0.0462,$$

$$I_{12} = (E_1 - I_{ab} \cdot r_{ab})/r_{bc} = (1.1 - 0.1062 \cdot 9.8)/0.98 = 0.0604,$$

$$I_1 = I_{11} + I_{12} = 0.8980 + 0.0604 = 0.9584.$$

1-27 (2.45). In an electric circuit fig. 1.32, it is necessary to calculate a short-circuit current in points $K1$, $K2$, $K3$ as well as the other branch currents. Numerical data are: $E_1 = 1.1$; $E_2 = 1.05$; $E_3 = 0.8$; $E_4 = 0.7$; $r_1 = 0.05$; $r_2 = 0.1$; $r_3 = 2$; $r_4 = 2.5$; $r_5 = 0.2$; $r_6 = 0.2$; $r_7 = 0.4$; $r_8 = 0.6$; $r_9 = 0.1$; $r_{10} = 0.5$; $r_{11} = 1$.

Figure 1.32. See file “Album”

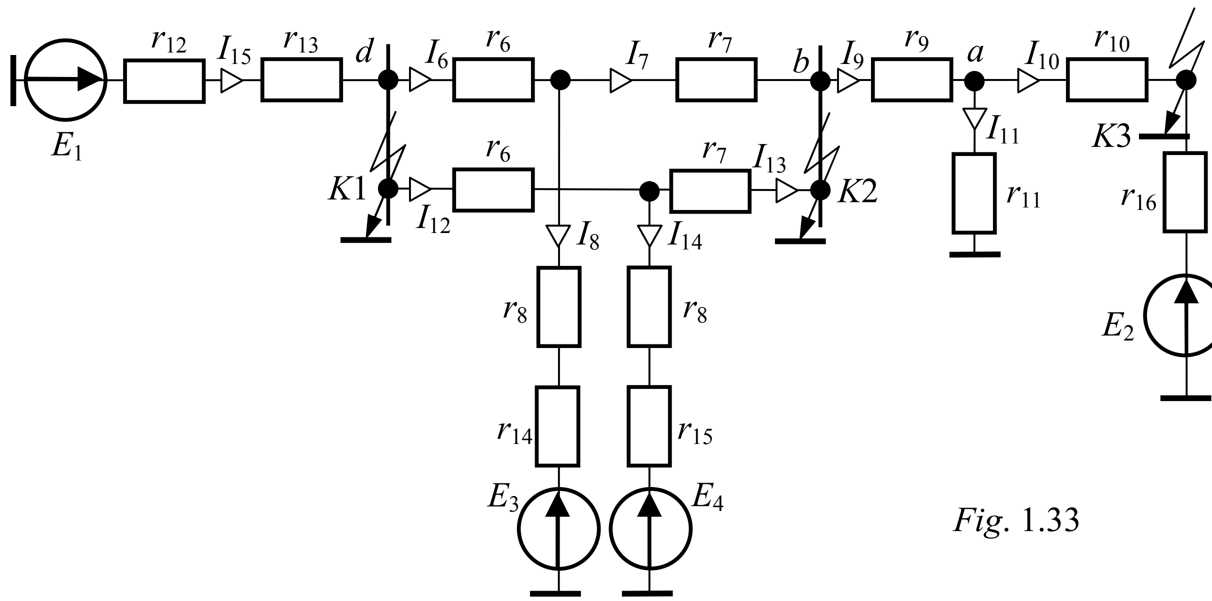


Fig. 1.33

Solution. Similarly to problem 1-24, let's simplify the scheme fig. 1.32 and obtain scheme fig. 1.33:

- substitute parallel resistors r_5 by a single one $r_{13} = ½r_5 = 0.1$;
- group of three identical branches E_1-r_1 is presented by an equivalent one E_1-r_{12}: $r_{12} = r_1/3 = 0.0167$;
- the same is done with groups E_2-r_2, E_3-r_3 and E_4-r_4:

$r_{14} = ½r_3 = 1$; $r_{15} = r_4/3 = 0.8333$; $r_{16} = ½r_2 = 0.05$.

1. Let's deal with the simplifications in scheme fig. 1.33 with the short-circuit in point $K1$. The branches r_{11}–$(r_{10}+r_{16})$ are substituted by single one E_5-r_{23}:

$$r_{23} = \frac{r_{11}\cdot(r_{10}+r_{16})}{r_{11}+r_{10}+r_{16}} = \frac{1\cdot(0.5+0.5)}{1+0.5+0.05} = 0.3548;$$

$$E_5 = \frac{E_2\cdot(r_{10}+r_{16})^{-1}}{(r_{10}+r_{16})^{-1}+r_{11}^{-1}} = \frac{1.05/0.55}{0.55^{-1}+1^{-1}} = 0.6774.$$

The resistance stars r_6-r_7-(r_8+r_{14}) and r_6-r_7-(r_8+r_{15}) are replaced by the equivalent triangles r_{17}-r_{19}-r_{20} и r_{18}-r_{21}-r_{22}, respectively:

$$r_{17} = r_6 + (r_8 + r_{14}) + \frac{r_6\cdot(r_8+r_{14})}{r_7} = 0.2 + 0.6 + 1 + \frac{0.2\cdot(0.6+1)}{0.4} = 2.6,$$

$$r_{19} = r_6 + r_7 + \frac{r_6\cdot r_7}{r_8+r_{14}} = 0.2 + 0.4 + \frac{0.2\cdot 0.4}{0.6+1} = 0.65,$$

$$r_{20} = r_7 + (r_8 + r_{14}) + \frac{r_7\cdot(r_8+r_{14})}{r_6} = 0.4 + 1.6 + \frac{0.4\cdot(0.6+1)}{0.2} = 5.2,$$

$$r_{18} = r_6 + (r_8 + r_{15}) + \frac{r_6\cdot(r_8+r_{15})}{r_7} = 0.2 + 0.6 + 0.8333 + \frac{0.2\cdot(0.6+0.8333)}{0.4} = 2.35,$$

$$r_{21} = r_6 + r_7 + \frac{r_6\cdot r_7}{r_8+r_{15}} = 0.2 + 0.4 + \frac{0.2\cdot 0.4}{0.6+0.8333} = 0.6558,$$

$$r_{22} = r_7 + (r_8 + r_{15}) + \frac{r_7\cdot(r_8+r_{15})}{r_6} = 0.4 + 1.4333 + \frac{0.4\cdot(0.6+0.8333)}{0.2} = 4.70.$$

Similar to the way used in problem 1-26 (fig. 1.29), we obtain scheme fig. 1.34, which we keep on simplifying into scheme fig. 1.35.

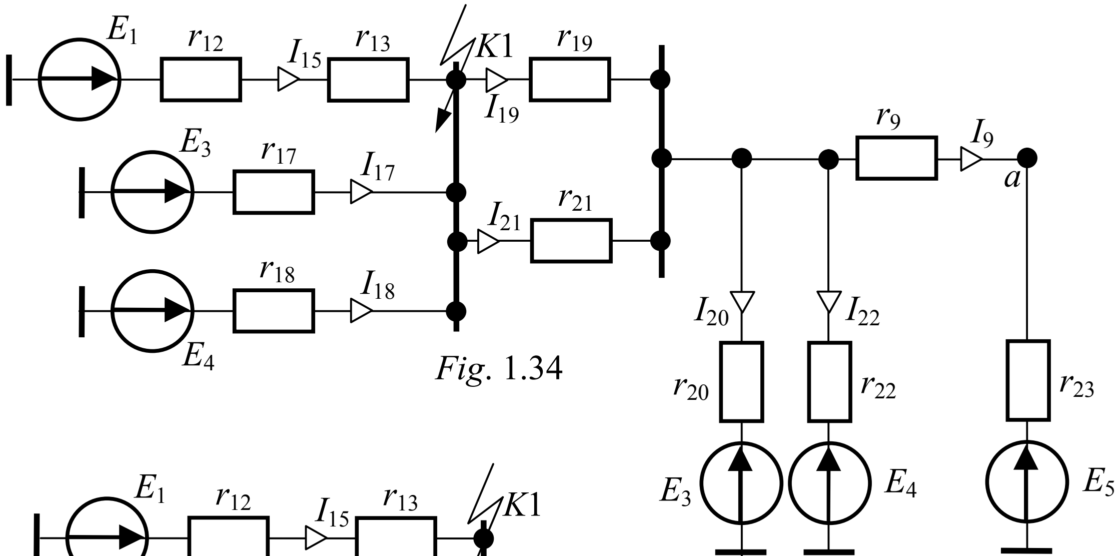


Fig. 1.34

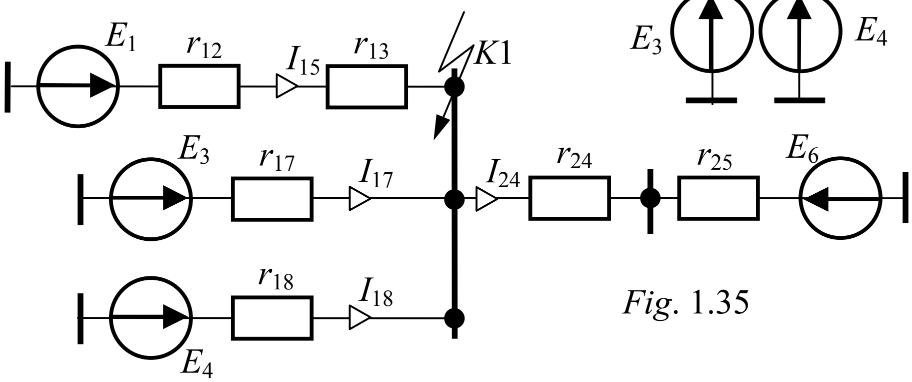


Fig. 1.35

$$r_{24} = \frac{r_{19} \cdot r_{21}}{r_{19} + r_{21}} = \frac{0.65 \cdot 0.6558}{0.65 + 0.6558} = 0.3264,$$

$$E_6 = \frac{E_3 \cdot r_{20}^{-1} + E_4 \cdot r_{22}^{-1} + E_5 \cdot (r_{23} + r_9)^{-1}}{r_{20}^{-1} + r_{22}^{-1} + (r_{23} + r_9)^{-1}} =$$

$$= \frac{0.8/5.2 + 0.7/4.7 + 0.6774/(0.3548 + 0.1)}{5.2^{-1} + 4.7^{-1} + 0.4548^{-1}} = 0.6883;$$

$$r_{25} = \left(r_{20}^{-1} + r_{22}^{-1} + (r_{23} + r_9)^{-1}\right)^{-1} = \left(5.2^{-1} + 4.7^{-1} + 0.4548^{-1}\right)^{-1} = 0.3840.$$

Calculate the currents under the schemes fig. 1.33-1.35:

$$I_{15} = E_1/(r_{12} + r_{13}) = 1.1/(0.0167 + 0.1) = 9.426,$$

$$I_{17} = E_3/r_{17} = 0.8/2.6 = 0.3077,$$

$$I_{18} = E_4/r_{18} = 0.7/2.35 = 0.2979,$$

$$I_{24} = -E_6/(r_{24} + r_{25}) = -0.6883/(0.3264 + 0.3840) = -0.9689,$$

$$I_{19} = I_{24} \cdot \frac{r_{21}}{r_{19} + r_{21}} = -0.9689 \cdot \frac{0.6558}{0.65 + 0.6558} = -0.4866,$$

$$I_{21} = I_{24} \cdot \frac{r_{19}}{r_{19} + r_{21}} = -0.9689 \cdot \frac{0.65}{0.65 + 0.6558} = -0.4823,$$

$$I_{20} = -(E_3 + I_{24} \cdot r_{24})/r_{20} = -(0.8 - 0.9689 \cdot 0.3264)/5.2 = -0.0930,$$

$$I_{22} = -(E_4 + I_{24} \cdot r_{24})/r_{22} = -(0.7 - 0.9689 \cdot 0.3264)/4.7 = -0.0816,$$

$$I_9 = -(E_5 + I_{24} \cdot r_{24})/(r_{23} + r_9) = -(0.6774 - 0.9689 \cdot 0.3264)/(0.3548 + 0.1) = -0.7941,$$

$$\varphi_a = -I_{24} \cdot r_{24} - I_9 \cdot r_9 = 0.9689 \cdot 0.3264 + 0.7941 \cdot 0.1 = 0.3957,$$

$$I_{11} = \varphi_a/r_{11} = 0.3957/1 = 0.3957,$$
$$I_{10} = -(E_2 - \varphi_a)/(r_{10} + r_{16}) = -(1.05 - 0.3957)/(0.5 + 0.05) = -1.190,$$
$$I_2 = -½I_{10} = 0.5\cdot 1.190 = 0.595,$$
$$I_8 = I_{20} - I_{17} = -0.093 - 0.3077 = -0.4007,$$
$$I_{14} = I_{22} - I_{18} = -0.0816 - 0.2979 = -0.3795,$$
$$I_6 = -(E_3 + I_8\cdot(r_8 + r_{14}))/r_6 = -(0.8 - 0.4007\cdot(0.6 + 1))/0.2 = -0.7944,$$
$$I_7 = I_6 - I_8 = -0.7944 + 0.4007 = -0.3937,$$
$$I_{12} = -(E_4 + I_{14}\cdot(r_8 + r_{15}))/r_6 = -(0.7 - 0.3795\cdot(0.6 + 0.8333))/0{,}2 = -0.7803,$$
$$I_{13} = I_{12} - I_{14} = -0.7803 + 0.3795 = -0{,}4008,$$
$$I_1 = I_{15}/3 = 9.426/3 = 3.142,$$
$$I_5 = I_{15}/2 = 9.426/2 = 4.713,$$
$$I_3 = I_8/2 = -0.4007/2 = -0.2004,$$
$$I_4 = I_{14}/3 = -0.3795/3 = -0{,}1265,$$
$$I_{K3} = 2\cdot I_5 - I_6 - I_{12} = 2\cdot 4.713 + 0.7944 + 0.7803 = 11.0.$$

Verification: $I_{S1} = I_{15} + I_{17} + I_{18} - I_{24} = 9.426 + 0.3077 + 0.2979 + 0.9689 = 11.$

Circuit calculation with **the short-circuits** in points ***K*2** and ***K*3** is fulfilled in the way considered above, that's why let's only show the necessary schemes and calculation results.

2. While calculating the scheme with short-circuit in point $K2$ of fig. 1.33, the schemes fig. 1.36 and 1.37 are used. For these schemes, we have the following:

$$r_{17} = 2.6,\ r_{19} = 0.65,\ r_{20} = 5.2,\ r_{18} = 2.35,\ r_{21} = 0.6558,\ r_{22} = 4.70,$$
$$r_{23} = 0.3548,\ E_5 = 0.6774;$$

$$E_6 = \frac{E_3\cdot r_{20}^{-1} + E_4\cdot r_{22}^{-1} + E_5\cdot(r_{23} + r_9)^{-1}}{r_{20}^{-1} + r_{22}^{-1} + (r_{23} + r_9)^{-1}} = 0.6883,$$

$$r_{25} = \left(r_{20}^{-1} + r_{22}^{-1} + (r_{23} + r_9)^{-1}\right)^{-1} = 0.3840, \quad r_{24} = \frac{r_{19}\cdot r_{21}}{r_{19} + r_{21}} = 0.3264,$$

$$E_7 = \frac{E_3\cdot r_{17}^{-1} + E_4\cdot r_{18}^{-1} + E_1\cdot(r_{12} + r_{13})^{-1}}{r_{17}^{-1} + r_{18}^{-1} + (r_{12} + r_{13})^{-1}} = 1.0695,$$

$$r_{26} = \left(r_{17}^{-1} + r_{18}^{-1} + (r_{12} + r_{13})^{-1}\right)^{-1} = 0.1066.$$

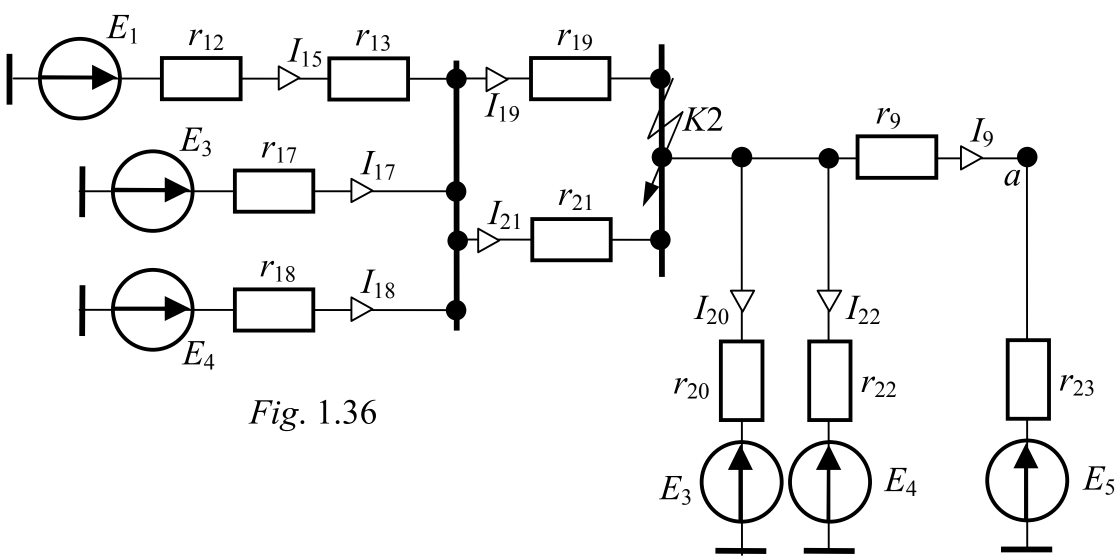


Fig. 1.36

The schemes fig. 1.33, 1.36, 1.37 give the possibility to calculate the currents:

$I_{24} = E_7/(r_{26} + r_{24}) = 2.470,\ \ I_{25} = -E_6/r_{25} = -1.7924,$

$I_{20} = -E_3/r_{20} = -0.1538,\ \ I_{22} = -E_4/r_{22} = -0.1489,\ \ I_9 = -E_5/(r_{23} + r_9) = -1.4894,$

$$I_{19} = I_{24}\cdot\frac{r_{21}}{r_{19} + r_{21}} = 1.2405,\ \ I_{21} = I_{24}\cdot\frac{r_{19}}{r_{19} + r_{21}} = 1.2295,$$

$I_{15} = (E_1 - I_{24}\cdot r_{24})/(r_{12} + r_{13}) = 2.5175,$

$I_{17} = (E_3 - I_{24}\cdot r_{24})/r_{17} = -0{,}0024,\ \ I_{18} = (E_4 - I_{24}\cdot r_{24})/r_{18} = -0.0452,$

$\varphi_a = -I_9\cdot r_9 = 0.1489,\ \ I_{11} = \varphi_a/r_{11} = 0.1489,$

$I_{10} = -(E_2 - \varphi_a)/(r_{10} + r_{16}) = -1.6384,\ \ I_2 = -½I_{10} = 0.8192,$

$I_8 = I_{20} - I_{17} = -0.1514,\ \ I_{14} = I_{22} - I_{18} = -0.1037,$

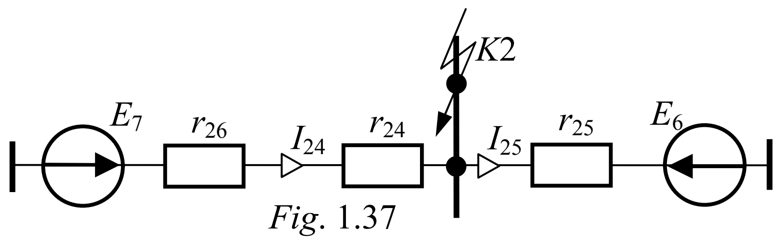


Fig. 1.37

$I_7 = (E_3 + I_8\cdot(r_8 + r_{14}))/r_7 = 1.3944,\ \ I_6 = I_7 + I_8 = 1.243,$

$I_{13} = (E_4 + I_{14}\cdot(r_8 + r_{15}))/r_7 = 1.3784,\ \ I_{12} = I_{13} + I_{14} = 1.2747,$

$I_1 = I_{15}/3 = 0.8392,\ \ \ I_5 = I_{15}/2 = 1.2788,$

$I_3 = I_8/2 = -0.0757,\ \ I_4 = I_{14}/3 = -0.0346,$

$I_{S2} = I_7 + I_{13} - I_9 = 4.2622.$

3. While calculating the scheme with short-circuit in point *K*3 of fig. 1.33, the schemes fig. 1.38-1.41 are used. For these schemes, we have the following:

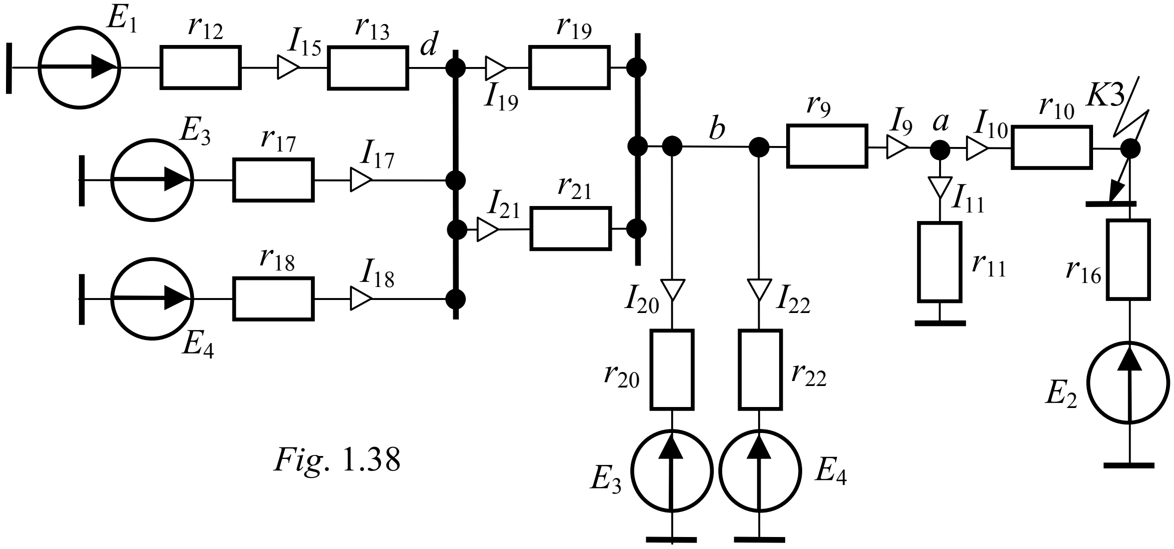


Fig. 1.38

$r_{17} = 2.6,\ r_{19} = 0.65,\ r_{20} = 5.2,\ r_{18} = 2.35,\ r_{21} = 0.6558,\ r_{22} = 4.70.$

$$r_{24} = \frac{r_{19}\cdot r_{21}}{r_{19} + r_{21}} = 0.3264,\ \ E_7 = \frac{E_3\cdot r_{17}^{-1} + E_4\cdot r_{18}^{-1} + E_1\cdot(r_{12} + r_{13})^{-1}}{r_{17}^{-1} + r_{18}^{-1} + (r_{12} + r_{13})^{-1}} = 1.0695,$$

$$r_{26} = \left(r_{17}^{-1} + r_{18}^{-1} + (r_{12} + r_{13})^{-1}\right)^{-1} = 0.1066,$$

$$E_8 = \frac{E_3\cdot r_{20}^{-1} + E_4\cdot r_{22}^{-1} + E_7\cdot(r_{24} + r_{26})^{-1}}{r_{20}^{-1} + r_{22}^{-1} + (r_{24} + r_{26})^{-1}} = 1.0214,$$

$$r_{27} = \left(r_{20}^{-1} + r_{22}^{-1} + (r_{24} + r_{26})^{-1}\right)^{-1} = 0.3684,$$

$$E_9 = \frac{E_8 \cdot (r_{27} + r_9)^{-1}}{r_{11}^{-1} + (r_{27} + r_9)^{-1}} = 0.6956, \;\; r_{28} = \left(r_{11}^{-1} + (r_{27} + r_9)^{-1}\right)^{-1} = 0.3190.$$

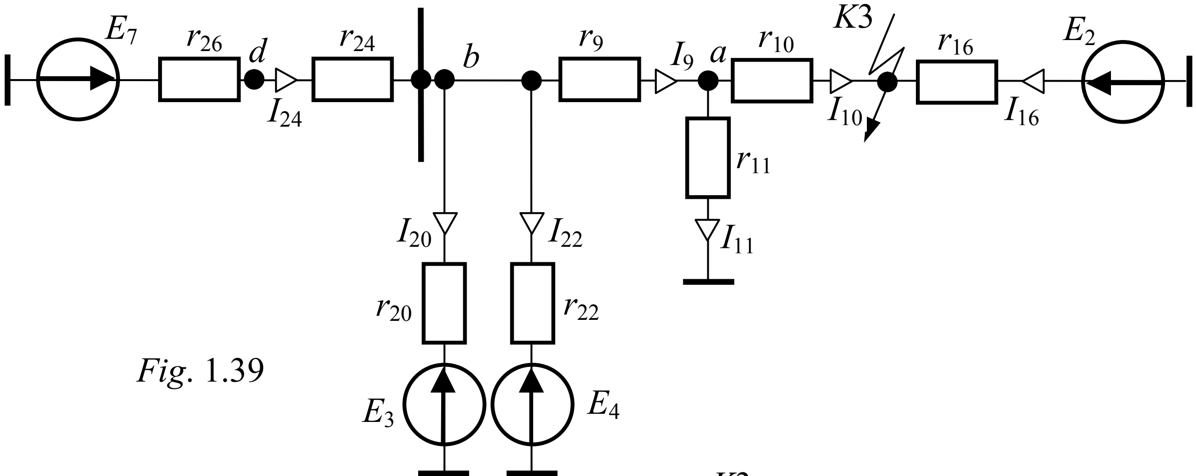


Fig. 1.39

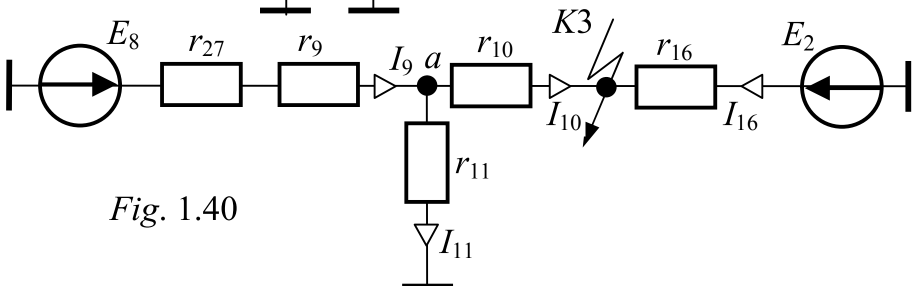


Fig. 1.40

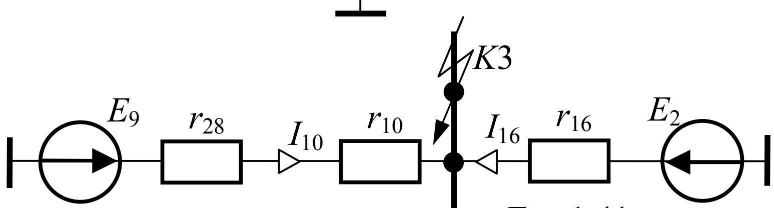


Fig. 1.41

The schemes fig. 1.33, 1.38-1.41 give the possibility to calculate the currents:

$I_{10} = E_9/(r_{28} + r_{10}) = 0.8493, \;\; I_{16} = E_2/r_{16} = 21,$

$\varphi_a = I_{10} \cdot r_{10} = 0.4247, \;\; I_{11} = \varphi_a/r_{11} = 0.4247, \;\; I_9 = I_{10} + I_{11} = 1.274,$

$\varphi_b = \varphi_a + I_9 \cdot r_9 = 0.5521, \;\; I_{24} = (E_7 - \varphi_b)/(r_{26} + r_{24}) = 1.1949,$

$I_{20} = -(E_3 - \varphi_b)/r_{20} = -0.0477, \;\; I_{22} = -(E_4 - \varphi_b)/r_{22} = -0.0315,$

$$I_{19} = I_{24} \cdot \frac{r_{21}}{r_{19} + r_{21}} = 0.6001, \;\; I_{21} = I_{24} \cdot \frac{r_{19}}{r_{19} + r_{21}} = 0.5948,$$

$\varphi_d = \varphi_b + I_{24} \cdot r_{24} = 0.9421, \;\; I_{15} = (E_1 - \varphi_d)/(r_{12} + r_{13}) = 1.353,$

$I_{17} = (E_3 - \varphi_d)/r_{17} = -0.0547, \;\; I_{18} = (E_4 - \varphi_d)/r_{18} = -0{,}1030,$

$I_8 = I_{20} - I_{17} = 0.007, \;\; I_{14} = I_{22} - I_{18} = 0.0715,$

$I_7 = (E_3 + I_8 \cdot (r_8 + r_{14}) - \varphi_b)/r_7 = 0.6478, \;\; I_6 = I_7 + I_8 = 0.6548,$

$I_{13} = (E_4 + I_{14} \cdot (r_8 + r_{15}) - \varphi_b)/r_7 = 0.6260, \;\; I_{12} = I_{13} + I_{14} = 0.6975,$

$I_1 = I_{15}/3 = 0.451, \;\; I_5 = I_{15}/2 = 0.6765,$

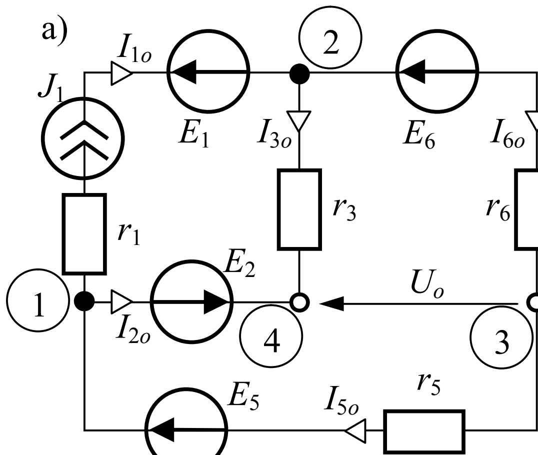
$I_3 = I_8/2 = 0.0035, \;\; I_4 = I_{14}/3 = 0.0238,$

$I_2 = I_{16}/2 = 10.5, \;\; I_{S3} = I_{16} + I_{10} = 21.8493.$

1.6. EQUIVALENT GENERATOR METHOD

1-28 (2.46). Determine current I_4 in the scheme fig. 1.18,a (problem 1-15) by the equivalent generator method.

Solution. Firstly, we disjoin branch with resistance r_4 and obtain the no-load condition of the equivalent generator as regards to the terminals «3» and «4», it being shown in fig. 1.42,a. In fig. 1.42,b the passive part of the remaining scheme is shown, which is suitable for calculation of the inner resistance of the equivalent generator.



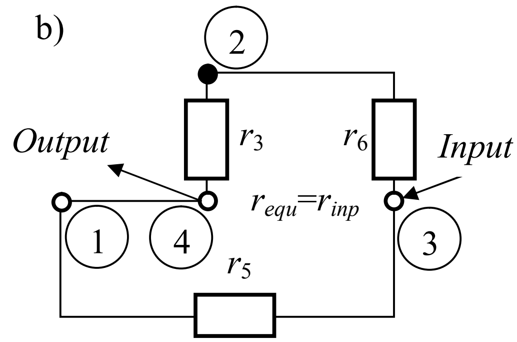


Fig. 1.42

Under Kirchhoff's voltage law for the bottom loop of the scheme fig. 1.42,a, we have:
$$U_o = -E_2 - E_5 + I_{5o} \cdot r_5.$$

Current I_{5o} may be calculated by the mesh current method
$$I_{5o} \cdot (r_5 + r_3 + r_6) - J_1 \cdot r_3 = E_5 + E_2 - E_6,$$

from here $I_{5o} = \dfrac{120 + 100 - 60 + 4 \cdot 40}{30 + 40 + 20} = 3.556\ A,$

and $U_o = -100 - 120 + 3.556 \cdot 30 = -113.3\ V.$

In accordance with the scheme fig. 1.42,b, the input resistance concerning terminals «3» and «4» is

$$r_{34inp} = r_{equ} = \frac{(r_3 + r_6) \cdot r_5}{r_3 + r_6 + r_5} = \frac{(40 + 20) \cdot 30}{40 + 20 + 30} = 20\ Ohm.$$

Required current is $I_4 = \dfrac{U_o}{r_{equ} + r_4} = \dfrac{-113.3}{20 + 60} = -1.417\ A,$

which coincides with the above-obtained result.

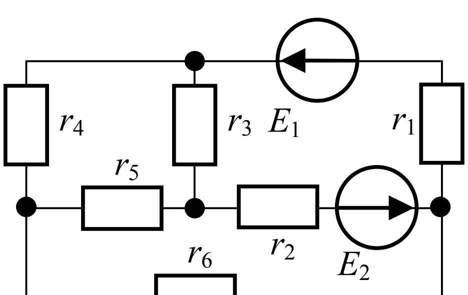


Fig. 1.43

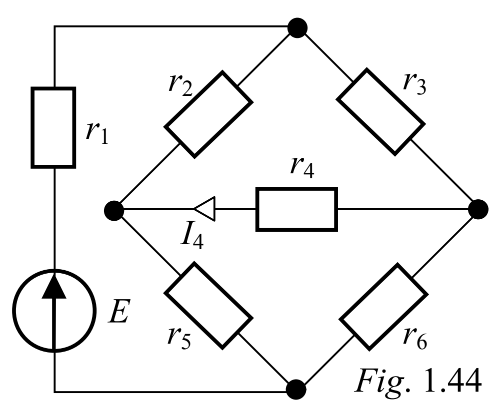


Fig. 1.44

1-29 (2.50). In scheme fig. 1.43, determine the current through resistance r_2, if:
$E_1 = 72\ V,\ E_2 = 14\ V,\ r_1 = r_6 = 10\ Ohm,\ r_2 = 25\ Ohm,\ r_3 = r_5 = 40\ Ohm,\ r_4 = 20\ Ohm.$
Answers: $U_o = 50\ V,\ R_{inp} = 25\ Ohm,\ I_2 = 1\ A.$

1-30. Determine the current I_4 in the bridge diagonal (fig. 1.44), if $E = 40\ V$, $r_1 = 80\ Ohm,\ r_2 = 100\ Ohm,\ r_3 = r_6 = 60\ Ohm,\ r_4 = 49{,}5\ Ohm,\ r_5 = 20\ Ohm.$
Answers: $U_o = 20\ V,\ R_{inp} = 50.5\ Ohm,\ I_4 = 0.2\ A.$

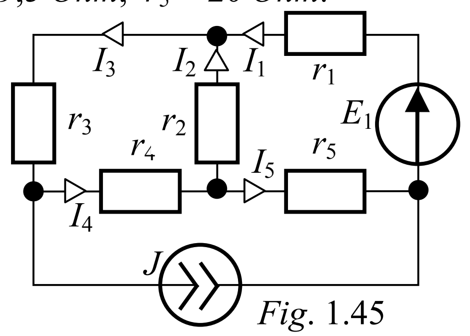


Fig. 1.45

1.7. THE SUPERPOSITION METHOD

1-31 (2.51). Determine the branch currents in the scheme fig. 1.45, if $E_1 = 16\ V$,

$J = 1\ A,\ r_1 = r_2 = r_3 = r_4 = r_5 = 6\ Ohm.$

Solution. 1. Assume the positive directions of the branch currents.

2. Imagine that in the circuit there is only one acting source of emf E_1 (fig. 1.46), determine the branch currents:

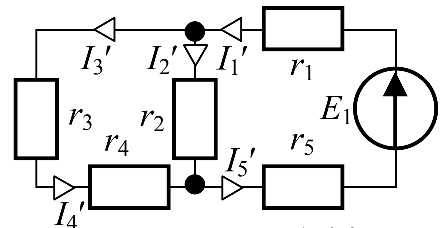


Fig. 1.46

$$I_5' = I_1' = \frac{E_1}{r_1 + r_5 + \dfrac{r_2\cdot(r_4+r_3)}{r_2+r_4+r_3}} = \frac{16}{6+6+\dfrac{6\cdot(6+6)}{6+6+6}} = 1\ A.$$

Current I_2' is determined under the current dispersion rule: $I_2' = I_1'\cdot\dfrac{r_4+r_3}{r_4+r_3+r_2} = 1\cdot\dfrac{12}{18} = 0.667\ A.$

Current I_3' is determined under Kirchhoff's current law

$$I_4' = I_3' = I_1' - I_2' = 1 - 0.667 = 0.333\ A.$$

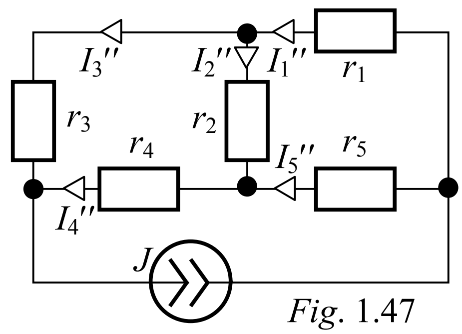


Fig. 1.47

3. Secondly, imagine that in the circuit there is only one acting source of current J (fig. 1.47), determine the branch currents.

To find currents in scheme fig. 1.47, we transform the resistance triangle r_2-r_3-r_4 into the equivalent star.Take into account the triangle resistances are identical:

$$r_{23} = r_{24} = r_{34} = \frac{r_2 r_3}{r_2+r_3+r_4} = \frac{36}{18} = 2\ Ohm.$$

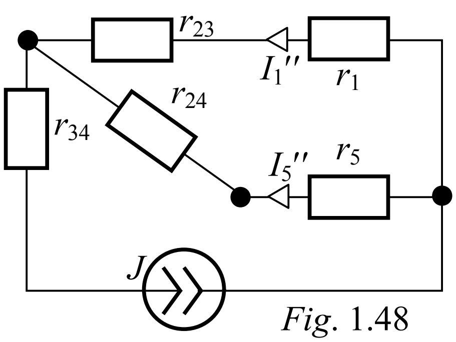


Fig. 1.48

As a result of transformation, there is a scheme fig. 1.48, from where we find the currents I_1'' and I_5'':

$$I_1'' = J\cdot\frac{r_{24}+r_5}{r_{24}+r_5+r_{23}+r_1} = 1\cdot\frac{2+6}{2+6+2+6} = 0.5\ A,$$

$$I_5'' = J - I_1'' = 1 - 0.5 = 0.5\ A.$$

From scheme fig. 1.47 according to Kirchhoff's voltage law for a loop r_1-r_2-r_5, we determine the current I_2'':

$$r_1\cdot I_1'' + r_2\cdot I_2'' - r_5\cdot I_5'' = 0,\quad \text{from here}\quad I_2'' = \frac{r_5 I_5'' - r_1 I_1''}{r_2} = 0.$$

Under Kirchhoff's current law we determine the remaining currents:

$I_4'' = I_2'' + I_5'' = 0.5\ A, \qquad I_3'' = I_1'' - I_2'' = 0.5\ A.$

4) In accordance with the superposition principle, we determine full currents in the circuit branches

$I_1 = I_1' + I_1'' = 1 + 0,.5 = 1.5\ A, \qquad I_2 = -I_2' - I_2'' = 0,.67\ A,$

$I_3 = I_3' + I_3'' = 0.333 + 0.5 = 0.833\ A,$

$I_4 = I_4' - I_4'' = 0.333 - 0.5 = -0.167\ A,$

$I_5 = I_5' - I_5'' = 1 - 0.5 = 0.5\ A.$

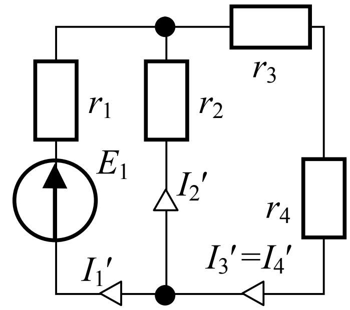


Fig. 1.49

1-32 (2.52). Using the conditions of problem 1.13 (fig. 1.16), determine the currents by the superposition method.

Solution. 1) Determine the currents from the source E_1 action (fig. 1.49):

$$I_1' = \frac{E_1}{\dfrac{(r_3+r_4)\cdot r_2}{r_3+r_4+r_2}+r_1} = \frac{120}{\dfrac{(5+15)\cdot 20}{5+15+20}+20} = 4\ A,$$

$$I_3' = I_4' = I_1'\cdot\frac{r_2}{r_2+r_3+r_4} = 4\cdot\frac{20}{20+5+15} = 2\ A,$$

$I_2' = I_3' - I_1' = 2 - 4 = -2\ A.$

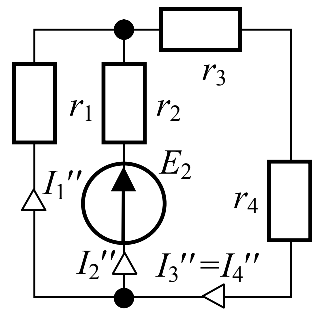


Fig. 1.50

2) Determine the currents from the source E_2 action (fig. 1.50). Since $r_1 = r_2$, the schemes in fig. 1.49 and fig. 1.50 are identical, and because of $E_2 = 0.5E_1$, the component currents of scheme fig. 1.50 are two times less than corresponding currents of scheme fig. 1.49:

$I_2'' = 0.5I_1' = 2\ A, \qquad I_1'' = 0.5I_2' = -1\ A,$

$I_3'' = I_4'' = 0.5I_3' = 1\ A.$

3) Determine the currents from the current source J action (fig. 1.51):

$$I_4''' = J\cdot\frac{r_3+\dfrac{r_1r_2}{r_1+r_2}}{r_4+r_3+\dfrac{r_1r_2}{r_1+r_2}} = 4\cdot\frac{5+20/2}{15+5+20/2} = 2\ A,$$

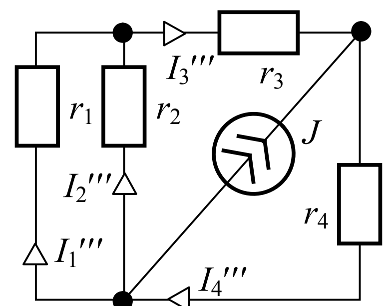


Fig. 1.51

$I_3''' = I_4''' - J = 2 - 4 = -2\ A.$

As $r_1 = r_2$, then $I_1''' = I_2''' = 0,5I_3''' = -1\ A.$

4) In accordance with the superposition principle, the full currents are:

$I_1 = I_1' + I_1'' + I_1''' = 4 - 1 - 1 = 2\ A, \qquad I_3 = I_3' + I_3'' + I_3''' = 2 + 1 - 2 = 1\ A,$

$I_2 = I_2' + I_2'' + I_2''' = -2 + 2 - 1 = -1\ A, \qquad I_4 = I_4' + I_4'' + I_4''' = 2 + 1 + 2 = 5\ A.$

2. NONLINEAR ELECTRIC AND MAGNETIC D-C CIRCUITS

2.1. NONLINEAR ELECTRIC CIRCUITS

2-1 (9.1). In fig. 2.1,a there is a volt-ampere characteristic of an incandescent lamp, in fig. 2.1,b – that of a stabilitron, in fig. 2.1,c – that of a tunnel diode. For working points *A*, *B*, *C*, determine both static and differential resistances of the elements.

Solution. Let's draw the secant (sec) (from the coordinate origin) and tangent (tang) lines through the given points. For fig. 2.1,a, we have:

$$r_{stA}=\frac{U_A}{I_A}=\frac{3}{0.1}=30\ Ohm,\qquad r_{dA}=\frac{dU}{dI}\Big|_A=lim\frac{\Delta U}{\Delta I}=\frac{10}{0.22}=45.5\ Ohm;$$

$$r_{stB}=\frac{U_B}{I_B}=\frac{20}{0.24}=83.3\ Ohm,\qquad r_{\partial B}=\frac{dU}{dI}\Big|_B=\frac{40}{0.16}=250\ Ohm;$$

$$r_{stC}=\frac{U_C}{I_C}=\frac{50}{0.32}=156.3\ Ohm,\qquad r_{\partial C}=\frac{dU}{dI}\Big|_C=\frac{50}{0.13}=385\ Ohm.$$

For fig. 2.1,b, we have:

$$r_{stA}=\frac{2.5}{0.4}=6.25\ Ohm,\qquad r_{dA}=\frac{0.1}{0.4}=0.25\ Ohm;$$

$$r_{stB}=\frac{-5}{-0}=\infty\ Ohm,\qquad r_{dB}=\frac{+5}{0}=\infty\ Ohm;$$

$$r_{stC}=\frac{-10}{-0.4}=25\ Ohm,\qquad r_{dC}=\frac{0}{0.4}=0\ Ohm.$$

For fig. 2.1,c, we have:

$$r_{stA}=\frac{0.04}{3\cdot10^{-3}}=13.3\ Ohm,\qquad r_{dA}=\frac{0.04}{3\cdot10^{-3}}=13.3\ Ohm>0;$$

$$r_{stB}=\frac{0.3}{2\cdot10^{-3}}=150\ Ohm,\qquad r_{dB}=\frac{\Delta U}{\Delta I}=\frac{0.50-0}{(0-5.2)\cdot10^{-3}}=-96.2\ Ohm<0;$$

$$r_{stC}=\frac{0.8}{0.3\cdot10^{-3}}=267\ Ohm,\qquad r_{dC}=\frac{0.3}{3\cdot10^{-3}}=100\ Ohm>0.$$

Note, in the negative-going part of the volt-ampere characteristic, differential resistance is negative. Static resistance is always positive.

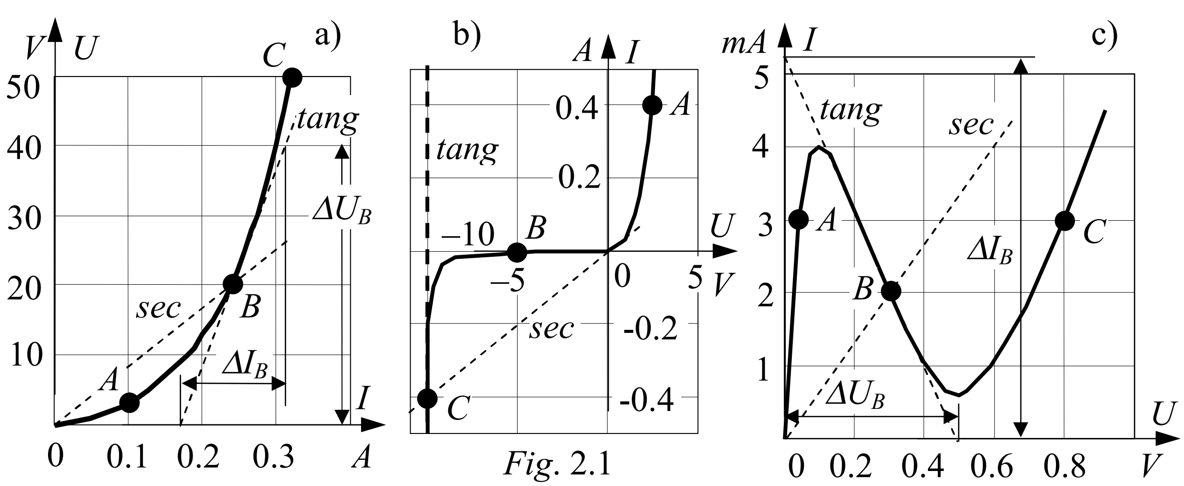


Fig. 2.1

2-2 (9.2). VAC of a nonlinear resistor is set by the formula

$$I = 0.1 \cdot U + 0.015 \cdot U^3, \text{ where } U\,[V],\ I\,[A].$$

Determine static and differential resistance at voltage $U = 1\ V$.

Answers: $R_{st} = \dfrac{U}{I} = \dfrac{1}{0.115} = 8.7\ Ohm$; $\quad \dfrac{dI}{dU} = 0.1 + 0.045 \cdot U^2$,

$$R_{dif} = \frac{dU}{dI} = \frac{1}{0.1 + 0.045U^2} = \frac{1}{0.145} = 6.9\ Ohm.$$

2-3 (9.3). Calculate the circuit current of fig. 2.2, check up the power balance, if

$$U = 100\ V, \quad r_2 = 20\ Ohm,$$

volt-ampere characteristics of nonlinear elements are set by the tables:

Table 2.1

U_1, *V*	0	15	20	23	30
I_1, *A*	0	0.5	1	2	4

Table 2.2

U_3, *V*	0	5	15	40	80
I_3, *A*	0	1	2	3	4

Solution. Let's employ the diagram method of calculation of a nonlinear electric circuit, using the equation system generated beforehand:

under Kirchhoff's voltage law $U_1 + U_2 + U_3 = U$;

moreover their currents are the same $I_1 = I_2 = I_3 = I$, as the resistances are connected in series;

correlations between currents and voltages of the circuit parts with nonlinear resistances are given in the tables:

$U_1 = f_1(I_1)$ – table 1, $\quad U_3 = f_2(I_3)$ – table 2,

however, for linear resistance in accordance with Ohm's law, we have analytical dependence $U_2 = I_2 \cdot r_2$.

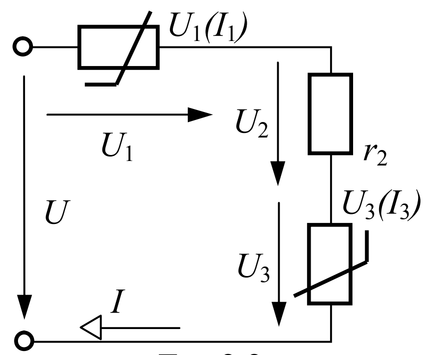


Fig. 2.2

The diagram constructions corresponding to above-mentioned equation system are shown in fig. 2.3 and are presented in the Cartesian coordinate system *I(U)*.

Rewrite the initial equation which was written under Kirchhoff's voltage law, leaving the voltages across nonlinear elements in the left side of the equation and carrying the linear correlation $U_2 = I \cdot r_2$ into the right side of the equation:

$$U_1 + U_3 = U - I \cdot r_2.$$

Construct auxiliary dependences (fig. 2.3)

$(U_1 + U_3)(I)$ and $(U - I \cdot r_2)(I)$,

intersection point *A* gives the problem solution:

$I = 2.5\ A, \quad U_1 = 25\ V, \quad U_3 = 25\ V.$

Verification of the power balance:

- the source power

$$P_G = U \cdot I = 100 \cdot 2.5 = 250\ W;$$

- summary power of resistances

$$\Sigma P_C = U_1 \cdot I + U_3 \cdot I + I^2 \cdot r_2 =$$
$$= 25 \cdot 2.5 + 25 \cdot 2.5 + 2.5^2 \cdot 20 = 250\ W.$$

Power balance $P_G = \Sigma P_C$ is true,

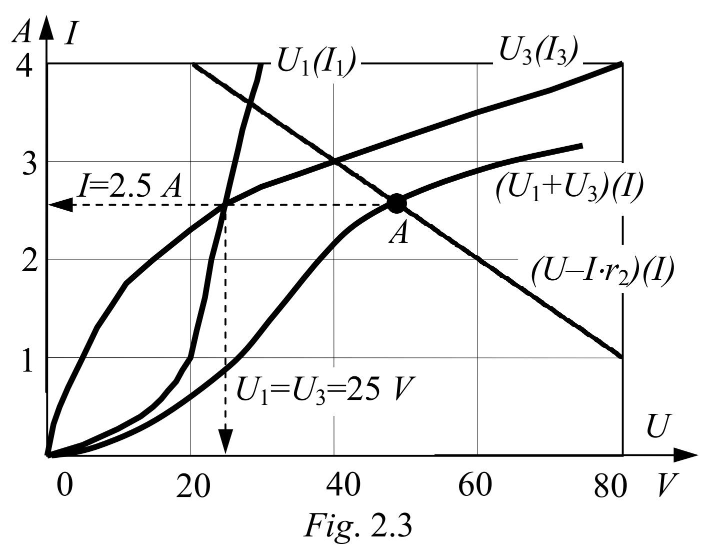


Fig. 2.3

the problem has been solved correctly.

2-4 (9.4). Calculate the currents in the scheme fig. 2.4 by the diagram method as well as the source voltage; verify the power balance; if

$$I_{13} = 5\ A, \quad r_3 = 14\ Ohm,$$

volt-ampere characteristics of the nonlinear resistors are given in tables 2.3 and 2.4.

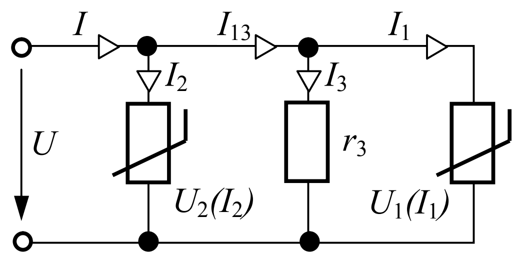


Fig. 2.4

Table 2.3

U_1, *V*	10	20	30	40	50	70
I_1, *A*	2	3	2	1	2	4.8

Table 2.4

U_2, *V*	10	20	40	60	80
I_2, *A*	1.25	2	2.5	3	3.3

Solution. For case of parallel connection, let's rewrite the initial equations under Kirchhoff's current law, adding the coupling equations between currents and voltages:

$$I_{13} = I_1 + I_3, \qquad I = I_2 + I_{13},$$

$$I_1 = f_1(U_1), \quad I_2 = f_2(U_2), \quad U_3 = I_3 \cdot r_3, \text{ moreover } U = U_1 = U_2 = U_3.$$

The first of the above-written equations is solved by presenting it firstly in the following form $\quad I_1 = I_{13} - \dfrac{U}{r_3}$,

the left part of which is the current of the nonlinear element $I_1 = f_1(U)$ set in table 2.3, and the right part presents the linear dependence on voltage U: $\quad I_{13} - \dfrac{U}{r_3} = 5 - \dfrac{U}{14}$.

All above-mentioned dependences are presented in fig. 2.5,a together with the solution of the first of the presented equations.

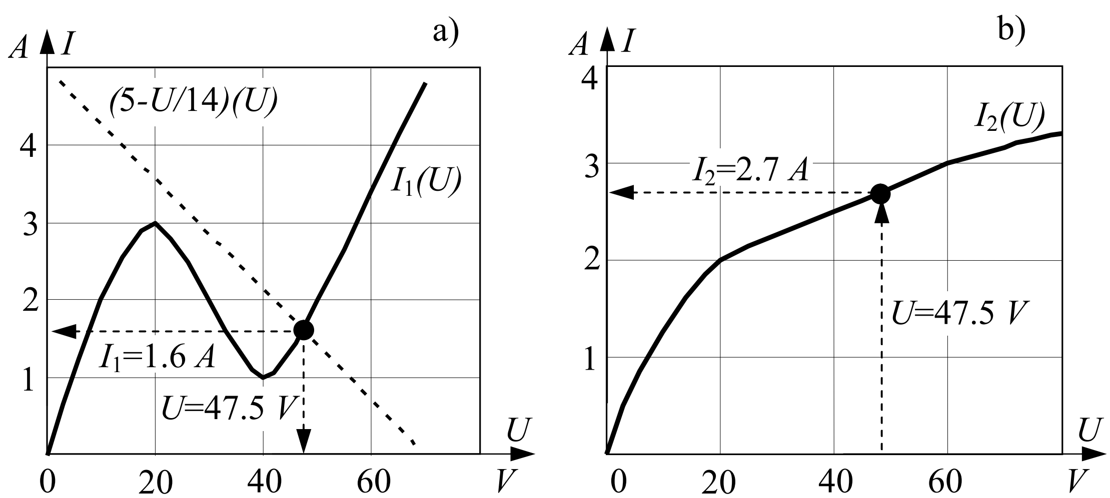


Fig. 2.5

Since the diagram method possesses appreciable inaccuracy, let's verify the result by equality $\quad I_1 = I_{13} - \dfrac{U}{r_3}$.

$$1.6 \overset{?}{=} (5 - \frac{47.5}{14}) = (5 - 3.393) = 1.607, \text{ where } I_3 = 3.393\ A.$$

Results differ in the 4th digit, which means the sufficient accuracy of the solution.

In fig. 2.5,b the determination of current I_2 is presented.

The generator current is $\qquad I = I_2 + I_{13} = 2.7 + 5 = 7.7\ A$.

The generator power is $P_G = U \cdot I = 47.5 \cdot 7.7 = 365.8\ W$.

Summary power of consumers is

$$\Sigma P_C = U \cdot I_2 + I_3^2 \cdot r_3 + U \cdot I_1 = 47.5 \cdot 2.7 + 3.393^2 \cdot 14 + 47.5 \cdot 1.6 = 365.4\ W.$$

$P_G \approx \Sigma P_C$ – is fulfilled.

2-5 (9.5). Calculate the currents, verify the power balance in the scheme fig. 2.6, if $U = 80\ V$, $r_2 = 10\ Ohm$, volt-ampere characteristics of the nonlinear resistors being set in tables 2.5 and 2.6:

Table 2.5

U_1, V	20	35	46	50	53	54
I_1, A	2	4	6	8	10	12

Table 2.6

U_3, V	0	5	20	40	60	90
I_3, A	0	2	4	5.2	6	6.5

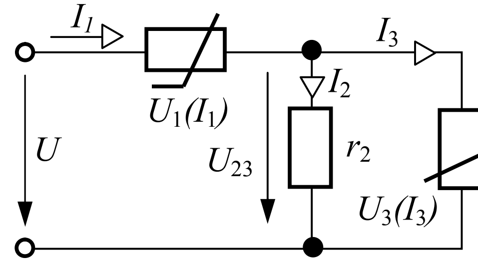


Fig. 2.6

Solution. In coordinate system *U(I)* (fig. 2.7), we construct volt-ampere characteristics of the elements connected in parallel $I_3(U_3)$ and $U_2 = I_2 \cdot r_2$ as well as an auxiliary characteristic $(I_2 + I_3)(U_{23}) = (I_1)(U_{23})$, because $I_1 = (I_2 + I_3)$.

Under Kirchhoff's voltage law

$U_1 + U_{23} = U$ or $U_{23} = U - U_1$.

We draw another auxiliary characteristic

$$U - U_1 = U - f_1(I_1),$$

having assumed a number of values of the current I_1. The calculation results are tabulated in table 2.7.

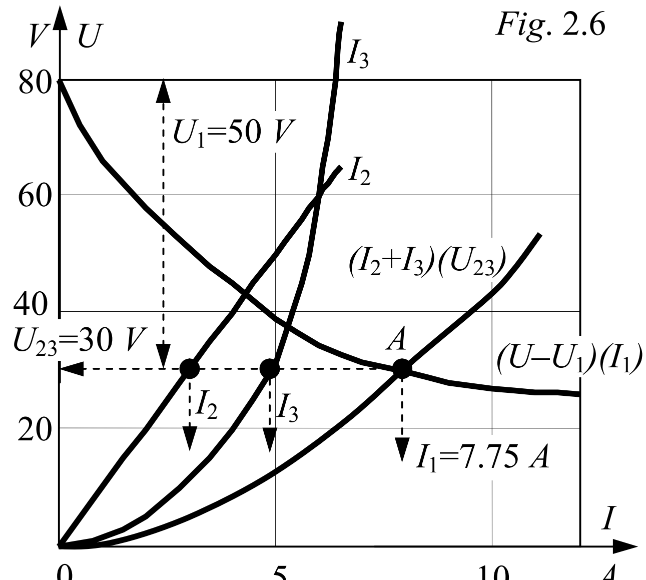


Fig. 2.7

Table 2.7

I_1, A	0	2	4	6	8	10	12
U_1, V	0	20	35	46	50	53	54
$U - U_1 = 80 - U_1$, V	80	60	45	34	30	27	26

Point *A* of intersection of two auxiliary curves sets the problem solution:

$$I_1 = 7.75\ A,\quad U_{23} = 30\ V,\quad I_2 = 3\ A,\quad I_3 = 4.75\ A,\quad U_1 = 50\ V.$$

Verification of the power balance: $P_G = U \cdot I_1 = 80 \cdot 7.75 = 625\ W$;

$$\Sigma P_C = U_1 \cdot I_1 + I_2^2 \cdot r_2 + U_{23} \cdot I_3 = 50 \cdot 7.75 + 3^2 \cdot 10 + 30 \cdot 4.75 = 620\ W.$$

2-6 (9.8). In fig. 2.8,a, there is a scheme with three identical nonlinear elements, their volt-ampere characteristics being set in table 2.8. EMF of the voltage sources are $E_1 = 100\ V$, $E_2 = 10\ V$, $E_3 = 20\ V$. Determine all the branch currents, do not take into account the inner resistances of the sources.

Table 2.8

$\pm U$, V	0	5	20	30	50	70	100
$\pm I$, mA	0	10	30	39	50	55	60

Solution. Since the branches of scheme are connected in parallel to two nodes *A* and *B*, let's employ the method of the junction voltage, having assumed arbitrary directions of the branch currents and the junction voltage U_{AB} (shown if figure).

For loops including voltage U_{AB} and one of the branches, in accordance with Kirchhoff's voltage law the following three equations occur:

$$U_{AB} + U_1 = E_1, \quad U_{AB} + U_2 = E_2, \quad U_{AB} - U_3 = -E_3.$$

From these equations we express the voltage U_{AB}:

$$U_{AB} = E_1 - U_1, \quad U_{AB} = E_2 - U_2, \quad U_{AB} = -E_3 + U_3.$$

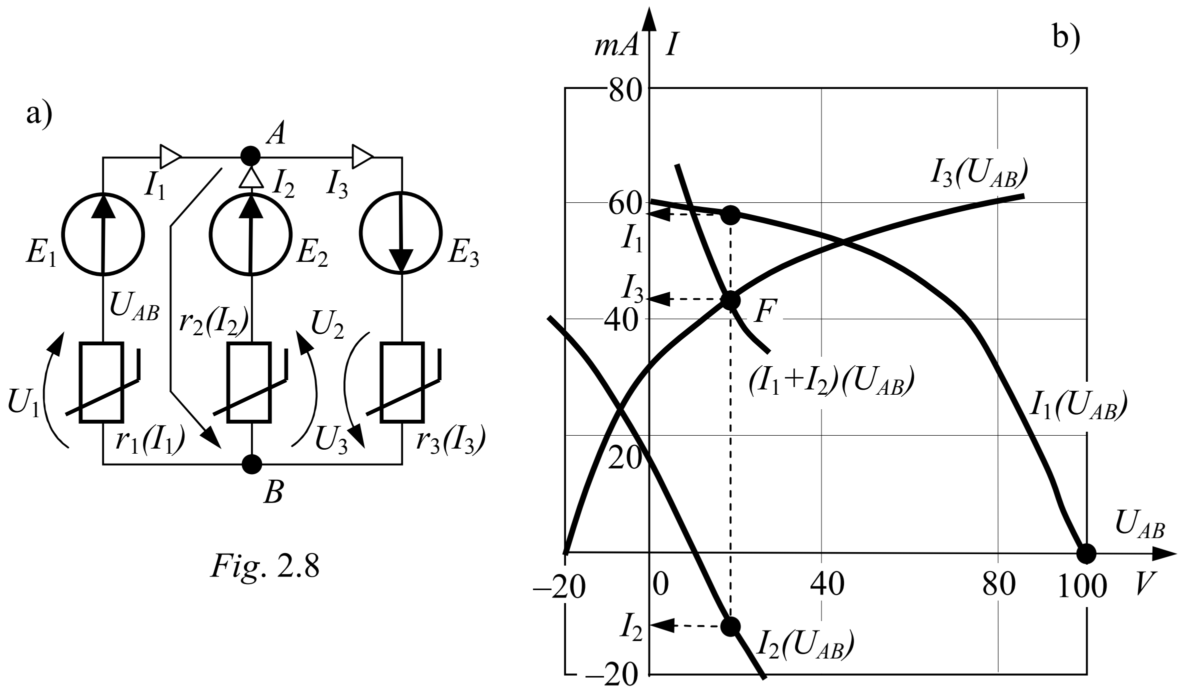


Fig. 2.8

On the ground of the latter expressions it is convenient to compute the current dependence of the separate branch on the junction voltage in the following order:

- assume an arbitrary value of the branch current and in accordance with volt-ampere characteristic of the element we determine the voltage across the nonlinear element corresponding to this value ;
- voltage U_{AB} corresponding to the chosen current value is calculated under the corresponding equation in accordance with Kirchhoff's voltage law.

Dependences $I_1(U_{AB})$, $I_2(U_{AB})$, $I_3(U_{AB})$ are shown in fig. 2.8,b.

In accordance with Kirchhoff's current law there is an equation $I_1 + I_2 = I_3$, which is to be true for the scheme under consideration at single voltage value U_{AB}. Let's draw the auxiliary curve $(I_1+I_2)(U_{AB})$ and find the point *F* of intersection of this curve with dependence $I_3(U_{AB})$.

Point *F* sets the problem solution:

$$U_{AB} = 18\ V, \quad I_1 = 58\ mA, \quad I_2 = -15\ mA, \quad I_3 = 43\ mA,$$

voltages across the elements are $U_1 = 82\ V$, $U_2 = -8\ V$, $U_3 = 38\ V$.

Algebraic sum of powers of the generators is:

$$\Sigma P_G = E_1 \cdot I_1 + E_2 \cdot I_2 + E_3 \cdot I_3 = 100 \cdot 58 + 10 \cdot (-15) + 20 \cdot 43 = 6510\ mW;$$

sum of the consumer powers is:

$$\Sigma P_C = U_1 \cdot I_1 + U_2 \cdot I_2 + U_3 \cdot I_3 = 82 \cdot 58 + (-8) \cdot (-15) + 38 \cdot 43 = 6510\ mW.$$

Power balance is true.

2-7 (9.11). In fig. 2.9,a there is a scheme with a nonlinear element *(NE)*, its volt-ampere characteristic being set in table 2.9.

Table 2.9

U_4, *V*	0	20	40	60	80	100
I_4, *A*	0	2.8	4	4.9	5.4	5.7

Parameters of the linear elements are as follows

$$r_1 = r_2 = r_3 = 10\ Ohm,\quad E_5 = 50\ V,\quad J = 5\ A.$$

Determine all the branch currents and verify the power balance.

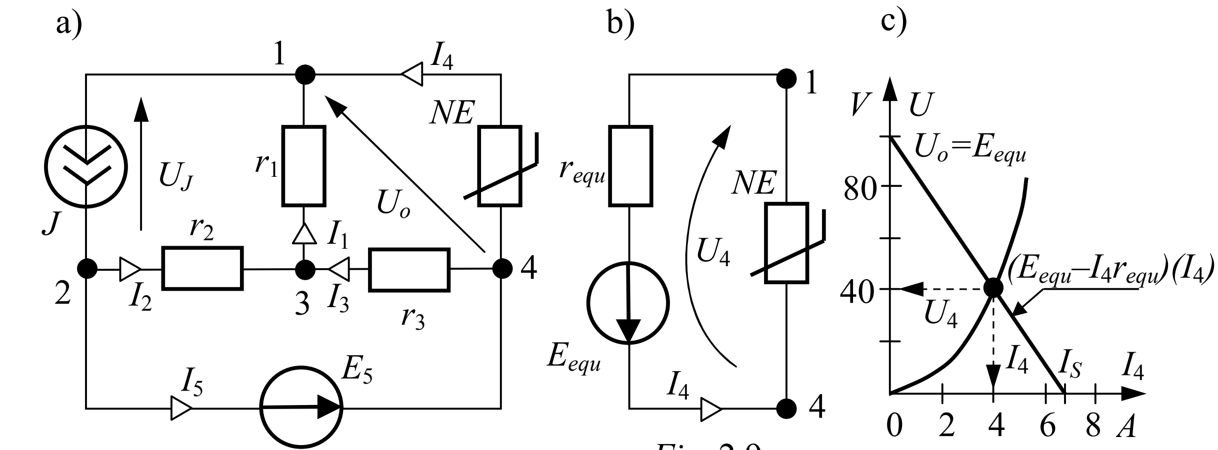


Fig. 2.9

Solution. Firstly, we calculate the current I_4 of the nonlinear element, previously having substituted the linear part of the scheme concerning terminals of the nonlinear element (1-4) by the equivalent generator.

Furthermore $U_o = E_{equ} = I_{3o} \cdot r_3 + I_{1o} \cdot r_1$, where the currents with subscript $_o$ (no-load or open-circuit condition) are the currents of the scheme elements which occur after disconnection of the branch with nonlinear element. In a new simpler scheme there is $I_{1o} = J = 5\ A$, and in accordance with the mesh current method

$$I_{3o} = \frac{E_5 + J \cdot r_2}{r_2 + r_3} = \frac{50 + 5 \cdot 10}{10 + 10} = 5\ A,$$

and $U_o = 5 \cdot 10 + 5 \cdot 10 = 100\ V$.

Inner resistance of the equivalent generator is equal to the input resistance of the corresponding passive linear scheme concerning terminals 1-4:

$$r_{equ} = r_{14inp} = r_1 + \frac{r_2 \cdot r_3}{r_2 + r_3} = 10 + \frac{10 \cdot 10}{10 + 10} = 15\ Ohm.$$

For an equivalent scheme (fig. 2.9,b), on the one hand, $U_4(I_4)$ is the VAC of *NE*, on the other hand, $U_4 = E_{equ} - I_4 \cdot r_{equ}$ is VAC of active two-pole network.

Solution of the system of two latter equations is presented in fig. 2.9,c, from where $U_4 = 40\ V$, $I_4 = 4\ A$.

Let's recur to initial scheme fig. 2.9,a and calculate the other currents:

$$I_1 = J - I_4 = 5 - 4 = 1\ A,$$

$$I_3 = \frac{U_4 - I_1 r_1}{r_3} = \frac{40 - 1 \cdot 10}{10} = 3\ A,$$

$$I_5 = I_3 + I_4 = 3 + 4 = 7\ A,$$

$$I_2 = I_1 - I_3 = 1 - 3 = -2\ A,$$

voltage across the terminals of the current source is as follows

$$U_J = I_2 \cdot r_2 + I_1 \cdot r_1 = -2 \cdot 10 + 1 \cdot 10 = -10\ V.$$

Verification of the power balance:

$\Sigma P_G = U_J \cdot J + E_5 \cdot I_5 = -10 \cdot 5 + 50 \cdot 7 = 300\ W$;

$\Sigma P_C = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 + I_3^2 \cdot r_3 + U_4 \cdot I_4 = 1^2 \cdot 10 + 2^2 \cdot 10 + 3^2 \cdot 10 + 40 \cdot 4 = 300\ W.$

2-8 (9.16). Circuit parameters of fig. 2.10,a are as follows: $r_1 = 10\ Ohm$, $r_2 = 20\ Ohm$, input voltage $U_1 = 24\ V$, VAC of Zener diode is approximated with three straight segments (see fig. 2.10,b). Given scheme is the simplest voltage stabilizer U_2 for the load r_2. Determine the currents and load voltage, calculate the voltage stabilization factor.

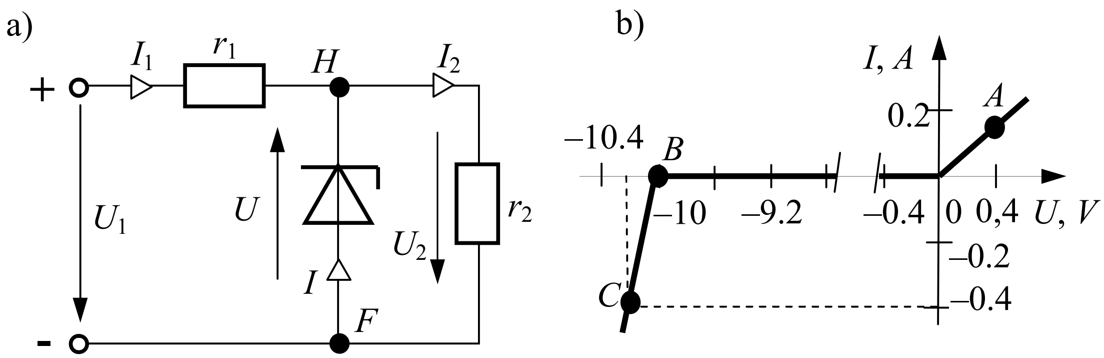


Fig. 2.10

Solution. First, calculate the differential resistance of diode in the working segment *BC*:

$$r_{dif} = \frac{0.2}{0.4} = 0.5\ Ohm.$$

Here, the voltage across diode is negative and if it is less than 10 *V*, there is no diode current $I = 0$. Then $\quad I_1 = I_2 = \dfrac{U_1}{r_1 + r_2}, \quad U = -U_2 = -\dfrac{U_1}{r_1 + r_2} \cdot r_2 < -10\ V = U_0.$

From here, we find minimum input voltage when scheme works as the voltage stabilizer:

$$U_{1min} = -U_0 \cdot \frac{r_1 + r_2}{r_2} = 10 \cdot \frac{10 + 20}{20} = 15\ V.$$

The given voltage is $U_1 = 24\ V > U_{1min}$ then Zener diode works in the segment *BC*, the circuit of fig. 2.11,a being its equivalent scheme while the calculation scheme is as in fig. 2.11,b. It is calculated by method of two nodes:

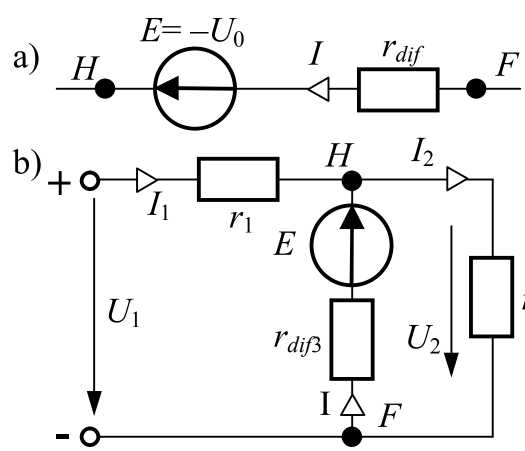


Fig. 2.11

$$U_2 = \frac{\dfrac{U_1}{r_1} + \dfrac{E}{r_{dif}}}{\dfrac{1}{r_1} + \dfrac{1}{r_{dif}} + \dfrac{1}{r_2}} = \frac{\dfrac{24}{10} + \dfrac{10}{0.5}}{\dfrac{1}{10} + \dfrac{1}{0.5} + \dfrac{1}{20}} = 10.42\ V,$$

$$I_1 = \frac{U_1 - U_2}{r_1} = \frac{24 - 10.42}{10} = 1.358\ A, \quad I_2 = \frac{U_2}{r_2} = \frac{10.42}{20} = 0.521\ A,$$

$$I = I_2 - I_1 = -0.837\ A.$$

A passive stabilizer scheme for augmentations is presented in fig. 2.12, its calculation gives

$$\Delta U_2 = \frac{\Delta U_1}{r_1 + \dfrac{r_2 \cdot r_{dif}}{r_2 + r_{dif}}} \cdot \frac{r_2 \cdot r_{dif}}{r_2 + r_{dif}} =$$

$$= \Delta U_1 \cdot \frac{r_2 \cdot r_{dif}}{r_1 \cdot r_2 + r_1 \cdot r_{dif} + r_2 \cdot r_{dif}} =$$

$$= \Delta U_1 \cdot \frac{20 \cdot 0.5}{10 \cdot 20 + 10 \cdot 0.5 + 20 \cdot 0.5} = \Delta U_1 \cdot 0.0465.$$

Voltage stabilization factor is

$$k_{st} = \frac{\Delta U_1 / U_1}{\Delta U_2 / U_2} = \frac{\Delta U_1 / 24}{0.0465 \cdot \Delta U_1 / 10.42} = 9.34.$$

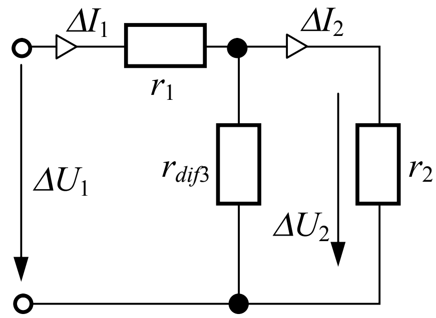


Fig. 2.12

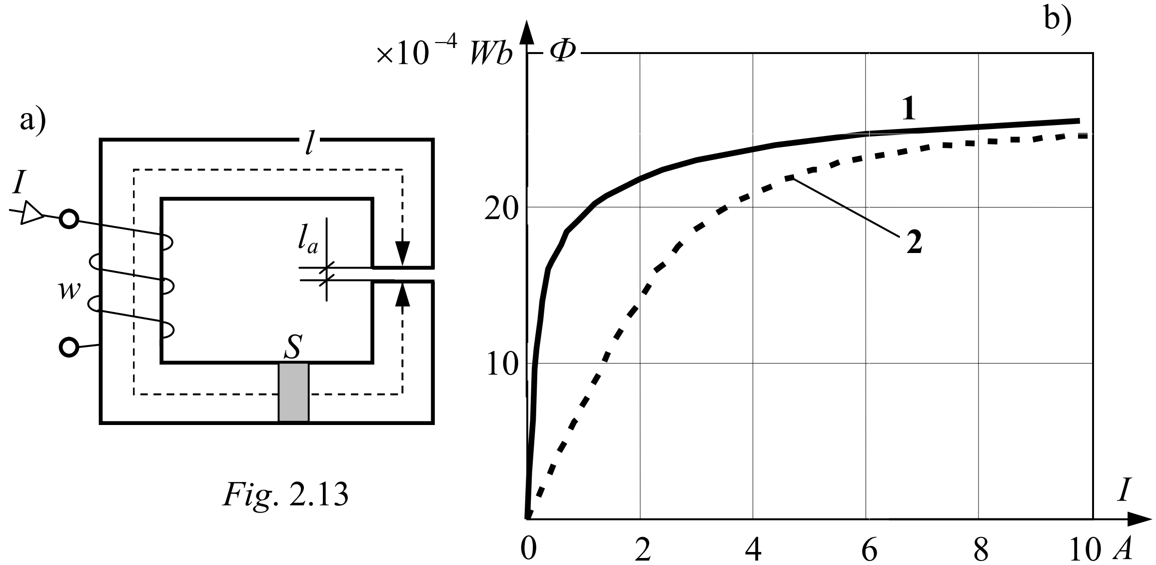


Fig. 2.13

2.2. D-C MAGNETIC CIRCUITS

2-9 (9.19). In fig. 2.13,a there is a choke-coil with the turn number w = 500, the wire being wound around the core made of electric steel 1512, the magnetization curve of which is presented in table 2.10. The core mean length is l = 100 *cm*, steel cross-section S = 16 cm^2. Air gap is absent l_a = 0. Calculate and construct the Weber-ampere characteristic of the coil.

Table 2.10. Magnetization curve of steel 1512

B, *T*	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
H, *A/cm*	0.25	0.5	0.65	0.8	0.95	1.15	1.5	2	3

Continuation of table 2.10

B, *T*	1.2	1.3	1.35	1.4	1.45	1.5	1.55	1.6	1.65
H, *A/cm*	4.65	7.4	9	12	15	22	33	49	90

Solution. Weber-ampere characteristic of the coil *Φ(I)* is calculated on the ground of the magnetization curve and geometrical size of the ferromagnetic core: having assumed, for instance, B = 1.2 *T* with the corresponding value H = 4.65 *A/cm*, we obtain

$$\Phi = B{\cdot}S = 1.2{\cdot}16{\cdot}10^{-4} = 19.2{\cdot}10^{-4}\ Wb, \qquad I = \frac{Hl}{w} = \frac{4.65\cdot 100}{500} = 0.93\ A.$$

Tabulate the calculation results (table 2.11).

Table 2.11

B, T	0.4	0.6	0.8	1	1.1	1.2	1.3	1.4	1.5	1.6
H, A/cm	0.5	0.8	1.15	2	3	4.65	7.4	12	22	49
Φ,× 10^{-4} Wb	6.4	9.6	12.8	16	17.6	19.2	20.8	22.4	24	25.6
I, A	0.1	0.16	0.23	0.4	0.6	0.93	1.48	2.4	4.4	9.8

Weber-ampere characteristic of the coil without the air gap is presented in fig. 2.13,b, curve **1**.

2-10 (9.20). In the core of the coil studied in problem 2-9 the air gap $l_a = 1.2\ mm$ long is made. Construct the Weber-ampere characteristic of the coil with air gap.

Solution. In comparison with the previous problem, the coil current changes even with the same flux values because in accordance with Ampere's law the new current value is as follows $I = \dfrac{Hl + H_a l_a}{W}$.

Magnetic intensity in the air gap is

$$H_a = \frac{B}{\mu_0} = \frac{B}{4\pi\cdot 10^{-7}} = 0.8{\cdot}10^6{\cdot}B, \quad \text{where} \quad H_a[A/m],\ B[T]$$

or $\quad H_a = 0.8{\cdot}10^4{\cdot}B, \quad$ where $H_a[A/cm]$, $B[T]$.

For example, for induction $B = 1.2\ T \quad H = 4.65\ A/cm$,

$$H_a = 0.8{\cdot}10^4{\cdot}1.2 = 9.6{\cdot}10^3\ A/cm,$$

new current

$$I = \frac{Hl}{W} + \frac{H_a l_a}{W} = I' + I'' = \frac{4.65\cdot 100}{500} + \frac{9.6\cdot 10^3\cdot 1.2\cdot 10^{-1}}{500} = 0.93 + 2.304 = 3.324\ A.$$

Weber-ampere characteristic of the coil in the table form gets view of the table 2.12.

Table 2.12

Φ, ×10^{-4} Wb	6.4	9.6	12.8	16	17.6	19.2	20.8	22.4	24	25.6
I, A	0.87	1.31	1.77	2.32	2.71	3.32	3.98	5.09	7.28	12.9

This Weber-ampere characteristic is presented in fig. 2.13,b, curve **2**.

While calculating the magnetic circuits, a reluctance of the part of the magnetic circuit is usually involved

$$R_m = \frac{U_m}{\Phi} = \frac{Hl}{BS} = \frac{Hl}{\mu\cdot\mu_0\cdot HS} = \frac{l}{\mu\cdot\mu_0\cdot S}\ [1/H].$$

In the example under consideration, the reluctance of the portion of the magnetic core of length $l = 100\ cm$, cross-section $S = 16\ cm^2$ at $B = 1.2\ T$, $H = 4.65\ A/cm$ is

$$R_{mC} = \frac{4.65\cdot 100}{1.2\cdot 16\cdot 10^{-4}} = 24.22{\cdot}10^4\ 1/H,$$

and the reluctance of the air gap of length $l_a = 1.2\ mm$, cross-section $S = 16\ cm^2$ at $B = 1.2\ T$

$$R_{ma} = \frac{l_a}{\mu_0\cdot S} = \frac{1.2\cdot 10^{-3}}{4\pi\cdot 10^{-7}\cdot 16\cdot 10^{-4}} = 59.71{\cdot}10^4\ 1/H.$$

Thus, at $B = 1.2\ T$ the reluctance of the air gap of length $1.2\ mm$ is greater than reluctance of the core of length $1\ m$ by $\frac{R_{Ma}}{R_{MC}} = \frac{59.71}{24.22} = 2.5$ times.

2-11 (9.21). In the air gap of the electromagnet (fig. 2.14) it is necessary to create a magnetic field with magnetic induction $B = 1.1\ T$. Core and armature are made of electric steel 1512, the magnetization curve of which is presented in table 2.10, they have identical cross-section $S_C = S_{am} = S = 20\ cm^2$, the core length is $l_C = 80\ cm$, the armature length is $l_{am} = 30\ cm$. Number of turns of the coil is $w = 800$. The length of one air gap is $l_a = 0.4\ mm$. Determine:

- current through the coil;
- the coil inductance;
- attraction force between the core and armature;
- the electromagnet force of separation.

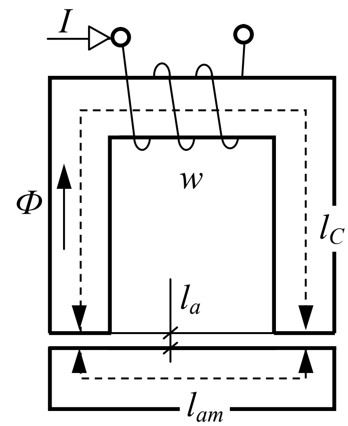


Fig. 2.14

Solution. In accordance with Ampere's law, the necessary magnetization force of the coil is $Iw = H_C \cdot l_C + H_{am} \cdot l_{am} + 2 \cdot H_a \cdot l_a$.

The coil magnetic flux is $\Phi = B \cdot S_a$.

Since $S_a = S_C = S$, then $B_a = B_C = B$ and by the magnetization curve at $B = 1.1\ T$ we find $H_C = H_{am} = 3\ A/cm$,

$$H_a = 0.8 \cdot 10^4 \cdot B = 0.8 \cdot 10^4 \cdot 1.1 = 8.8 \cdot 10^3\ A/cm,$$

$$Iw = 3 \cdot 80 + 3 \cdot 30 + 2 \cdot 8.8 \cdot 10^3 \cdot 0.4 \cdot 10^{-1} = 1034\ A,$$

The coil full current is $I = \frac{Iw}{w} = \frac{1034}{800} = 1.29\ A.$

The coil flux-linkage is $\Psi = B \cdot S \cdot w = 1.1 \cdot 20 \cdot 10^{-4} \cdot 800 = 1.76\ Wb,$

the coil inductance is $L = \frac{\Psi}{I} = \frac{1.76}{1.29} = 1.36\ H.$

We calculate the attractive force of the electromagnet for a single air gap by the method of feasible shifts (see subject «Applied mechanics»). the magnetic field energy in the air gap is $W_a = \frac{B_a H_a}{2} V_a = \frac{B_a H_a}{2} \cdot S_a \cdot l_a.$

At infinitesimal displacement of the armature, the air gap length changes at constant values of B_a, H_a. The force of attraction of the armature to the core per a single air gap is

$$F_1 = -\frac{dW_a}{dl_a} = -\frac{B_a H_a}{2} \cdot S_a.$$

Electromagnet attractive force for two air gaps is

$$F = |2 \cdot F_1| = B_a \cdot H_a \cdot S_a = \frac{B_a^2}{\mu_0} \cdot S_a = 0.8 \cdot 10^6 \cdot B_a^2 \cdot S_a.$$

In example under consideration

$$F = 0.8 \cdot 10^6 \cdot 1.1^2 \cdot 20 \cdot 10^{-4} = 1936\ N \approx 198\ kgF.$$

Let's determine the electromagnet force of separation when there is no air gap $l_a = 0$ and the magnetization force is $Iw = 1034\ A$.

Then at $S_C = S_{am} = S$ and homogeneous material of the core

$$H_C = H_{am} = \frac{Iw}{l_{am} + l_C} = \frac{1034}{80 + 30} = 9.4\ A/cm.$$

By the magnetization curve, the corresponding magnetic induction is $B = 1.357\ T$, the electromagnet force of separation is

$$F = 0.8\cdot10^6\cdot1.357^2\cdot20\cdot10^{-4} = 2946\ H \approx 300\ kgF.$$

2-12 (9.22). Left magnetic core (fig. 2.15) possesses the air gap $l_a = 0.11\ cm$, the right one is without the air gap. The core mean length is $l_1 = l_2 = 60\ cm$, cross-section $S_1 = S_2 = S_a = 6\ cm^2$. Magnetic flux through the left core is $\Phi_1 = 6.6\cdot10^{-4}\ Wb$. Material of the cores is electric steel 1512, its magnetization curve being set by table 2.10.

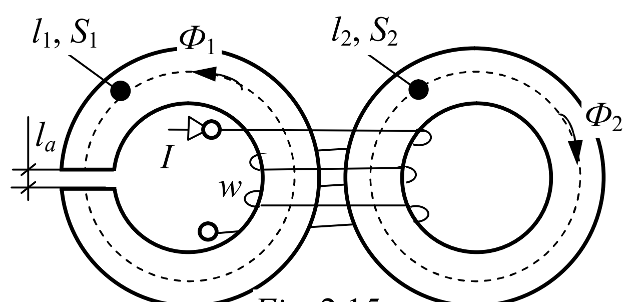


Fig. 2.15

Determine mmf of the coil and magnetic flux Φ_2.

Answers: $Iw = 1148\ A$, $\Phi_2 = 8.88\cdot10^{-4}\ Wb$.

2-13 (9.24). A core of a branched magnetic circuit fig. 2.16,a is made of the electric steel 1512. The core sizes are $l_1 = 60\ cm$, $S_1 = 6\ cm^2$, $l_2 = 20\ cm$, $S_2 = 12\ cm^2$, $l_3 = 40\ cm$, $S_3 = S_a = 8\ cm^2$, $l_a = 1\ mm$. Number of the coil turns is $w = 400$. It is necessary to create a magnetic field in the air gap, its magnetic induction being $B_a = 1.1\ T$. Determine the coil current.

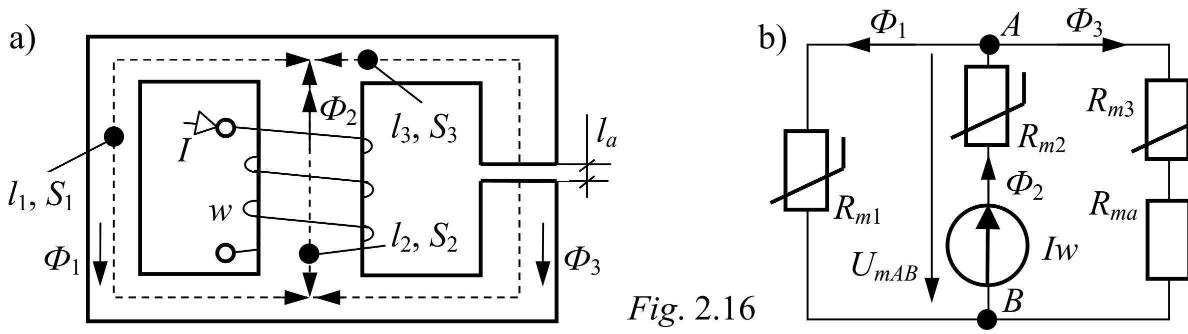


Fig. 2.16

Solution. For easier imagination, let's draw the electric equivalent scheme (fig. 2.16,b).

Magnetic flux is $\Phi_3 = B_a\cdot H_a = 1.1\cdot8\cdot10^{-4} = 8.8\cdot10^{-4}\ Wb$,

magnetic voltage is $U_{mAB} = H_3\cdot l_3 + H_a\cdot l_a$.

By the magnetization curve, we find: since $B_3 = B_a = 1.1\ T$ then $H_3 = 3\ A/cm$,

$$H_a = 0.8\cdot10^4\cdot B_a = 0.8\cdot10^4\cdot1.1 = 0.88\cdot10^4\ A/cm,$$

$$U_{mAB} = 3\cdot40 + 0.88\cdot10^4\cdot0.1 = 1000\ A.$$

The magnetic field intensity in the first branch is

$$H_1 = \frac{U_{mAB}}{l_1} = \frac{1000}{60} = 16.7\ A/cm,$$

the magnetic induction corresponding to that intensity is $B_1 = 1.462\ T$,

the magnetic flux through the first leg is $\Phi_1 = B_1\cdot S_1 = 1.462\cdot6\cdot10^{-4} = 8.77\cdot10^{-4}\ Wb$.

The magnetic flux through the middle leg is

$$\Phi_2 = \Phi_1 + \Phi_3 = (8.77+8.8)\cdot10^{-4} = 17.57\cdot10^{-4}\ Wb,$$

its magnetic induction is $B_2 = \frac{\Phi_2}{S_2} = \frac{17.57 \cdot 10^{-4}}{12 \cdot 10^{-4}} = 1.464\ T,$

corresponding magnetic intensity is $H_2 = 16.96\ A/cm$.

Under Ampere's law $Iw = H_2 \cdot l_2 + U_{mAB}$,

from here $I = \frac{16.96 \cdot 20 + 1000}{400} = 3.35\ A.$

2-14 (9.27). Magnetic circuit fig. 2.17,a is made of steel 1512 and has the following sizes: $l_1 = 40\ cm$, $l_2 = 12\ cm$, $l_3 = 30\ cm$, $S_1 = S_3 = 4\ cm^2$, $S_2 = 2\ cm^2$. Exciting force is $Iw = 1800\ A$, the flux through the first leg is $\Phi_1 = 5.8 \cdot 10^{-4}\ Wb$. Determine the length of the air gap l_a.

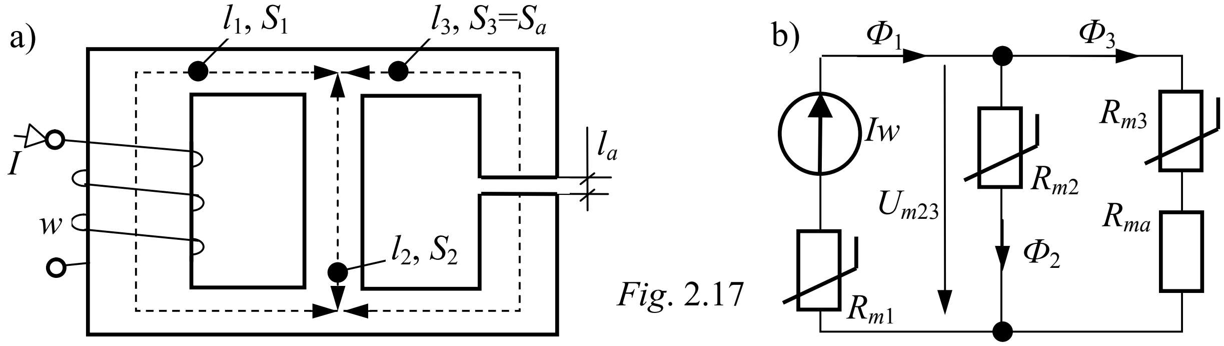


Fig. 2.17

Solution. The magnetic circuit scheme corresponds to series-parallel connection of branches (fig. 2.17,b). On the ground of Kirchhoff's voltage law, we write down

$$U_{m23} = Iw - H_1 \cdot l_1.$$

To find H_1 by the magnetization curve, one should determine

$$B_1 = \frac{\Phi_1}{S_1} = \frac{5.8 \cdot 10^{-4}}{4 \cdot 10^{-4}} = 1.45\ T,$$

and $H_1 = 15\ A/cm$ corresponds to it, then $U_{m23} = 1800 - 15 \cdot 40 = 1200\ A$.

On the other hand, $U_{m23} = H_2 \cdot l_2$, from here $H_2 = \frac{U_{m23}}{l_2} = \frac{1200}{12} = 100\ A/cm,$

and on the ground of the magnetization curve, we obtain $B_2 = 1.66\ T$.

The magnetic flux is $\Phi_2 = B_2 \cdot S_2 = 1.66 \cdot 2 \cdot 10^{-4} = 3.32 \cdot 10^{-4}\ Wb$.

On the ground of Kirchhoff's first law

$$\Phi_3 = \Phi_1 - \Phi_2 = (5.8 - 3.32) \cdot 10^{-4} = 2.48 \cdot 10^{-4}\ Wb,$$

and induction $B_3 = B_a = \frac{\Phi_3}{S_3} = \frac{2.48 \cdot 10^{-4}}{4 \cdot 10^{-4}} = 0.62\ T,$

which the magnetic intensity in the third lag corresponds to

$$H_3 = 0.83\ A/см,$$

and that in the air gap is $H_a = 0.8 \cdot 10^4 \cdot B_. = 0.8 \cdot 10^4 \cdot 0.62 = 4960\ A/cm$.

The magnetic voltage drop across the reluctance of the air gap is as follows

$$H_a \cdot l_a = U_{m23} - H_3 \cdot l_3 = 1200 - 0.83 \cdot 30 = 1175\ A,$$

from here the air gap length is $l_a = \frac{H_a \cdot l_a}{H_a} = \frac{1175}{4960} = 0.237\ cm.$

2-15 (9.30). A magnetic circuit core (fig. 2.18) is made of electric steel 1512, its magnetization curve being given in table 2.10. The core sizes are $l_1 = l_3 = 33\ cm$, $l_2 = 11\ cm$, $l_a = 0.1\ cm$, $S_1 = 12\ cm^2$, $S_2 = 24\ cm^2$, $S_3 = 16\ cm^2$.

Exciting forces are $I_1w_1 = 500\ A$, $I_2w_2 = 1000\ A$, $I_3w_3 = 750\ A$.

Determine magnetic fluxes of all the core parts.

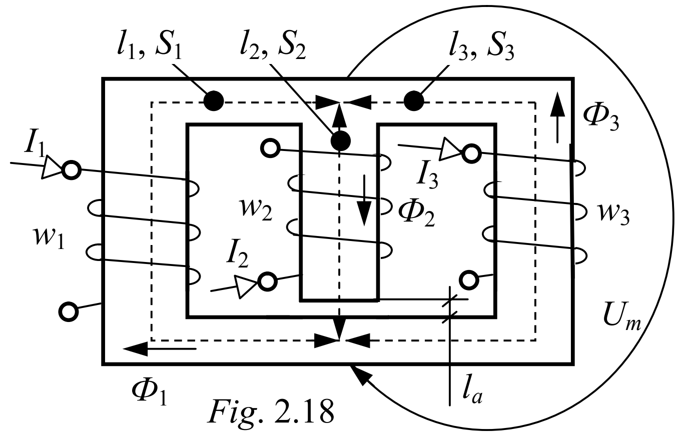


Fig. 2.18

Solution. Assume arbitrary directions of the magnetic fluxes and the junction voltage and generate the equations by the method of two nodes:

$$U_m = I_1w_1 - H_1 \cdot l_1,$$
$$U_m = -I_2w_2 + H_2 \cdot l_2 + H_a \cdot l_a,$$
$$U_m = I_3w_3 - H_3 \cdot l_3.$$

We tabulate the calculation of the flux dependencies on the junction voltage (table 2.13).

Table 2.13

B	H	H_a	$H_1 \cdot l_1 = H_3 \cdot l_3$	Φ_1	$U_m = I_1w_1 - H_1 \cdot l_1$	Φ_2	$U_m = -I_2w_2 + H_2 \cdot l_2 + H_a \cdot l_a$	Φ_3	$U_m = I_3w_3 - H_3 \cdot l_3$
T	A/cm	$\times 10^3$ A/cm	A	$\times 10^{-4}$ Wb	A	$\times 10^{-4}$ Wb	A	$\times 10^{-4}$ Wb	A
0.4	0.5	3.2	16.5	4.8	484	9.6	–674.5	6.4	734
0.8	1.15	6.4	38	9.6	462	19.2	–347	12.8	712
1	2	8	66	12	434	24	–178	16	684
1.1	3	8.8	99	13.2	401	26.4	–87	17.6	651
1.2	4.65	9.6	153.5	14.4	346	28.8	11	19.2	597
1.3	7.4	10.4	244	15.6	256	31.2	121	20.8	506
1.4	12	11.2	396	16.8	104	33.6	252	22.4	354
1.45	15	11.6	495	17.4	5	34.8	325	23.2	255
1.5	22	12	726	18	–226	36	442	24	24
1.6	49	12.8	1617	19.2	–1117	38.4	819	25.6	–867

Graphical calculation of the magnetic circuit taking into account Kirchhoff's first law $\Phi_1 + \Phi_3 = \Phi_2$ at single value of U_m is presented in fig. 2.19.

The intersection point A $(\Phi_1+\Phi_3)(U_m) = \Phi_2(U_m)$ sets the value of the junction voltage U_m as well as magnetic flux Φ_2, and points B and C give fluxes Φ_1 and Φ_3:

$$U_m = 400\ A,\ \ \Phi_2 = 35.6 \cdot 10^{-4}\ Wb,\ \ \Phi_1 = 13.2 \cdot 10^{-4}\ Wb,\ \ \Phi_3 = 22.4 \cdot 10^{-4}\ Wb.$$

Voltages across the branch reluctances are:

$$U_{m1} = H_1 \cdot l_1 = I_1w_1 - U_m = 500 - 400 = 100\ A,$$
$$U_{m2} = H_2 \cdot l_2 + H_a \cdot l_a = I_2w_2 + U_m = 1000 + 400 = 1400\ A,$$
$$U_{m3} = H_3 \cdot l_3 = I_3w_3 - U_m = 750 - 400 = 350\ A.$$

Verification of the energy balance:

$$\Sigma\, Iw \cdot \Phi = (500 \cdot 13.2 + 1000 \cdot 35.6 + 750 \cdot 22.4) \cdot 10^{-4} = 5.9\ J,$$

$$\Sigma U_m \cdot \Phi = (100 \cdot 13.2 + 1400 \cdot 35.6 + 350 \cdot 22.4) \cdot 10^{-4} = 5.9\ J.$$

Since $\Sigma Iw \cdot \Phi = \Sigma U_m \cdot \Phi$, then the problem is solved correctly.

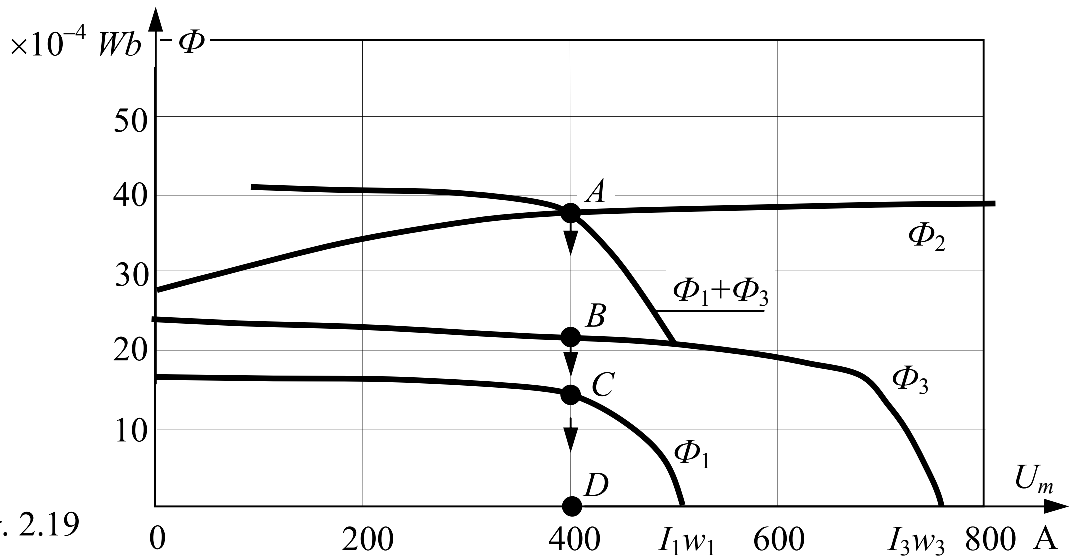


Fig. 2.19

2-16 (9.31). A magnetic circuit fig. 2.20 is made of steel 1512. The core sizes are $l_1 = l_3 = 44\ cm$, $l_2 = 22\ cm$, $l_a = 0.4\ mm$, $S_1 = S_3 = 20\ cm^2$, $S_2 = 10\ cm^2$.

Exciting forces are $I_1w_1 = 400\ A$, $I_2w_2 = 200\ A$, while fluxes are $\Phi_1 = \Phi_2$.

Find the flux Φ_3 as well as the exciting force I_3w_3.

Methodical instructions. Using the method of two nodes it is recommended to calculate and to construct the dependences $\Phi_1(U_{mAB})$ and $\Phi_2(U_{mAB})$, their intersection point determines the values of U_{mAB}, Φ_1 and Φ_2, then it is possible to find I_3w_3 with the aid of Kirchhoff's voltage law (see fig. 2.21).

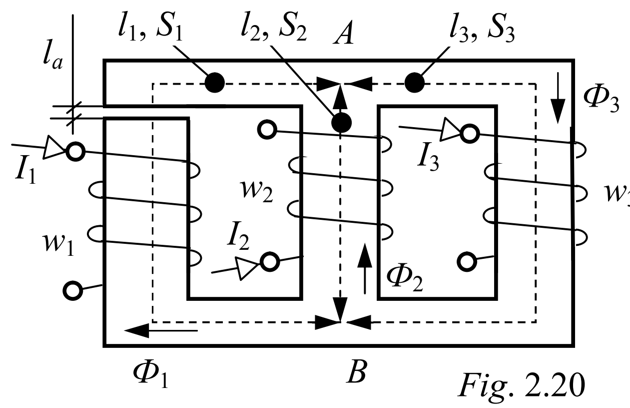


Fig. 2.20

$$U_{mAB} = I_1w_1 - H_1 \cdot l_1 - H_B \cdot l_B, \quad U_{mAB} = -I_2w_2 - H_2 \cdot l_2, \quad I_3w_3 = U_{mAB} - H_3 \cdot l_3.$$

Answers: $\Phi_1 = \Phi_2 = -15{,}5 \cdot 10^{-4}\ Wb$, $U_{mAB} = 690\ A$, $\Phi_3 = -31 \cdot 10^{-4}\ Wb$, $I_3w_3 = 2140\ A$.

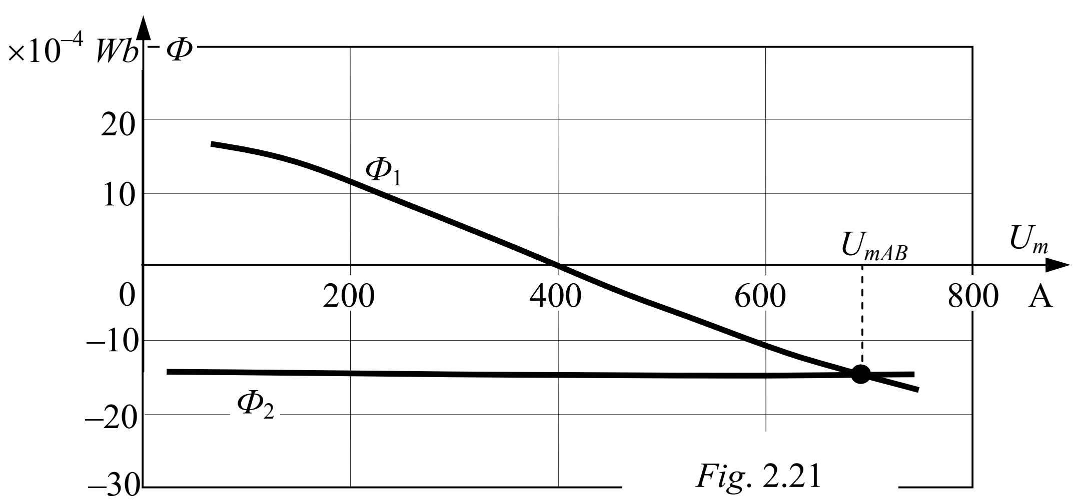


Fig. 2.21

3. LINEAR A-C CIRCUITS

3.1. CALCULATION OF SIMPLE CIRCUITS

3-1 (3.1). In fig. 3.1 the voltage and current oscillograms are presented. It is necessary to write down the expressions of their instantaneous values as well as to define the effective values.

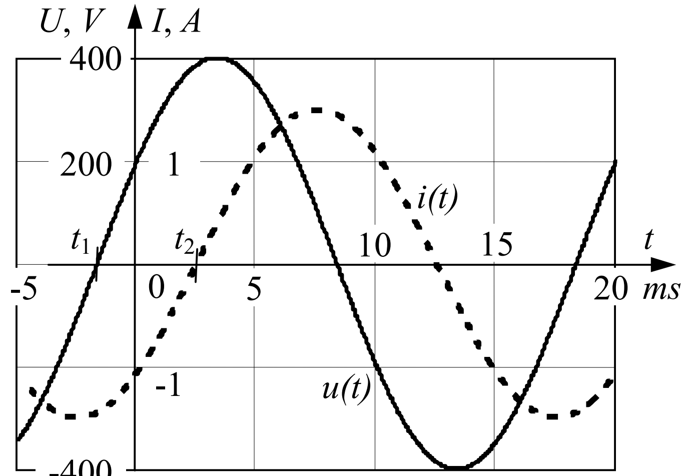


Fig. 3.1

Solution. By oscillogram, we determine the period $T = 20\ ms$, hence, the frequency is $f = 1/T = 50\ Hz$, and angular velocity is $\omega = 2\pi f = 314\ rad/s$.

Initial phases of voltage and current in degrees, respectively, are:

$\psi_u = -t_1\cdot(360/T) = 1.67\cdot(360/20) = 30°$, $\psi_i = -t_2\cdot(360/T) = -2{,}5\cdot(360/20) = -45°$.

Amplitudes: $U_m = 400\ V$, $I_m = 1.5\ A$.

Finally, $u(t) = 400\cdot sin(314t + 30°)\ V$, $i(t) = 1.5\cdot sin(314t - 45°)\ A$.

Effective values: $U = \dfrac{U_m}{\sqrt{2}} = \dfrac{400}{\sqrt{2}} = 282\ V$, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{1.5}{\sqrt{2}} = 1.061\ A$.

3-2 (3.2). A circuit r, L with parameters $r = 35\ Ohm$, $L = 80\ mH$ is supplied from a source of sinusoidal voltage of frequency $f = 50\ Hz$. The source voltage amplitude is $U_m = 200\ V$, its initial phase being $\psi_u = -20°$. Compute instantaneous and effective values of the current. Draw the circuit phasor diagram. Find active, reactive powers and volt-amperes of the circuit. Draw the power triangle.

Solution. Let's imagine the calculation scheme of the circuit (fig. 3.2,a).

Write down the instantaneous value of the circuit voltage:

$$u(t) = U_m\cdot sin(\omega t + \psi_u) = 200\cdot sin(\omega t - 20°)\ V.$$

Angular velocity is $\omega = 2\pi f = 2\pi\cdot 50 = 314\ rad/s$.

The circuit reactance is $x_L = \omega L = 314\cdot 80\cdot 10^{-3} = 25.12\ Ohm$.

In accordance with Kirchhoff's voltage law for the circuit loop we have $u = u_r + u_L$ or in phasor form $\underline{U} = \underline{U}_r + \underline{U}_L$. On the ground of this expression, the phasor diagram is drawn (fig. 3.2,b).

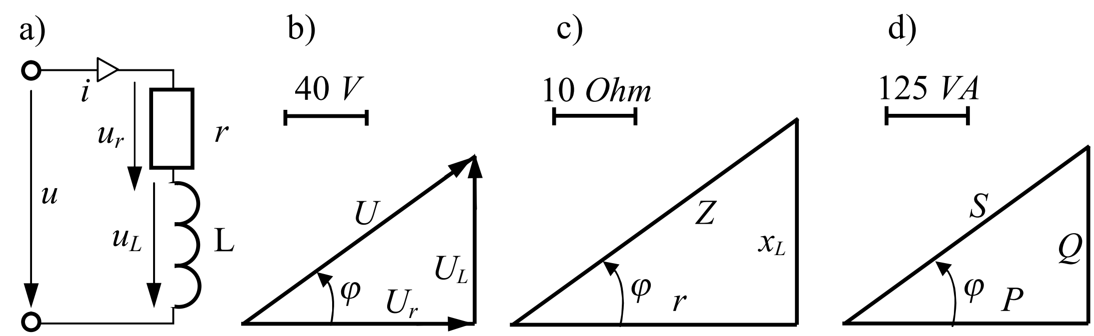


Fig. 3.2

The circuit impedance triangle is presented in fig. 3.2,c. This is a right triangle, it allowing to obtain the following:

the circuit impedance $Z = \sqrt{r^2 + x_L^2} = \sqrt{35^2 + 25.12^2} = 43.1$ *Ohm*,

the phase shift between the current and voltage $\varphi = arctg\frac{x_L}{r} = \text{arctg}\frac{25.12}{35} = 35.67°$.

Under Ohm's law for amplitudes, we find $I_m = \frac{U_m}{Z} = \frac{200}{43.1} = 4.64$ *A*.

The initial phase of the current sinusoid is $\psi_i = \psi_u - \varphi = -20° - 35.67° = -55.67°$.

The instantaneous current value is $i(t) = 4.64 \cdot sin(314t - 55.67°)$ *A*.

The effective current value is $I = \frac{I_m}{\sqrt{2}} = \frac{4.64}{\sqrt{2}} = 3.28$ *A*.

The effective voltages across different portions of the circuit are:

- across resistance $U_r = I \cdot r = 3.28 \cdot 35 = 115$ *V*;
- across reactance $U_L = I \cdot x_L = 3.28 \cdot 25.12 = 82.4$ *V*;
- at the circuit input (network voltage) $U = \frac{U_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4$ *V*.

The circuit active power is $P = U \cdot I \cdot cos\varphi = I^2 \cdot r = 3.28^2 \cdot 35 = 376.5$ *W*.

The reactive power is $Q = U \cdot I \cdot sin\varphi = I^2 \cdot x_L = 3.28^2 \cdot 25.12 = 270.2$ *VAr*.

The circuit volt-amperes is $S = U \cdot I = 141.4 \cdot 3.28 = 464$ *VA*.

The power triangle is shown in fig. 3.2,d.

Note, on the ground of any triangle of fig. 3.2 it is possible to compute the power factor $cos\varphi = \frac{r}{Z} = \frac{U_r}{U} = \frac{P}{S} = \frac{376.5}{464} = 0.811 = cos35.67°$,

earlier it being calculated on the ground of the circuit impedance triangle.

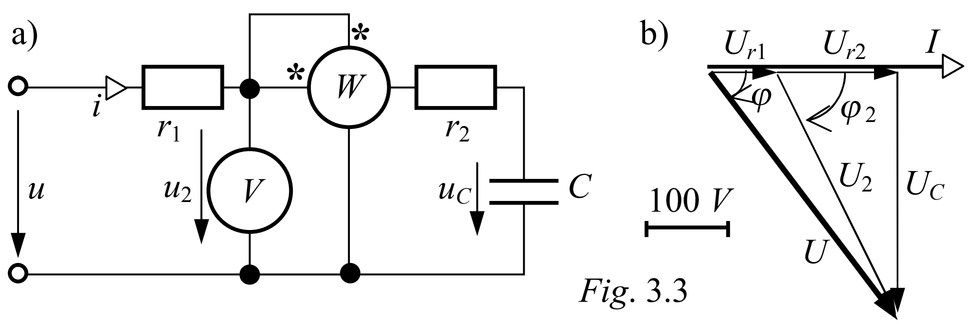


Fig. 3.3

3-3 (3.3). Sinusoidal current $i(t) = 10 \cdot sin(\omega t + 15°)$ *A* of frequency $f = 400$ *Hz* flows in circuit fig. 3.3,a. Resistances are $r_1 = 10$ *Ohm*, $r_2 = 20$ *Ohm*, capacitance is $C = 10$ *μF*.

Determine the instantaneous network voltage $u(t)$ and the voltage across a capacitor $u_C(t)$. Find the voltmeter and wattmeter readings. Construct a circuit phasor diagram.

Solution. Reactance is as follows $x_C = \frac{1}{\omega C} = \frac{10^6}{2\pi \cdot 400 \cdot 10} = 39.81$ Ohm.

Voltage amplitude across the capacitance is

$$U_{Cm} = I_m \cdot x_C = 10 \cdot 39.81 = 398.1 \text{ } V.$$

Effective values are $I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07$ *A*, $U_C = \frac{U_{Cm}}{\sqrt{2}} = \frac{398.1}{\sqrt{2}} = 281.5$ *V*.

Effective voltages across resistances are:

$$U_{r1}= I\cdot r_1 = 7.07\cdot 10 = 70.7\ V, \qquad U_{r2} = I\cdot r_2 = 7.07\cdot 20 = 141.4\ V.$$

Kirchhoff's voltage law in a phasor form at a single current I has a view $\underline{U}_{r1}+\underline{U}_{r2}+\underline{U}_C = \underline{U}$, in correspondence with it the circuit phasor diagram can be constructed (fig. 3.3,b).

From the voltage right triangle (exterior triangle) we find

$$U=\sqrt{(Ir_1+Ir_2)^2+U_C^2}=\sqrt{(70.7+141.4)^2+281.5^2}=352\ V,$$

$$\varphi = arctg\frac{-U_C}{U_{r1}+U_{r2}} = \text{arctg}\frac{-281.5}{212.2} = -53°.$$

The instantaneous value of the circuit voltage is

$$u(t)= U_m\cdot sin(\omega t+\psi_i+\varphi) = U\cdot\sqrt{2}\cdot sin(\omega t+15°+(-53°)) = 352\sqrt{2}\cdot sin(\omega t - 38°)\ V.$$

The voltage across a capacitor lags the current by 90°, its instantaneous value being $u_C(t)= 398.1\cdot sin(\omega t - 75°)\ V$.

The voltage across portion r_2-C, supplied to a voltmeter and wattmeter, is calculated by the voltage triangle U_2-U_{r2}-U_C:

$$U_2 = U_W=\sqrt{(U_{r2})^2+U_C^2}=\sqrt{141.4^2+281.5^2}=315\ V.$$

The voltmeter of scheme fig. 3.3,a measures the effective voltage $U_2 = 315\ V$.

The wattmeter's reading is $P_W = U_W\cdot I_W\cdot cos\left(U_W \hat{,} I_W\right)$.

In our example $I_W = I$, that's why
$P_W = U_2\cdot I\cdot cos\varphi_2 = I\cdot(U_2\cdot cos\varphi_2)= I\cdot U_{r2}= I\cdot I\cdot r_2 = I^2\cdot r_2 = P_2$ – active power, consumed by resistance r_2, and $P_2 = 7.07^2\cdot 20 = 1000\ W$.

3-4 (3.4). Determine the current and voltages in an electric circuit fig. 3.4,a, if: the coil resistance is $r_c = 4\ Ohm$, the coil reactance is $x_c = 6\ Ohm$, the rheostat resistance is $R = 2\ Ohm$, the capacitance is $x_C = 14\ Ohm$, A-C network voltage is $U = 50\ V$. Draw the circuit phasor diagram.

Solution. The circuit current I is measured by ammeter A:

$$I=\frac{U}{Z_{□□}}=\frac{U}{\sqrt{(r_c+R)^2+(x_c-x_C)^2}}=\frac{50}{\sqrt{(4+2)^2+(6-14)^2}}=5\ A.$$

The circuit phase shift is $\varphi = arctg\dfrac{x_c - x_C}{r_c+R} = arctg\dfrac{6-14}{4+2} = -53.13° < 0.$

Voltmeter V measures the input voltage $U = 50\ V$.

The coil voltage is measured by voltmeter V_1:

$$U_c=\sqrt{(Ir_c)^2+(Ix_c)^2} = I\cdot Z_c = I\cdot\sqrt{(r_c)^2+(x_c)^2} = 5\cdot\sqrt{4^2+6^2} = 36\ V.$$

The rheostat voltage: $U_R = I\cdot R = 5\cdot 2 = 10\ V$,

the capacitor voltage: $U_C = I\cdot x_C = 5\cdot 14 = 70\ V$.

One can see the circuit phasor diagram in fig. 3.4,b.

The wattmeter measures the circuit active power

$P = U\cdot I\cdot cos\varphi = I_2^2\cdot(r_c + R) = 5^2\cdot(4 + 2) = 150\ W$.

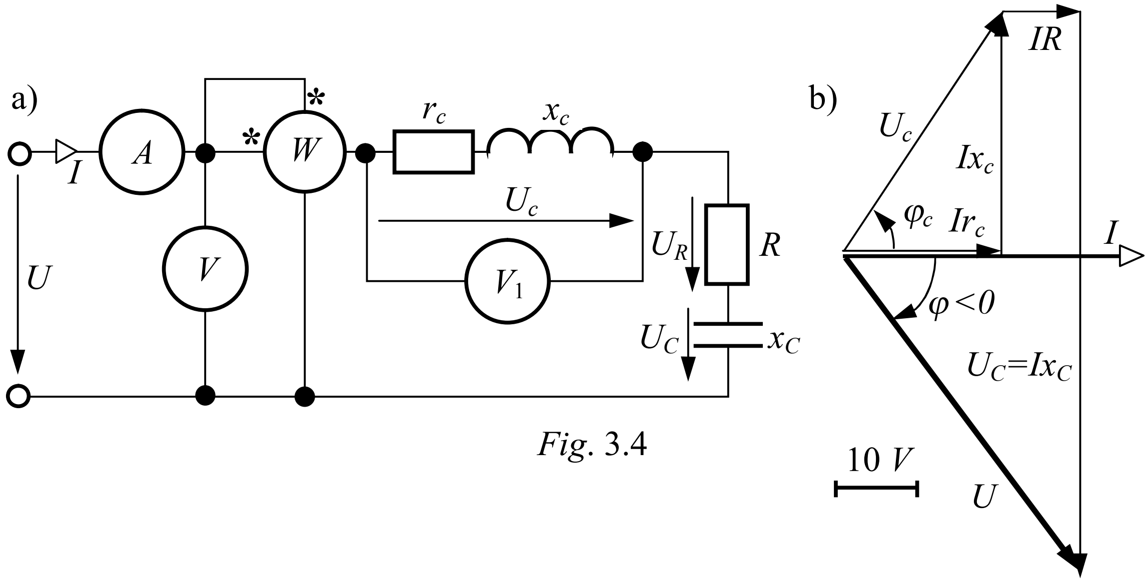


Fig. 3.4

Pay attention, in series A-C circuit with dissimilar reactive elements (inductive and capacitive), the voltage across the reactance may be greater than input voltage:

$$U_C = 70\ V > U = 50\ V.$$

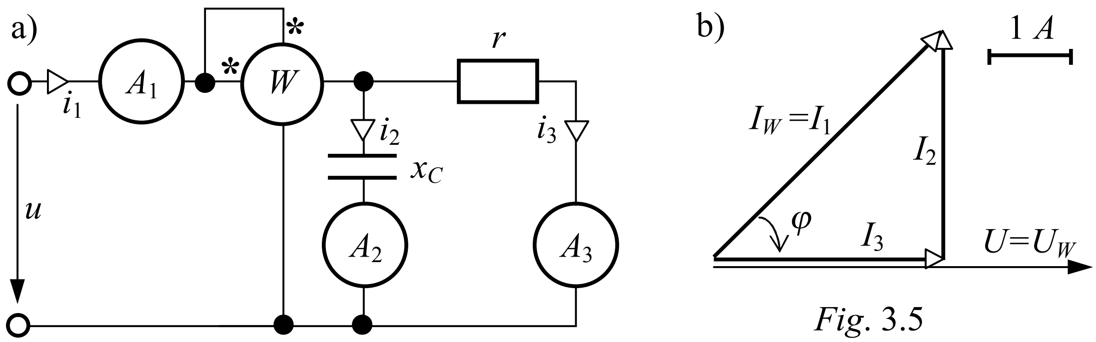


Fig. 3.5

3-5 (3.6). Determine the instruments' reading in scheme fig. 3.5,a as well as the instantaneous current value i_1 in common part of the scheme, construct a phasor diagram, if $u(t) = 200\cdot sin(\omega t + 25°)\ V$, $r = 50\ Ohm$, $x_C = 50\ Ohm$.

Solution. Instruments react upon the effective values of the quantities. Effective values of the currents in parallel branches are as follows

$$I_2 = \frac{U}{x_C} = \frac{U_m}{\sqrt{2}x_C} = \frac{200}{\sqrt{2}\cdot 50} = 2\sqrt{2} = 2.83\ A,$$

$$I_3 = \frac{U}{r} = \frac{200}{\sqrt{2}\cdot 50} = 2\sqrt{2} = 2.83\ A.$$

As the current through the resistance i_3 is in phase with voltage, and the current through the capacitance i_2 leads the voltage by 90°, and in accordance with Kirchhoff's current law there is the expression $i_1 = i_2 + i_3$, i.e. $\overline{I}_1 = \overline{I}_2 + \overline{I}_3$, then the current triangle at the phasor diagram is a right one (fig. 3.5,b); from here

$$I_1 = \sqrt{I_3^2 + I_2^2} = \sqrt{\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = 4\ A,$$

the phase shift between current i_1 and voltage u at the circuit input is negative and equal to $\varphi = -\,arctg\frac{I_2}{I_3} = -\,arctg 1 = -\,45°.$

The instantaneous current value is

$$i_1(t) = I_{1m}\cdot sin(\omega t + \psi_u - \varphi) = 4\sqrt{2}\cdot sin(\omega t + 70°)\ A.$$

The ammeters readings are

$$A_1 \rightarrow I_1 = 4\ A, \qquad A_2 \rightarrow I_2 = 2.83\ A, \qquad A_3 \rightarrow I_3 = 2.83\ A.$$

The wattmeter reading is

$$P_W = U_W\cdot I_W\cdot cos\left(U_W \hat{,} I_W\right) = U\cdot I_1\cdot cos\varphi = \frac{200}{\sqrt{2}}\cdot 4\cdot cos 45° = 400\ W.$$

Note, in scheme fig. 3.5,a the wattmeter measures the active power of the circuit part situated to the right from wattmeter. In accordance with Joule's effect in this circuit part, the power is spent in the resistance r_3 only, furthermore

$P_3 = I_3^2\cdot r_3 = \left(2\sqrt{2}\right)^2\cdot 50 = 400\ W$, it coinciding with the wattmeter reading.

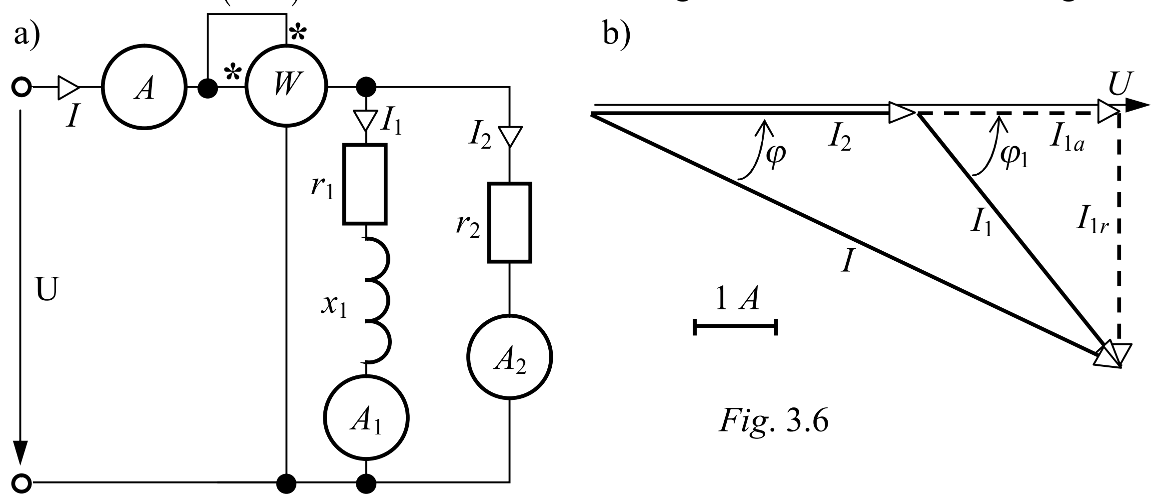


Fig. 3.6

3-6 (3.7). Find the instruments' reading in scheme fig. 3.6,a, construct a phasor diagram, if $U = 200\ B$, $r_1 = 30\ Ohm$, $x_1 = 40\ Ohm$, $r_2 = 50\ Ohm$.

Check up the balances of active and reactive powers.

Solution. We determine the currents of parallel branches by Ohm's law and find corresponding ammeters' reading:

$$I_2 = \frac{U}{Z_2} = \frac{U}{r_2} = \frac{200}{50} = 4\ A, \qquad \rightarrow A_2,$$

$$I_1 = \frac{U}{Z_1} = \frac{U}{\sqrt{r_1^2 + x_1^2}} = \frac{200}{\sqrt{30^2 + 40^2}} = 4\ A, \qquad \rightarrow A_1.$$

Current i_2 in the resistance is in phase with voltage u, current i_1 lags it by the angle φ_1 because there is an inductance in this branch, and the angle φ_1 itself is determined from the impedance triangle for this branch

$$\varphi_1 = arctg\frac{x_1}{r_1} = arctg\frac{40}{30} = 53.13°.$$

Furthermore $cos\varphi_1 = \dfrac{r_1}{Z_1} = \dfrac{30}{50} = 0.6, \qquad sin\varphi_1 = \dfrac{x_1}{Z_1} = \dfrac{40}{50} = 0.8.$

Everything mentioned above is taken into account when constructing the circuit phasor diagram (fig. 3.6,b).

In example under consideration the current skew-angular triangle $\underline{I}_1$, $\underline{I}_2$, $\underline{I}$ appears. The problem of the skew-angular triangle calculation is reduced to the right triangle calculation if to project the current phasor system onto two orthogonally related directions: one of them being parallel to the voltage phasor (these current components are called the *active* ones) and the other one being perpendicular to the voltage phasor (these components are termed the *reactive* ones).

Furthermore $I_{2a} = I_2 = 4\ A$, $I_{2r} = 0$,

$$I_{1a} = I_1 \cdot cos\varphi_1 = 4\cdot 0.6 = 2.4\ A, \quad I_{1r} = I_1 \cdot sin\varphi_1 = 4\cdot 0.8 = 3.2\ A.$$

From the exterior right triangle, the summary current of the parallel branches is determined, it being measured by ammeter A:

$$I = \sqrt{\left(\sum I_a\right)^2 + \left(\sum I_r\right)^2} = \sqrt{(4+2.4)^2 + 3.2^2} = 7.16\ A.$$

Then $\varphi = arctg\dfrac{\sum I_r}{\sum I_a} = arctg\dfrac{3.2}{6.4} = 26.57°$,

$$cos\varphi = \frac{\sum I_a}{I} = \frac{6.4}{7.16} = 0.447, \quad sin\varphi = \frac{\sum I_r}{I} = \frac{3.2}{7.16} = 0.224.$$

The wattmeter reading is $P_W = U \cdot I \cdot cos\varphi = U \cdot \Sigma I_a = 200\cdot 6.4 = 1280\ W$ – this is the source active power P_G.

The summary active power of the consumers is calculated under the formula of Joule effect: $\Sigma P_C = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 = 4^2\cdot 30 + 4^2\cdot 50 = 1280\ W$.

The equality $P_G = \Sigma P_C$ is true, the active power balance being reached.

Reactive powers:

- that of generator $Q_G = U \cdot I \cdot sin\varphi = U \cdot I_r = 200\cdot 3.2 = 640\ VAr$;
- that of consumers $\Sigma Q_C = I_1^2 \cdot x_1 = 4^2\cdot 40 = 640\ VAr$,

which means the fulfillment of the reactive power balance.

3.2. CALCULATION OF THE SERIES-PARALLEL CONNECTION OF ELEMENTS BY THE PHASOR-DIAGRAM METHOD

3-7 (3.19). A circuit fig. 3.7 is supplied with voltage $U = 220\ V$. When capacitor C is not connected, the instruments read: $A \to I = 2\ A$; $W \to P = 40\ W$. It is required to determine the possible minimum ammeter's reading after the capacitor connection as well as the capacitor capacity.

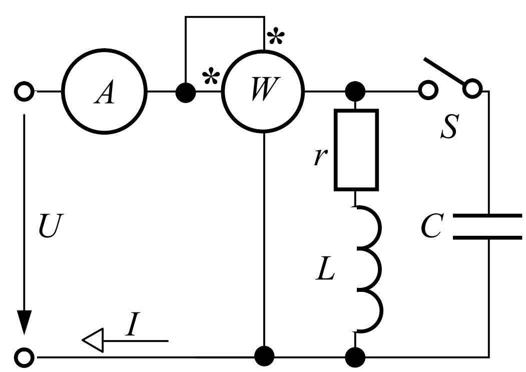


Fig. 3.7

Solution. When the capacitor is not connected, from the instruments' reading we determine the impedance, resistance and reactance of the branch r, L:

$$Z = \frac{U}{I} = \frac{220}{2} = 110\ Ohm; \quad r = \frac{P}{I^2} = \frac{40}{2^2} = 10\ Ohm;$$

$$x_L = \sqrt{Z^2 - r^2} = \sqrt{110^2 - 10^2} = 109.5\ Ohm.$$

The minimum current value in the common part of a circuit happens when there is a circuit current resonance after the capacitor has been connected. In this case, the current

I possesses the active component only: $I = I_a = U\cdot g$, where g – the circuit conductance, it being equal to the conductance of the branch r, L, namely:

$$g = \frac{r}{Z^2} = \frac{10}{110^2} = 2.26\cdot10^{-4}\ S.$$

Then $I = U\cdot g = 220\cdot2.26\cdot10^{-4} = 0.182\ A$.

We determine a value of capacity from the condition that its susceptance should be equal to the susceptance of the branch r, L, i.e.

$$\omega C = \frac{x_L}{Z^2},\quad \text{from here}\quad C = \frac{x_L}{\omega\cdot Z^2} = \frac{109.5}{314\cdot110^2} = 2.88\cdot10^{-5}\ F = 28.8\ \mu F.$$

3-8 (3.20). In the scheme (fig. 3.8,a), determine all the branch currents as well as the voltages across all the circuit parts, make up a balance of the active and reactive powers, draw the circuit complete phasor diagram, write down the instantaneous current values, if $u(t) = U_m\cdot sin(\omega t + \psi_u)$;

$U_m = 600\ V$; $\psi_u = -90°$; $r_1 = 10\ Ohm$, $r_3 = x_2 = x_3 = 20\ Ohm$, $x_4 = 20\ Ohm$.

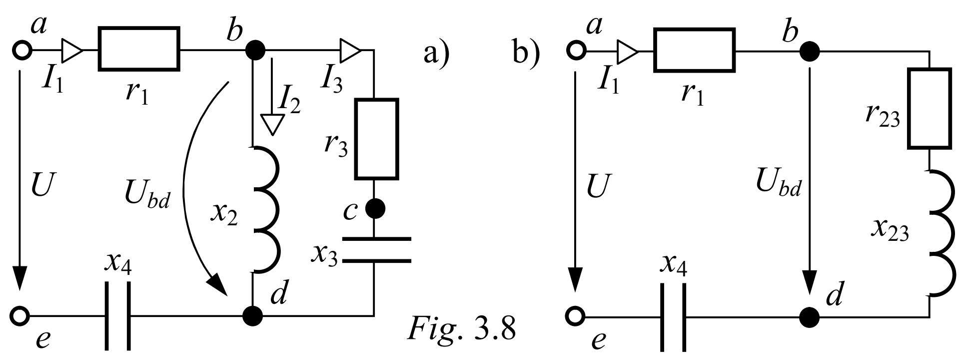


Fig. 3.8

Solution. Substitute the branched part of the initial scheme by an equivalent branch with parameters r_{23}, x_{23}; for that we calculate conductance and susceptance (with account of the reactance character) of the parallel branches:

$$g_2 = \frac{r_2}{r_2^2 + x_2^2} = \frac{0}{0 + 20^2} = 0; \qquad b_2 = \frac{x_2}{r_2^2 + x_2^2} = \frac{20}{0 + 20^2} = 0.05\ S\ (ind.);$$

$$g_3 = \frac{r_3}{r_3^2 + x_3^2} = \frac{20}{20^2 + 20^2} = 0.025\ S; \qquad b_3 = \frac{x_3}{r_3^2 + x_3^2} = \frac{20}{20^2 + 20^2} = 0.025\ S\ (cap.);$$

$$g_{23} = g_2 + g_3 = 0 + 0.025 = 0.025\ S; \qquad b_{23} = |b_2 - b_3| = 0.05 - 0.025 = 0.025\ S\ (ind.);$$

$$r_{23} = \frac{g_{23}}{g_{23}^2 + b_{23}^2} = \frac{0.025}{0.025^2 + 0.025^2} = 20\ Ohm;$$

$$x_{23} = \frac{b_{23}}{g_{23}^2 + b_{23}^2} = \frac{0.025}{0.025^2 + 0.025^2} = 20\ Ohm\ (ind.).$$

The equivalent scheme to calculate the current through the common portion of the circuit is presented in fig. 3.8,b: $i_1(t) = I_{1m}\cdot sin(\omega t + \psi_u - \varphi_{inp})$;

where $$I_{1m} = \frac{U_m}{\sqrt{(r_1 + r_{23})^2 + (x_{23} - x_4)^2}} = \frac{600}{\sqrt{(10+20)^2 + (20-50)^2}} = 10\sqrt{2}\ A;$$

$$\varphi_{inp} = arctg\frac{x_{23} - x_4}{r_1 + r_{23}} = arctg\frac{20 - 50}{10 + 20} = -45°.$$

Hence, the instantaneous current value is as follows

$$i_1(t) = 10\sqrt{2}\cdot sin(\omega t - 90° + 45°) = 10\sqrt{2}\cdot sin(\omega t - 45°)\ A,$$

the effective current value being $I_1 = \dfrac{I_{1m}}{\sqrt{2}} = 10\ A$.

The voltage across the parallel branches is $u_{bd}(t) = U_{bdm}\cdot sin(\omega t + \psi_u - \varphi_{inp} + \varphi_{23})$;

where $U_{bdm} = I_{1m}\cdot\sqrt{r_{23}^2 + x_{23}^2} = 10\sqrt{2}\cdot\sqrt{20^2 + 20^2} = 400\ V$;

$$\varphi_{23} = arctg\frac{x_{23}}{r_{23}} = arctg\frac{20}{20} = 45°,$$

or $u_{bd}(t) = 400\cdot sin(\omega t - 90° + 45° + 45°) = 400\cdot sin(\omega t)\ V$.

Effective values of the voltage across the equivalent circuit portion and currents of the parallel branches are

$$U_{bd} = \frac{U_{bdm}}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282\ V; \qquad I_2 = \frac{U_{bd}}{x_2} = \frac{282}{20} = 14.1\ A;$$

$$I_3 = \frac{U_{bd}}{\sqrt{r_3^2 + x_3^2}} = \frac{282}{\sqrt{20^2 + 20^2}} = 10\ A.$$

The phase shifts for the second and the third branches are

$$\varphi_2 = arctg\frac{x_2}{r_2} = arctg\frac{20}{0} = 90°,\ \ \varphi_3 = arctg\frac{-x_3}{r_3} = arctg\frac{-20}{20} = -45°.$$

The instantaneous current values of the parallel branches are

$$i_2(t) = I_2\sqrt{2}\cdot sin(\omega t + \psi_{ubd} - \varphi_2) = 20\cdot sin(\omega t - 90°)\ A;$$

$$i_3(t) = I_3\sqrt{2}\cdot sin(\omega t + \psi_{ubd} - \varphi_3) = 10\sqrt{2}\cdot sin(\omega t + 45°)\ A;$$

where $\psi_{ubd} = \psi_u - \varphi_{inp} - \varphi_{23} = -90° + 45° + 45° = 0$.

Let's verify the active power balance: $U\cdot I_1\cdot cos\varphi_{inp} = I_1^2\cdot r_1 + I_3^2\cdot r_3$.

$$\frac{600}{\sqrt{2}}\cdot 10\cdot cos(-45°) = 10^2\cdot 10 + 10^2\cdot 20 \quad \text{or} \quad 3000\ W = 3000\ W.$$

Let's verify the reactive power balance:

$$U\cdot I_1\cdot sin\varphi_{inp} = I_1^2\cdot(-x_4) + I_2^2\cdot x_2 + I_3^2\cdot(-x_3).$$

$$\frac{600}{\sqrt{2}}\cdot 10\cdot sin(-45°) = 10^2\cdot(-50) + (10\sqrt{2})^2\cdot 20 + 10^2\cdot(-20) \quad \text{or} \quad -3000\ VAr = -3000\ VAr.$$

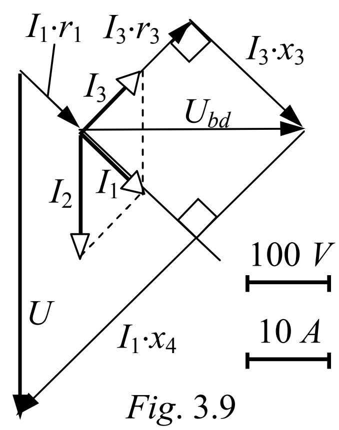


Fig. 3.9

Conclusion: balances of the active and reactive powers are true which means the problem is solved correctly, it allowing to draw the complete phasor diagram (fig. 3.9).

Calculate the unknown voltages across different circuit elements:

$U_{ab} = I_1\cdot r_1 = 10\cdot 10 = 100\ V$; $\quad U_{bc} = I_3\cdot r_3 = 10\cdot 20 = 200\ V$;

$U_{ca} = I_3\cdot x_3 = 10\cdot 20 = 200\ V$; $\quad U_{de} = I_1\cdot x_4 = 10\cdot 50 = 500\ V$.

Phasor diagram construction begins with a choice of the voltage and current scales. Then, a voltage phasor across the parallel branches $\underline{U}_{bd}$ is drawn arbitrarily, the current phasors $\underline{I}_2$, $\underline{I}_3$ and $\underline{I}_1$ being drawn by angles φ_2, φ_3, φ_{23} to the voltage phasor, respectively. Doing this take into account the expression $\underline{I}_1 = \underline{I}_2 + \underline{I}_3$.

The other voltage phasors are drawn in correspondence with the equations under Kirchhoff's voltage law and with account of the order of the element location in a scheme (see fig. 3.8,b): $\underline{U}_{ab} + \underline{U}_{bd} + \underline{U}_{de} = \underline{U}$.

3-9 (3.21). Determine the effective values of all voltages and currents in scheme fig. 3.10,a, do verification of the active and reactive power balance, construct the circuit complete phasor diagram, if: $U = 300$ *V*; $r_2 = 30$ *Ohm*, $x_2 = 10$ *Ohm*; $x_3 = 33.33$ *Ohm*; $x_4 = 2$ *Ohm*; $x_5 = 20$ *Ohm*; $r_6 = 10$ *Ohm*; $x_1 = 8$ *Ohm*; $r_1 = 4$ *Ohm*.

Solution. In the scheme under consideration there are two circuit forks: the second and the third branches are connected in parallel in the portion *bc*, they can be substituted by an equivalent branch r_{23}-x_{23} (fig. 3.10,b); in the portion *de* there are two parallel branches – 5th and 6th which can be substituted by an equivalent circuit r_{56}-x_{56}. The substitution is performed on the ground of the correlations between the conductance and susceptance of the parallel branches:

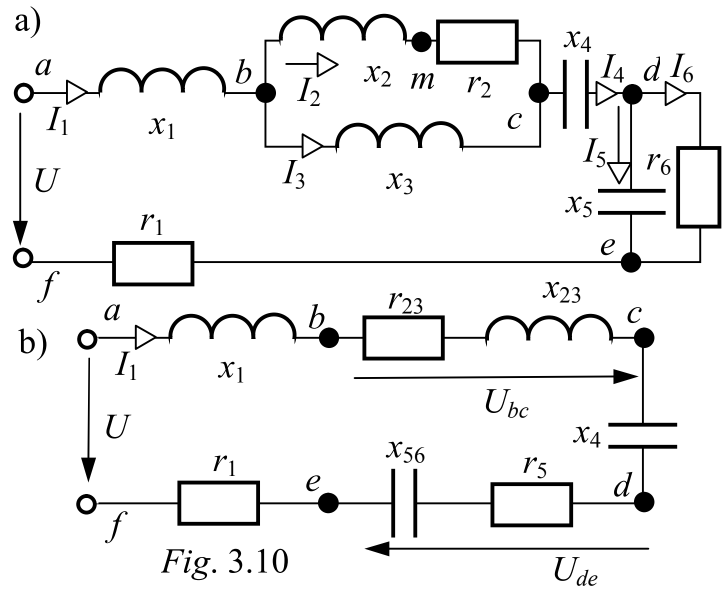


Fig. 3.10

$$g_2 = \frac{r_2}{r_2^2 + x_2^2} = \frac{30}{30^2 + 10^2} = 0.03\ S;$$

$$b_2 = \frac{x_2}{r_2^2 + x_2^2} = \frac{10}{30^2 + 20^2} = 0.01\ S\ (ind.);$$

$$g_3 = 0; \qquad b_3 = \frac{1}{x_3} = \frac{1}{33.33} = 0.03\ S\ (ind.);$$

$$g_{23} = g_2 + g_3 = 0.03 + 0 = 0.03\ S; \qquad b_{23} = b_2 + b_3 = 0.01 + 0.03 = 0.04\ S\ (ind.);$$

$$r_{23} = \frac{g_{23}}{g_{23}^2 + b_{23}^2} = \frac{0.03}{0.03^2 + 0.04^2} = 12\ Ohm;$$

$$x_{23} = \frac{b_{23}}{g_{23}^2 + b_{23}^2} = \frac{0.04}{0.03^2 + 0.04^2} = 16\ Ohm\ (ind.).$$

$$g_5 = 0; \qquad g_6 = \frac{1}{r_6} = \frac{1}{10} = 0.1\ S; \qquad b_5 = \frac{1}{x_5} = \frac{1}{20} = 0.05\ См\ (cap.); \qquad b_6 = 0;$$

$$g_{56} = g_5 + g_6 = 0 + 0.1 = 0.1\ S; \qquad b_{56} = b_5 + b_6 = 0.05 + 0 = 0.05\ S\ (cap.);$$

$$r_{56} = \frac{g_{56}}{g_{56}^2 + b_{56}^2} = \frac{0.1}{0.1^2 + 0.05^2} = 8\ Ohm; \quad x_{56} = \frac{b_{56}}{g_{56}^2 + b_{56}^2} = \frac{0.05}{0.1^2 + 0.05^2} = 4\ Ohm\ (cap.).$$

The circuit input impedance found by the equivalent scheme (fig. 3.10,b) is

$$Z_{inp} = \sqrt{(r_{23} + r_{56} + r_1)^2 + (x_1 + x_{23} - x_4 - x_{56})^2} =$$

$$= \sqrt{(12 + 8 + 4)^2 + (8 + 16 - 2 - 4)^2} = 30\ Ohm;$$

$$cos\varphi_{inp} = \frac{r_{23} + r_{56} + r_1}{Z_{inp}} = \frac{12 + 8 + 4}{30} = 0.8;$$

$$sin\varphi_{inp} = \frac{x_1 + x_{23} - x_4 - x_{56}}{Z_{inp}} = \frac{8+16-2-4}{30} = 0.6.$$

The current of the common circuit part is $I_1 = I_4 = \frac{U}{Z_{inp}} = \frac{300}{30} = 10\ A,$

the voltages across the forks are $U_{bc} = I_1 \cdot \sqrt{r_{23}^2 + x_{23}^2} = 10\sqrt{12^2 + 16^2} = 200\ V,$

$U_{de} = I_1 \cdot \sqrt{r_{56}^2 + x_{56}^2} = 10\sqrt{8^2 + 4^2} = 40\sqrt{5}\ V,$

currents through the other branches are

$I_2 = \frac{U_{bc}}{\sqrt{r_2^2 + x_2^2}} = \frac{200}{\sqrt{30^2 + 10^2}} = 2\sqrt{10}\ A,$ $I_5 = \frac{U_{de}}{x_5} = \frac{40\sqrt{5}}{20} = 2\sqrt{5}\ A,$

$I_3 = \frac{U_{bc}}{x_3} = \frac{200}{33.33} = 6\ A,$ $I_6 = \frac{U_{de}}{r_6} = \frac{40\sqrt{5}}{10} = 4\sqrt{5}\ A.$

Let's check up the power balance. The active power balance for scheme fig. 3.10,a is described by an equation $U \cdot I_1 \cdot cos\varphi_{inp} = I_1^2 \cdot r_1 + I_2^2 \cdot r_2 + I_6^2 \cdot r_6,$

$300 \cdot 10 \cdot 0.8 = 10^2 \cdot 4 + \left(2\sqrt{10}\right)^2 \cdot 30 + \left(4\sqrt{5}\right)^2 \cdot 30$

or $3000\ W = 3000\ W$ – it's true.

The circuit reactive power balance is as follows

$$U \cdot I_1 \cdot sin\varphi_{inp} = I_1^2 \cdot x_1 + I_2^2 \cdot x_2 + I_3^2 x_3 - I_4^2 \cdot x_4 - I_5^2 \cdot x_5,$$

$300 \cdot 10 \cdot 0.6 = 10^2 \cdot 8 + \left(2\sqrt{10}\right)^2 \cdot 10 + 6^2 \cdot 33.33 - 10^2 \cdot 2 - \left(2\sqrt{5}\right)^2 \cdot 20$

or $1800\ VAr = 800 + 400 + 1200 - 200 - 400\ VAr$ – it's true.

Since both power balances are true the problem is solved correctly, it allowing to draw a phasor diagram.

As in scheme fig. 3.10,a there are two forks, initially the phasor diagram is drawn for the series equivalent circuit fig. 3.10,b and its construction begins from the arbitrary choice of the direction of the current phasor $\underline{I}_1$ of the series circuit (horizontally, to the right) (fig. 3.11).

Let's write down an equation under Kirchhoff's voltage law in a phasor form observing the principle: the voltage drops across the scheme elements strictly follow the elements location and each voltage phasor is supplied with a subscript corresponding to the scheme points:

$$\overline{U}_{ab} + \overline{I}_1 \cdot r_{23} + \overline{I}_1 \cdot x_{23} + \overline{U}_{cd} + \overline{I}_1 \cdot r_{56} + \overline{I}_1 \cdot x_{56} + \overline{U}_{ef} = \overline{U} = \overline{U}_{af}.$$

At the same time $\overline{U}_{ab} = \overline{I}_1 \cdot x_1 = 10 \cdot 8 = 80\ V$ and this voltage phasor leads the current $\overline{I}_1$ by 90°;

$I_1 \cdot r_{23} = 10 \cdot 12 = 120\ V,$ $I_1 \cdot x_{23} = 10 \cdot 16 = 160\ V,$

$U_{cd} = I_1 \cdot x_4 = 10 \cdot 2 = 20\ V,$ $I_1 \cdot r_{56} = 10 \cdot 8 = 80\ V,$

$I_1 \cdot x_{56} = 10 \cdot 4 = 40\ V,$ $I_1 \cdot r_1 = U_{ef} = 10 \cdot 4 = 40\ V.$

In fig. 3.11, the voltage drop phasors $\overline{I}_1 \cdot r_{23}$, $\overline{I}_1 \cdot x_{23}$, $\overline{I}_1 \cdot r_{56}$, $\overline{I}_1 \cdot x_{56}$ are shown by dash arrows, because these voltages are absent in the initial scheme.

Then, pass on to the construction of the current phasors I_2 and I_3. Current I_3 is perpendicular to the voltage U_{bc}, while $\overline{I}_2=\overline{I}_1-\overline{I}_3$.

Current I_6 is set in parallel to phasor U_{de}, while current $\overline{I}_5=\overline{I}_1-\overline{I}_6$ is set in correspondence with Kirchhoff's current law.

Then, the voltage drop phasors $\overline{U}_{bm}=\overline{I}_2\cdot x_2$ and $\overline{U}_{mc}=\overline{I}_2\cdot r_2$ are set as regard to the current $\underline{I}_2$.

One may see a complete view of the phasor diagram in fig. 3.11. In this diagram, the voltage phasor $\overline{U}_{md}$ is also shown.

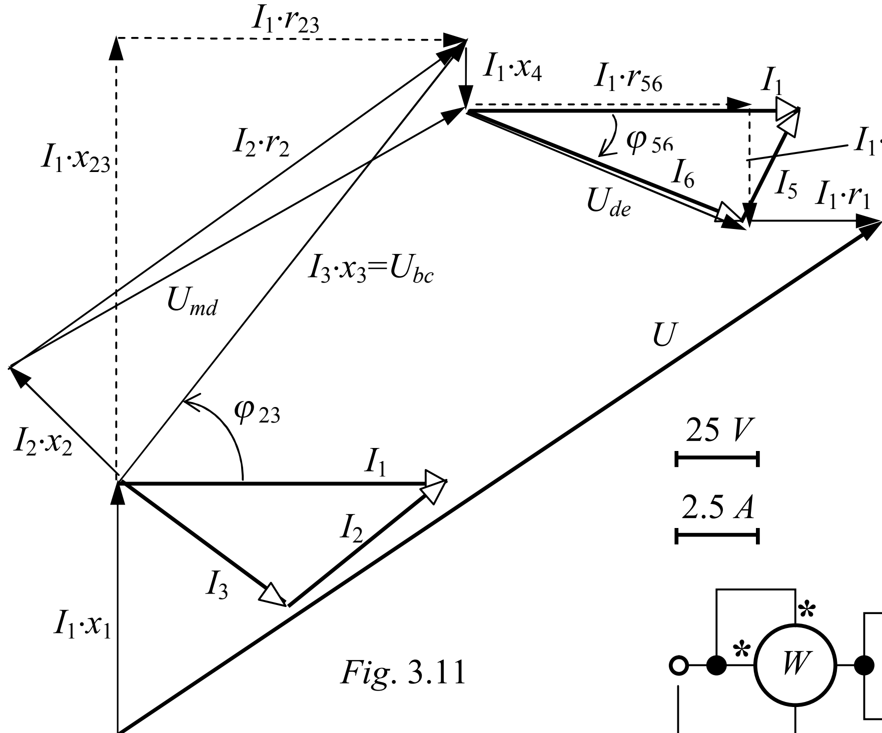


Fig. 3.11

3-10 (3.25). In scheme fig. 3.12, it is known:

$U = 300\ V$; $r_1 = 16\ Ohm$; $x_1 = 24\ Ohm$;
$r_2 = 16.67\ Ohm$; $r_3 = 5\ Ohm$; $x_3 = 15\ Ohm$.

Determine the instrument readings.

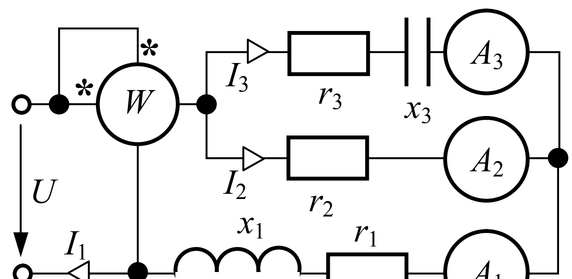


Fig. 3.12

Solution. 1. Substitute an equivalent impedance connection r_{23} and x_{23} for parallel connection of the branches 2 and 3:

$$g_2=\frac{1}{r_2}=\frac{1}{16.67}=0.06\ S;\quad b_2=0;\qquad Z_3=\sqrt{r_3^2+x_3^2}=\sqrt{5^2+15^2}=\sqrt{250}=5\sqrt{10}\ \ Ohm;$$

$$g_3=\frac{r_3}{Z_3^2}=\frac{5}{250}=0.02\ S;\qquad\qquad b_3=\frac{x_3}{Z_3^2}=\frac{15}{250}=0.06\ S;$$

$$g_{23}=g_2+g_3=0.06+0.02=0.08\ S;\quad b_{23}=|b_2-b_3|=|0-0.06|=0.06\ S\ \text{(cap.)}$$

$$Y_{23}=\sqrt{g_{23}^2+b_{23}^2}=\sqrt{0.08^2+0.06^2}=0.1\ S;\quad Z_{23}=\frac{1}{Y_{23}}=10\ Ohm;$$

$$r_{23}=\frac{g_{23}}{Y_{23}^2}=\frac{0.08}{0.1^2}=8\ Ohm;\qquad\qquad x_{23}=\frac{b_{23}}{Y_{23}^2}=\frac{0.06}{0.1^2}=6\ Ohm\ \text{(cap.)}$$

2. Input circuit impedance and its power factor are:

$$Z=\sqrt{(r_1+r_{23})^2+(x_1-x_{23})^2}=\sqrt{(16+8)^2+(24-6)^2}=30\ Ohm;$$

$$cos\varphi=\frac{r_1+r_{23}}{Z}=\frac{24}{30}=0.8.$$

The effective current value through the common circuit portion (the first ammeter's reading): $I_1=\frac{U}{Z}=\frac{300}{30}=10\ A.$

The voltage across the terminals of the parallel branches is

$$U_{23}=Z_{23}\cdot I_1=10\cdot 10=100\ V.$$

The readings of the second and the third ammeters are

$$I_2=\frac{U_{23}}{r_2}=\frac{100}{16.67}=6\ A; \qquad I_3=\frac{U_{23}}{Z_3}=\frac{100}{5\sqrt{10}}=2\sqrt{10}\ A.$$

3. Wattmeter is to measure the circuit active power. Its reading is

$$P=U\cdot I_1\cdot cos\varphi=300\cdot 10\cdot 0.8=2400\ W.$$

Resistors active power

$$P_R=r_1\cdot I_1^2+r_2\cdot I_2^2+r_3\cdot I_3^2=16\cdot 10^2+16.67\cdot 6^2+5\cdot\left(2\sqrt{10}\right)^2=2400\ W$$

is equal to the source active power, it means the active power balance is true.

3-11 (3.22). In scheme fig. 3.13,a it is known: $u(t)=100\sqrt{2}\cdot sin(\omega t+30°)\ V$;
$r_1=5\ Ohm$; $x_{C1}=8\ Ohm$; $r_2=3\ Ohm$; $x_{C2}=10\ Ohm$; $x_L=4\ Ohm$.

Determine the currents as well as the circuit power factor, construct the circuit complete phasor diagram. Problem is to be solved by the method of proportional quantities. In addition, you should answer the questions: what should be the meaning of x_{C2} to reach the current resonance? What should be the meaning of x_{C1} to reach the voltage resonance?

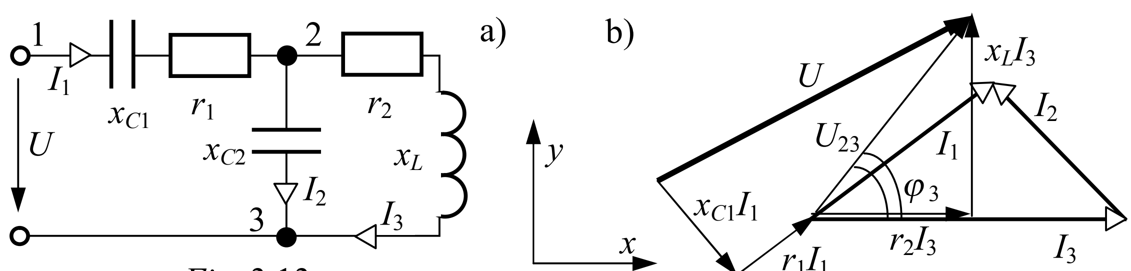


Fig. 3.13

Solution. 1. Let's sketch a phasor diagram (fig. 3.13,b). It is necessary to start the construction with the circuit portion which is the furthest from the source (that is the third branch). As there is a series impedance connection, initially we draw the current phasor I_3. Then the diagram is constructed in the backward direction: from the scheme end to the source bearing in mind Kirchhoff's laws and rules of the diagram construction.

2. The calculations are performed in the same order as the phasor diagram is constructed.

Let $I_3=1\ A$, i.e. $i_3(t)=\sqrt{2}\cdot sin(\omega t)\ A$. Then

$$U_{r2}=r_2\cdot I_3=3\ V, \quad U_{xL}=x_L\cdot I_3=4\ V, \quad I_2=\frac{U_{23}}{x_{C2}}=\frac{5}{10}=0.5\ A.$$

The current phasor projections onto axes x and y are as follows:

$I_{3x} = I_3 = 1\ A,\quad I_{3y} = 0;\quad \varphi_3 = arctg\frac{x_L}{r_2} = arctg\frac{4}{3} = 53.1°,$

$I_{2x} = I_2 \cdot cos(\varphi_3 + 90°) = 0.5\cdot(-0.8) = -0.4\ A,\quad I_{2y} = I_2\cdot sin(\varphi_3 + 90°) = 0.5\cdot 0.6 = 0.3\ A.$

Determine the first current by its projections:

$I_{1x} = I_{2x} + I_{3x} = -0.4 + 1 = 0.6\ A,\qquad I_{1y} = I_{2y} + I_{3y} = 0.3\ A,$

$I_1 = \sqrt{I_{1x}^2 + I_{1y}^2} = \sqrt{0.6^2 + 0.3^2} = 0.671\ A.$

The first current phase is $\psi_{i1} = arctg\frac{I_{1y}}{I_{1x}} = arctg\frac{0.3}{0.6} = 26.6°.$

Determine the calculated value of the input voltage by its projections:

$U_{xC1} = x_{C1}\cdot I_1 = 8\cdot 0.671 = 5.36\ V;\quad U_{r1} = r_1\cdot I_1 = 5\cdot 0.671 = 3.35\ V,$

$U_x = U_{xC1}\cdot cos(90° - \psi_{i1}) + U_{r1}\cdot cos(\psi_{i1}) + U_{r2} =$

$= 5.36\cdot cos(90° - 26.6°) + 3.35\cdot cos(26.6°) + 3 = 8.40\ V,$

$U_y = U_{xC1}\cdot sin(\psi_{i1} - 90°) + U_{r1}\cdot sin(\psi_{i1}) + U_{xL} =$

$= 5.36\cdot sin(26.6° - 90°) + 3.35\cdot sin(26.6°) + 4 = 0.707\ V,$

$U_{calc} = \sqrt{U_x^2 + U_y^2} = \sqrt{8.40^2 + 0.707^2} = 8.43\ V,$

$\psi_{Ucalc} = arctg\frac{U_y}{U_x} = arctg\frac{0.707}{8.43} = 4.8°.$

3. The recalculation coefficients are:

$k = \frac{U}{U_{calc}} = \frac{100}{8.43} = 11.86;\quad \Delta\psi = \psi_U - \psi_{Ucalc} = 30° - 4.8° = 25.2°.$

4. The obtained answers are
$i_1(t) = 7.96\sqrt{2}\cdot sin(\omega t + 51.8°)\ A;$
$i_2(t) = 5.93\sqrt{2}\cdot sin(\omega t + 168.4°)\ A;$
$i_3(t) = 11.86\sqrt{2}\cdot sin(\omega t + 25.2°)\ A.$

5. The current resonance condition is: $b_2 = b_3.$

$b_2 = \frac{1}{x_{C2}};\quad b_3 = \frac{x_L}{r_2^2 + x_L^2} = \frac{4}{25}\ S;\quad x_{C2} = \frac{1}{b_2} = \frac{1}{b_3} = \frac{25}{4} = 6.25\ Ohm.$

6. The voltage resonance condition is $x_{C1} = x_{23},$

however $Z_{23} = \frac{U_{23}}{I_1} = \frac{5}{0.671} = 7.46\ Ohm,$

$x_{23} = Z_{23}\cdot sin(\varphi_3 - \psi_{i1}) = 7.46\cdot sin(53{,}1° - 26.6°) = 3.33\ Ohm.$

Thus, the voltage resonance is observed at condition of $x_{C1} = 3.33\ Ohm.$

3-12 (3.24). Determine the instruments' reading by method of phasor diagrams in scheme fig. 3.14, if:

$U = 200\ V;\ x_1 = 8\ Ohm;\ x_2 = 10\ Ohm;$
$r_3 = 5\ Ohm;\ x_3 = 15\ Ohm;\ r_1 = 6\ Ohm.$

Construct a circuit phasor diagram.

Answers: $A_1 \to 10\ A;\ V \to 224\ V;$
$A_2 \to 22{,}4\ A;\ A_3 \to 14{,}14\ A;\ W \to 1000\ W.$

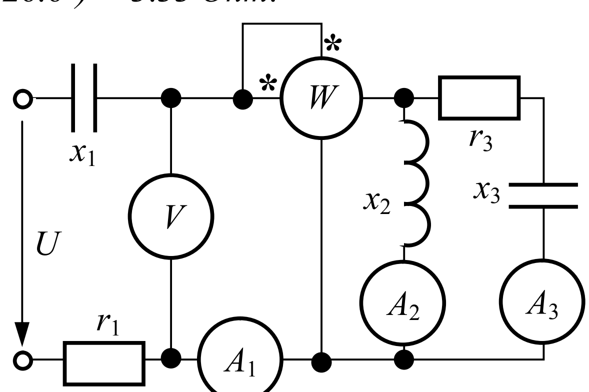


Fig. 3.14

3.3. COMPLEX-NOTATION (SYMBOLIC) METHOD

3-13 (3.29). Solve the problem 3.2 by complex-notation method.

Solution. Initial data are presented by complex numbers:

the complex circuit voltage amplitude is $\underline{U}_m = U_m \cdot e^{j\psi u} = 200 \cdot e^{-j20°}$ *V*;

the complex circuit *r-L* impedance is

$$\underline{Z} = r + j\omega L = 35 + j2\pi \cdot 50 \cdot 80 \cdot 10^{-3} = 35 + j25.12 = 43.1 \cdot e^{j35.67°} \ Ohm.$$

Calculate the complex current amplitude by Ohm's law

$$\underline{I}_m = \frac{\underline{U}_m}{\underline{Z}} = \frac{200 \cdot e^{-j20}}{43.1 \cdot e^{j35.67}} = 4.64 \cdot e^{-j55.67°} \ A.$$

The current instantaneous value is

$$i(t) = \text{Im}\left[\underline{I}_m \cdot e^{j\omega t}\right] = \text{Im}\left[4.64 \cdot e^{-j55.67} \cdot e^{j\omega t}\right] =$$

$$= \text{Im}\left[4.64 \cdot e^{j(\omega t - 55.67)}\right] = 4.64 \cdot sin(\omega t - 55.67°) \ A.$$

The complex power at the circuit input is

$$\underline{S} = \frac{\underline{U}_m \cdot \underline{I}_m^*}{2} = \frac{200 \cdot e^{-j20} \cdot 4{,}64 \cdot e^{j55{,}67}}{2} = 464 \cdot e^{j35.67°} = 377 + j270.5 \ \ BA = P + jQ.$$

3-14 (3.30). In scheme fig. 3.15, calculate the effective and instantaneous values of the voltage across the capacitor, if

$$U = 380 \ V; \quad r = 1 \ kOhm; \quad C = 2 \ \mu F.$$

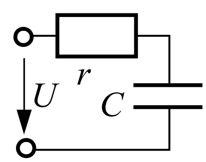


Fig. 3.15

Solution. Let's combine the input voltage phasor with the real axis. Then

$$\underline{U} = U = 380 \ V.$$

The complex circuit impedance is

$$\underline{Z} = r - jx_C = r - j\frac{1}{\omega C} = 1000 - j\frac{1}{314 \cdot 2 \cdot 10^{-6}} = 1000 - j1592 = 1880 \cdot e^{-j57.87°} \ Ohm.$$

Under Ohm's law, we determine the circuit current complex

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{380}{1880 \cdot e^{-j57.87}} = 0.202 \cdot e^{j57.87°} \ A.$$

Complex voltage across the capacitance is

$$\underline{U}_C = \underline{I} \cdot (-jx_C) = 0.202 \cdot e^{j57.87°} \cdot (-j1592) = 321.8 \cdot e^{-j32.13°} \ V.$$

The instantaneous value of the voltage across the capacitance is

$$u_C(t) = \text{Im}\left[\sqrt{2}\underline{U}_C \cdot e^{j\omega t}\right] = 455.1 \cdot sin(\omega t - 32.13°) \ V.$$

3-15 (3.31). In scheme fig. 3.16, determine the ammeter reading, if

$$u(t) = 300 \cdot sin(\omega t - 32.13°) \ V; \quad r_1 = 12 \ Ohm; \quad r_2 = x_{L1} = 16 \ Ohm; \quad x_{L2} = 20 \ Ohm;$$
$$x_{C2} = 32 \ Ohm; \quad r_3 = x_{L3} = 100 \ Ohm; \quad x_{C1} = 12.5 \ Ohm.$$

Solution. Let's determine the complex impedances of the parallel branches:

$\underline{Z}_1 = r_1 + jx_{L1} = 12 + j16$ *Ohm*; $\quad \underline{Z}_2 = r_2 + j(x_{L2} - x_{C2}) = 16 - j12$ *Ohm*;

$\underline{Z}_3 = r_3 = 100$ *Ohm*; $\quad \underline{Z}_4 = -jx_{C1} = -j12.5$ *Ohm*; $\quad \underline{Z}_5 = jx_{L3} = j100$ *Ohm*.

The complex circuit admittance is

$$\underline{Y} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} + \frac{1}{\underline{Z}_4} = \frac{1}{12 + j16} + \frac{1}{16 - j12} + \frac{1}{100} + \frac{1}{j12{,}5} + \frac{1}{j100} =$$

$= 0.08 + j0.06 = 0.1 \cdot e^{j36.9°}\ S.$

The complex input voltage corresponding to its sinusoid is as follows $\underline{U} = \frac{300}{\sqrt{2}} \cdot e^{-j30°}\ V.$

We determine the current through the common circuit part by Ohm's law

$\underline{I} = \underline{Y} \cdot \underline{U} = 0.1 \cdot e^{j36.9°} \cdot \frac{300}{\sqrt{2}} \cdot e^{-j30°} = 21.21 \cdot e^{j6.9°}\ A.$

Finally, ammeter reads 21.21 *A*.

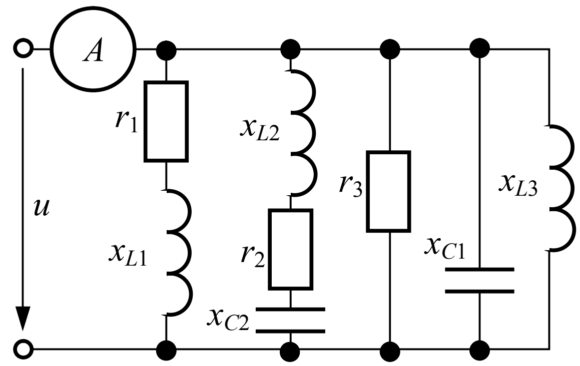


Fig. 3.16

3-16 (3.32). In condition of the problem 3.12, determine the instrument reading with the aid of the complex-notation method.

Solution. Let's combine an input voltage phasor with the real axis, it means

$$\underline{U} = U = 200\ V.$$

Determine the complex branch impedances

$\underline{Z}_1 = r_1 - jx_1 = 6 - j8\ Ohm;\quad \underline{Z}_2 = jx_2 = j10\ Ohm;\quad \underline{Z}_3 = r_3 - jx_3 = 5 - j15\ Ohm.$

The complex circuit impedance is as follows

$$\underline{Z} = \underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = 6 - j8 + \frac{j10 \cdot (5 - j15)}{j10 + 5 - j15} = 16 + j12\ Ом.$$

The source current is $\underline{I}_1 = \frac{\underline{U}}{\underline{Z}} = \frac{200}{16 + j12} = 8 - j6 = 10 \cdot e^{-j36.9°}\ A.$

The currents through the parallel branches are

$$\underline{I}_2 = \underline{I}_1 \cdot \frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = 10 \cdot e^{-j36.9°} \cdot \frac{5 - j15}{5 - j5} = 10 - j20 = 22.36 \cdot e^{-j63.4°}\ A;$$

$$\underline{I}_3 = \underline{I}_1 \cdot \frac{\underline{Z}_2}{\underline{Z}_2 + \underline{Z}_3} = 10 \cdot e^{-j36.9°} \cdot \frac{j10}{5 - j5} = -2 + j14 = 14.14 \cdot e^{j98.1°}\ A.$$

The voltage across the parallel branches is measured by a voltmeter and is supplied to a wattmeter, it being

$$\underline{U}_W = \underline{I}_2 \cdot \underline{Z}_2 = 22.36 \cdot e^{-j63.4°} \cdot j10 = 223.6 \cdot e^{j26.6°} = 200 + j100\ V.$$

The wattmeter reading is

$$P_W = \mathrm{Re}[\underline{U}_W \cdot \underline{I}_1^*] = \mathrm{Re}[223{,}6 \cdot e^{j26.6°} \cdot 10 \cdot e^{j36.9°}] = 1000\ W.$$

Hence, the instrument readings are:

$A_1 \to 10\ A;\quad A_2 \to 22.36\ A;\quad A_3 \to 14.14\ A;\quad V \to 223.6\ V;\quad W \to 1000\ W.$

3-17 (3.33). In scheme fig. 3.17,a it is known: $E_1 = E_2 = 100\ V$, furthermore, E_2 leads E_1 by 90°; $J = 5\ A$, furthermore, the current of this source is in opposite phase to E_2; $r = x_C = 10\ Ohm$; $x_L = 20\ Ohm$.

It is necessary to determine the currents in all the branches as well as the wattmeter reading, work out the reactive power balance and construct the topographic diagram for loop 1-2-3-1.

Solution. Let's combine the phasor E_1 with the real axis, then the source complexes are as follows:

$$\underline{E}_1 = 100\ V;\quad \underline{E}_2 = 100 \cdot e^{j90°} = j100\ V;\quad \underline{J} = 5 \cdot e^{-j90°} = -j5\ A.$$

Since there are two nodes in the scheme, then a rational method to determine the currents is method of two nodes: $\underline{U}_{12} = \dfrac{\underline{E}_1\underline{Y}_1 - \underline{E}_2\underline{Y}_2 - \underline{J}}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}$,

where the branch complex admittances are $\underline{Y}_1 = \dfrac{1}{r} = \dfrac{1}{10} = 0.1\ S$;

$$\underline{Y}_2 = \frac{1}{jx_L - jx_C} = \frac{1}{j20 - j10} = \frac{1}{j10} = -j0.1\ S; \qquad \underline{Y}_3 = \frac{1}{-jx_C} = \frac{1}{-j10} = j0.1\ S.$$

Then $\underline{U}_{12} = \dfrac{100 \cdot 0{,}1 - j100 \cdot (-j0{,}1) + j5}{0{,}1 - j0{,}1 + j0{,}1} = j50\ V.$

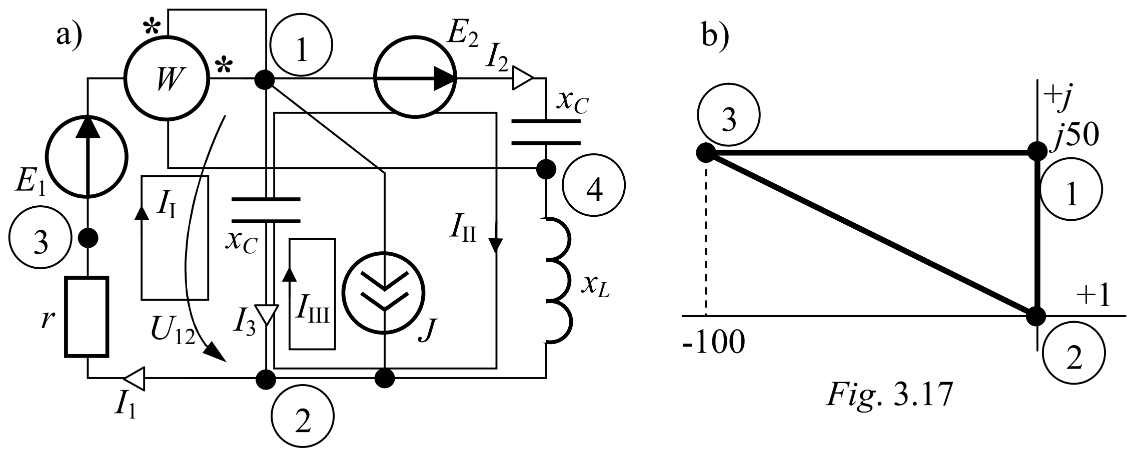


Fig. 3.17

Determine the currents by Ohm's law: $\underline{I}_1 = \dfrac{\underline{E}_1 - \underline{U}_{12}}{r} = \dfrac{100 - j50}{10} = 10 - j5\ A$;

$\underline{I}_2 = (\underline{E}_2 + \underline{U}_{12})\cdot\underline{Y}_2 = (j100 + j50)\cdot(-j0.1) = 15\ A$;

$\underline{I}_3 = \underline{U}_{12}\cdot\underline{Y}_3 = j50\cdot j0.1 = -5\ A$.

One may verify the correctness of the current determination by Kirchhoff's current law for node 1: $-\underline{I}_1 + \underline{I}_2 + \underline{I}_3 + \underline{J} = 0$ or $-10 + j5 + 15 - 5 - j5 = 0$.

The wattmeter reading is: $P_W = \mathrm{Re}[\underline{U}_{14}\cdot(-\underline{I}_1^*)]$.

Under Kirchhoff's voltage law we find

$$\underline{U}_{14} = -\underline{E}_2 + \underline{I}_2\cdot(-jx_C) = -j100 + 15\cdot(-j10) = -j250\ V.$$

Then $P_W = \mathrm{Re}[-j250\cdot(-10 - j5)] = -1250\ W$.

The reactive power balance:

the source reactive power is

$Q_G = \mathrm{Im}[\underline{E}_1\cdot\underline{I}_1^* + \underline{E}_2\cdot\underline{I}_2^* + (-\underline{U}_{12})\cdot\underline{J}^*] = \mathrm{Im}[100\cdot(10 + j5) + j100\cdot 15 - j15\cdot j5] = 2000\ VAr$;

the consumer reactive power is

$Q_C = I_2^2\cdot(x_L - x_C) + I_3^2\cdot(-x_C) = 15^2\cdot 10 - 5^2\cdot 10 = 2000\ VAr.$

So, the reactive power balance $Q_G = Q_C$ is true.

While drawing the topographic diagram, assume $\underline{\varphi}_2 = 0$, then

$\underline{\varphi}_1 = \underline{I}_3\cdot(-jx_C) = \underline{U}_{12} = j50\ V$; $\quad \underline{\varphi}_3 = \underline{\varphi}_1 - \underline{E}_1 = -100 + j50\ V$; $\quad \underline{\varphi}_2 = \underline{\varphi}_3 + \underline{I}_1\cdot r = 0.$

In fig. 3.17,b there is the required diagram.

Note, the current calculation may be fulfilled by the mesh current method having solved the following equation system

$$\begin{cases} \underline{I}_{\mathrm{I}}\cdot(r - jx_C) - \underline{I}_{\mathrm{II}}\cdot(-jx_C) - \underline{J}\cdot(-jx_C) = \underline{E}_1; \\ -\underline{I}_{\mathrm{I}}\cdot(-jx_C) + \underline{I}_{\mathrm{II}}\cdot(jx_L - 2jx_C) + \underline{J}\cdot(-jx_C) = \underline{E}_2. \end{cases}$$

3-18 (3.34). In scheme fig. 3.18 it is known: $\underline{E}_1 = 380\ V$, $\underline{E}_2 = j100\ V$, $\underline{J} = 10\cdot e^{j45°}\ A$, $r_1 = 10\ Ohm$, $x_1 = 14.29\ Ohm$, $r_3 = 6\ Ohm$, $x_2 = 3.3\ Ohm$.

Calculate the currents by the method of two nodes, make the checking calculation by the mesh current method, work out the power balance, construct the circuit topographic diagram combined with the current phasor diagram.

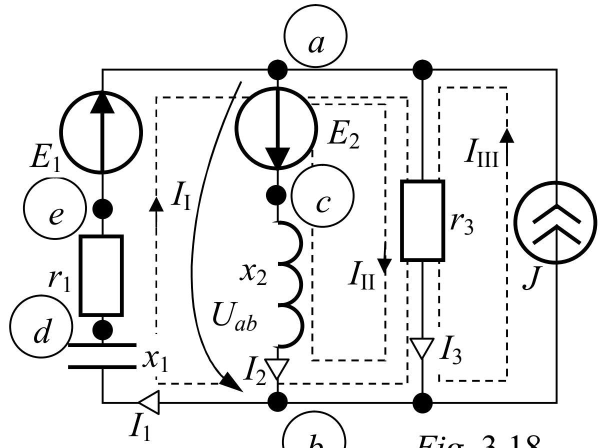


Fig. 3.18

Solution. 1. The complex branch impedances are:

$\underline{Z}_1 = r_1 - jx_1 = 10 - j14.29\ Ohm$;

$\underline{Z}_2 = jx_2 = j3.3\ Ohm$; $\underline{Z}_3 = r_3 = 6\ Ohm$.

2. The current calculation by the method of two nodes is:

$$\underline{U}_{ab} = \frac{\underline{E}_1\underline{Z}_1^{-1} - \underline{E}_2\underline{Z}_2^{-1} + \underline{J}}{\underline{Z}_1^{-1} + \underline{Z}_2^{-1} + \underline{Z}_3^{-1}} =$$

$$= \frac{380\cdot(10 - j14.29)^{-1} - j100\cdot(j3.3)^{-1} + 7.07 + j7.07}{(10 - j14.29)^{-1} + (j3.3)^{-1} + 6^{-1}} =$$

$$= 83.60\cdot e^{j165.4°} = -80.89 + j21.09\ V.$$

$$\underline{I}_1 = \frac{\underline{E}_1 - \underline{U}_{ab}}{\underline{Z}_1} = \frac{380 + 80.89 - j21.09}{10 - j14.29} = 26.45\cdot e^{j52.4°} = 16.14 + j20.96\ A;$$

$$\underline{I}_2 = \frac{\underline{E}_2 + \underline{U}_{ab}}{\underline{Z}_2} = \frac{j100 - 80.89 + j21.09}{j3.3} = 44.13\cdot e^{j33.7°} = 36.70 + j24.51\ A;$$

$$\underline{I}_3 = \frac{\underline{U}_{ab}}{\underline{Z}_3} = \frac{-80.89 + j21.09}{6} = 13.93\cdot e^{j165.4°} = -13.48 + j3.52\ A.$$

3. The equation system for the loop currents is:

$$\begin{cases}(\underline{Z}_1 + \underline{Z}_3)\cdot\underline{I}_{I} + \underline{Z}_3\cdot\underline{I}_{II} = \underline{E}_1 - \underline{Z}_3\cdot\underline{J};\\ \underline{Z}_3\cdot\underline{I}_{I} + (\underline{Z}_2 + \underline{Z}_3)\cdot\underline{I}_{II} = -\underline{E}_2 - \underline{Z}_3\cdot\underline{J}.\end{cases} \quad \begin{cases}(10 - j14.29+6)\cdot\underline{I}_{I} + 6\cdot\underline{I}_{II} = 380 - 6\cdot(7.07 + j7.07);\\ 6\cdot\underline{I}_{I} + (j3.3 + 6)\cdot\underline{I}_{II} = -j100 - 6\cdot(7.07 + j7.07).\end{cases}$$

The solution of the system is: $\underline{I}_{I} = 26.45\cdot e^{j52.4°}\ A$; $\underline{I}_{II} = -44.13\cdot e^{j33.8°}\ A$.

The branch currents determined through the loop currents are as follows:

$\underline{I}_1 = \underline{I}_{I} = 26.45\cdot e^{j52.4°}\ A$; $\underline{I}_2 = -\underline{I}_{II} = 44.13\cdot e^{j33.7°}\ A$;

$\underline{I}_3 = \underline{I}_{I} + \underline{I}_{II} + \underline{J} = 16.14 + j20.96 - 36.70 - j24.51 + 7.07 + j7.07 = -13.49 + j3.52\ A$.

4. The complex source power is:

$$\underline{S}_G = \underline{E}_1\cdot \overset{*}{I}_1 + \underline{E}_2\cdot \overset{*}{I}_2 + \underline{U}_{ab}\cdot \overset{*}{J} =$$

$$= 380\cdot(16.14 + j20.96) + j100\cdot(36.70 + j24.51) + (-80.89 + j21.09)\cdot(7.07 - j7.07) =$$

$$= 8162 - j3573\ VA.$$

The active and reactive power of consumers are:

$P_C = I_1^2\cdot r_1 + I_3^2\cdot r_3 = 26.45^2\cdot 10 + 13.93^2\cdot 6 = 8160\ W$;

$Q_C = I_1^2\cdot(-x_1) + I_2^2\cdot x_2 = -26.45^2\cdot 14.29 + 44.13^2\cdot 3.3 = -3571\ VAr$.

Since $P_C \approx \mathrm{Re}(\underline{S}_G)$ and $Q_C \approx \mathrm{Im}(\underline{S}_G)$, it means the power balance is true.

5. Let's calculate the potential complexes of the different circuit points. Assume $\underline{\varphi}_b = 0$, then $\underline{\varphi}_a = \underline{U}_{ab} = -80.89 + j21.09\ V$.

The other potentials are
$\underline{\varphi}_c = \underline{\varphi}_a + \underline{E}_2 = -80.82 + j121{,}09$ V;
$\underline{\varphi}_e = \underline{\varphi}_a - \underline{E}_1 = -460.89 + j21.09$ V;
$\underline{\varphi}_d = -\underline{I}_1 \cdot (-jx_1) = 378.0 \cdot e^{j142.4°}$ V.

One may see the diagram in fig. 3.19.

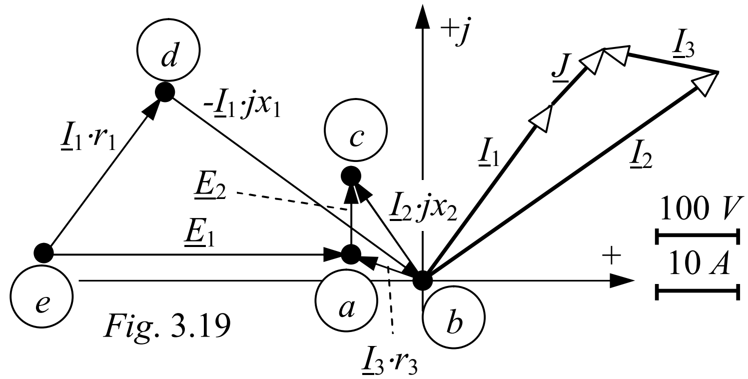


Fig. 3.19

3.4. CIRCUITS WITH MUTUAL INDUCTANCE

3-19 (3.47). Mark similar terminals in scheme fig. 3.20,a.

Solution. Let's mark the upper terminal of the first coil possessing w_1 number of turns by the sign *, the bottom terminal being left without a sign. Further, we choose the arbitrary direction of the first coil current i_1 (in the figure it is directed inside the terminal with sign *).

With the aid of the right hand rule (or the right-hand screw rule), we find the direction of the magnetic flux Φ_{11} of self-induction of the first coil. At the same time we show the direction of the magnetic flux of mutual inductance Φ_{12} (subscript «1» – field by the current i_1, subscript «2» – action upon the second coil).

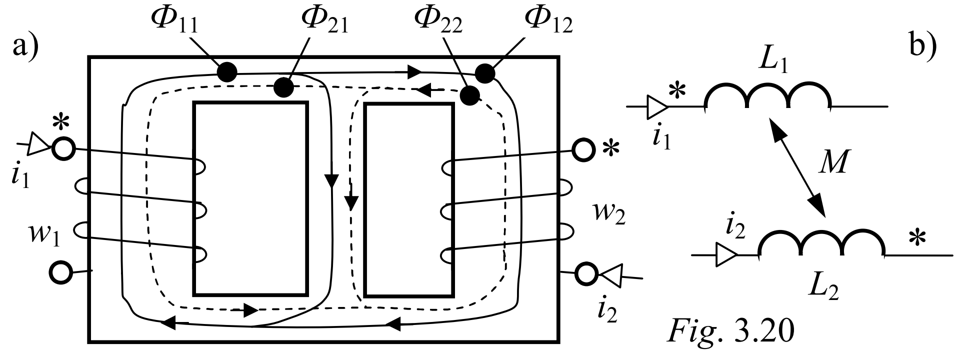


Fig. 3.20

In order to find the similar terminal (*) of the second coil, we choose the arbitrary direction of the second coil current. For instance, the current i_2 enters into the bottom terminal, then we find the directions of fluxes Φ_{22} and Φ_{21}.

In scheme fig. 3.20,a it happens the magnetic fluxes of self-induction and mutual induction are subtracted (flow in opposite direction) in each coil. So, the currents i_1 and i_2 are directed differently as regards the similar terminals, the terminal (*) of the second coil being situated overhead.

Note, the position of similar terminals does not depend on the chosen directions of the currents while marking, furthermore, their position is specified only by the magnetic core construction and the winding path.

At further investigation of the scheme including windings w_1 and w_2, the scheme fig. 3.20,a is replaced by the scheme fig. 3.20,b.

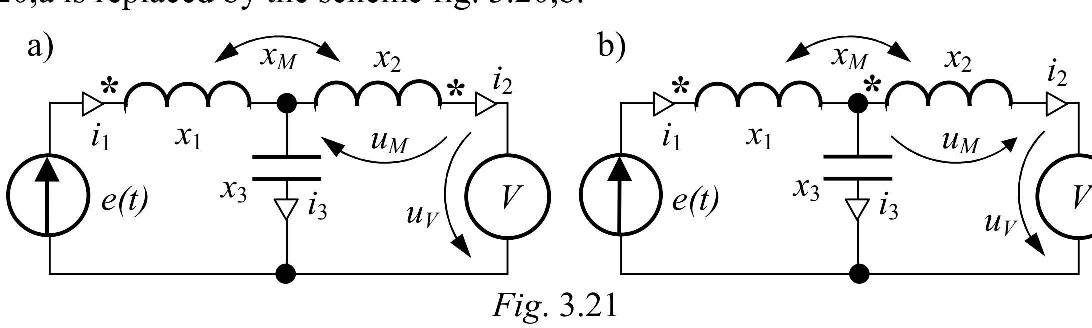


Fig. 3.21

3-20 (3.48). In fig. 3.21,a there are inductively coupled coils which are connected in a node with similar terminals, while in fig. 3.21,b the same coils are connected in a node with unsimilar terminals. Determine the voltmeter reading in both cases, if: $x_1 = 20$ *Ohm*, $x_2 = 10$ *Ohm*, the coil coupling coefficient is $k = 0.5$, capacitive reactance is $x_3 = 10$ *Ohm*, emf $e(t) = 100\sqrt{2}\,sin(\omega t)\ V$.

Solution. Perform the calculations in complex form. The source complex emf in both schemes is $\underline{E} = E\cdot e^{j\psi e} = 100\ V$.

The voltmeter impedance is infinite, that's why the voltmeter current in both schemes is $i_2 = 0$, while the current complexes are

$$\underline{I}_1 = \underline{I}_3 = \frac{\underline{E}}{jx_1 - jx_3} = \frac{100}{j20 - j10} = -j10\ A.$$

Voltage $\underline{U}_M = \underline{I}_1\cdot jx_M$, induced in the second coil by the alternating current $\underline{I}_1$, has the same direction as the current $\underline{I}_1$ as regard to the similar terminals, which is reflected in both schemes fig. 3.21,a and 3.21,b. The mutual inductive reactance is as follows

$$x_M = k\cdot\sqrt{x_1 x_2} = 0.5\sqrt{20\cdot 10} = 5\sqrt{2}\ Ohm.$$

One can calculate a voltage across the voltmeter terminals with the aid of Kirchhoff's voltage law. In scheme fig. 3.21,a $\underline{U}_V - \underline{I}_3\cdot(-jx_3) - \underline{U}_M = 0$, from here

$$\underline{U}_V = (-j10)\cdot(-j10) + (-j10)\cdot j5\sqrt{2} = -100 + 10\cdot 7.07 = -29.3\ V,$$

while the voltmeter reading is $U_V = 29.3\ V$.

In the scheme fig. 3.21,b $\underline{U}_V - \underline{I}_3\cdot(-jx_3) + \underline{U}_M = 0$, from here

$$\underline{U}_V = (-j10)\cdot(-j10) - (-j10)\cdot j5\sqrt{2} = -100 - 10\cdot 7.07 = -170.7\ V,$$

while the voltmeter reading is $U_V = 170.7\ V$.

Note, this problem illustrates the practical way to clue the similar terminals of the transformer windings. It is sufficient just to compare but the voltmeter reading for two cases of the transformer secondary winding connection and to conclude.

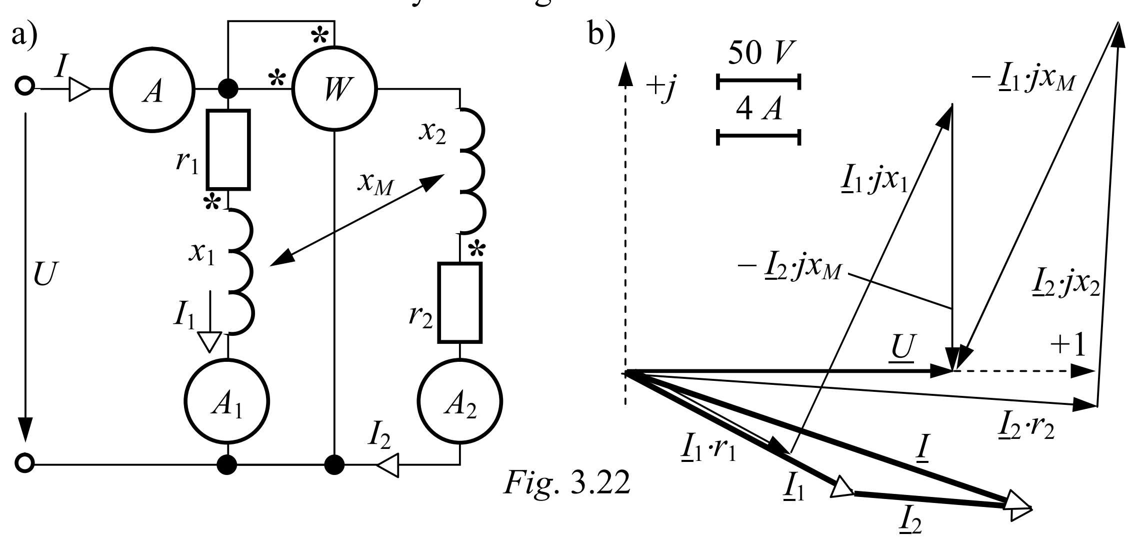


Fig. 3.22

3-21 (3.49). The scheme parameters fig. 3.22,a are $r_1 = 10$ *Ohm*, $x_1 = 20$ *Ohm*, $r_2 = 40$ *Ohm*, $x_2 = 30$ *Ohm*, $x_M = 20$ *Ohm*, the supply voltage is $U = 220\ V$. Find the instrument readings. Construct the circuit phasor diagram. Write down a power balance equation. Determine the active power transmitted through the magnetic field from one branch into another.

Solution. Assume $\underline{U} = 220\ V$. The complex branch impedances are

$\underline{Z}_1 = r_1 + jx_1 = 10 + j20\ Ohm$, $\underline{Z}_2 = r_2 + jx_2 = 40 + j30\ Ohm$, $\underline{Z}_M = jx_M = j20\ Ohm$.

Let's employ the Kirchhoff's equation system to calculate the currents:

$\underline{I} = \underline{I}_1 + \underline{I}_2$; $\underline{I}_1\cdot\underline{Z}_1 - \underline{I}_2\cdot\underline{Z}_M = \underline{U}$; $\underline{I}_2\cdot\underline{Z}_2 - \underline{I}_1\cdot\underline{Z}_M = \underline{U}$.

Solving the equation system of two latter equations, we obtain:

$$\underline{I}_1 = \underline{U}\cdot\frac{\underline{Z}_2 + \underline{Z}_M}{\underline{Z}_1\underline{Z}_2 - \underline{Z}_M^2} = \frac{220\cdot(40 + j30 + j20)}{(10 + j20)(40 + j30) - (j20)^2} = 12.6\cdot e^{-j28.36°} = 11.09 - j5.99\ A;$$

$$\underline{I}_2 = \underline{U}\cdot\frac{\underline{Z}_1 + \underline{Z}_M}{\underline{Z}_1\underline{Z}_2 - \underline{Z}_M^2} = \frac{220\cdot(10 + j20 + j20)}{(10 + j20)(40 + j30) - (j20)^2} = 8.11\cdot e^{-j3.74°} = 8.09 - j0.53\ A;$$

$$\underline{I} = 11.09 - j5.99 + 8.09 - j0.53 = 19.18 - j6.52 = 20.26\cdot e^{-j18.77°}\ A.$$

Verify the power balance. The source power is

$$\underline{S}_G = P_G + jQ_G = \underline{U}\cdot\underline{I}^* = 220\cdot(19.18 + j6.52) = 4220 + j1434\ VA.$$

Total active power of the consumers is

$$\Sigma P_C = I_1^2\cdot r_1 + I_2^2\cdot r_2 = 12.6^2\cdot 10 + 8.11^2\cdot 40 = 4219\ W,$$

total reactive power is

$$\Sigma Q_C = I_1^2\cdot x_1 + I_2^2\cdot x_2 - 2\cdot I_1\cdot I_2\cdot x_M\cdot cos(\psi_{i1} - \psi_{i2}) =$$
$$= 12.6^2\cdot 20 + 8.11^2\cdot 30 - 2\cdot 12.6\cdot 8.11\cdot 20\cdot cos(-28.36° + 3.74°) = 1433\ VAr.$$

Since both power balances are true, the problem is solved correctly.

Note, the component of the reactive power $-2\cdot I_1\cdot I_2\cdot x_M\cdot cos(\psi_{i1} - \psi_{i2}) = -3716\ VAr$ denotes the reactive power demand to create the magnetic flux necessary for the active power transmission, while a sign «minus» of this power points to the mutual weakening (demagnetization) of the self-induction fluxes by the mutual induction fluxes (see fig. 3.20).

The wattmeter reading is $P_W = \mathrm{Re}(\underline{U}\cdot\underline{I}_2^*) = \mathrm{Re}[220\cdot(8.09 + j0.53)] = 1780\ W$.

The wattmeter measures the active power delivered to the second branch by the «electric manner»: $P_{2inp} = \mathrm{Re}(\underline{U}\cdot\underline{I}_2^*) = 1780\ W$.

The active power consumed by the second branch is determined in accordance with Joule's law $P_{2C} = I_2^2\cdot r_2 = 8.11^2\cdot 40 = 2631\ W$.

Difference of these powers is delivered into the first branch through the magnetic field $P_{2\to1} = P_{2inp} - P_{2C} = 1780 - 2631 = -851\ W$.

A sign «minus» in the result obtained indicates that in reality there is an active power transmission from the first branch into the second one ($P_{1\to2} = +851\ W$) to cover the power shortage (consumption greater than delivering).

Note, there is another way to calculate this active power transmitted:

$$P_{1\to2} = -I_1\cdot I_2\cdot x_M\cdot sin(\psi_{i1} - \psi_{i2}) = -12.6\cdot 8.11\cdot 20\cdot sin(-28.36° + 3.74°) = +851\ W.$$

To construct a phasor diagram, initially we draw the current phasors (complexes) $\underline{I}_1$ and $\underline{I}_2$ (fig. 3.22,b). Then the voltage drops are calculated

$I_1\cdot r_1 = 12.6\cdot 10 = 126\ V$, $I_1\cdot x_1 = 12.6\cdot 20 = 252\ V$, $I_2\cdot x_M = 8.11\cdot 20 = 162.2\ V$,

$I_2\cdot r_2 = 8.11\cdot 40 = 324.4\ V$, $I_2\cdot x_2 = 8.11\cdot 30 = 243.3\ V$, $I_1\cdot x_M = 12.6\cdot 20 = 252\ V$.

Starting from the coordinate origin, we perform the operations of the phasor addition in accordance with the initial Kirchhoff's equation system:

$$\begin{cases} \underline{I}_1\cdot r_1 + \underline{I}_1\cdot jx_1 - \underline{I}_2\cdot jx_M = \underline{U}, \\ \underline{I}_2\cdot r_2 + \underline{I}_2\cdot jx_2 - \underline{I}_1\cdot jx_M = \underline{U}. \end{cases}$$

The phasor diagram is presented in fig. 3.22,b.

3-22 (3.50). When switch P is open, the voltmeter reading in scheme fig. 3.23 is 150 V. The scheme parameters are $U = 300\ V$,

$r_1 = 12\ Ohm$, $x_{1L} = 10\ Ohm$,

$x_{1C} = 26\ Ohm$, $r_2 = 4\ Ohm$,

$x_2 = 20\ Ohm$, $r = 12\ Ohm$,

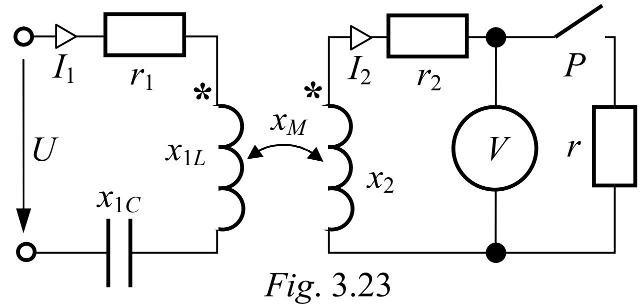


Fig. 3.23

Find the scheme currents when the switch is closed, construct a circuit phasor diagram.

Solution. Determine x_M from no-load condition of the single-phase transformer $I_{2o} = 0$ when the switch is open.

$$\underline{I}_{1o} = \frac{\underline{U}}{r_1 + jx_{1L} - jx_{1C}} = \frac{300}{12 + j(10 - 26)} = 15 \cdot e^{j53.13°}\ A;$$

$U_{2o} = I_{1o} \cdot x_M$ – the voltmeter reading, from here $\quad x_M = \dfrac{U_{2o}}{I_{1o}} = \dfrac{150}{15} = 10\ Ohm.$

Let's check up a relationship resulting from the mutual induction phenomenon $x_M \le \sqrt{x_1 x_2}$: $\ 10 < \sqrt{10 \cdot 20}$ – it's true.

When the switch is closed, then in accordance with Kirchhoff's voltage law for the primary and secondary loops of the transformer, we have

$$\underline{I}_1 \cdot \underline{Z}_1 - \underline{I}_2 \cdot \underline{Z}_M = \underline{U}; \qquad \underline{I}_2 \cdot \underline{Z}_2 - \underline{I}_1 \cdot \underline{Z}_M = 0,$$

where the complex impedance of the primary loop is

$$\underline{Z}_1 = r_1 + jx_{1L} - jx_{1C} = 12 - j16\ Ohm,$$

that of the secondary loop is $\quad \underline{Z}_2 = r_2 + r + jx_2 = 16 + j20\ Ohm,$

mutual impedance is $\quad \underline{Z}_M = jx_M = j10\ Ohm,$

the complex voltage is $\quad \underline{U} = 300\ V.$

Solving the equation system, we find:

$$\underline{I}_1 = \frac{\underline{U} \cdot \underline{Z}_2}{\underline{Z}_1 \underline{Z}_2 - \underline{Z}_M^2} = \frac{300 \cdot (16 + j20)}{(12 - j16)(16 + j20) - (j10)^2} = 12.55 \cdot e^{j52.83°} = 7.582 + j10\ A;$$

$$\underline{I}_2 = \frac{\underline{U} \cdot \underline{Z}_M}{\underline{Z}_1 \underline{Z}_2 - \underline{Z}_M^2} = \frac{300 \cdot j10}{(12 - j16)(16 + j20) - (j10)^2} = 4.9 \cdot e^{j91.5°} = -0.128 + j4.9\ A.$$

Let's check up the power balance on the ground of the current calculations.

$$\underline{S}_G = P_G + jQ_G = \underline{U} \cdot \overset{*}{\underline{I}}_1 = 300 \cdot (7.582 - j10) = 2275 - j3000\ \ VA.$$

Total active power of the consumers is

$$\Sigma P_C = I_1^2 \cdot r_1 + I_2^2 \cdot (r_2 + r) = 12.55^2 \cdot 10 + 4.9^2 \cdot 16 = 2274\ W \approx P_G.$$

Total reactive power of the consumers is

$$\Sigma Q_C = I_1^2 \cdot (x_{1L} - x_{1C}) + I_2^2 \cdot x_2 - 2 \cdot \mathrm{Im}(\underline{I}_2 \cdot jx_M \cdot \overset{*}{\underline{I}}_1) =$$

$$= 12.55^2 \cdot (10 - 26) + 4.9^2 \cdot 20 - 2 \cdot \mathrm{Im}(4.9 \cdot e^{j91.5°} \cdot 10 \cdot e^{j90°} \cdot 12.55 \cdot e^{-j52.83°}) =$$

$$= -2040 - 2 \cdot \mathrm{Im}(615 \cdot e^{j128.67°}) = -2040 - 960 = -3000\ VAr = Q_G.$$

Note, a transformer has no “electric” way to transmit energy from the primary coil into the secondary one, that’s why the active power transmission is executed but through the magnetic field, the following equality being kept

$$P_{1\to2} = -\,\mathrm{Re}(\underline{I}_2 \cdot jx_M \cdot \underline{I}_1^*) = I_2^2 \cdot (r_2 + r).$$

Let’s make sure:

$$P_{1\to2} = -\,\mathrm{Re}(4.9 \cdot e^{j91.5°} \cdot 10 \cdot e^{j90°} \cdot 12.55 \cdot e^{-j52.83°}) = -\,\mathrm{Re}(615 \cdot e^{j128.67°}) = 384.3\ W \approx$$
$$\approx 4.9^2 \cdot 16 = 384.2\ W.$$

The voltmeter reading at the closed switch is

$$U_V = I_2 \cdot r = 4.9 \cdot 12 = 58.8\ V.$$

To construct a phasor diagram, we calculate the voltage drops:

$I_1 \cdot r_1 = 12.55 \cdot 12 = 150.6\ V$; $\quad I_1 \cdot x_{1L} = 12.55 \cdot 10 = 125.5\ V$;

$I_1 \cdot x_{1C} = 12.55 \cdot 26 = 326.3\ V$; $\quad I_2 \cdot x_M = 4.9 \cdot 10 = 49\ V$;

$I_2 \cdot r_2 = 4.9 \cdot 4 = 19.6\ V$; $\quad I_2 \cdot x_2 = 4.9 \cdot 20 = 98\ V$;

$I_2 \cdot r_H = 4.9 \cdot 12 = 58.8\ V$; $\quad I_1 \cdot x_M = 12.55 \cdot 10 = 125.5\ V$.

We start constructing a phasor diagram with the current phasors $\underline{I}_1$ and $\underline{I}_2$ on the complex plane in accordance with the calculated complexes (fig. 3.24).

The voltage drop phasors are performed in accordance with Kirchhoff’s voltage law for loops $\quad \underline{I}_1 \cdot r_1 + \underline{I}_1 \cdot jx_{1L} - \underline{I}_2 \cdot jx_M + \underline{I}_1 \cdot (-jx_{1C}) = \underline{U}$;

$$\underline{I}_2 \cdot r_H + \underline{I}_2 \cdot r_2 + \underline{I}_2 \cdot jx_2 - \underline{I}_1 \cdot jx_M = 0.$$

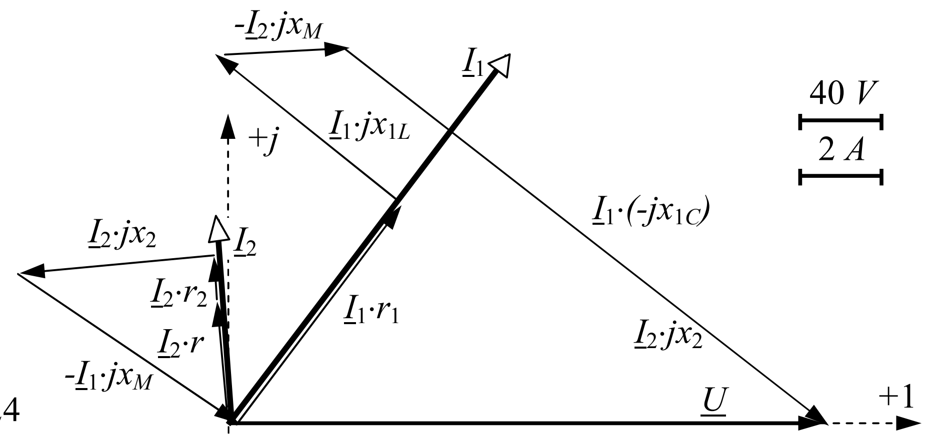


Fig. 3.24

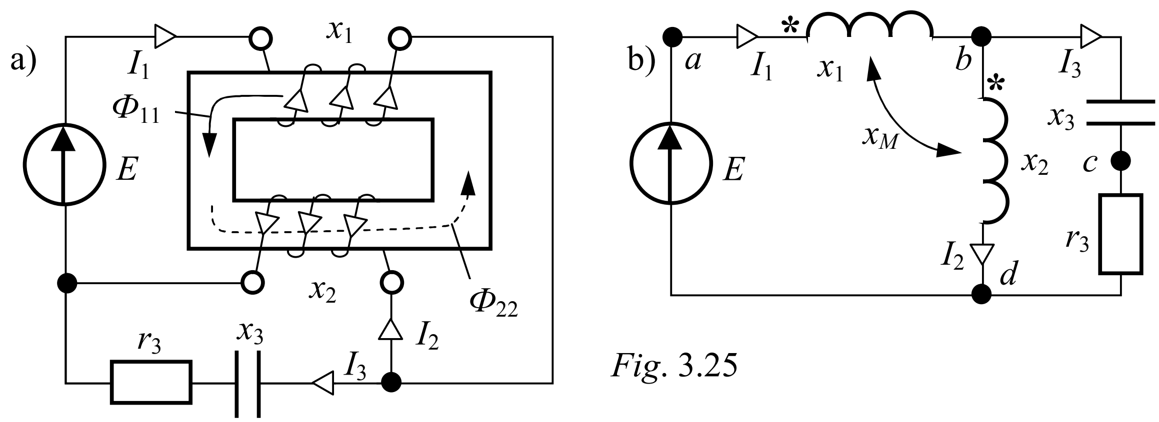


Fig. 3.25

3-23 (3.51). Analyze the state of the circuit with an autotransformer (fig. 3.25,a), find the active power transmitted through the magnetic field, construct a circuit phasor diagram, if $x_1 = 40\ Ohm$, $x_2 = 80\ Ohm$, $x_M = 50\ Ohm$, $r_3 = 40\ Ohm$, $x_3 = 20\ Ohm$, $E = 220\ V$.

Solution. When the similar terminals have been marked with the aid of the magnetic fluxes Φ_{11} and Φ_{22}, the electric circuit scheme for calculation acquires the form in fig. 3.25,b, its state is defined by Kirchhoff's equation system

$$\begin{cases} \underline{I}_1 = \underline{I}_2 + \underline{I}_3; \\ \underline{I}_1 \cdot jx_1 + \underline{I}_2 \cdot jx_M + \underline{I}_3 \cdot (-jx_3) + \underline{I}_3 \cdot r_3 = \underline{E}; \\ \underline{I}_2 \cdot jx_2 + \underline{I}_1 \cdot jx_M - \underline{I}_3 \cdot (-jx_3) - \underline{I}_3 \cdot r_3 = 0. \end{cases}$$

The circuit phasor diagram will be constructed in accordance with this equation system.

To avoid solving of the equation system given above, we eliminate the mutual inductance and in this way we obtain the equivalent scheme fig. 3.26,a and the complex equivalent scheme corresponding to it shown in fig. 3.26,b.

The elimination is carried out in accordance with the following rule: if the inductively coupled elements x_1 and x_2 converge in node b with their unsimilar terminals then the positive calculated reactances $+x_M = +\omega M$ are added to the inductive elements x_1 and x_2 whereas the negative calculated inductive reactance $-x_M = -\omega M$ is introduced into the circuit part which is common for both inductive elements x_1 and x_2 (in the problem under consideration this is branch №3) .

Assume the circuit complex emf fig. 3.26,b to be equal to

$$\underline{E} = E \cdot e^{j\psi_E} = 220\ V.$$

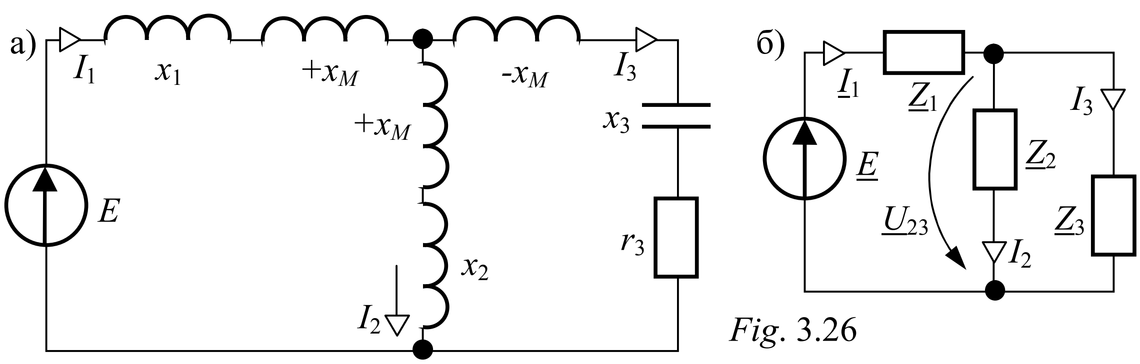


Fig. 3.26

The complex branch impedances are

$$\underline{Z}_1 = jx_1 + jx_M = j40 + j50 = j90\ Ом,$$

$$\underline{Z}_2 = jx_2 + jx_M = j80 + j50 = j130\ Ом,$$

$$\underline{Z}_3 = r_3 - jx_3 - jx_M = 40 - j20 - j50 = 40 - j70 = 80.62 \cdot e^{-j60.25°}\ Ом.$$

The input circuit impedance is

$$\underline{Z}_{inp} = \underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = j90 + \frac{j130 \cdot 80.62e^{-j60.25}}{j130 + 40 - j70} = 132.4 \cdot e^{j10.89°}\ Ohm,$$

the fork impedance being

$$\underline{Z}_{23} = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{j130 \cdot 80{,}62e^{-j60.25}}{j130 + 40 - j70} = 145.3 \cdot e^{-j26.56°}\ Ohm.$$

Input current is $\underline{I}_1 = \dfrac{\underline{E}}{\underline{Z}_{inp}} = \dfrac{220}{132.4e^{j10.89}} = 1.662 \cdot e^{-j10.89°}\ A,$

the fork voltage being $\underline{U}_{23} = \underline{I}_1 \cdot \underline{Z}_{23} = 1.662 \cdot e^{-j10.89°} \cdot 145.3 \cdot e^{-j26.56°} = 241.5 \cdot e^{-j37.45°}\ V,$

the currents of the parallel branches being

$$\underline{I}_2 = \frac{\underline{U}_{23}}{\underline{Z}_2} = \frac{241.5e^{-j37.45}}{j130} = 1.858 \cdot e^{-j127.45°}\ A,$$

$$\underline{I}_3 = \frac{\underline{U}_{23}}{\underline{Z}_3} = \frac{241.5e^{-j37.45}}{82.62e^{-j60.25}} = 3.0 \cdot e^{j22.8°}\ A.$$

To make sure the currents of the initial scheme fig 3.25,a are calculated correctly, let's check up the power balance:

- the generator power is

$\underline{S}_G = \underline{E} \cdot \overset{*}{\underline{I}}_1 = 220 \cdot 1.662 \cdot e^{j10.89°} = 359 + j69.1\ VA = P_G + jQ_G.$

- the consumer active power is

$\Sigma P_C = I_3^2 \cdot r_3 = 3^2 \cdot 40 = 360\ W \approx P_G,$

- the consumer reactive power is

$\Sigma Q_C = I_1^2 \cdot x_1 + I_2^2 \cdot x_2 + 2 \cdot I_1 \cdot I_2 \cdot x_M \cdot cos(\psi_{i1} - \psi_{i2}) - I_3^2 \cdot x_3 =$

$= 1.662^2 \cdot 40 + 1.858^2 \cdot 80 + 2 \cdot 1.662 \cdot 1.858 \cdot 50 \cdot cos(-10.89° + 127.45°) - 3^2 \cdot 20 =$

$= 110.5 + 276.2 - 138.1 - 180 = 68.6\ VAr \approx Q_G.$

The active power transmitted from the first branch into the second one is

$P_{1\to2} = \mathrm{Re}(\underline{I}_2 \cdot jx_M \cdot \overset{*}{\underline{I}}_1) = I_1 \cdot I_2 \cdot x_M \cdot sin(\psi_{i1} - \psi_{i2}) =$

$= 1.662 \cdot 1.858 \cdot 50 \cdot sin(-10.89° + 127.45°) = 138.1\ W.$

Pay attention, no active powers are consumed in the first and the second branches of the scheme with an autotransformer, $P_{1C} = P_{2C} = 0$, as there is no resistance in the autotransformer windings ($r_1 = r_2 = 0$), correspondingly, there is no heat loss either.

The voltage across the portion *ab* of scheme fig. 3.25,b is

$\underline{U}_{ab} = \underline{I}_1 \cdot jx_1 + \underline{I}_2 \cdot jx_M = 1.662 \cdot e^{-j10.89°} \cdot 40 \cdot e^{j90°} + 1.858 \cdot e^{-j127.45°} \cdot 50 \cdot e^{j90°} = 86.76 \cdot e^{j5.83°}\ V,$

and that across the portion *bd*

$\underline{U}_{bd} = \underline{I}_2 \cdot jx_2 + \underline{I}_1 \cdot jx_M = \underline{I}_3 \cdot (r_3 - jx_3) = 3 \cdot e^{j22,8°} \cdot (40 - j20) = 134.2 \cdot e^{-j3.77°}\ V.$

The power delivered to the first coil (portion *ab*) is

$\underline{S}_{1inp} = \underline{U}_{ab} \cdot \overset{*}{\underline{I}}_1 = 86.76 \cdot e^{j5.83°} \cdot 1.662 \cdot e^{j10.89°} = 138.1 + j41.5\ VA = P_{1inp} + jQ_{1inp}.$

Since there is no heat loss at this portion, then the total active power delivered P_{1inp} is to be transmitted into the second branch

$$P_{1\to2} = P_{1inp} - I_1^2 \cdot r_1 = 138.1 - 0 = 138.1\ W,$$

this fact is checkrd in the following way:

$\underline{S}_{2inp} = \underline{U}_{bd} \cdot \overset{*}{\underline{I}}_2 = 134.2 \cdot e^{-j3.77°} \cdot 1.858 \cdot e^{j127.45°} = -138.1 + j207.5\ \ VA = P_{2inp} + jQ_{2inp}.$

As $P_{2C} = I_2^2 \cdot r_2 = 0$, then the active power transmitted from the second branch into the first one is as follows $P_{2\to1} = P_{2inp} - P_{2C} = -138.1 - 0 = -138.1\ W$;

however, the sum is equal to zero: $P_{1\to2} + P_{2\to1} = 138.1 - 138.1 = 0.$

To construct a phasor diagram, one need find the voltage drops:

$\underline{I}_1 \cdot x_1 = 1.662 \cdot 40 = 66.5\ V$;
$\underline{I}_2 \cdot x_M = 1.858 \cdot 50 = 92.9\ V$;
$\underline{I}_2 \cdot x_2 = 1.858 \cdot 80 = 150.6\ V$;
$\underline{I}_1 \cdot x_M = 1.662 \cdot 50 = 83.1\ V$;
$\underline{I}_3 \cdot x_3 = 3 \cdot 20 = 60\ V$;
$\underline{I}_3 \cdot r_3 = 3 \cdot 40 = 19.6\ V$,
the currents being presented in an algebraic form

$\underline{I}_1 = 1.63 - j0.31\ A$;
$\underline{I}_2 = -1.13 - j1.47\ A$;
$\underline{I}_3 = 2.76 + j1.16\ A$.

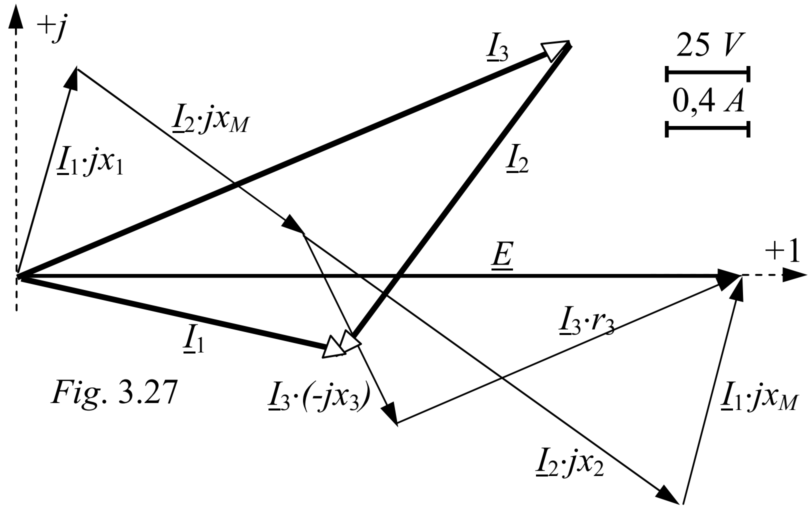


Fig. 3.27

On the complex plane (fig. 3.27), we construct the current phasors in accordance with Kirchhoff's current law, this equation is taken from the initial equation system; then we draw the voltage drop phasors in accordance with the initial equations under Kirchhoff's voltage law.

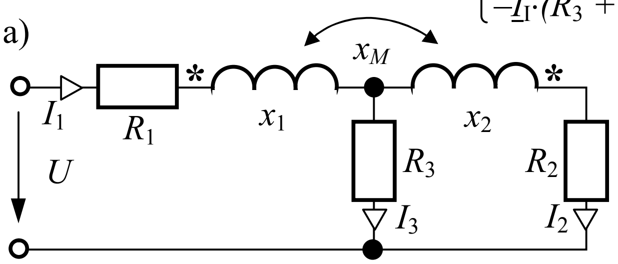
3-24 (3.52). In scheme fig. 3.28,a, calculate the currents, if $U = 100\ V$, $R_1 = 5\ Ohm$, $R_2 = x_1 = 15\ Ohm$, $R_3 = x_M = 10\ Ohm$, $x_2 = 25\ Ohm$.

Solution. Let's solve the problem by the mesh current method, having eliminated first the inductively coupled elements (fig. 3.50,b). The equation system is:

$$\begin{cases} \underline{I}_{\mathrm{I}} \cdot (R_1 + jx_1 - jx_M + jx_M + R_3) - \underline{I}_{\mathrm{II}} \cdot (R_3 + jx_M) = \underline{U}; \\ -\underline{I}_{\mathrm{I}} \cdot (R_3 + jx_M) + \underline{I}_{\mathrm{II}} \cdot (R_3 + jx_M - jx_M + jx_2 + R_2) = 0. \end{cases}$$

$$\begin{cases} \underline{I}_{\mathrm{I}} \cdot (15 + j15) - \underline{I}_{\mathrm{II}} \cdot (10 + j10) = 100; \\ -\underline{I}_{\mathrm{I}} \cdot (10 + j10) + \underline{I}_{\mathrm{II}} \cdot (25 + j25) = 0. \end{cases}$$

a)



b)

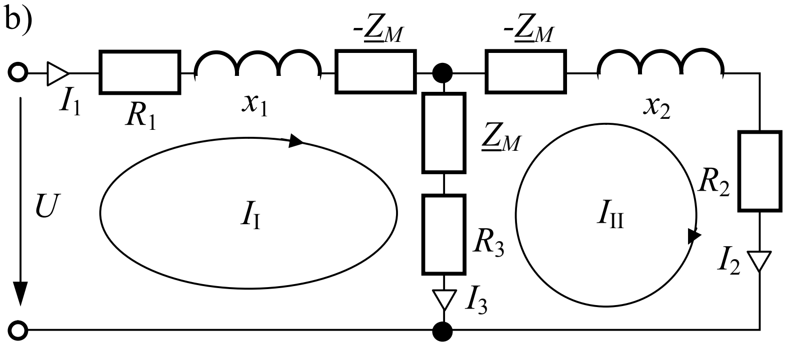


Fig. 3.28

The system solution results are as follows:

$\underline{I}_{\mathrm{I}} = \Delta_{\mathrm{I}}/\Delta = 6.43 \cdot e^{-j45°}\ A$; $\underline{I}_{\mathrm{II}} = \Delta_{\mathrm{II}}/\Delta = 2.57 \cdot e^{-j45°}\ A$.

The branch currents are determined through the loop currents by the superposition principle: $\underline{I}_1 = \underline{I}_{\mathrm{I}} = 6{,}43 \cdot e^{-j45°}\ A$;
$\underline{I}_2 = \underline{I}_{\mathrm{II}} = 2{,}57 \cdot e^{-j45°}\ A$;
$\underline{I}_3 = \underline{I}_{\mathrm{I}} - \underline{I}_{\mathrm{II}} = 3{,}87 \cdot e^{-j45°}\ A$.

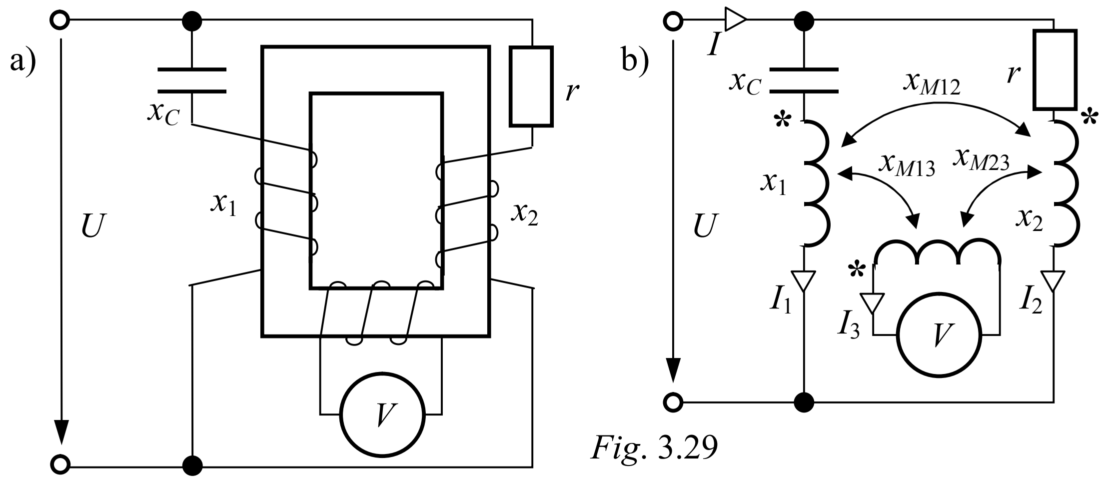


Fig. 3.29

3-25 (3.53). In scheme fig. 3.29,a, determine the currents, construct a phasor diagram, find the active power transmitted from the first coil into the second one as well as the voltmeter reading. Determine the circuit input impedance with the help of Ohm's law and by means of mutual inductance elimination. Numerical data are: $U = 100\ V$, $x_C = x_1 = 20\ Ohm$, $x_2 = 10\ Ohm$, $r = 10\ Ohm$, $x_{M12} = x_{M23} = 10\ Ohm$, $x_{M13} = 5\ Ohm$.

Solution. Determine the similar terminals of the coils and construct a scheme for calculation (fig. 3.29,b). Let's solve the problem by Kirchhoff's equation method. As the infinite impedance of the voltmeter $Z_V = \infty$, there is no current through the third coil $I_3 = 0$. Thus, the current I_3 has no influence and is not taken into account while solving the equations. The equation system has a view:

$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2; & (3.1)\\ \underline{I}_1\cdot(-jx_C + jx_1) + \underline{I}_2\cdot jx_{M12} = \underline{U}; & (3.2)\\ \underline{I}_1\cdot jx_{M12} + \underline{I}_2\cdot(r + jx_2) = \underline{U}; & (3.3)\\ \underline{I}_1\cdot jx_{M13} + \underline{I}_2\cdot jx_{M23} = \underline{U}_V. & (3.4) \end{cases}$$

Solve the equations (3.2) and (3.3) concerning $\underline{I}_1$ and $\underline{I}_2$:

$$\begin{cases} \underline{I}_1\cdot 0 + \underline{I}_2\cdot j10 = 100;\\ \underline{I}_1\cdot j10 + \underline{I}_2\cdot(10 + j10) = 100; \end{cases} \qquad \underline{I}_2 = 100/j10 = -j10\ A; \qquad \underline{I}_1 = \frac{100 + j10(10 + j10)}{j10} = 10\ A.$$

From (3.1) it follows that $\underline{I} = 10 + (-j10) = 14{,}14\cdot e^{-j45°}\ A$.

To construct a phasor diagram (fig. 3.30,a), let's find the voltage drops across all the elements:

$\underline{U}_C = \underline{I}_1\cdot(-jx_C) = 10\cdot(-j20) = -j200\ V$; $\underline{U}_r = \underline{I}_2\cdot r = -j10\cdot 10 = -j100\ V$;

$\underline{U}_{X1} = \underline{I}_1\cdot jx_1 = 10\cdot j20 = j200\ V$; $\underline{U}_{X2} = \underline{I}_2\cdot jx_2 = -j10\cdot j10 = 100\ V$;

$\underline{U}_{XM12} = \underline{I}_2\cdot jx_{M12} = -j10\cdot j10 = 100\ V$; $\underline{U}_{XM21} = \underline{I}_1\cdot jx_{M12} = 10\cdot j10 = j100\ V$.

Active power consumed by the first branch is as follows

$$P_1 = \mathrm{Re}(\underline{U}\cdot \overset{*}{\underline{I}}_1) = \mathrm{Re}(100\cdot 10) = 1000\ W.$$

The power loss in the first branch is absent: $\Delta P_1 = r_1\cdot I_1^2 = 0$.

The active power transmitted from the first branch into the second one through the magnetic field is as follows $P_{1\to 2} = P_1 - \Delta P_1 = 1000 - 0 = 1000\ W$.

The voltage across the voltmeter in accordance with (3.4) is

$$\underline{U}_V = -10\cdot j5 - (-j10)\cdot j10 = -j50 - 100 = 111.8\cdot e^{j26{,}6°}\ V.$$

The voltmeter reading is $V \rightarrow U_V = 111.8\ V$.

The input impedance found by Ohm's law is $\underline{Z} = \dfrac{\underline{U}}{\underline{I}} = \dfrac{100}{14.14 \cdot e^{-j45}} = 7.07 \cdot e^{j45^\circ}\ Ohm.$

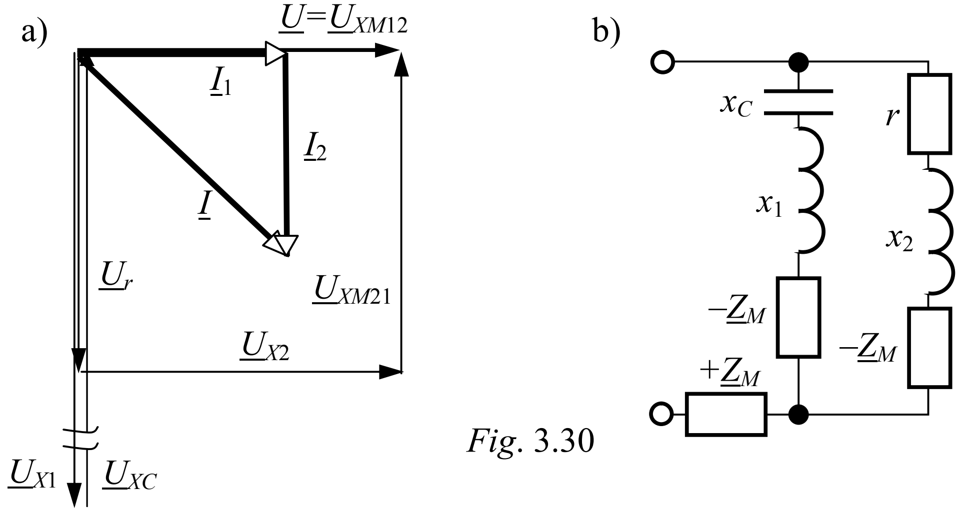


Fig. 3.30

After the elimination of the mutual inductance, we obtain a scheme fig. 3.30,b. Here

$$\underline{Z}_M = jx_{M12} = j10\ Ohm.$$

The circuit input impedance is

$$\underline{Z} = \underline{Z}_M + \frac{(-jx_C + jx_1 - \underline{Z}_M)(r + jx_2 - \underline{Z}_M)}{-jx_C + jx_1 - \underline{Z}_M + r + jx_2 - \underline{Z}_M} =$$

$$= j10 + \frac{-j10 \cdot 10}{10 - j10} = j10 + 5 - j5 = 7.07 \cdot e^{j45^\circ}\ Ohm.$$

One may see the answer is the same as at the calculation by Ohm's law.

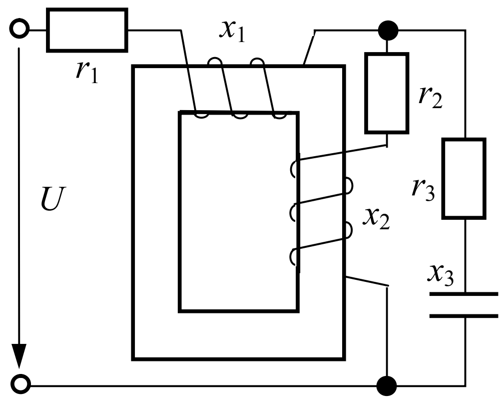


Fig. 3.31

3-26 (3.54). In scheme fig. 3.31, it is required to establish an equation system using Kirchhoff's laws method. Calculate the currents by means of the mutual inductance elimination. Determine the currents by the mesh current method. Determine the power transmitted from the fist coil into the second one through the magnetic field. Numerical data are as follows: $U = 220\ V$, $r_1 = 6\ Ohm$, $x_1 = 10\ Ohm$, $r_2 = 12\ Ohm$, $x_2 = 18\ Ohm$, $r_3 = 18\ Ohm$, $x_3 = 20\ Ohm$, $x_M = 8\ Ohm$.

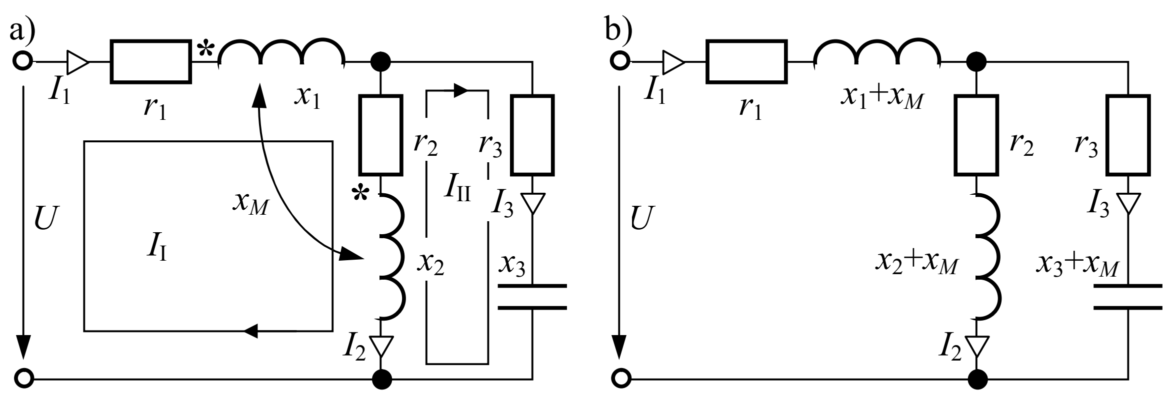


Fig. 3.32

Solution. 1. Let's establish a computation scheme (fig. 3.32,a). The equation system to find the currents by Kirchhoff's laws is:

$$\begin{cases} \underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0; \\ r_1\cdot\underline{I}_1 + jx_1\cdot\underline{I}_1 + jx_M\cdot\underline{I}_2 + r_2\cdot\underline{I}_2 + jx_2\cdot\underline{I}_2 + jx_M\cdot\underline{I}_1 = \underline{U}; \\ -r_2\cdot\underline{I}_2 - jx_2\cdot\underline{I}_2 - jx_M\cdot\underline{I}_1 + r_3\cdot\underline{I}_3 - jx_3\cdot\underline{I}_3 = 0. \end{cases}$$

2. Eliminate the mutual inductance (fig. 3.32,b) and compute the circuit obtained.

$\underline{Z}_1 = r_1 + jx_1 + jx_M = 6 + j18 = 18.97\cdot e^{j71.6°}$ *Ohm*,

$\underline{Z}_2 = r_2 + jx_2 + jx_M = 12 + j26 = 28.64\cdot e^{j65.2°}$ *Ohm*,

$\underline{Z}_3 = r_3 - jx_3 - jx_M = 18 - j28 = 33.29\cdot e^{-j57.3°}$ *Ohm*.

$$\underline{Z}_{23} = \frac{\underline{Z}_2\cdot\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3} = \frac{28.64\cdot e^{j65.2}\cdot 33.29\cdot e^{-j57.3}}{12+j26+18-j28} = 31.70\cdot e^{j11.8°} = 31.04 + j6.47 \text{ Ohm}.$$

The circuit input impedance and currents are

$\underline{Z}_{inp} = \underline{Z}_1 + \underline{Z}_{23} = 6 + j18 + 31.04 + j6.47 = 37.04 + j24.47 = 44.39\cdot e^{j33.5°}$ *Ohm*.

$$\underline{I}_1 = \frac{\underline{U}}{\underline{Z}_{inp}} = \frac{220}{44.39e^{j33.5}} = 4.96\cdot e^{-j33.5°} \text{ A},$$

$$\underline{I}_2 = \underline{I}_1\cdot\frac{\underline{Z}_{23}}{\underline{Z}_2} = 4.96\cdot e^{-j33.5°}\cdot\frac{31.7\cdot e^{j11.8}}{28.64\cdot e^{j65.2}} = 5.49\cdot e^{-j86.9°} \text{ A},$$

$$\underline{I}_3 = \underline{I}_1\cdot\frac{\underline{Z}_{23}}{\underline{Z}_3} = 4.96\cdot e^{-j33.5°}\cdot\frac{31.7\cdot e^{j11.8}}{33.29\cdot e^{-j57.3}} = 4.72\cdot e^{j35.6°} \text{ A}.$$

3. The equation system by the mesh current method is as follows:

$$\begin{cases} (r_1 + jx_1 + r_2 + jx_2 + j2x_M)\cdot\underline{I}_{\text{I}} - (r_2 + jx_2 + jx_M)\cdot\underline{I}_{\text{II}} = \underline{U}; \\ -(r_2 + jx_2 + jx_M)\cdot\underline{I}_{\text{I}} + (r_2 + jx_2 + r_3 - jx_3)\cdot\underline{I}_{\text{II}} = 0; \end{cases}$$

$$\begin{cases} (18 + j44)\cdot\underline{I}_{\text{I}} - (12 + j26)\cdot\underline{I}_{\text{II}} = 220; \\ -(12 + j26)\cdot\underline{I}_{\text{I}} + (30 - j2)\cdot\underline{I}_{\text{II}} = 0. \end{cases}$$

Its solution is $\underline{I}_{\text{I}} = 4.96\cdot e^{-j33.5°}$ *A*, $\underline{I}_{\text{II}} = 4.72\cdot e^{j35.6°}$ *A*.

The branch currents are: $\underline{I}_1 = \underline{I}_{\text{I}} = 4.96\cdot e^{-j33.5°}$ *A*, $\underline{I}_3 = \underline{I}_{\text{II}} = 4.72\cdot e^{j35.6°}$ *A*,

$\underline{I}_2 = \underline{I}_{\text{I}} - \underline{I}_{\text{II}} = 4.96\cdot e^{-j33.5°} - 4.72\cdot e^{j35.6°} = 5.49\cdot e^{-j86.9°}$ *A*.

4. The active power transmitted from the first coil into the second one by a magnetic field is as follows:

$$P_{1\to2} = x_M\cdot I_1\cdot I_2\cdot \sin(\psi_{i1} - \psi_{i2}) = 8\cdot4.96\cdot5.49\cdot\sin(-33.5° + 86.9°) = 174.9 \text{ W}.$$

3-27 (3.56). Mark similar terminals of the coils fig. 3.33.

Answer: There are three pairs of similar terminals, namely:

upper terminal of the 1st coil and bottom one of the 2nd;

upper terminal of the 2nd coil and bottom one of the 3rd;

upper terminal of the 1st coil and bottom one of the 3rd.

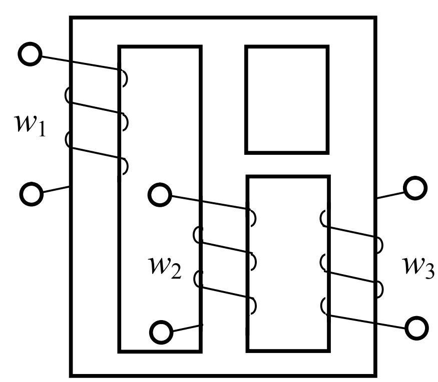


Fig. 3.33

4. THREE-PHASE CIRCUITS

4.1. CALCULATION OF THE SYMMETRICAL THREE-PHASE CIRCUITS

4-1 (4.1). A symmetrical three-phase network of voltage $U = 380\ V$ supplies with energy a balanced load whose phases are connected in star (fig. 4.1,a), its phase impedance being $r = 12\ Ohm$, $x = 16\ Ohm$.

Find the instrument readings, construct of a phasor diagram.

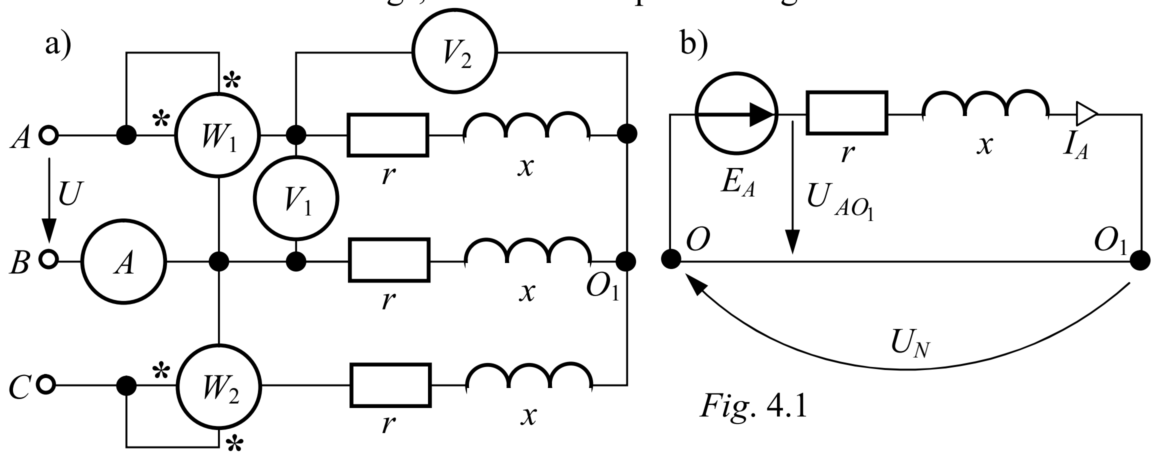


Fig. 4.1

Solution. When one talks about the voltage of a three-phase circuit, it means on default the line-to-line network voltage. When line-to-neutral or simply phase voltage is given, it is expressly stated.

In conditions of the present problem, the network voltage is $U = 380\ V$, moreover, because of the symmetry: $U_{AB} = U_{BC} = U_{CA} = U = 380\ V$.

Phase voltages are $\sqrt{3}$ times less than line voltages:

$U_A = U_B = U_C = U/\sqrt{3} = 380/\sqrt{3} = 220\ V.$

Assume the generator phases are Y-connected, then the generator phase voltages are $E_A = E_B = E_C = U_{ph} = 220\ V$.

In a symmetrical system Y-Y, the junction voltage is absent $U_N = 0$ and the circuit state calculation is made by the equivalent scheme for but a single phase (fig. 4.1,b).

Assume $\underline{E}_A = 220\ V$, the current in the equivalent scheme is

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_A} = \frac{220}{12 + j16} = 11 \cdot e^{-j53.13°}\ A,$$

furthermore, the consumer phase voltage is $\underline{U}_{AO_1} = \underline{E}_A - \underline{U}_N = \underline{E}_A = \underline{U}_A$.

Write down the results of computation of the working condition of other phases (B and C) on the ground of the idea of the symmetrical three-phase system of the direct sequential order of phases

$$\underline{E}_B = \underline{U}_{BO_1} = \underline{U}_B = \underline{E}_A \cdot e^{-j120°} = 220 \cdot e^{-j120°}\ V;$$

$$\underline{E}_C = \underline{U}_{CO_1} = \underline{U}_C = \underline{E}_A \cdot e^{j120°} = 220 \cdot e^{j120°}\ V;$$

similarly for currents $\underline{I}_B = \underline{I}_A \cdot e^{-j120°} = 11 \cdot e^{-j173.13°}\ A;$

$$\underline{I}_C = \underline{I}_A \cdot e^{j120°} = 11 \cdot e^{j66.87°}\ A.$$

The circuit phasor diagram is presented in fig. 4.2; the potential of the generator neutral point is zero $\underline{\varphi}_O = 0$ (fig. 4.1,b); the generator windings are Y-connected.

$\underline{\varphi}_A = \underline{E}_A = 220\ V$; $\underline{\varphi}_B = 220 \cdot e^{-j120°}\ V$; $\underline{\varphi}_C = 220 \cdot e^{j120°}\ V$; $\underline{\varphi}_{O_1} = \underline{\varphi}_O + \underline{U}_N = 0$;

$$\underline{U}_{AO} = \underline{\varphi}_A - \underline{\varphi}_O = \underline{E}_A = \underline{U}_{AO_1} = \underline{\varphi}_A - \underline{\varphi}_{O_1} = \underline{U}_A = 220\ V;$$

similarly $\underline{U}_B = \underline{\varphi}_B - \underline{\varphi}_O = 220 \cdot e^{-j120°}\ V$; $\underline{U}_C = \underline{\varphi}_C - \underline{\varphi}_O = 220 \cdot e^{j120°}\ V$;

the phase currents are to be oriented concerning their phase voltages (they are shifted by the angle $\varphi = arctg\frac{x}{r} = arctg\frac{16}{12} = 53.13°$).

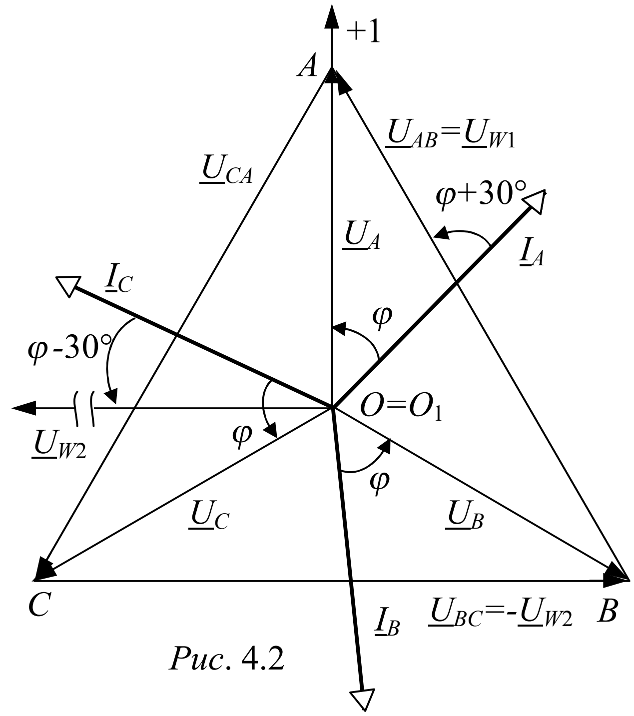


Рис. 4.2

Line voltages are:

$\underline{U}_{AB} = \underline{\varphi}_A - \underline{\varphi}_B = \underline{U}_A \cdot \sqrt{3} \cdot e^{j30°} = 380 \cdot e^{j30°}\ V$;

$\underline{U}_{BC} = \underline{U}_{AB} \cdot e^{-j120°} = 380 \cdot e^{-j90°}\ V$;

$\underline{U}_{CA} = \underline{U}_{AB} \cdot e^{j120°} = 380 \cdot e^{j150°}\ V$.

Voltages and currents of the wattmeters coils are:

$\underline{U}_{W1} = \underline{U}_{AB} = 380 \cdot e^{j30°}\ V$;

$\underline{I}_{W1} = \underline{I}_A = 11 \cdot e^{-j53.13°}\ A$;

$\underline{U}_{W2} = -\underline{U}_{BC} = 380 \cdot e^{j90°}\ V$;

$\underline{I}_{W2} = \underline{I}_C = 11 \cdot e^{j66.87°}\ A$.

The instrument readings:

- ammeter *A* measures the line current I_B, its reading being $I_B = 11\ A$;
- voltmeter V_1 measures the line voltage $U_{AB} = 380\ V$;
- voltmeter V_2 measures the phase voltage of the consumer $U_A = 220\ V$;
- the wattmeters readings are:

$$P_{W1} = \mathrm{Re}[\underline{U}_{W1} \cdot \overset{*}{\underline{I}}_{W1}] = \mathrm{Re}[\underline{U}_{AB} \cdot \overset{*}{\underline{I}}_A] = U_{AB} \cdot I_A \cdot cos(\varphi + 30°) =$$
$$= 380 \cdot 11 \cdot cos(53.13° + 30°) = 500\ W;$$

$$P_{W2} = \mathrm{Re}[\underline{U}_{W2} \cdot \overset{*}{\underline{I}}_{W2}] = \mathrm{Re}[\underline{U}_{CB} \cdot \overset{*}{\underline{I}}_C] = U_{CB} \cdot I_C \cdot cos(\varphi - 30°) =$$
$$= 380 \cdot 11 \cdot cos(53.13° - 30°) = 3846\ W.$$

Let's pay attention to the sum of these two wattmeters readings:

$$P_{W1} + P_{W2} = U \cdot I \cdot cos(\varphi + 30°) + U \cdot I \cdot cos(\varphi - 30°) =$$
$$= U \cdot I \cdot cos30° \cdot cos\varphi - U \cdot I \cdot sin30° \cdot sin\varphi + U \cdot I \cdot cos30° \cdot cos\varphi + U \cdot I \cdot sin30° \cdot sin\varphi =$$
$$= 2 \cdot U \cdot I \cdot cos30° \cdot cos\varphi = 2 \cdot U \cdot I \cdot \frac{\sqrt{3}}{2} \cdot cos\varphi = \sqrt{3} \cdot U \cdot I \cdot cos\varphi = P$$

– active power of the balanced consumer. In the given example, it is

$$\sqrt{3} \cdot U \cdot I \cdot cos\varphi = \sqrt{3} \cdot 380 \cdot 11 \cdot cos(53.13°) = 4346 = 500 + 3846\ W.$$

4-2 (4.2). Analyze the symmetrical mode of the star-connection (fig. 4.3), if $r = 40\ Ohm$, $x = 80\ Ohm$, a network voltage $U = 380\ V$.

Construct a circuit phasor diagram.

Answers: $I_C = 2.46\ A$; $U_B = 220\ V$; $U_{BC} = 380\ V$;

$P_{W1} = -56\ Bm$; $P_{W2} = 780\ W$; $P_{W1} + P_{W2} = 3 \cdot I^2 \cdot r = 726\ W$.

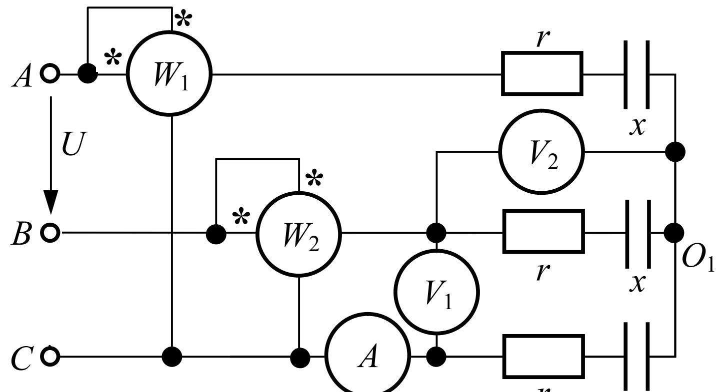


Fig. 4.3

4-3 (4.3). The phases of the consumer of the problem 4.2 are Δ-connected (fig. 4.4). Analyze the scheme working mode. Find the ratio between the line currents $I_Δ$ of scheme fig. 4.4 and the line currents I_Y of fig. 4.3.

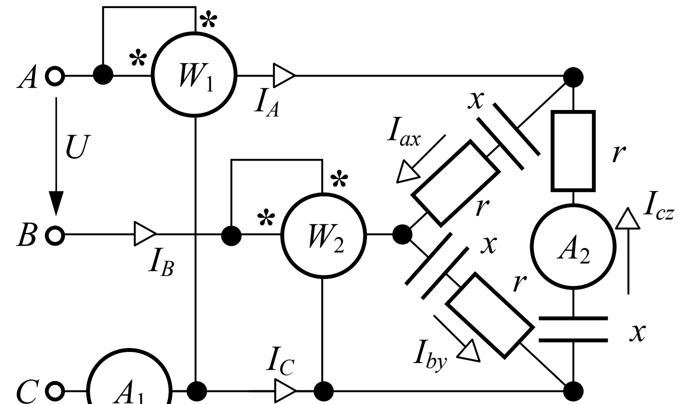


Fig. 4.4

Solution. At Δ-connection of the consumer, its line voltages are equal to the phase voltages. Assume $\underline{U}_{AB} = U = 380\ V$. Then

$$\underline{U}_{AX} = \underline{U}_{AB} = 380\ V; \qquad \underline{U}_{BY} = 380 \cdot e^{-j120°}\ V; \qquad \underline{U}_{CZ} = 380 \cdot e^{j120°}\ V.$$

Phase currents are determined by Ohm's law:

$$\underline{I}_{ax} = \frac{\underline{U}_{AX}}{\underline{Z}} = \frac{380}{40 - j80} = 4.25 \cdot e^{j63.44°}\ A;$$

$$\underline{I}_{by} = \frac{\underline{U}_{BY}}{\underline{Z}} = \underline{I}_{ax} \cdot e^{-j120°} = 4.25 \cdot e^{-j56.56°}\ A;$$

$$\underline{I}_{cz} = \frac{\underline{U}_{CZ}}{\underline{Z}} = \underline{I}_{ax} \cdot e^{j120°} = 4.25 \cdot e^{j183.44°}\ A.$$

Line currents are found under Kirchhoff's current law:

$$\underline{I}_A = \underline{I}_{ax} - \underline{I}_{cz} = \sqrt{3}\,\underline{I}_{ax} \cdot e^{-j30°} = 7.36 \cdot e^{j33.44°}\ A;$$

$$\underline{I}_B = \underline{I}_{by} - \underline{I}_{ax} = \sqrt{3}\,\underline{I}_{by} \cdot e^{-j30°} = \underline{I}_A \cdot e^{-j120°} = 7.36 \cdot e^{-j86.56°}\ A;$$

$$\underline{I}_C = \underline{I}_{cz} - \underline{I}_{by} = \sqrt{3}\,\underline{I}_{cz} \cdot e^{-j30°} = \underline{I}_A \cdot e^{j120°} = 7.36 \cdot e^{j153.44°}\ A.$$

A phasor diagram of the voltages and currents of the *Δ*-connection is constructed in such a way as to present the correlation between the line currents and the phase currents in a convenient manner. For that, the phase voltage phasors of *Δ*-connection are shown originating from one and the same point (fig. 4.5,a), the phase currents are to be oriented concerning these voltages, the line currents are determined in accordance with Kirchhoff's current law. On the ground of the correlations between the currents presented in the phasor diagram, it is possible to obtain the above-written correlations for the line current calculation.

Note, the traditional way of the phasor diagram construction is allowable (the phasor diagram can be made in the form of the complex potential diagram) (fig. 4.2), the triangle phase currents being oriented concerning the network line voltages, which are equal to the triangle phase voltages as it is shown in fig. 4.5,b. However, such construction demands to transport the phase currents with the opposite sign while constructing the line currents.

On the ground of the calculations, we write down the ammeters readings:

- line current $I_A = 7.36\ A$ is measured by ammeter A_1;
- phase current of the Δ-connection is measured by the ammeter A_2, its reading being $I_{cz} = 4.25\ A$.

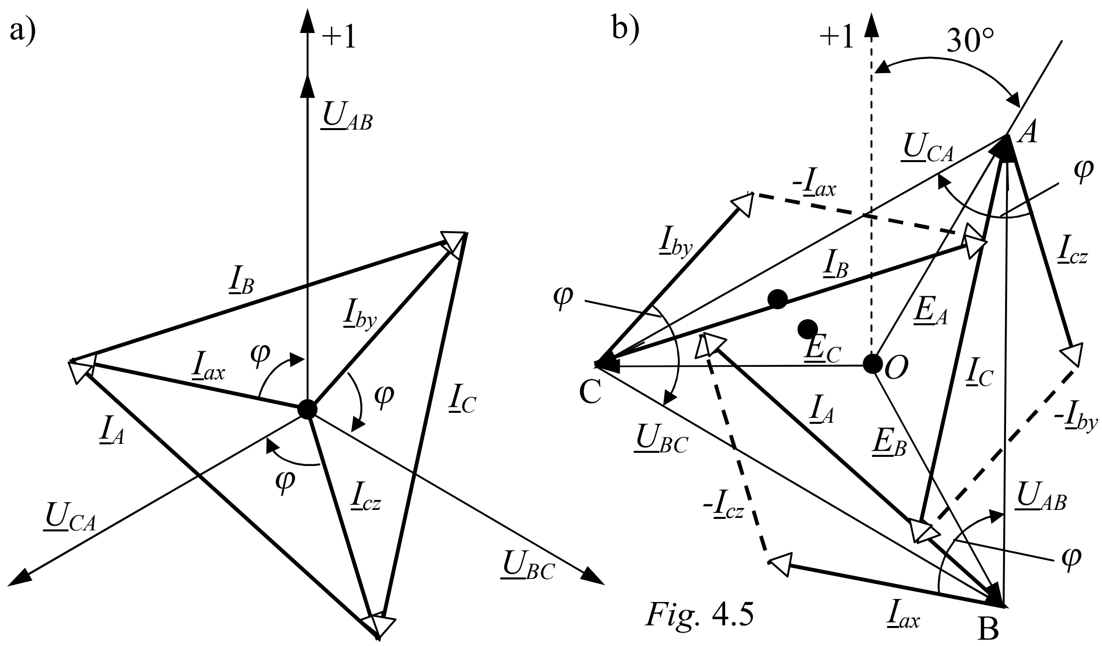


Fig. 4.5

At the star-connection of the same impedances, the line currents are $I_Y = 2.46\ A$ (problem 4.2). Thus, when reconnecting the impedances from the star into delta at the same line voltage, a line current increases by $\dfrac{I_\Delta}{I_Y} = \dfrac{7.36}{2.46} = 3$ times.

The wattmeters readings in scheme fig. 4.4 are:

$$P_{W1} = \text{Re}[\underline{U}_{AC} \cdot \overset{*}{\underline{I}}_A] = \text{Re}[-380 \cdot e^{j120°} \cdot 7.36 \cdot e^{-j33.44°}] = -168\ W;$$

$$P_{W2} = \text{Re}[\underline{U}_{BC} \cdot \overset{*}{\underline{I}}_B] = \text{Re}[380 \cdot e^{-j120°} \cdot 7.36 \cdot e^{j86.56°}] = 2334\ W.$$

The sum of the readings of two wattmeters $P_{W1} + P_{W2} = 2167\ W$ is equal to the active power of the three-phase consumer: $P = 3 \cdot I_{ph} \cdot r_{ph} = 3 \cdot 4.25^2 \cdot 40 = 2167\ W$.

4-4 (4.4). Find the instrument readings in scheme fig. 4.6,a, if

$$r = 76\ Ohm, \quad x = 44\ Ohm, \quad U = 380\ V.$$

Solution. Compose an equivalent scheme of the three-phase circuit under consideration but for a single phase *(A)* (fig. 4.6,b) and determine its parameters:

- a phase voltage of the source $\underline{U}_A = \dfrac{U}{\sqrt{3}} = \dfrac{380}{\sqrt{3}} = 220\ V$,

- a phase impedance of the star, equivalent to the triangle $r_Y = \dfrac{r}{3} = \dfrac{76}{3}\ Ohm$.

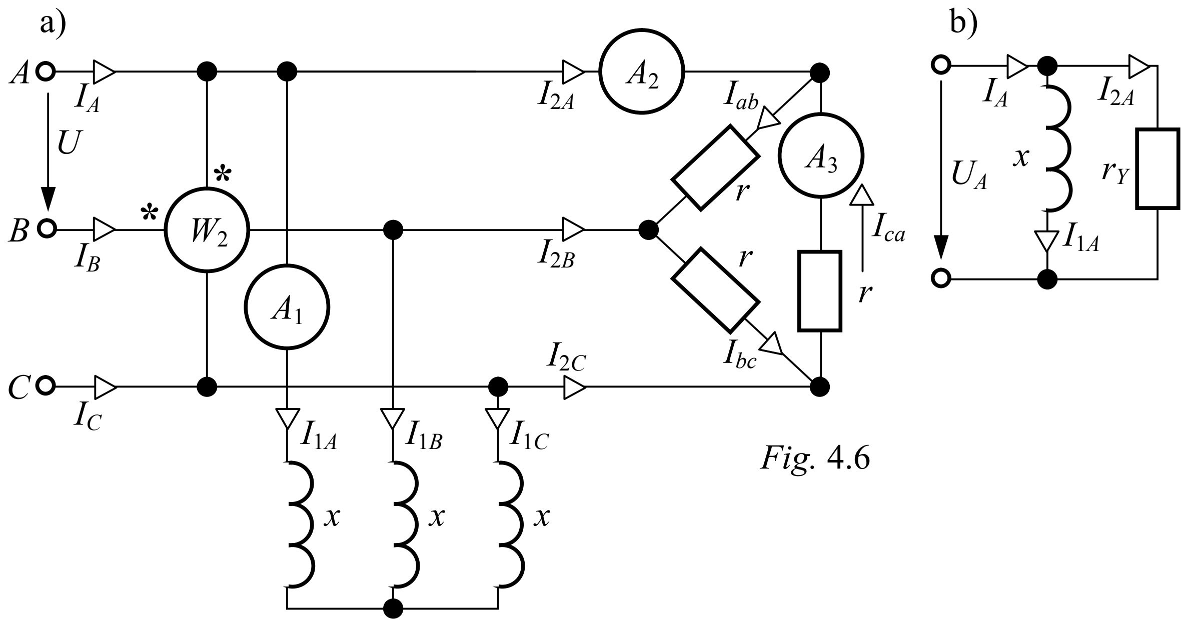


Fig. 4.6

The further calculation is performed in complex form, the source voltage being assumed $\underline{U}_A = 220\ V$:

- a phase current of the 1st consumer Y-connected is $\underline{I}_{1A} = \dfrac{\underline{U}_A}{jx} = \dfrac{220}{j44} = -j5\ A$;

- a line current of the Δ-connection is $\underline{I}_{2A} = \dfrac{\underline{U}_A}{r_Y} = \dfrac{220 \cdot 3}{76} = \dfrac{380\sqrt{3}}{76} = 5\sqrt{3}\ A.$

The equivalent scheme fig. 4.6,b does not allow to calculate the phase current of the symmetrical triangle, it being $\sqrt{3}$ times less than a line current and being not in phase with it (see solution of the problem 4.3):

$\underline{I}_{ab} = \dfrac{\underline{I}_{2A}}{\sqrt{3}} \cdot e^{j30°} = \dfrac{5\sqrt{3}}{\sqrt{3}} \cdot e^{j30°} = 5 \cdot e^{j30°}\ A;$

$\underline{I}_{bc} = \underline{I}_{ab} \cdot e^{-j120°} = 5 \cdot e^{-j90°}\ A;$

$\underline{I}_{ca} = \underline{I}_{ab} \cdot e^{j120°} = 5 \cdot e^{j150°}\ A.$

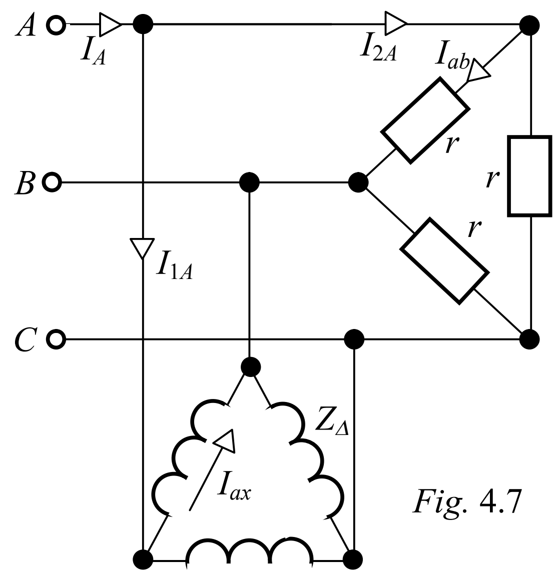


Fig. 4.7

The generator line current is

$$\underline{I}_A = \underline{I}_{1A} + \underline{I}_{2A} = -j5 + 5\sqrt{3} = 10 \cdot e^{-j30°}\ A,$$

The current of the wattmeter current coil is

$$\underline{I}_W = \underline{I}_B = \underline{I}_A \cdot e^{-j120°} = 10 \cdot e^{-j150°}\ A.$$

Voltage across the wattmeter potential coil is

$$\underline{U}_W = \underline{U}_{AC} = \sqrt{3}\,\underline{U}_A \cdot e^{-j30°} = 380 \cdot e^{-j30°}\ V.$$

The wattmeter reading is

$$P_W = \mathrm{Re}[\underline{U}_W \cdot \overset{*}{\underline{I}}_W] = \mathrm{Re}[380 \cdot e^{-j30°} \cdot 10 \cdot e^{j150°}] = -1900\ W.$$

The ammeters readings are:

$$A_1 \rightarrow I_{1A} = 5\ A, \quad A_2 \rightarrow I_{2A} = 5\sqrt{3}\ A, \quad A_3 \rightarrow I_{CA} = 5\ A.$$

Note, for scheme fig. 4.6,a another equivalent scheme (fig. 4.7) is possible, where star-connection of the 1[st] consumer is replaced by the equivalent Δ-connection, furthermore, $\underline{Z}_\Delta = 3\cdot\underline{Z}_Y = 3\cdot j44 = j132\ Ohm$.

Working mode of each triangle is calculated separately: at the phase voltage $\underline{U}_A = 220\ V$ the line voltage is $\underline{U}_{AB} = \underline{U}_A\sqrt{3}\,e^{j30^\circ} = 380\cdot e^{j30^\circ}\ V$.

Currents are $\underline{I}_{ab} = \dfrac{\underline{U}_{AB}}{r} = \dfrac{380\cdot e^{j30}}{76} = 5\cdot e^{j30^\circ}\ A,$

$$\underline{I}_{2A} = \underline{I}_{ab}\sqrt{3}\,e^{-j30^\circ} = 5\sqrt{3}\ A, \qquad \underline{I}_{ax} = \frac{\underline{U}_{AB}}{\underline{Z}_\Delta} = \frac{380\cdot e^{j30}}{j132} = \frac{5}{\sqrt{3}}\cdot e^{-j60^\circ}\ A,$$

$$\underline{I}_{1A} = \underline{I}_{ax}\sqrt{3}\,e^{-j30^\circ} = 5\cdot e^{-j90^\circ} = -j5\ A$$

And so on as in the text above.

4-5 (4.5). A source of the line voltage $U = 660\ V$ (fig. 4.8,a) supplies some loads through the reactors $x_0 = 25\ Ohm$, namely: a Y-connected motor each phase of which possesses the impedance $r = x = 50\ Ohm$; three-phase oil-filled capacitor used to improve the installation power factor, its phases being Δ-connected and possessing the reactance $x_C = 300\ Ohm$.

Determine the voltage across the motor terminals and its power. Find out the voltage loss.

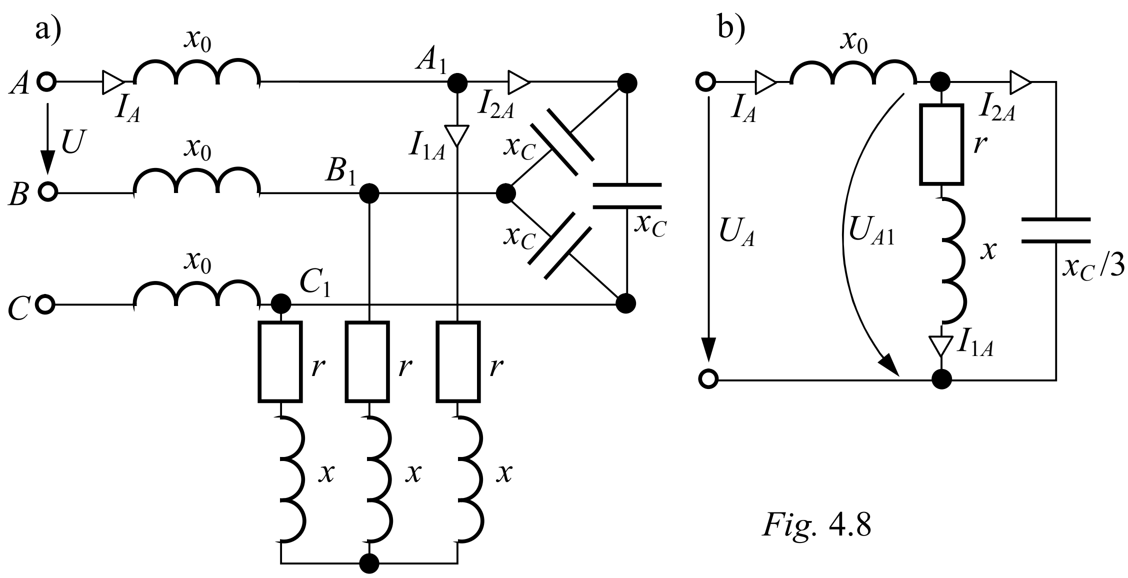


Fig. 4.8

Solution. Calculate the three-phase system by the equivalent single-phase diagram (fig. 4.8b).

A generator phase voltage is $U_A = \dfrac{U}{\sqrt{3}} = \dfrac{660}{\sqrt{3}} = 380\ V.$

Impedance of the star-connection equivalent to the Δ-connection is

$$x_Y = \frac{x_C}{3} = \frac{300}{3} = 100\ Ohm.$$

Assume $\underline{U}_A = U_A = 380\ V$, then

$$\underline{I}_A = \frac{\underline{U}_A}{jx_0 + \dfrac{(r+jx)(-jx_Y)}{r+jx-jx_Y}} = \frac{380}{j25 + \dfrac{(50+j50)(-j100)}{50+j50-j100}} = \frac{380}{j25+100} = 3.687 \cdot e^{-j14.04°}\ A,$$

the motor phase voltage is

$$\underline{U}_{A1} = \underline{I}_A \cdot \frac{(r+jx)(-jx_Y)}{r+jx-jx_Y} = 3.687 \cdot e^{-j14.04°} \cdot 100 = 368.7 \cdot e^{-j14.04°}\ V.$$

A line voltage across the motor terminals (fig. 4.8,a) is

$$U_{A1B1} = \sqrt{3} \cdot U_{A1} = \sqrt{3} \cdot 368.7 = 638.6\ V.$$

The voltage loss is $\Delta U = U_{AB} - U_{A1B1} = 660 - 638.6 = 21.4\ V$, which equals in a percentage term from the network voltage

$$\Delta U\% = \frac{\Delta U}{U_{AB}} \cdot 100 = \frac{21.4}{660} \cdot 100 = 3.24\%.$$

Note, according to the Regulations for Operation of Consumer Electrical Installations (ПТЭ), a voltage loss must not exceed 5%. So, a scheme comprising reactors to confine the short-currents at the consumer terminals complies with ПТЭ referring the allowable voltage loss.

A motor current is $\underline{I}_{1A} = \dfrac{\underline{U}_{A1}}{r+jx} = \dfrac{368.7 \cdot e^{-j14.04}}{50+j50} = 5.215 \cdot e^{-j59.04°}\ A.$

The active power of the motor is

$$P = \sqrt{3} \cdot U_{A1B1} \cdot I_{1A} \cdot cos(\varphi_1) = 3 \cdot I_{1A}^2 \cdot r = 3 \cdot 5.215^2 \cdot 50 = 4079\ W.$$

4-6 (4.6). Voltage of the symmetrical three-phase network is $U = 380\ V$ (fig. 4.9). The parameters of the circuit under study are $\underline{Z}_1 = 1.5\ Ohm$, $\underline{Z}_2 = 1 + j2\ Ohm$, $\underline{Z}_3 = -j6\ Ohm$, $\underline{Z}_4 = 21 + j12\ Ohm$. Determine the line and the phase currents of the consumers. Find out the line and phase voltages of each consumer.

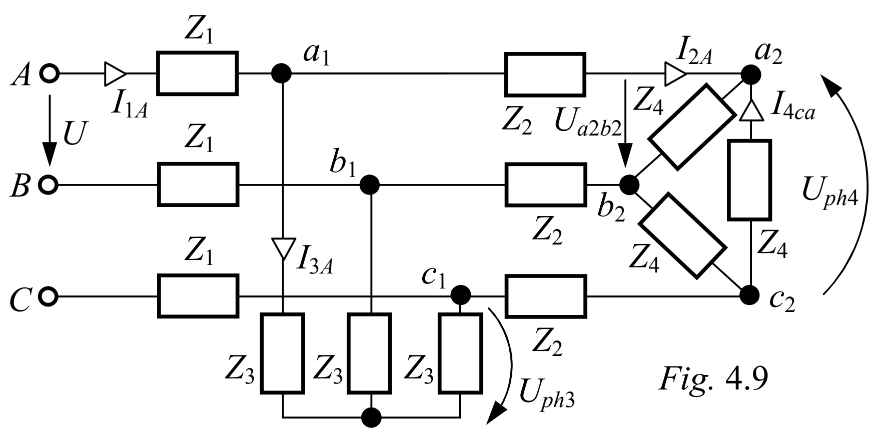


Fig. 4.9

Answers: $I_{3A} = 32.4\ A$; $I_{4ca} = 11.23\ A$; $I_{2A} = 19.45\ A$; $I_{1A} = 25.93\ A$;
$U_{ph3} = 194.5\ V$; $U_{a1b1} = 337\ V$; $U_{a2b2} = U_{ph4} = 272\ V$.

4-7 (4.7). The electric energy is delivered by two transmission lines *TL*1 and *TL*2 to the plant substation from two three-phase sources located in the different geographical points (fig. 4.10).

Voltages at the line input are equal to each other and are 6.3 *kV*, they are in phase in the no-load condition (they have one and the same initial phase).

Phase resistance and reactance of the first and the second lines are, respectively: $r_1 = 0.5$ *Ohm*, $x_1 = 0.3$ *Ohm*, $r_2 = 0.4$ *Ohm*, $x_2 = 0.6$ *Ohm*.

Nominal voltage at the substation bus-bars is $U_n = 6$ *kV*, installed power of the motors is $P_n = 4000$ *kW* at $cos\varphi_n = 0.8$ $(\varphi_n > 0)$, reactive power of the static capacitors is $Q_n = 2500$ *kVAr*.

Determine the voltage across the substation bus-bars, the line currents, active and reactive powers of the sources as well as their volt-amperes.

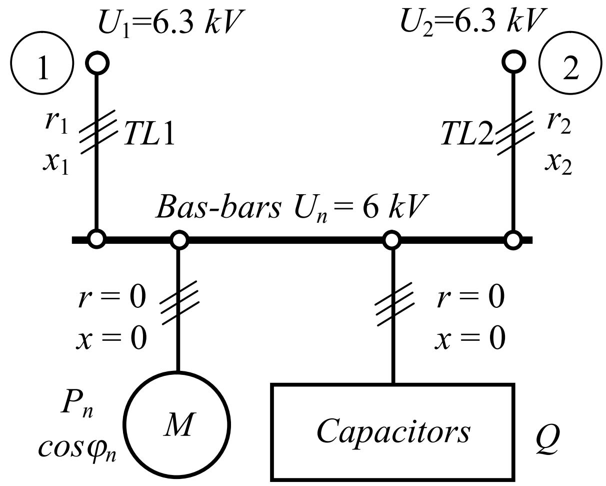


Fig. 4.10

Solution. Let's draw an equivalent scheme of the three-phase system for a single phase (fig. 4.11). Phase emf are calculated through the given line voltages at the beginnings of the transmission lines $U_1 = U_2$: $E_1 = E_2 = \dfrac{U_1}{\sqrt{3}} = \dfrac{6300}{\sqrt{3}} = 3638$ *V*.

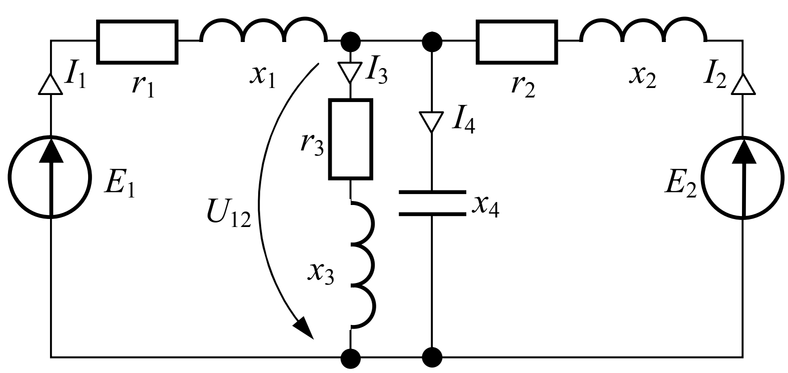


Fig. 4.11

As these voltages are in phase to each other at the parallel work of sources then they have identical initial phases at the corresponding phases. Assume $\psi_{e1} = \psi_{e2} = 0$, then the emf complexes are $\underline{E}_1 = \underline{E}_2 = E_1 = 3638$ *V*.

Complex impedances of the lines per phase are $\underline{Z}_1 = r_1 + jx_1 = 0.5 + j0.3$ *Ohm*, $\underline{Z}_2 = r_2 + jx_2 = 0.4 + j0.6$ *Ohm*.

The parameters of the different taps may be calculated by their nominal data.

For tap 3, we have $P_n = \sqrt{3} \cdot U_n \cdot I_n \cdot cos\varphi_n$, from here

$$I_n = \frac{P_n}{\sqrt{3} U_n cos\varphi_n} = \frac{4000}{\sqrt{3} \cdot 6 \cdot 0.8} = 481\ A;$$

phase impedance at the *Y*-connected load is $Z_3 = \dfrac{U_n}{\sqrt{3} I_n} = \dfrac{6000}{\sqrt{3} \cdot 481} = 7.2$ *Ohm*;

its resistance is $r_3 = Z_3 \cdot cos\varphi_n = 7.2 \cdot 0.8 = 5.76$ *Ohm*;

its reactance is $x_3 = Z_3 \cdot sin\varphi_n = Z_3 \cdot \sqrt{1 - cos^2 \varphi_n} = 7.2 \cdot 0.6 = 4.32$ *Ohm*;

its complex impedance is $\underline{Z}_3 = r_3 + jx_3 = 5.76 + j4.32 = 7.2 \cdot e^{j36.87°}$ *Ohm*.

For capacitors, we have $Q_n = 3 \cdot I_{4n} \cdot x_4 = \sqrt{3} \cdot U_n \cdot I_{4n}$, from here

$I_{4n} = \dfrac{2500}{\sqrt{3} \cdot 6} = 240.6$ *A*; $x_4 = \dfrac{U_n}{\sqrt{3} I_{4n}} = \dfrac{6000}{\sqrt{3} \cdot 240.6} = 14.4$ *Ohm*; $\underline{Z}_4 = -jx_4 = -j14.4$ *Ohm*.

Calculate the scheme fig. 4.11 by method of two nodes.

$$\underline{U}_{12} = \frac{\dfrac{\underline{E}_1}{\underline{Z}_1} + \dfrac{\underline{E}_2}{\underline{Z}_2}}{\underline{Z}_1^{-1} + \underline{Z}_2^{-1} + \underline{Z}_3^{-1} + \underline{Z}_4^{-1}} =$$

$$= 3638 \cdot \frac{\dfrac{1}{0.5 + j0.3} + \dfrac{1}{0.4 + j0.6}}{\dfrac{1}{0.5 + j0.3} + \dfrac{1}{0.4 + j0.6} + \dfrac{1}{-j14.4} + \dfrac{1}{5.76 + j4.32}} =$$

$$= 3530 - j72.8 = 3531 \cdot e^{-j1.18°}\ V;$$

$$\underline{I}_1 = \frac{\underline{E}_1 - \underline{U}_{12}}{\underline{Z}_1} = \frac{3638 - 3530 + j72.8}{0.5 + j0.3} = 223.1 + j11.77 = 223.4 \cdot e^{j3.02°}\ A;$$

$$\underline{I}_2 = \frac{\underline{E}_2 - \underline{U}_{12}}{\underline{Z}_2} = \frac{3638 - 3530 + j72.8}{0.4 + j0.6} = 166.8 - j65.5 = 180.3 \cdot e^{-j22.23°}\ A;$$

$$\underline{I}_3 = \frac{\underline{U}_{12}}{\underline{Z}_3} = \frac{3530 - j72.8}{5.76 + j4.32} = 386.2 - j302.2 = 490.4 \cdot e^{-j38.05°}\ A;$$

$$\underline{I}_4 = \frac{\underline{U}_{12}}{\underline{Z}_4} = \frac{3530 - j72.8}{-j14.4} = 5 + j245 = 245.1 \cdot e^{j88.1°}\ A.$$

The line voltage at the substation bus-bars is

$$U_{12l} = \sqrt{3}\, U_{12} = \sqrt{3} \cdot 3531 = 6115\ B = 6.115\ кB.$$

The currents in lines are $I_1 = 223.4\ A$; $I_2 = 180.3\ A$.

The complex powers of the sources (at the line input) are

$$\underline{S}_1 = 3 \cdot \underline{E}_1 \cdot \overset{*}{\underline{I}}_1 = 3 \cdot 3638 \cdot (223.1 - j11.78) \cdot 10^{-3} = 2435 - j128.5\ kVA,$$

$$\underline{S}_2 = 3 \cdot \underline{E}_2 \cdot \overset{*}{\underline{I}}_2 = 3 \cdot 3638 \cdot (166.8 + j65.5) \cdot 10^{-3} = 1685 + j747.5\ kVA,$$

from here $P_1 = 2435\ kW$, $Q_1 = -128.5\ kVAr$, $S_1 = 2438\ kVA$;
$P_2 = 1685\ kW$, $Q_2 = 747.5\ kVAr$, $S_2 = 1843\ kVA$.

4.2. UNBALANCED THREE-PHASE CIRCUITS

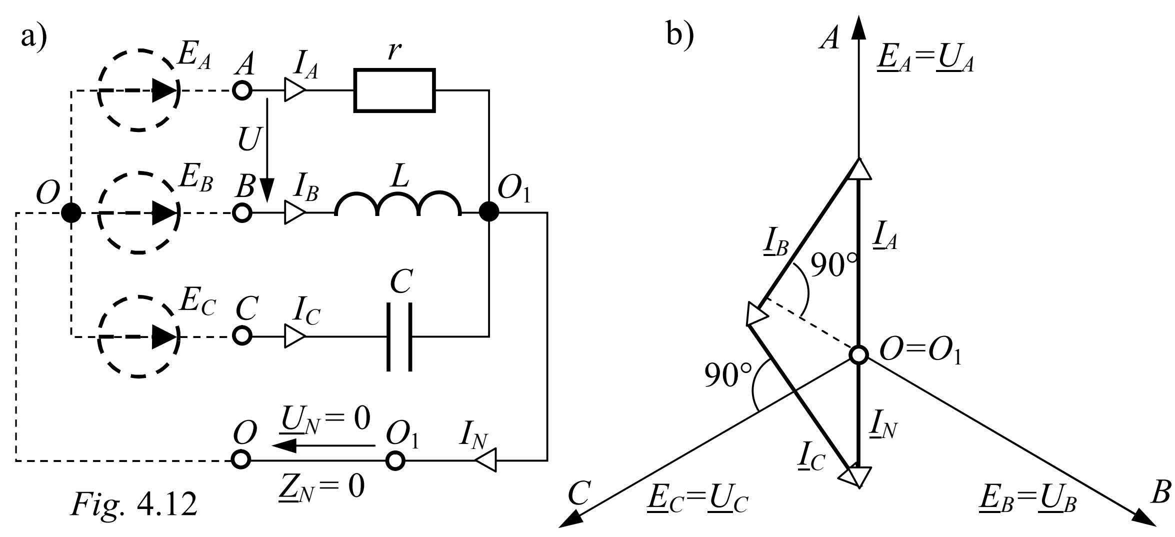


Fig. 4.12

4-8 (4.13). Determine the currents of the four-wire circuit (fig. 4.12,a) as well as the voltages across the phases of the unbalanced consumer supplied from the symmetrical three-phase network of voltage $U = 380$ *V*, if $r = \omega L = \dfrac{1}{\omega C} = 44$ *Ohm*.

Find out the active and reactive powers of the unbalanced consumer, construct a phasor diagram.

Solution. On default, one may assume the phases of the symmetrical generator are *Y*-connected with the neutral point *O* brought out (in fig. 4.12,a it is shown by dash lines). Since the neutral conductor impedance is zero $\underline{Z}_N = 0$, then the potentials are $\underline{\varphi}_0 = \underline{\varphi}_{01} = 0$ and $\underline{U}_N = \underline{\varphi}_0 - \underline{\varphi}_{01} = 0$.

In this case the phase voltages of the unbalanced consumer are equal to the phase emf of the symmetrical generator $\underline{U}_A = \underline{E}_A - \underline{U}_N = \underline{E}_A$, similarly $\underline{U}_B = \underline{E}_B$, $\underline{U}_C = \underline{E}_C$.

The phase emf is $E = \dfrac{U}{\sqrt{3}} = \dfrac{380}{\sqrt{3}} = 220$ *V*.

Having assumed $\underline{E}_A = 220$ *V*, we obtain $\underline{E}_B = 220 \cdot e^{-j120°}$ *V*, $\underline{E}_C = 220 \cdot e^{j120°}$ *V*.

Under Ohm's law, the currents are $\underline{I}_A = \dfrac{\underline{U}_A}{\underline{Z}_A} = \dfrac{220}{44} = 5$ *A*;

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_B} = \frac{220 \cdot e^{-j120}}{44 \cdot e^{j90}} = 5 \cdot e^{-j210°} A; \quad \underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{220 \cdot e^{j120}}{44 \cdot e^{-j90}} = 5 \cdot e^{j210°} A,$$

furthermore, in accordance with Kirchhoff's current law we have

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C = 5 \cdot (1 + e^{-j210°} + e^{j210°}) = 5 \cdot (1 - \sqrt{3}) = -3.64 A.$$

The consumer active power is

$$P = P_A + P_B + P_C = \sum_1^3 I_q^2 r_q = I_A^2 \cdot r = 5^2 \cdot 44 = 1100 W.$$

The reactive power is determined as the algebraic sum of the powers of the consumer three phases:

$$Q = Q_A + Q_B + Q_C = \sum_1^3 I_q^2 x_q = I_B^2 \cdot \omega L - I_C^2 \cdot \frac{1}{\omega C} = 5^2 \cdot 44 - 5^2 \cdot 44 = 0.$$

The circuit phasor diagram is presented in fig. 4.12,b.

4-9 (4.14). Solve the problem 4.8 for the open neutral conductor.

Solution. Let's perform a scheme of installation for calculation (fig. 4.13,a).

Compute the junction voltage (voltage of the neutral displacement)

$$\underline{U}_N = \frac{\dfrac{\underline{E}_A}{\underline{Z}_A} + \dfrac{\underline{E}_B}{\underline{Z}_B} + \dfrac{\underline{E}_C}{\underline{Z}_C}}{\dfrac{1}{\underline{Z}_A} + \dfrac{1}{\underline{Z}_B} + \dfrac{1}{\underline{Z}_C}} = 220 \cdot (1 + e^{-j210°} + e^{j210°}) = -160 V.$$

The consumer phase voltages are

$\underline{U}_A = \underline{E}_A - \underline{U}_N = 220 + 160 = 380$ *V*, $\quad U_A = 380$ *V*;

$\underline{U}_B = \underline{E}_B - \underline{U}_N = 220 \cdot e^{-j120°} + 160 = 50 - j190$ *V*, $\quad U_B = 196.5$ *V*;

$\underline{U}_C = \underline{E}_C - \underline{U}_N = 220 \cdot e^{j120°} + 160 = 50 + j190$ *V*, $\quad U_C = 196.5$ *V*.

The consumer phase currents are equal to the line currents:

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{380}{44} = 8.64\ A, \qquad I_A = 8.64\ A;$$

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_B} = \frac{50 - j190}{j44} = -\,4.32 - j1.14\ A, \qquad I_B = 4.47\ A;$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{50 + j190}{-\,j44} = -\,4.32 + j1.14\ A, \qquad I_C = 4.47\ A.$$

Calculations are checked in accordance with Kirchhoff's current law:

$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$, i.e. it's true.

The consumer active power is $P = \sum_{1}^{3} P_q = I_A^2 \cdot r = 8.64^2 \cdot 44 = 3285\ W.$

The reactive power is $Q = \sum_{1}^{3} Q_q = I_B^2 \cdot \omega L - I_C^2 \cdot \frac{1}{\omega C} = 4.47^2 \cdot 44 - 4.47^2 \cdot 44 = 0.$

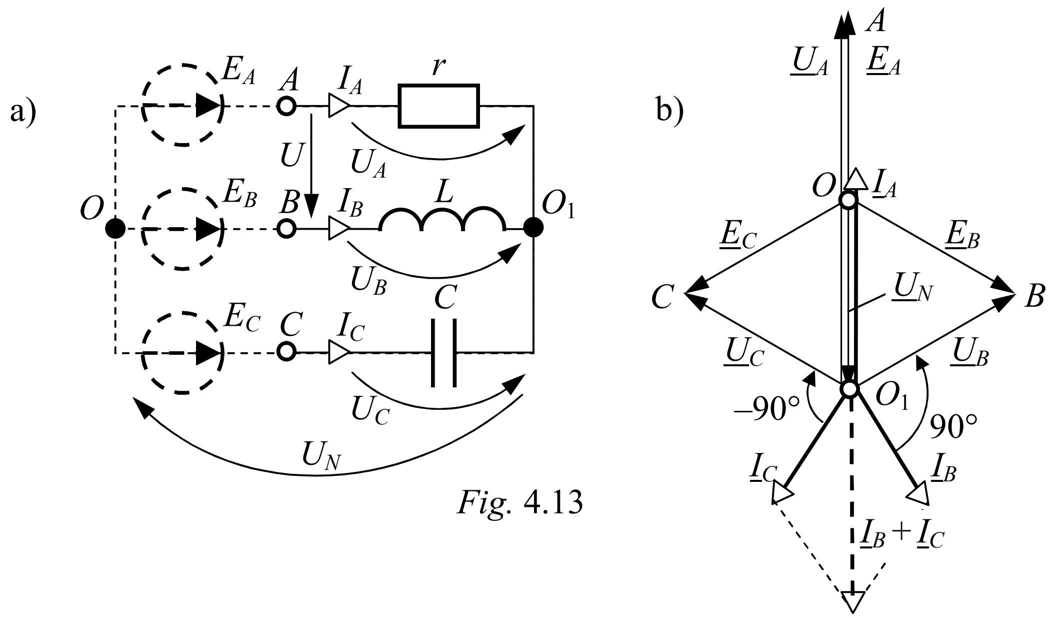


Fig. 4.13

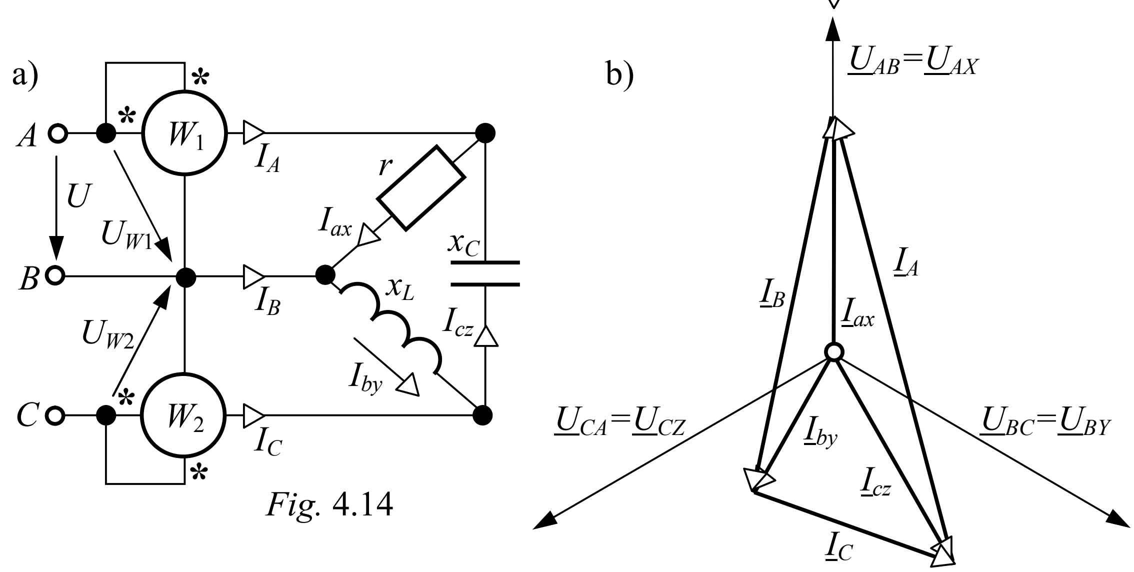


Fig. 4.14

4-10 (4.15). Compute the currents of the unbalanced triangle (fig. 4.14,a), construct a phasor diagram, if $U = 380\ V$, $r = x_C = 100\ Ohm$, $x_L = 100\sqrt{2}\ Ohm$.

Defind the wattmeters readings, compare them with the active power of the unbalanced three-phase consumer.

Solution. When an impedance triangle is connected to a three-phase generator, the generator line voltages are equal to the load phase voltages. Assume

$$\underline{U}_{AB} = 380\ V = \underline{U}_{AX},\ \text{then}$$

$$\underline{U}_{BY} = \underline{U}_{BC} = 380 \cdot e^{-j120°}\ V, \quad \underline{U}_{CZ} = \underline{U}_{CA} = 380 \cdot e^{j120°}\ V.$$

We find the triangle phase currents by Ohm's law:

$$\underline{I}_{ax} = \frac{\underline{U}_{AX}}{\underline{Z}_{AX}} = \frac{380}{100} = 3.8\ A; \quad \underline{I}_{by} = \frac{\underline{U}_{BY}}{\underline{Z}_{BY}} = \frac{380 \cdot e^{-j120}}{100\sqrt{2}e^{j90}} = 1.9\sqrt{2} \cdot e^{-j210°}\ A;$$

$$\underline{I}_{cz} = \frac{\underline{U}_{CZ}}{\underline{Z}_{CZ}} = \frac{380 \cdot e^{j120}}{100e^{-j90}} = 3.8 \cdot e^{j210°}\ A.$$

We find the line currents by Kirchhoff's current law

$\underline{I}_A = \underline{I}_{ax} - \underline{I}_{cz} = 3.8 - 3.8 \cdot e^{j210°} = 3.8 + 3.3 + j1.9 = 7.1 + j1.9 = 7.35 \cdot e^{j15°}\ A;$

$\underline{I}_B = \underline{I}_{by} - \underline{I}_{ax} = -2.3 + j1.34 - 3.8 = -6.13 + j1.34 = -6.28 \cdot e^{-j12.33°}\ A;$

$\underline{I}_C = \underline{I}_{cz} - \underline{I}_{by} = -3.3 - j1.9 + 2.33 - j1.34 = -0.97 - j3.24 = -3.38 \cdot e^{j73.33°}\ A.$

A phasor diagram for the impedance triangle is presented in fig. 4.14,b.

The wattmeters readings are

$$P_{W1} = \text{Re}[\underline{U}_{AB} \cdot \overset{*}{\underline{I}}_A] = \text{Re}[380 \cdot (7.1 - j1.9)] = 2698\ W,$$

$$P_{W2} = \text{Re}[\underline{U}_{CB} \cdot \overset{*}{\underline{I}}_C] = \text{Re}[-380 \cdot e^{-j120°} \cdot (-3.38 \cdot e^{-j73.33°})] = -1250\ W.$$

The active power of the unbalanced load is

$P = P_{AX} + P_{BY} + P_{CZ} = I_{ax}^2 \cdot r = 3.8^2 \cdot 100 = 1444\ W.$

Sum of readings of two wattmeters in the scheme is

$$P_{W1} + P_{W2} = 2698 - 1250 = 1448\ W = P.$$

Thus, the considered scheme of two wattmeters connection is a scheme to measure the active power in a three-phase three-wire system, the difference in the fourth digit of answer is the result of the number rounding up.

4-11 (4.16). The scheme given in problem 4.10 is supplied from the symmetrical source, the phases of which are *Δ*-connected. Assuming the source to be ideal (fig. 4.15,a), a phase emf to be equal to $\underline{E}_{AX} = 380\ V$, determine the generator phase currents.

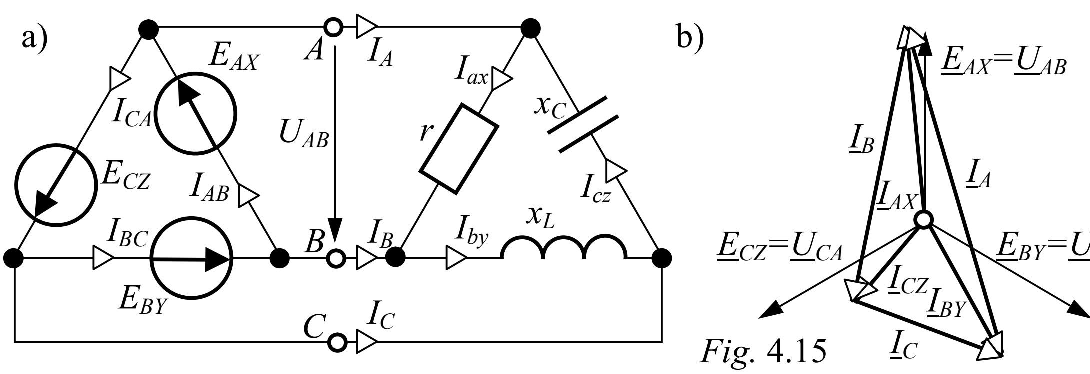


Fig. 4.15

Solution. Since the inner impedance of the phases of the three-phase source is equal to zero, then, at any load current, the line voltages are as follows:

$\underline{U}_{AB} = \underline{E}_{AX} = 380\ V,\ \underline{U}_{BC} = \underline{E}_{BY} = 380 \cdot e^{-j120^\circ}\ V,\ \ \underline{U}_{CA} = \underline{E}_{CZ} = 380 \cdot e^{j120^\circ}\ V;$

which coincides with the line voltages in the scheme given in problem 4.10, the solution of which gave the line currents

$$\underline{I}_A = 7.1 + j1.9\ A; \quad \underline{I}_B = -6.13 + j1.34\ A; \quad \underline{I}_C = -0.97 - j3.24\ A.$$

To compute the generator phase currents, let's establish Kirchhoff's equation system. By Kirchhoff's current law for nodes:

$$A)\ \underline{I}_{AB} - \underline{I}_{CA} = \underline{I}_A; \quad B)\ \underline{I}_{BC} - \underline{I}_{AB} = \underline{I}_B.$$

The equation by Kirchhoff's voltage law for a loop consisting of the generator phases is written taking into account that the inner impedance of a generator phase $\underline{Z}$ can be of any value, however, it is one and the same for each phase, it means that the generator is symmetrical . We obtain $\underline{I}_{AB} \cdot \underline{Z} + \underline{I}_{BC} \cdot \underline{Z} + \underline{I}_{CA} \cdot \underline{Z} = \underline{E}_{AX} + \underline{E}_{BY} + \underline{E}_{CZ}$.

For a symmetrical generator, there is an identity $\underline{E}_{AX} + \underline{E}_{BY} + \underline{E}_{CZ} \equiv 0$, that's why $\underline{Z} \cdot (\underline{I}_{AB} + \underline{I}_{BC} + \underline{I}_{CA}) \equiv 0$ at any inner impedance $\underline{Z}$.

Thus, to compute the source phase currents, it is necessary to solve an equation system

$$\begin{cases} \underline{I}_{AB} - \underline{I}_{CA} = \underline{I}_A; \\ \underline{I}_{BC} - \underline{I}_{AB} = \underline{I}_B; \\ \underline{I}_{AB} + \underline{I}_{BC} + \underline{I}_{CA} = 0; \end{cases}$$

from here taking into account $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$, we have:

$$\underline{I}_{AB} = \frac{\underline{I}_A - \underline{I}_B}{3}; \quad \underline{I}_{BC} = \frac{\underline{I}_B - \underline{I}_C}{3}; \quad \underline{I}_{CA} = \frac{\underline{I}_C - \underline{I}_A}{3}.$$

For an example under consideration, we have:

$$\underline{I}_{AB} = \frac{7.1 + j1.9 + 6.13 - j1.34}{3} = 4.41 + j0.19\ A, \qquad I_{AB} = 4.41\ A;$$

$$\underline{I}_{BC} = \frac{-6.13 + j1.34 + 0.97 + j3.24}{3} = -1.71 + j1.53\ A, \qquad I_{BC} = 2.3\ A;$$

$$\underline{I}_{CA} = \frac{-0.97 - j3.24 - 7.1 - j1.9}{3} = -2.69 - j1.71\ A, \qquad I_{CA} = 3.19\ A.$$

Note, the source phase currents differ from the load phase currents which are in accordance with problem 4.10: $I_{ax} = 3.8\ A$; $I_{by} = 2.7\ A$; $I_{cz} = 3.8\ A$.

A phasor diagram of the source currents is presented in fig. 4.15,b.

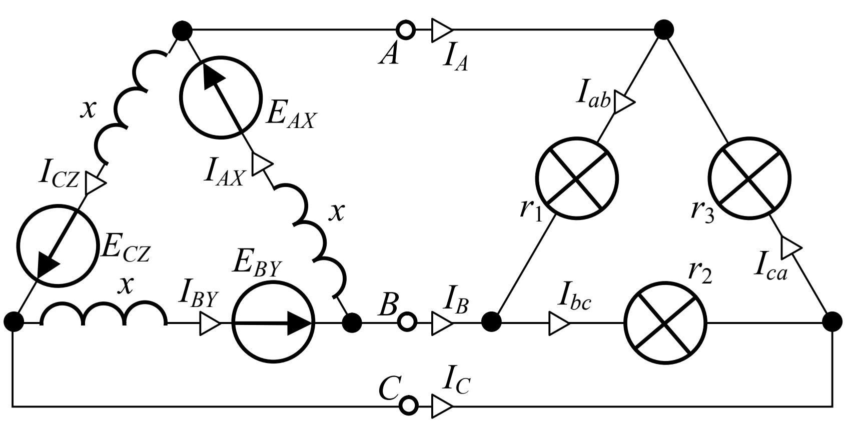


Fig. 4.16

4-12 (4.17). The three-phase system *Δ-Δ* (fig. 4.16) is applied for the energy supply of the lighting in dangerous mediums (coal dust and slack or methane in mines, flour-milling dust, woodworking shops, paint-and-lacquer industry, etc.).

The generator is symmetrical, its phase emf being $E_{AX} = E_{BY} = E_{CZ} = 127\ V$, inner phase impedance being $x = 9\ Ohm$. The load is unbalanced, furthermore, $r_1 = 20\ Ohm$, $r_2 = 30\ Ohm$, $r_3 = 50\ Ohm$. Analyze the circuit state.

Solution. Let's transform the active three-pole network into an equivalent star-connection (fig. 4.17): $E_A = E_B = E_C = \dfrac{E_{AX}}{\sqrt{3}} = \dfrac{127}{\sqrt{3}} = 73.3\ V$, $x_Y = \dfrac{x}{3} = \dfrac{9}{3} = 3\ Ohm$.

Do the same with the load. The impedances of the equivalent star are

$$r_A = \frac{r_1 \cdot r_3}{r_1 + r_2 + r_3} = \frac{20 \cdot 50}{20 + 30 + 50} = 10\ Ohm;$$

$$r_B = \frac{r_1 \cdot r_2}{r_1 + r_2 + r_3} = \frac{20 \cdot 30}{100} = 6\ Ohm; \qquad r_C = \frac{r_2 \cdot r_3}{r_1 + r_2 + r_3} = \frac{30 \cdot 50}{100} = 15\ Ohm.$$

Assume $\underline{E}_A = 73.3\ V$, then $\underline{E}_B = 73.3 \cdot e^{-j120°}\ V$, $\underline{E}_C = 73.3 \cdot e^{j120°}\ V$.

The junction voltage is

$$\underline{U}_N = \frac{\dfrac{\underline{E}_A}{r_A + jx_Y} + \dfrac{\underline{E}_B}{r_B + jx_Y} + \dfrac{\underline{E}_C}{r_C + jx_Y}}{\dfrac{1}{r_A + jx_Y} + \dfrac{1}{r_B + jx_Y} + \dfrac{1}{r_C + jx_Y}} = \frac{\dfrac{73.3}{10 + j3} + \dfrac{73.3 \cdot e^{-j120}}{6 + j3} + \dfrac{73.3 \cdot e^{j120}}{15 + j3}}{\dfrac{1}{10 + j3} + \dfrac{1}{6 + j3} + \dfrac{1}{15 + j3}} =$$

$$= -17.05 \cdot e^{j61.99°} = -8 - j15.05\ V.$$

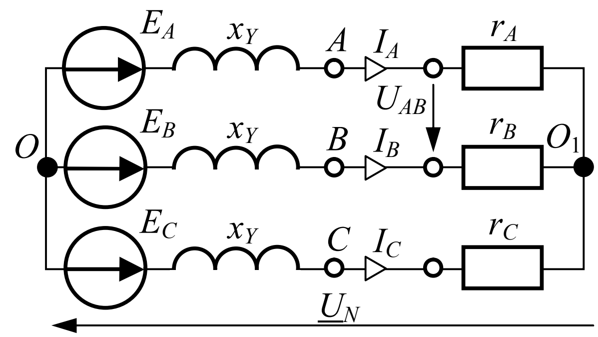


Fig. 4.17

The line currents are

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{r_A + jx_Y} = \frac{73.3 + 8 + j15.05}{10 + j3} =$$

$$= 7.87 - j0.857\ A;$$

$$\underline{I}_B = \frac{\underline{E}_B - \underline{U}_N}{r_B + jx_Y} =$$

$$= \frac{-36.65 - j63.48 + 8 + j15.05}{6 + j3} =$$

$$= -7.047 - j4.545\ A;$$

$$\underline{I}_C = \frac{\underline{E}_C - \underline{U}_N}{r_C + jx_Y} = \frac{-36.65 + j63.48 + 8 + j15.05}{15 + j3} = -0.829 + j5.397\ A.$$

The load line voltage is

$$\underline{U}_{AB} = \underline{I}_A \cdot r_A - \underline{I}_B \cdot r_B = (7.87 - j0.857) \cdot 10 - (-7.047 - j4.545) \cdot 6 =$$

$$= 121 + j18.7 = 122.4 \cdot e^{j8.79°}\ V.$$

The phase currents of the lighting load are (fig. 4.16)

$$\underline{I}_{ab} = \frac{\underline{U}_{AB}}{r_1} = \frac{121 + j18.7}{20} = 6.05 + j0.935 = 6.122 \cdot e^{j8.79°}\ A;$$

$$\underline{I}_{ca} = \underline{I}_{ab} - \underline{I}_A = 6.05 + j0.935 - 7.87 + j0.857 = -1.82 + j1.792 = 2.554 \cdot e^{j135.44°}\ A;$$

$$\underline{I}_{bc} = \underline{I}_{ab} + \underline{I}_B = 6.05 + j0.935 - 7.047 - j4.545 = -0.997 - j3.61 = 3.745 \cdot e^{-j105.44°}\ A.$$

The line voltages are

$$U_{BC} = I_{bc} \cdot r_2 = 112.4\ V, \quad U_{CA} = I_{ca} \cdot r_3 = 127.7\ V.$$

The active power of the lighting load and generator is

$$P = P_{AB} + P_{BC} + P_{CA} = I_{ab}^2 \cdot r_1 + I_{bc}^2 \cdot r_2 + I_{ca}^2 \cdot r_3 =$$

$$= 6.122^2 \cdot 10 + 3.745^2 \cdot 30 + 2.554^2 \cdot 50 = 1122\ W = 1.122\ kW.$$

The phase currents of the source (see problem 4.11)

$$\underline{I}_{AX} = \frac{\underline{I}_A - \underline{I}_B}{3} = 4.972 + j1.229 = 5.122 \cdot e^{j13.88°}\ A;$$

$$\underline{I}_{BY} = \frac{\underline{I}_B - \underline{I}_C}{3} = -2.073 - j3.314 = -3.91 \cdot e^{j57.97°}\ A;$$

$$\underline{I}_{CZ} = \frac{\underline{I}_C - \underline{I}_A}{3} = -2.9 + j2.085 = -3.572 \cdot e^{-j35.71°}\ A.$$

The reactive power of the generator (and of the whole circuit) is

$$Q = Q_{AX} + Q_{BY} + Q_{CZ} = I_{AX}^2 \cdot x + I_{BY}^2 \cdot x + I_{CZ}^2 \cdot x =$$
$$= 9 \cdot (5.122^2 + 3.91^2 + 3.572^2) = 488.5\ VAr = 0.4885\ kVAr.$$

The source volt-amperes are

$$S = \sqrt{P^2 + Q^2} = \sqrt{1.122^2 + 0.4885^2} = 1.224\ kVA.$$

The source power factor is $cos\varphi = \dfrac{P}{S} = \dfrac{1.122}{1.224} = 0.917.$

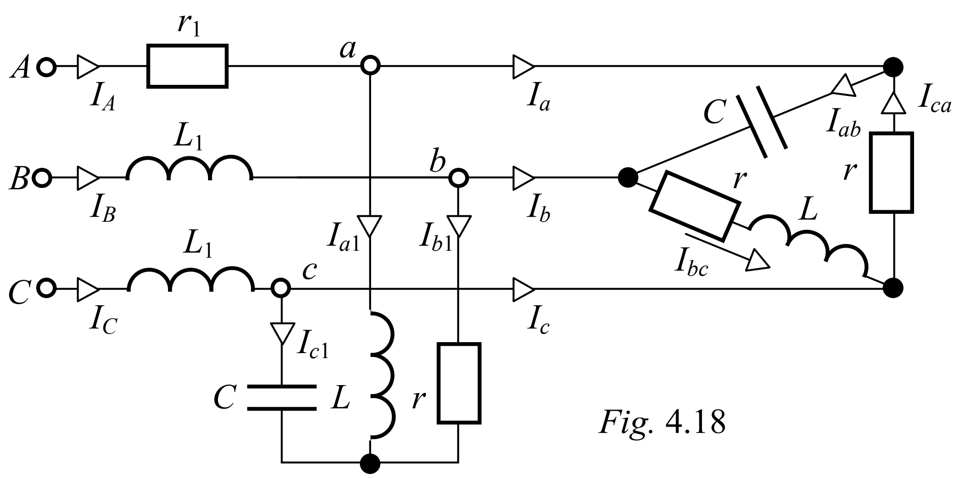


Fig. 4.18

4.13 (4.18). A three-phase circuit (fig. 4.18) is connected to a symmetrical generator of voltage $U = 660\ V$. The circuit parameters are

$$r = \omega L = \frac{1}{\omega C} = 10\ Ohm, \quad r_1 = \omega L_1 = 5\ Ohm.$$

Compute the line and phase currents of all the circuit parts.

Solution. Let's modify the scheme by means of the method of equivalent simplifications. Initially, substitute the equivalent impedance triangle for the non-balanced star without a neutral conductor:

$$\underline{Z}_{ab1} = j\omega L + r + \frac{r \cdot j\omega L}{-j\dfrac{1}{\omega C}} = j10 + 10 + \frac{10 \cdot j10}{-j10} = j10\ Ohm,$$

$$\underline{Z}_{bc1} = r - j\frac{1}{\omega C} + \frac{r \cdot \left(-j\dfrac{1}{\omega C}\right)}{j\omega L} = 10 - j10 + \frac{10 \cdot (-j10)}{j10} = -j10\ Ohm,$$

$$\underline{Z}_{ca1} = j\omega L - j\frac{1}{\omega C} + \frac{j\omega L \cdot \left(-j\frac{1}{\omega C}\right)}{r} = j10 - j10 + \frac{j10 \cdot (-j10)}{10} = j10 \ Ohm.$$

Henceforward, two impedance triangles are connected in parallel (fig. 4.19,a).

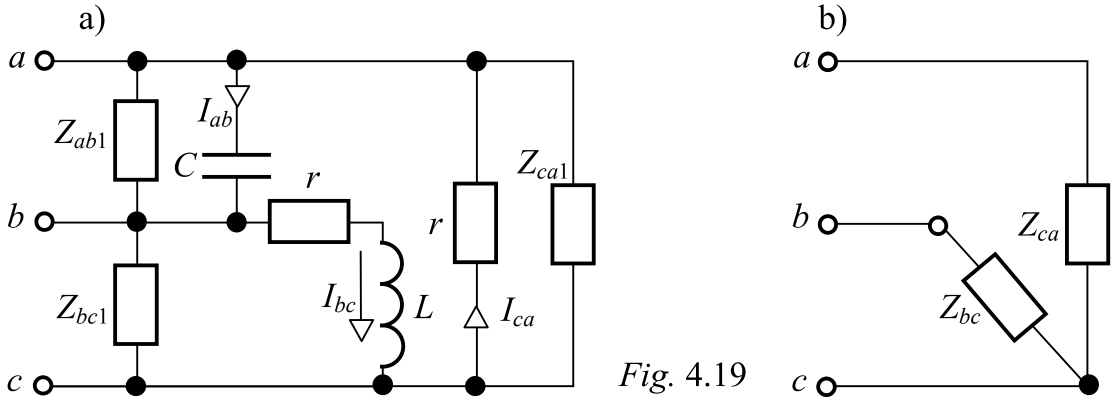


Fig. 4.19

These two triangles may be replaced by the equivalent one with the following phase impedances

$\underline{Z}_{ab} = \dfrac{\underline{Z}_{ab1} \cdot \left(-j\frac{1}{\omega C}\right)}{\underline{Z}_{ab1} - j\frac{1}{\omega C}} = \dfrac{j10(-j10)}{j10 - j10} = \infty$, consequently, there is the current resonance in this parallel lossless loop, its input impedance being infinite (open circuit as regards to the rest of the scheme);

$$\underline{Z}_{bc} = \frac{\underline{Z}_{bc1} \cdot (j\omega L + r)}{\underline{Z}_{bc1} + j\omega L + r} = \frac{-j10(j10+10)}{-j10 + j10 + 10} = 10 - j10 \ Ohm;$$

$$\underline{Z}_{ca} = \frac{\underline{Z}_{ca1} \cdot r}{\underline{Z}_{ca1} + r} = \frac{10 \cdot 10}{10+10} = 5 \ Ohm.$$

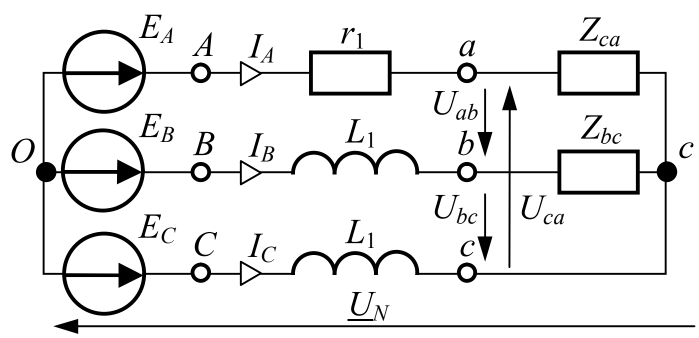


Fig. 4.20

As a result, we have an equivalent scheme of the load connected to the terminals *a-b-c*, it being presented in fig. 4.19,b.

Imagine, the coils of the symmetrical three-phase source are *Y*-connected with emf

$$E = \frac{U}{\sqrt{3}} = \frac{660}{\sqrt{3}} = 380 \ V = \underline{E}_A.$$

We obtain the transformed scheme fig. 4.20. For this scheme

$$\underline{U}_N = \frac{\frac{\underline{E}_A}{r_1 + \underline{Z}_{ca}} + \frac{\underline{E}_B}{j\omega L_1 + \underline{Z}_{bc}} + \frac{\underline{E}_C}{j\omega L_1}}{\frac{1}{r_1 + \underline{Z}_{ca}} + \frac{1}{j\omega L_1 + \underline{Z}_{bc}} + \frac{1}{j\omega L_1}} = \frac{\frac{380}{5+5} + \frac{380 \cdot e^{-j120}}{j5 + 10 - j10} + \frac{380 \cdot e^{j120}}{j5}}{\frac{1}{5+5} + \frac{1}{j5+10-j10} + \frac{1}{j5}} = 200 + j225 \ V.$$

The generator line currents are

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{r_1 + \underline{Z}_{ca}} = \frac{380 - 200 - j225}{5 + 5} = 18 - j22.5\ A;$$

$$\underline{I}_B = \frac{\underline{E}_B - \underline{U}_N}{j\omega L_1 + \underline{Z}_{bc}} = \frac{-190 - j330 - 200 - j225}{j5 + 10 - j10} = -\ 39 - j55.5\ A;$$

$$\underline{I}_C = \frac{\underline{E}_C - \underline{U}_N}{j\omega L_1} = \frac{-190 + j330 - 200 - j225}{j5} = 21 + j78\ A.$$

The line voltages across the consumer terminals on the ground of the scheme fig. 4.20 are:

$\underline{U}_{ab} = \underline{I}_A \cdot \underline{Z}_{ca} - \underline{I}_B \cdot \underline{Z}_{bc} = (18 - j22.5) \cdot 5 - (-\ 39 - j55.5) \cdot (10 - j10) = 1035 + j52.5\ V$;

$\underline{U}_{bc} = \underline{I}_B \cdot \underline{Z}_{bc} = (-\ 39 - j55.5) \cdot (10 - j10) = -\ 945 - j165\ V$;

$\underline{U}_{ca} = -\ \underline{I}_A \cdot \underline{Z}_{ca} = -\ (18 - j22.5) \cdot 5 = -\ 90 + j112.5\ V$.

Further, recur to the initial scheme fig. 4.18 and work out the phase currents of the triangle

$$\underline{I}_{ab} = \frac{\underline{U}_{ab}}{-\ j\dfrac{1}{\omega C}} = \frac{1035 + j52.5}{-\ j10} = -\ 5.25 + j103.5\ A;$$

$$\underline{I}_{bc} = \frac{\underline{U}_{bc}}{r + j\omega L} = \frac{-\ 945 - j1655}{10 + j10} = -\ 55.5 + j39\ A;$$

$$\underline{I}_{ca} = \frac{\underline{U}_{ca}}{r} = \frac{-\ 90 + j112.5}{10} = -\ 9 + j11.25\ A.$$

The line currents of the triangle are

$$\underline{I}_a = \underline{I}_{ab} - \underline{I}_{ca} = 3.75 + j92.25\ A;$$
$$\underline{I}_b = \underline{I}_{bc} - \underline{I}_{ab} = -\ 50.25 - j64.5\ A;$$
$$\underline{I}_c = \underline{I}_{ca} - \underline{I}_{bc} = 46.5 - j27.75\ A.$$

The currents of the *Y*-connected consumer are computed under Kirchhoff's current law:

$$\underline{I}_{a1} = \underline{I}_A - \underline{I}_a = 14.25 - j114.75\ A;$$
$$\underline{I}_{b1} = \underline{I}_B - \underline{I}_b = 11.25 + j9\ A;$$
$$\underline{I}_{c1} = \underline{I}_C - \underline{I}_c = -\ 56.5 + j105.75\ A.$$

4.3. SPECIAL CASES OF UNBALANCED LOAD IN THREE-PHASE CIRCUITS

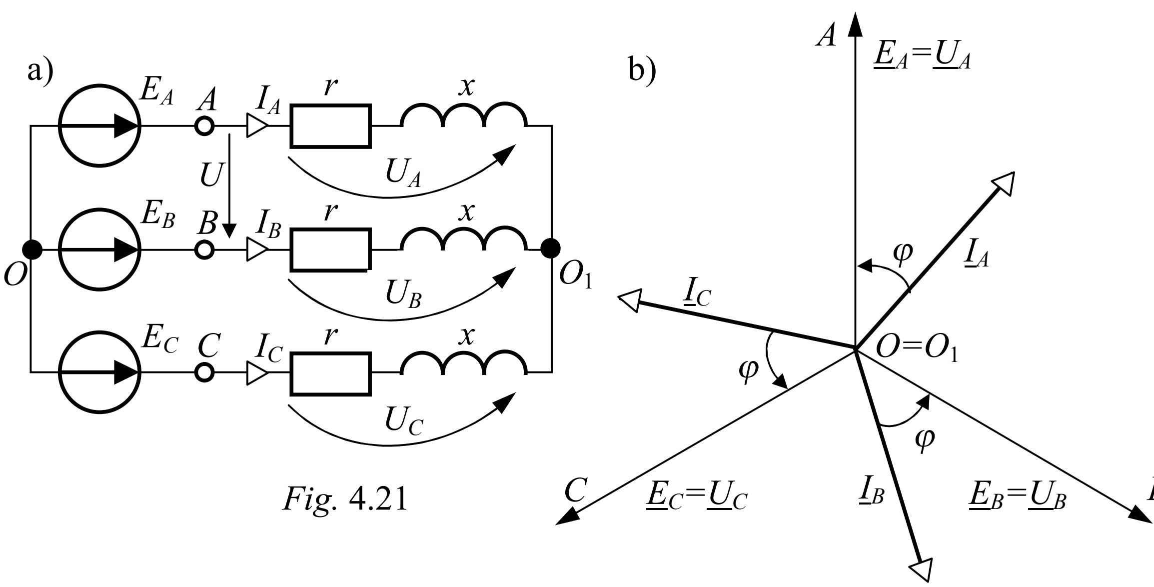


Fig. 4.21

4-14 (4.20). For a symmetrical three-phase system “star-star without a neutral conductor” (fig. 4.21,a), compute the working modes for the following cases:

- symmetrical mode;
- the line conductor A is broken;
- the short-circuit in phase A.

The scheme parameters are: $U = 380\ V$, $r = x = 20\ Ohm$.

Solution. In fig. 4.21,b, a phasor diagram of the **symmetrical mode** of the system Y-Y is presented. Furthermore, the voltage of the neutral displacement is absent $U_N = 0$.

The generator phase emf and the consumer phase voltages

$$\underline{E}_A = \underline{U}_A = \frac{U}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220\ V;\quad \underline{E}_B = \underline{U}_B = 220\cdot e^{-j120^\circ}\ V;\quad \underline{E}_C = \underline{U}_C = 220\cdot e^{j120^\circ}\ V;$$

as well as the currents $\underline{I}_A = \dfrac{\underline{U}_A}{\underline{Z}} = \dfrac{220}{20 + j20} = 7.78\cdot e^{-j45^\circ}\ A;$

$$\underline{I}_B = \underline{I}_A\cdot e^{-j120^\circ} = 7.78\cdot e^{-j165^\circ}\ A;\quad \underline{I}_C = \underline{I}_A\cdot e^{j120^\circ} = 7.78\cdot e^{j75^\circ}\ A$$

form the phasor symmetric systems.

When **the line conductor** A **breaks**, it is allowable to suppose that the additional impedance of the break $\underline{Z}_{br} = \infty$ is connected in series with the phase impedance $\underline{Z} = r + jx = 20 + j20\ Ohm$, the impedance of the branch A being equal to $\underline{Z}_A = \underline{Z}_{br} + \underline{Z} = \infty$. The junction voltage (it is termed a *voltage of the neutral displacement*) is as follows

$$\underline{U}_N = \frac{\dfrac{\underline{E}_A}{\underline{Z}_A} + \dfrac{\underline{E}_B}{\underline{Z}} + \dfrac{\underline{E}_C}{\underline{Z}}}{\dfrac{1}{\underline{Z}_A} + \dfrac{1}{\underline{Z}} + \dfrac{1}{\underline{Z}}} = \frac{\dfrac{1}{\underline{Z}}(\underline{E}_B + \underline{E}_C)}{\dfrac{2}{\underline{Z}}} = -\frac{\underline{E}_A}{2} = -\frac{220}{2} = -110\ V.$$

When calculating we take into account, that at the break of the wire A its conductance is zero $\dfrac{1}{\underline{Z}_A} = \dfrac{1}{\infty} = 0$ and the ratio $\dfrac{\underline{E}_A}{\underline{Z}_A} = \dfrac{\underline{E}_A}{\infty} = 0$; so, in the symmetrical system there is an identity

$$\underline{E}_A + \underline{E}_B + \underline{E}_C \equiv 0,\quad \text{from here}\quad \underline{E}_B + \underline{E}_C = -\underline{E}_A.$$

The voltage across the impedance $\underline{Z}_A$ $\quad \underline{U}_A = \underline{E}_A - \underline{U}_N = 1.5\cdot\underline{E}_A = 330\ V$ is the voltage between the break points of the wire A, but there is no current

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{\underline{U}_A}{\infty} = 0.$$

Voltages and currents of the fault-free phases are

$$\underline{U}_B = \underline{E}_B - \underline{U}_N = 220\cdot e^{-j120^\circ} + 110 = -j190\ V;\qquad \underline{I}_B = \frac{\underline{U}_B}{\underline{Z}} = \frac{-j190}{20 + j20} = 6.72\cdot e^{-j135^\circ}\ A;$$

$$\underline{U}_C = \underline{E}_C - \underline{U}_N = 220\cdot e^{j120^\circ} + 110 = j190\ V;\qquad \underline{I}_C = \frac{\underline{U}_C}{\underline{Z}} = \frac{j190}{20 + j20} = 6.72\cdot e^{j45^\circ}\ A.$$

Note, when the line conductor breaks a three-phase circuit turns into a single-phase one, that’s why the currents of the fault-free phases may be determined in a simpler way: $\quad \underline{I}_B = -\underline{I}_C = \dfrac{\underline{U}_{BC}}{2\underline{Z}} = \dfrac{\underline{E}_B - \underline{E}_C}{2\underline{Z}}.$

A phasor diagram of the given system *Y-Y* without the neutral conductor when the line conductor *A* is broken is presented in fig. 4.22,a.

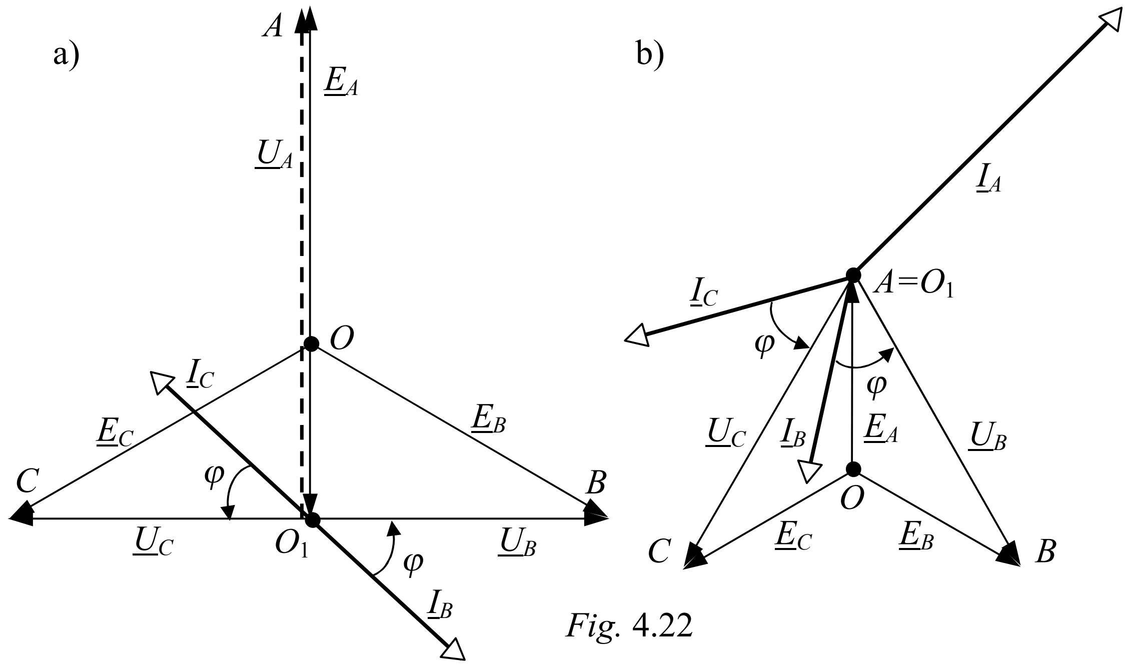


Fig. 4.22

Comparing the voltages across fault-free phases at the break of the line conductor *A* *($U_B = U_C$ = 190 V)* and at the symmetrical condition when all the voltages are $U_A = U_B = U_C = 220$ *V*, one may note the voltage decrease by 13.7%, which is inadmissible for the lighting supply.

When there is a **short-circuit of phase** *A* the scheme is calculated like the branched circuit with performance of the limit passage $\underline{Z}_A \to 0$:

$$\underline{U}_N = \frac{\dfrac{\underline{E}_A}{\underline{Z}_A} + \dfrac{\underline{E}_B}{\underline{Z}} + \dfrac{\underline{E}_C}{\underline{Z}}}{\dfrac{1}{\underline{Z}_A} + \dfrac{1}{\underline{Z}} + \dfrac{1}{\underline{Z}}} = \lim_{\underline{Z}_A \to 0} \frac{\dfrac{\underline{E}_A}{\underline{Z}_A}}{\dfrac{1}{\underline{Z}_A}} = \underline{E}_A.$$

Voltages and currents of the fault-free phases are

$$\underline{U}_B = \underline{E}_B - \underline{U}_N = \underline{E}_B - \underline{E}_A = -\underline{U}_{AB} = -380 \cdot e^{j30°} = 380 \cdot e^{-j150°} \; V;$$

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}} = \frac{380 \cdot e^{-j150}}{20 + j20} = 13.44 \cdot e^{-j195°} \; A;$$

$$\underline{U}_C = \underline{E}_C - \underline{U}_N = \underline{E}_C - \underline{E}_A = \underline{U}_{CB} = 380 \cdot e^{j150°} \; V;$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}} = \frac{380 \cdot e^{j150}}{20 + j20} = 13.44 \cdot e^{j105°} \; A.$$

A current of the short-circuited phase is

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_A} = \frac{0}{0} = -(\underline{I}_B + \underline{I}_C) = -\sqrt{3}\, \underline{I}_C \cdot e^{j30°} = -23.3 \cdot e^{j135°} = 23.3 \cdot e^{-j45°} \; A.$$

The expression for the current $\underline{I}_A$ follows the condition of the three-wire system

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0.$$

A circuit phasor diagram at the short-circuit in phase *A* is presented in fig. 4.22,b.

4-15 (4.21). For the symmetric system "star-star with a neutral conductor" ($\underline{Z}_N = 0$) (fig. 4.23,a), make calculations for three cases:

- symmetrical mode;
- the line conductor *B* is broken;
- short-circuited phase *B*, if: $U = 220$ *V*, $\underline{Z} = r = 20$ *Ohm*.

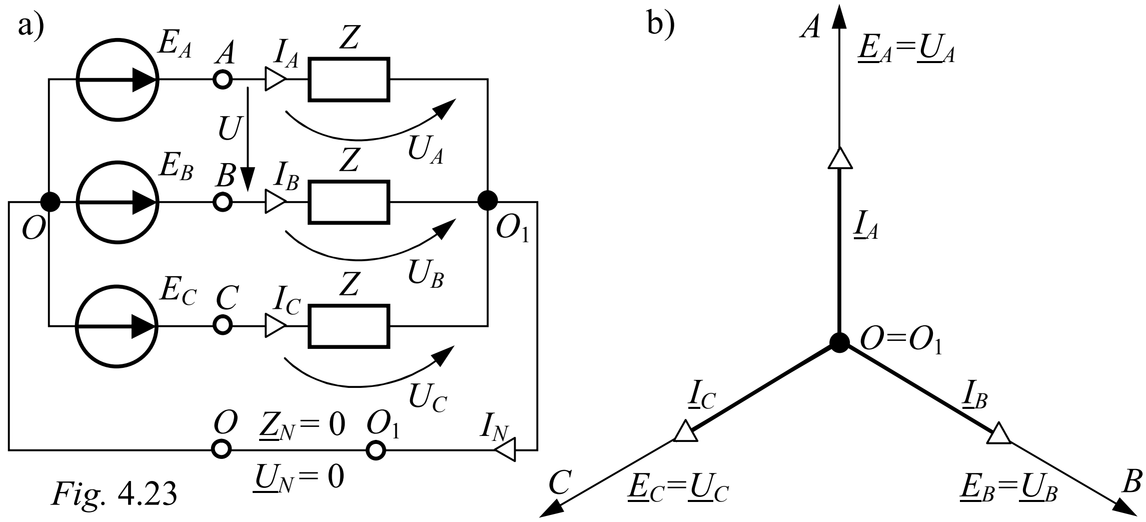


Fig. 4.23

Solution. There is a phasor diagram for **symmetrical mode** of scheme in fig. 4.23,b, furthermore, this time there is $\underline{U}_N = 0$ because of the zero impedance $\underline{Z}_N$.

Then, the phase voltages of the generator and load are equal to each other and are $\sqrt{3}$ times less than the line voltages: $U_{ph} = E = \dfrac{U}{\sqrt{3}} = \dfrac{220}{\sqrt{3}} = 127$ *V*.

Assume $\underline{E}_A = U_{ph} = 127$ *V*, then $\underline{E}_B = 127 \cdot e^{-j120°}$ *V*, $\underline{E}_C = 127 \cdot e^{j120°}$ *V*;

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}} = \frac{127}{20} = 6.35\,A;\ \underline{I}_B = \underline{I}_A \cdot e^{-j120°} = 6.35 \cdot e^{-j120°}\,A;\ \underline{I}_C = \underline{I}_A \cdot e^{j120°} = 6.35 \cdot e^{j120°}\,A.$$

When there is a **break of the line conductor** *B*, it is possible to suppose the break impedance $\underline{Z}_{br} = \infty$ is added in series to impedance $\underline{Z}$ in phase *B*, then the branch impedance has become $\underline{Z}_B = \underline{Z} + \underline{Z}_{br} = \infty$.

Under Kirchhoff's voltage law for loops "a branch – neutral conductor" when there is a break of the line conductor *B*, we have:

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}} = \frac{\underline{E}_A}{\underline{Z}} = \frac{127}{20} = 6.35\,A$$ – the same value as in the symmetrical mode,

$$\underline{I}_B = \frac{\underline{E}_B - \underline{U}_N}{\underline{Z}_B} = \frac{\underline{E}_B}{\underline{Z}_B} = \frac{\underline{E}_B}{\infty} = 0;$$

$$\underline{I}_C = \frac{\underline{E}_C - \underline{U}_N}{\underline{Z}} = \frac{\underline{E}_C}{\underline{Z}} = \frac{127 \cdot e^{j120}}{20} = 6.35 \cdot e^{j120°}\,A$$ – the same value as in the symmetrical mode.

The neutral conductor current is

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C = 6.35 + 0 + 6.35 \cdot e^{j120°} = -6.35 \cdot e^{-j120°} = 6.35 \cdot e^{j60°}\,A.$$

Note, $\underline{I}_N = -\underline{I}_{B\,sym}$. Because of this case, it is said *the neutral conductor takes the current of the fault phase upon itself*. A circuit phasor diagram at the break of the line conductor *B* is presented in fig. 4.24.

At the **short-circuited phase** B in a four-wire system, the current of the short-circuited loop $\underline{E}_B$ - $\underline{Z}_B$ - $\underline{Z}_N$ rises beyond all bounds. This is a *malfunction*. To protect against such a malfunction, a fuse is used in the line conductor B (as well as in the rest line conductors), it cuts out the faulted phase (it fuses), whereupon the scheme passes into he mode with broken phase B (fig. 4.23).

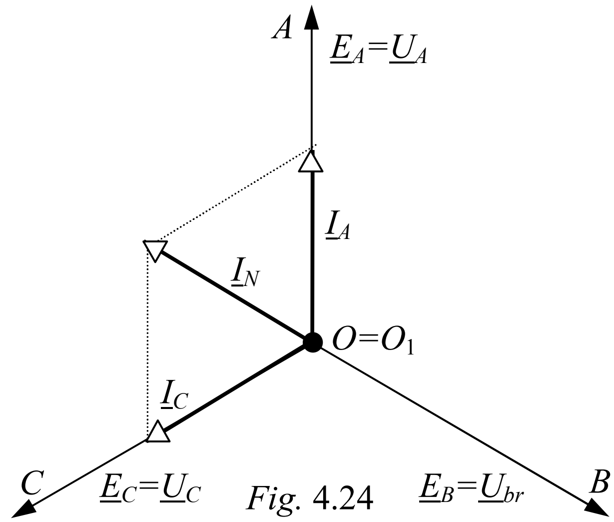


Fig. 4.24

4-16 (4.22). Compute a working mode of the balanced triangle (fig. 4.25,a) at $U = 660$ V, $x_C = 100$ Ohm for the following four cases, namely:

- symmetrical mode;
- the line conductor C is broken;
- the phase CZ is broken;
- the phase CZ is short-circuited.

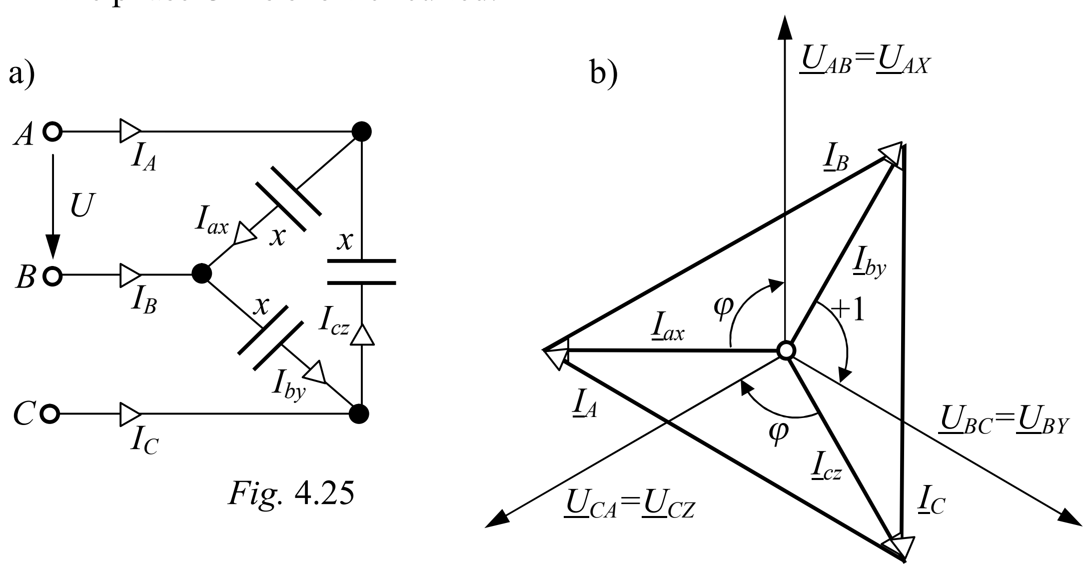


Fig. 4.25

Solution. Assume $\underline{U}_{AB} = U = 660$ V.

If the load phases are Δ-connected, the load line voltages and phase voltages are the same. In the **symmetrical mode**, we have

$$\underline{U}_{AX} = \underline{U}_{AB} = 660\ V,\quad \underline{U}_{BY} = \underline{U}_{BC} = 660\cdot e^{-j120°}\ V,\quad \underline{U}_{CZ} = \underline{U}_{CA} = 660\cdot e^{j120°}\ V.$$

The phase currents of triangle are

$$\underline{I}_{ax} = \frac{\underline{U}_{AX}}{\underline{Z}} = \frac{660}{-j100} = j6.6 = 6.6\cdot e^{j90°}\ A;$$

$$\underline{I}_{by} = \frac{\underline{U}_{BY}}{\underline{Z}} = \underline{I}_{ax}\cdot e^{-j120°} = 6.6\cdot e^{-j30°}\ A;\quad \underline{I}_{cz} = \frac{\underline{U}_{CZ}}{\underline{Z}} = \underline{I}_{ax}\cdot e^{j120°} = 6.6\cdot e^{j210°}\ A.$$

The line currents of triangle are

$$\underline{I}_A = \underline{I}_{ax} - \underline{I}_{cz} = \sqrt{3}\,\underline{I}_{ax}\cdot e^{-j30°} = 11.4\cdot e^{j60°}\ A;$$

$$\underline{I}_B = \underline{I}_A\cdot e^{-j120°} = 11.4\cdot e^{-j60°}\ A;\quad \underline{I}_C = \underline{I}_A\cdot e^{j120°} = 11.4\cdot e^{j180°} = -\,11.4\ A.$$

A phasor diagram of the balanced triangle is presented in fig. 4.25,b.

At the **broken line conductor** C, all the currents and the voltages of the load are forced by but the line voltage $\underline{U}_{AB}$.

The current $\underline{I}_{ax} = \dfrac{\underline{U}_{AX}}{\underline{Z}} = \dfrac{660}{-j100} = j6.6\ A$ – has previous value.

The current $\underline{I}_C = 0$, because the conductor C is broken; so, the currents

$$\underline{I}_{by} = \underline{I}_{cz} = -\frac{\underline{U}_{AB}}{2\underline{Z}} = -\frac{\underline{I}_{ax}}{2} = -j3{,}3\ A,$$

voltages across the phases are $\underline{U}_{BY} = \underline{U}_{CZ} = \underline{I}_{by} \cdot \underline{Z} = -\dfrac{\underline{U}_{AB}}{2} = -330\ V.$

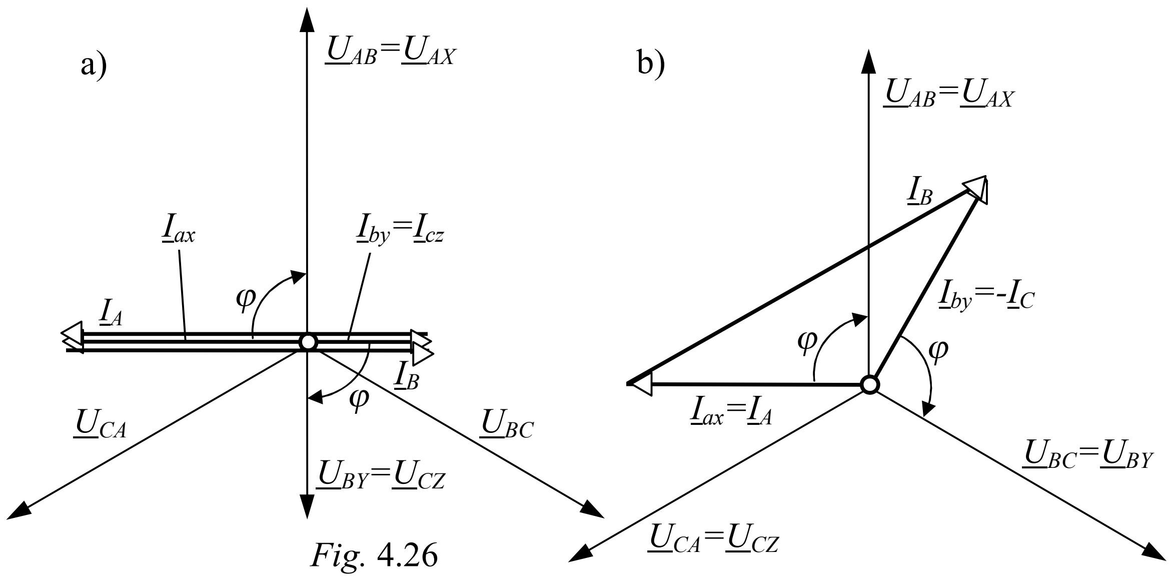


Fig. 4.26

The line currents are $\underline{I}_A = \underline{I}_{ax} - \underline{I}_{cz} = 1.5 \cdot \underline{I}_{ax} = j9.9\ A$;

$\underline{I}_B = \underline{I}_{by} - \underline{I}_{ax} = -1.5 \cdot \underline{I}_{ax} = -j9.9\ A$.

A phasor diagram of the impedance triangle at the break of the line conductor C is presented in fig. 4.26,a.

At the **break of the phase** CZ, its current is $\underline{I}_{cz} = 0$, while the phase currents $\underline{I}_{ax} = j6.6\ A$, $\underline{I}_{by} = 6.6 \cdot e^{-j30°}\ A$ are the same as in the symmetrical mode.

The line current $\underline{I}_A = \underline{I}_{ax} - \underline{I}_{cz} = \underline{I}_{ax} = j6.6\ A$, next line current $\underline{I}_B = \underline{I}_{by} - \underline{I}_{ax} = 11.4 \cdot e^{-j60°}\ A$ is the same as in the symmetrical mode, at last the line current $\underline{I}_C = -\underline{I}_{by} = 6.6 \cdot e^{j150°}\ A$.

A circuit phasor diagram at the break of the phase CZ is presented in fig. 4.26,b.

At the short-circuited phase CZ in loop «A – short-circuited phase CZ – B – source», there are no impedances, the current rising beyond all the bounds, a malfunction occurring; it is necessary to disconnect the wires A or C from the network.

4.4. METHOD OF SYMMETRIC COMPONENTS

4-17 (4.25). While connecting the secondary windings of the mains transformer, the start and the finish the winding BY were determined incorrectly. As a result, after the coils are Y-connected (fig. 4.27,a), a phasor system of emf takes a form fig. 4.27,b.

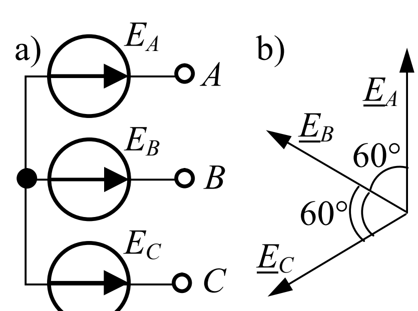


Fig. 4.27

Find the symmetric components of the above-presented non-symmetric system, if $E_A = E_B = E_C = 220\ V$.

Solution. Assume $\underline{E}_A = 220\ V$ (fig. 4.27,b), then

$$\underline{E}_B = 220\cdot e^{j60^\circ}\ V,\quad \underline{E}_C = 220\cdot e^{j120^\circ}\ V.$$

Zero sequence component is

$$\underline{E}_0 = \frac{1}{3}(\underline{E}_A + \underline{E}_B + \underline{E}_C) = \frac{1}{3}(220 + 220\cdot e^{j60^\circ} + 220\cdot e^{j120^\circ}) = \frac{440}{3}\cdot e^{j60^\circ} = 146.7\cdot e^{j60^\circ}\ V$$

Positive sequence component is

$$\underline{E}_1 = \frac{1}{3}(\underline{E}_A + a\cdot\underline{E}_B + a^2\cdot\underline{E}_C),\ \text{ where }\ a = e^{j120^\circ},\ \text{ from here}$$

$$\underline{E}_1 = \frac{220}{3}(1 + e^{j120^\circ}\cdot e^{j60^\circ} + e^{-j120^\circ}\cdot e^{j120^\circ}) = \frac{220}{3} = 73.33\ V.$$

Negative sequence component is

$$\underline{E}_2 = \frac{1}{3}(\underline{E}_A + a^2\cdot\underline{E}_B + a\cdot\underline{E}_C) = \frac{220}{3}(1 + e^{-j120^\circ}\cdot e^{j60^\circ} + e^{j120^\circ}\cdot e^{j120^\circ}) = 146.7\cdot e^{-j60^\circ}\ V.$$

Let’s verify the result of the phasor decomposition onto the symmetric components:

$$\underline{E}_A = \underline{E}_0 + \underline{E}_1 + \underline{E}_2 = 146.7\cdot e^{j60^\circ} + 73.33 + 146.7\cdot e^{-j60^\circ} = 220\ V;$$

$$\underline{E}_B = \underline{E}_0 + a^2\cdot\underline{E}_1 + a\cdot\underline{E}_2 = 146.7\cdot e^{j60^\circ} + 73.33\cdot e^{-j120^\circ} + 146.7\cdot e^{-j60^\circ}\cdot e^{j120^\circ} = 220\cdot e^{j60^\circ}\ V;$$

$$\underline{E}_C = \underline{E}_0 + a\cdot\underline{E}_1 + a^2\cdot\underline{E}_2 = 146.7\cdot e^{j60^\circ} + 73.33\cdot e^{j120^\circ} + 146.7\cdot e^{-j60^\circ}\cdot e^{-j120^\circ} = 220\cdot e^{j120^\circ}\ V.$$

4-18 (4.26). As the load at the end of the three-phase three-wire line is unbalanced (fig. 4.28,a), the line voltage phasors form a right triangle (fig. 4.28,b), its legs being $U_{AB} = U_{BC} = 360\ V$, furthermore, the sinusoid of the line voltage U_{AB} leads the voltage sinusoid U_{BC} by $\pi/2$.

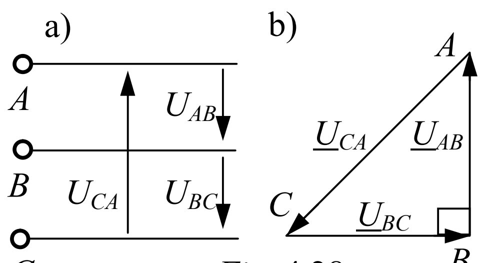


Fig. 4.28

Determine the symmetric components of the non-symmetric system of the line voltages. Find the unbalance factor.

Solution. Assume $\underline{U}_{AB} = U_{AB} = 360\ V$, then on the ground of fig. 4.28,b

$$\underline{U}_{BC} = \underline{U}_{AB}\cdot e^{-j90^\circ} = 360\cdot e^{-j90^\circ}\ V.$$

Since the line voltage phasors form a closed loop, then

$$\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} \equiv 0,\ \text{ and }\ \underline{U}_{CA} = -(\underline{U}_{AB} + \underline{U}_{BC}) = -(360 + 360\cdot e^{-j90^\circ}) =$$

$$= -360\cdot(1 - j) = 360\sqrt{2}\cdot e^{j135^\circ}\ V.$$

There is no zero sequence component in the line voltages, because

$$\underline{U}_0 = \frac{1}{3}(\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA}) = \frac{1}{3}\cdot 0 = 0.$$

The positive sequence component is

$$\underline{U}_1 = \frac{1}{3}(\underline{U}_{AB} + a\cdot\underline{U}_{BC} + a^2\cdot\underline{U}_{CA}) = \frac{1}{3}(360 + e^{j120^\circ}\cdot 360\cdot e^{-j90^\circ} + e^{-j120^\circ}\cdot 360\sqrt{2}\cdot e^{j135^\circ}) =$$

$$= 388 + j104 = 402\cdot e^{j15^\circ}\ V.$$

The negative sequence component is

$$\underline{U}_2 = \frac{1}{3}(\underline{U}_{AB} + a^2\cdot\underline{U}_{BC} + a\cdot\underline{U}_{CA}) = \frac{360}{3}(1 + e^{-j120^\circ}\cdot e^{-j90^\circ} + e^{j120^\circ}\cdot\sqrt{2}\cdot e^{j135^\circ}) =$$

$= -28 - j104 = 108 \cdot e^{-j105°}\ V.$

The unbalance factor is $k = \frac{U_2}{U_1} = \frac{108}{402} = 0.269$ or $k = 26.9\%$.

Note, according to the Regulations for Operation of Consumer Electrical Installations, the unbalance factor must not exceed 4%.

4-19 (4.27). *Y*-connected three-phase induction motor is supplied from the voltage system described in problem 4.18, each phase of the motor has the following impedance: at the positive phase sequence $\underline{Z}_1 = 8 + j6\ Ohm$, at the negative phase sequence it is $\underline{Z}_2 = 4.5 + j1\ Ohm$. Find the motor phase currents.

Solution. Suppose, the motor is connected to a non-symmetric generator, its winding being *Y*-connected (fig. 4.29,a).

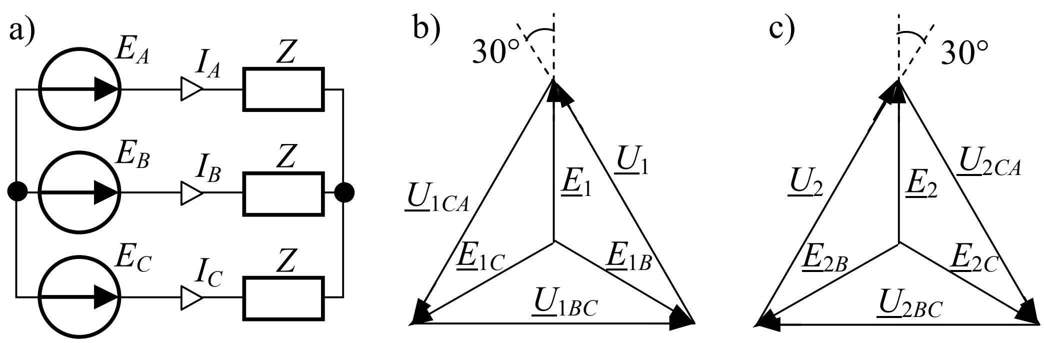


Fig. 4.29

Let's compute the symmetric components of the phase emf of the non-symmetric generator with the aid of the above-found symmetric components of the line voltages $\underline{U}_1$ and $\underline{U}_2$ and their correlations presented in the phasor diagrams fig. 4.29,b and 4.29,c:

$$\underline{E}_1 = \frac{\underline{U}_1}{\sqrt{3}} \cdot e^{-j30°} = \frac{402 \cdot e^{j15}}{\sqrt{3}} \cdot e^{-j30°} = 232 \cdot e^{-j15°}\ V;$$

$$\underline{E}_2 = \frac{\underline{U}_2}{\sqrt{3}} \cdot e^{j30°} = \frac{108 \cdot e^{-j105}}{\sqrt{3}} \cdot e^{j30°} = 62.4 \cdot e^{-j75°}\ V.$$

EMF of the non-symmetric generator are expressed through their symmetric components as follows: $\underline{E}_A = \underline{E}_1 + \underline{E}_2 = 232 \cdot e^{-j15°} + 62.4 \cdot e^{-j75°} = 268.7 \cdot e^{-j26,60°}\ V,$

$\underline{E}_B = a^2 \cdot \underline{E}_1 + a \cdot \underline{E}_2 = 232 \cdot e^{-j135°} + 62.4 \cdot e^{j45°} = 169.6 \cdot e^{-j135°}\ V,$

$\underline{E}_C = a \cdot \underline{E}_1 + a^2 \cdot \underline{E}_2 = 232 \cdot e^{j105°} + 62.4 \cdot e^{-j195°} = 268.7 \cdot e^{j116,6°}\ V.$

As a result of the transformation performed with the source emf (it is presented as a series connection of two symmetric systems – positive phase sequence and negative one), a scheme becomes symmetric one with respect to the symmetric components and then it is analyzed by the superposition method.

Computation of the positive and negative phase sequence current is:

$$\underline{I}_1 = \frac{\underline{E}_1}{\underline{Z}_1} = \frac{232 \cdot e^{-j15}}{8 + j6} = 23.2 \cdot e^{-j51.87°}\ A, \quad \underline{I}_2 = \frac{\underline{E}_2}{\underline{Z}_2} = \frac{62.4 \cdot e^{-j75}}{4.5 + j1} = 13.54 \cdot e^{-j87.53°}\ A.$$

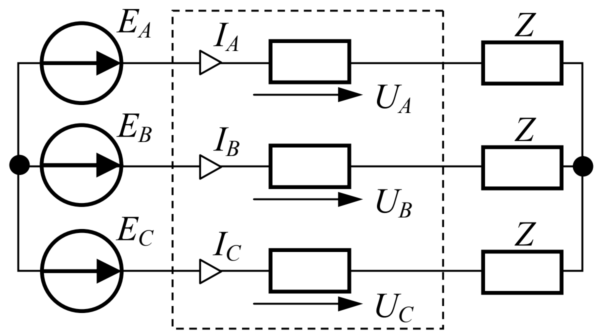
The motor phase currents are determined with account of the phenomenon that in the three-phase three-wire system there are no currents of zero sequence *(*$\underline{I}_0 = 0$*)*:

$\underline{I}_A = \underline{I}_1 + \underline{I}_2 = 23.2 \cdot e^{-j51.87°} + 13.54 \cdot e^{-j87.53°} = 35.10 \cdot e^{-j64.86°}\ A,$

$$\underline{I}_B = a^2\cdot\underline{I}_1 + a\cdot\underline{I}_2 = 23.2\cdot e^{-j171.87°} + 13.54\cdot e^{j32.47°} = 12.22\cdot e^{j160.95°}\ A,$$
$$\underline{I}_C = a\cdot\underline{I}_1 + a^2\cdot\underline{I}_2 = 23.2\cdot e^{j68.13°} + 13.54\cdot e^{-j207.53°} = 27.99\cdot e^{j96.90°}\ A.$$

4-20 (4.28). The motor described in problem 4.19 is connected to the symmetric three-phase system of voltage $U = 380\ V$. There is a break of the line conductor C in the supply circuit.

Carry out the same calculation as in problem 4.19 for a new working condition of the motor.



A circuit unknown until now

Fig. 4.30

Solution. There occurs a longitudinal symmetry breaking in a previously symmetrical three-phase circuit, which can be interpreted as a series additional connection of a consumer with unknown voltages $\underline{U}_A$, $\underline{U}_B$, $\underline{U}_C$ and currents $\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$. A scheme for calculation of the new working condition of the motor is presented in fig. 4.30.

Note, in the phases of till unknown connection there can be both passive and active circuit elements.

Carry out a formal decomposition of the non-symmetric systems of voltages and currents onto the symmetric components.

$$\begin{cases} \underline{U}_0 = \tfrac{1}{3}(\underline{U}_A + \underline{U}_B + \underline{U}_C), & \underline{I}_0 = \tfrac{1}{3}(\underline{I}_A + \underline{I}_B + \underline{I}_C), \\ \underline{U}_1 = \tfrac{1}{3}(\underline{U}_A + a\cdot\underline{U}_B + a^2\cdot\underline{U}_C), & \underline{I}_1 = \tfrac{1}{3}(\underline{I}_A + a\cdot\underline{I}_B + a^2\cdot\underline{I}_C), \\ \underline{U}_2 = \tfrac{1}{3}(\underline{U}_A + a^2\cdot\underline{U}_B + a\cdot\underline{U}_C), & \underline{I}_2 = \tfrac{1}{3}(\underline{I}_A + a^2\cdot\underline{I}_B + a\cdot\underline{I}_C). \end{cases} \quad (4.1)$$

Fistly, determine the symmetric components of the given emf system of the generator: under the conditions of the problem, the system remains symmetric and includes only positive phase sequence components even at the violation of the symmetric working condition of the scheme. Symmetric emf system does not contain the components of zero and negative sequences, i.e.

$$\underline{E}_2 = \underline{E}_0 = 0; \quad \underline{E}_1 = \frac{U}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220\ V.$$

With respect to the symmetric components, the whole scheme becomes a symmetric and it can be calculated using the equivalent schemes for a single phase with reference to each system.

An equivalent scheme of the positive phase sequence is presented in fig. 4.31,a, that one of the negative sequence is in fig. 4.31,b, and that one of the zero sequence is in fig. 4.31,c.

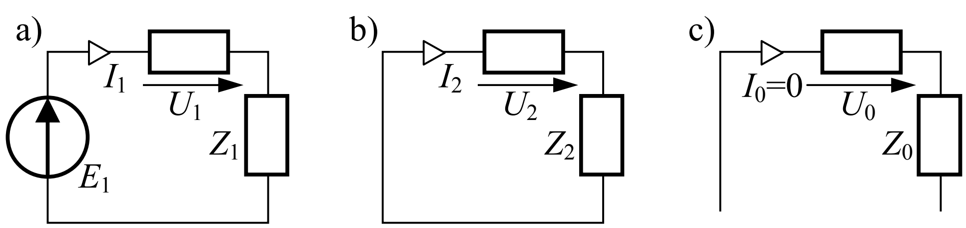


Fig. 4.31

A circuit break for three-wire scheme

In accordance with Kirchhoff's laws for equivalent schemes, we obtain 3 equations to find 6 unknown symmetric components of voltage and current:

$$\underline{I}_1\cdot\underline{Z}_1 + \underline{U}_1 = \underline{E}_1; \quad \underline{I}_2\cdot\underline{Z}_2 + \underline{U}_2 = 0; \quad \underline{I}_0 = 0. \qquad (4.2)$$

We develop the missing equations, knowing the characteristics of the non-symmetrical circuit section in accordance with fig. 4.32 when the line conductor *C* breaks:

$$\underline{\underline{U}_A = 0}; \quad \underline{\underline{U}_B = 0}; \quad \underline{U}_C \neq 0;$$

$$\underline{I}_A \neq 0; \quad \underline{I}_B \neq 0; \quad \underline{\underline{I}_C = 0}.$$

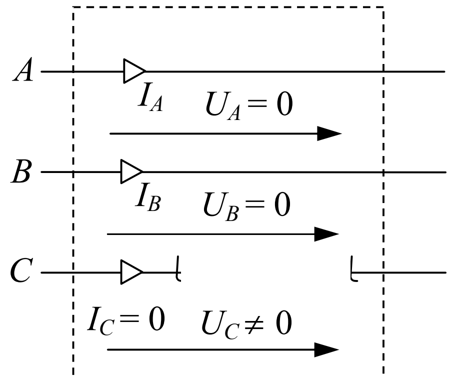


Fig. 4.32

Three underlined equations are determinate. Let's rewrite them, having replaced $\underline{U}_A$, $\underline{U}_B$, $\underline{I}_C$ by their symmetric components (unknown yet):

$$\begin{cases} \underline{U}_A = \underline{U}_0 + \underline{U}_1 + \underline{U}_2 = 0; \\ \underline{U}_B = \underline{U}_0 + a^2\cdot\underline{U}_1 + a\cdot\underline{U}_2 = 0; \qquad (4.3) \\ \underline{I}_C = \underline{I}_0 + a\cdot\underline{I}_1 + a^2\cdot\underline{I}_2 = 0. \end{cases}$$

From the system (4.1) considering that $\underline{U}_A = 0$, $\underline{U}_B = 0$, we obtain

$$\underline{U}_0 = \frac{1}{3}\underline{U}_C, \quad \underline{U}_1 = \frac{1}{3}a^2\cdot\underline{U}_C, \quad \underline{U}_2 = \frac{1}{3}a\cdot\underline{U}_C, \quad \text{from here } \frac{\underline{U}_1}{\underline{U}_2} = a. \qquad (4.4)$$

Take into account $\underline{I}_0 = 0$. On the ground of (4.3), we obtain

$$a\cdot\underline{I}_1 + a^2\cdot\underline{I}_2 = 0, \text{ from here } \quad \underline{I}_2 = -a^2\cdot\underline{I}_1 \qquad (4.5).$$

The other equations (4.2) are presented in the form

$\underline{U}_1 = \underline{E}_1 - \underline{I}_1\cdot\underline{Z}_1$, $\underline{U}_2 = -\underline{I}_2\cdot\underline{Z}_2$, from here taking into account (4.4), we have

$\dfrac{\underline{U}_1}{\underline{U}_2} = \dfrac{\underline{E}_1 - \underline{I}_1\underline{Z}_1}{-\underline{I}_2\underline{Z}_2} = a$, however, taking into account (4.5) it is $\dfrac{\underline{E}_1 - \underline{I}_1\underline{Z}_1}{a^2\underline{I}_1\underline{Z}_2} = a.$

As $a^3 = 1$, then $\underline{E}_1 - \underline{I}_1\cdot\underline{Z}_1 = \underline{I}_1\cdot\underline{Z}_2$, from here

$$\underline{I}_1 = \frac{\underline{E}_1}{\underline{Z}_1 + \underline{Z}_2} = \frac{220}{8 + j6 + 4.5 + j1} = 15.4\cdot e^{-j29.25°}\ A,$$

$$\underline{I}_2 = -a^2\cdot\underline{I}_1 = -e^{-j120°}\cdot 15.4\cdot e^{-j29.25°} = 15.4\cdot e^{j30.75°}\ A,$$

$$\underline{I}_0 = 0.$$

The motor phase currents are:

$\underline{I}_A = \underline{I}_1 + \underline{I}_2 = 15.4\cdot e^{-j29.25°} + 15.4\cdot e^{j30.75°} = 26.6\cdot e^{j0.75°}\ A,$

$\underline{I}_B = a^2\cdot\underline{I}_1 + a\cdot\underline{I}_2 = 15.4\cdot e^{-j149.25°} + 15.4\cdot e^{j150.75°} = 26.6\cdot e^{-j179.25°}\ A,$

$\underline{I}_C = a\cdot\underline{I}_1 + a^2\cdot\underline{I}_2 = 15.4\cdot e^{j90.75°} + 15.4\cdot e^{-j89.25°} = 0.$

4-20 (4.29). Two phases of a generator are closed through an ammeter (fig. 4.33). its impedance to currents of positive sequence is $\underline{Z}_1 = j8$ *Ohm*, that one of negative sequence is $\underline{Z}_2 = j2$ *Ohm*, the phase emf is $E = 100$ *V*. Determine the instrument readings.

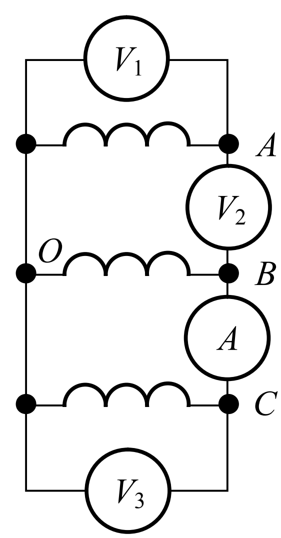


Fig. 4.33

Answer: as there is no neutral conductor $\underline{U}_0 = 0$, $\underline{I}_0 = 0$; the equation system

$$\begin{cases} \underline{U}_1 + \underline{I}_1\cdot\underline{Z}_1 = E, \\ \underline{U}_2 + \underline{I}_2\cdot\underline{Z}_2 = 0, \\ a^2\cdot\underline{U}_1 + a\cdot\underline{U}_2 = a\cdot\underline{U}_1 + a^2\cdot\underline{U}_2, \\ \underline{I}_1 + \underline{I}_2 = 0, \end{cases}$$

from the system $\underline{U}_1 = \underline{U}_2 = 20\ V$, $\underline{I}_1 = -\underline{I}_2 = -j10\ A$.

The instrument readings are: $V_1 \to U_A = 40\ V$, $V_2 \to U_{AB} = 60\ V$, $V_3 \to U_B = 20\ V$, $A \to I_B = 17.3\ A$.

4-21 (4.31). To limit the starting currents of a motor M (nominal line voltage is $U = 380\ V$, impedance of the positive sequence is $\underline{Z}_1 = 1 + j22\ Ohm$, that one of the negative sequence is $\underline{Z}_2 = 1 + j8\ Ohm$), three impedances (reactors) $\underline{Z} = j20\ Ohm$ are connected in series to it, they should be short-circuited after the motor start-up.

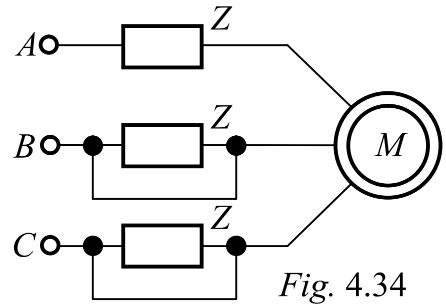


Fig. 4.34

Because of the switch malfunction, one of the impedances remains active after the motor tsart-up (fig. 4.34). Determine the ratio of the current components I_2/I_1 (negative and positive sequences).

Answer: as there is no neutral conductor $\underline{I}_0 = 0$;

the equation system is $\begin{cases} \underline{U}_1 + \underline{I}_1 \cdot \underline{Z}_1 = U/\sqrt{3}, \\ \underline{U}_2 + \underline{I}_2 \cdot \underline{Z}_2 = 0, \\ a^2 \cdot \underline{U}_1 + a \cdot \underline{U}_2 + \underline{U}_0 = 0, \\ a \cdot \underline{U}_1 + a^2 \cdot \underline{U}_2 + \underline{U}_0 = 0, \\ (\underline{I}_1 + \underline{I}_2) \cdot \underline{Z} = \underline{U}_1 + \underline{U}_2 + \underline{U}_0, \end{cases}$

from the system $\underline{U}_1 = \underline{U}_2 = \underline{U}_0 = 31.32 \cdot e^{-j0.53°}\ V$,

$\underline{I}_1 = 8.57 \cdot e^{-j87.31°}\ A$, $\quad \underline{I}_2 = 3.89 \cdot e^{j96.59°}\ A$; $\quad \underline{I}_2/\underline{I}_1 = \dfrac{-\underline{Z}}{3 \cdot \underline{Z}_2 + \underline{Z}} = 0.453 \cdot e^{-j176.10°}$.

4-22 (4.34). A generator with the following impedance to the different sequences $\underline{Z}_{G1} = j8\ Ohm$, $\underline{Z}_{G2} = j2\ Ohm$, $\underline{Z}_{G0} = j0.5\ Ohm$, the coils of which are connected in a star with the neutral earthed through the impedance $\underline{Z}_{gG} = 1 + j2\ Ohm$, generates the phase voltages $U_{ph} = 220\ V$ forming the symmetric three-phase system; the generator supplies a synchronous motor through a line with impedances $\underline{Z}_{l1} = \underline{Z}_{l2} = 1 + j2\ Ohm$, $\underline{Z}_{l0} = 1 + j4\ Ohm$, the coils of the motor are Y-connected and possesses the impedance $\underline{Z}_{M1} = j12\ Ohm$, $\underline{Z}_{M2} = j4\ Ohm$, $\underline{Z}_{M0} = j1\ Ohm$, its neutral being earthed through the impedance $\underline{Z}_{gM} = 2 + j1\ Ohm$ (fig. 4.35).

Determine the line currents in a symmetrical mode, the currents of the conductors B and C when the conductor A is broken as well as the voltage in the point of break.

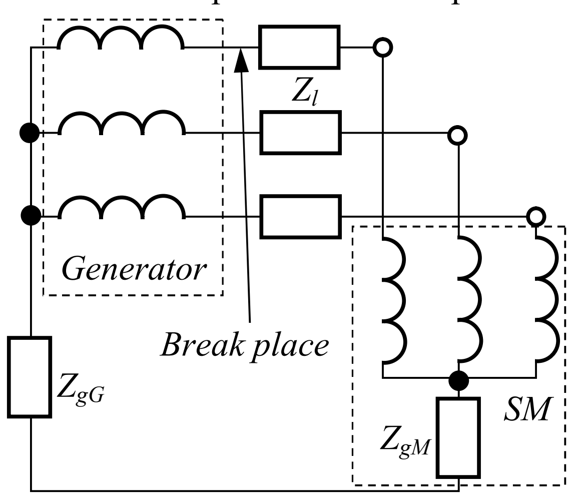


Fig. 4.35

Solution. In the symmetric mode, the neutral has no influence upon the circuit, that's why the calculation can be performed using a single-phase diagram. The generator coil, line and the motor coil are connected in series in one phase and only the positive sequence impedances are taken into account. Thus, total phase impedance is as follows

$$\underline{Z}_{ph} = \underline{Z}_{G1} + \underline{Z}_{l1} + \underline{Z}_{M1} = j8 + 1 + j2 + j12 = 1 + j22\ Ohm.$$

The line current is found by Ohm's law:

$$I_A = I_B = I_C = \frac{U_{ph}}{Z_{ph}} = \frac{220}{\sqrt{1^2 + 22^2}} = 9.99\ A.$$

When the line conductor A brakes, the circuit becomes non-symmetric. We imitate the spot of break by means of the involving the sources with voltages U_A, U_B, U_C, the currents I_A, I_B, I_C flowing through them (fig. 4.36). Above-mentioned voltages and currents form the non-symmetric phasor systems on the complex plane and can be decomposed into symmetric components. As a result, in accordance with the superposition principle, one non-symmetrical three-phase circuit fig. 4.42 splits into three symmetrical ones of the positive, negative and zero sequences (fig. 4.37,a, b, c); on the ground of Kirchhoff's voltage law, there are three equations for them:

$\underline{U}_1 + \underline{I}_1 \cdot \underline{Z}_1 = \underline{U}_\Phi$,

$\underline{U}_2 + \underline{I}_2 \cdot \underline{Z}_2 = 0$,

$\underline{U}_0 + \underline{I}_0 \cdot \underline{Z}_0 = 0$,

where $\underline{Z}_1 = \underline{Z}_{G1} + \underline{Z}_{l1} + \underline{Z}_{M1}$,

$\underline{Z}_2 = \underline{Z}_{G2} + \underline{Z}_{l2} + \underline{Z}_{M2}$,

$\underline{Z}_0 = \underline{Z}_{G0} + \underline{Z}_{l0} + \underline{Z}_{M0} + 3\underline{Z}_{gF} + 3\underline{Z}_{gM}$.

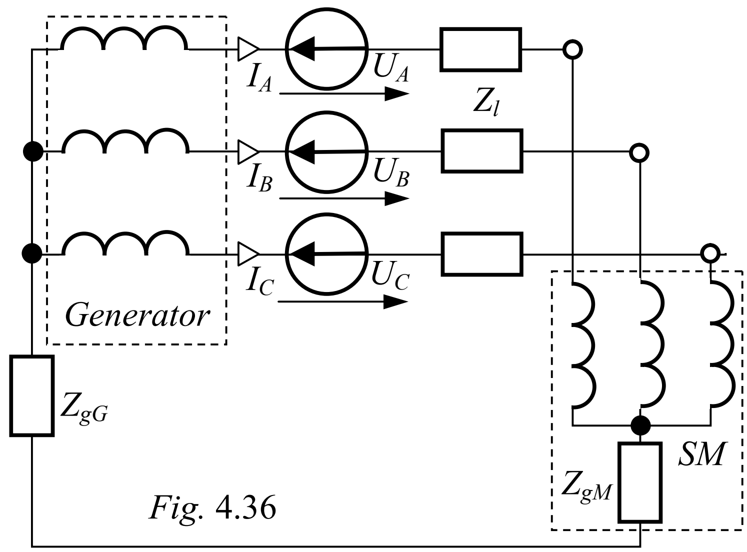


Fig. 4.36

Three missing equations are generated in accordance with the condition in the particular non-symmetrical place (the break place):

$$\underline{U}_B = a^2 \cdot \underline{U}_1 + a \cdot \underline{U}_2 + \underline{U}_0 = 0,$$
$$\underline{U}_C = a \cdot \underline{U}_1 + a^2 \cdot \underline{U}_2 + \underline{U}_0 = 0,$$
$$\underline{I}_A = \underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 0.$$

Solving the system with the aid of matrices, we obtain the required symmetric components; then we find all the quantities which are necessary to find in the problem.

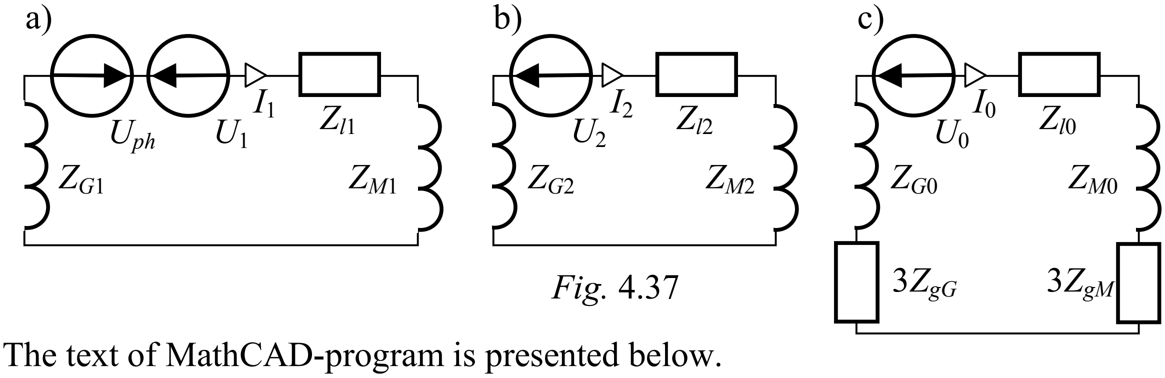


Fig. 4.37

The text of MathCAD-program is presented below.

ORIGIN:= 1 $\quad j := \sqrt{-1}$ – imaginary unit $\quad a := e^{j \cdot 120 \cdot deg}$

The initial data $\quad Uph := 220 \quad ZG1 := j \cdot 8 \quad ZG2 := j \cdot 2 \quad ZG0 := j \cdot 0.5$

$ZM1 := j \cdot 12 \quad ZM2 := j \cdot 4 \quad ZM0 := j \cdot 1 \quad ZL1 := 1 + j \cdot 2 \quad ZL2 := 1 + j \cdot 2 \quad ZL0 := 1 + j \cdot 4$

$ZgG := 1 + j \cdot 2 \quad ZgM := 2 + j \cdot 1$

The calculation of the impedances of the schemes of different sequences

$Z1 := ZG1+ZL1+ZM1 \quad Z1 = 1+j\cdot 22 \quad Z2 := ZG2+ZL2+ZM2 \quad Z2 = 1+j\cdot 8$

$Z0 := ZG0+ZL0+ZM0+3ZgG+3ZgM \quad Z0 = 10+j\cdot 14.5$

The equation system and its solution

$$K := \begin{bmatrix} 1 & 0 & 0 & Z1 & 0 & 0 \\ 0 & 1 & 0 & 0 & Z2 & 0 \\ 0 & 0 & 1 & 0 & 0 & Z0 \\ a^2 & a & 1 & 0 & 0 & 0 \\ a & a^2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \qquad L := \begin{bmatrix} Uph \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad X := K^{-1}\cdot L$$

Required symmetric components

$\underline{U}_1 = X_1 \quad \underline{U}_2 = X_2 \quad \underline{U}_0 = X_3 \quad \underline{I}_1 = X_4 \quad \underline{I}_2 = X_5 \quad \underline{I}_0 = X_6$

$$|\vec{X}| = \begin{bmatrix} 45.24 \\ 45.24 \\ 45.24 \\ 7.98 \\ 5.1 \\ 2.56 \end{bmatrix} \qquad \frac{arg(\vec{X})}{deg} = \begin{bmatrix} -10.4 \\ -10.4 \\ -10.4 \\ -84.7 \\ 86.7 \\ 114.2 \end{bmatrix}$$

$IB := a^2\cdot X_4 + a\cdot X_5 + X_6 \quad |IB| = 13.68 \quad arg(IB) = 166.7$

$IC := a\cdot X_4 + a^2\cdot X_5 + X_6 \quad |IC| = 10.87 \quad arg(IC) = 20.9$

$UA := X_1 + X_2 + X_3 \quad |UA| = 135.7 \quad arg(UA) = -10.4$

4-23 (4.35). The phase A is earthed at the end of the line (fig. 4.38). A generator phase emf is $E = 20\ kV$, the impedances of the different sequences are as follows: generator $\underline{Z}_{G1} = j9\ Ohm$, $\underline{Z}_{G2} = j1\ Ohm$, $\underline{Z}_{G0} = j0.5\ Ohm$; line $\underline{Z}_{l1} = \underline{Z}_{l2} = j1\ Ohm$, $\underline{Z}_{l0} = j2\ Ohm$; load $\underline{Z}_{L1} = j10\ Ohm$, $\underline{Z}_{L2} = j2\ Ohm$; impedance of the generator neutral grounding conductor $\underline{Z}_N = j0.5\ Ohm$. Determine:

1) all the phase currents of the generator and load;

2) voltages of the generator and load terminals concerning the ground.

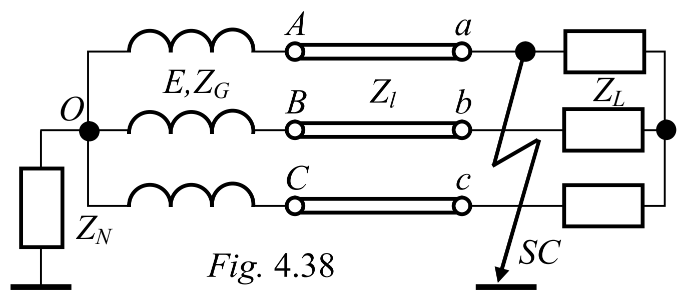


Fig. 4.38

Solution. In the problem under study, there is a so-called *transversal non-symmetry*. A place of non-symmetry is imitated by the involving of the sources with voltages U_a, U_b, U_c, the currents I_A, I_B, I_C flowing through them (fig. 4.39). Above-mentioned voltages and currents form the phasor non-symmetric systems on the complex plane and can be decomposed into the symmetric components. As a result, in accordance with the superposition principle, a single non-symmetrical three-phase circuit fig. 4.39 splits into three symmetrical ones of the positive, negative and zero sequences (fig. 4.40,a, b, c). Let's simplify the schemes fig. 4.40a, b, c, they being reduced to schemes fig. 4.41a, b, c. The parameters of the new schemes fig. 4.41 are determined under the formulae:

$$\underline{Z}_1 = \frac{\underline{Z}_{L1}\cdot(\underline{Z}_{G1}+\underline{Z}_{l1})}{\underline{Z}_{L1}+\underline{Z}_{G1}+\underline{Z}_{l1}};\quad \underline{Z}_2 = \frac{\underline{Z}_{L2}\cdot(\underline{Z}_{G2}+\underline{Z}_{l2})}{\underline{Z}_{L2}+\underline{Z}_{G2}+\underline{Z}_{l2}};\quad \underline{Z}_0 = \underline{Z}_{G0}+\underline{Z}_{l0}+3\underline{Z}_N;$$

$$\underline{E}_1 = \frac{\underline{E}\cdot(\underline{Z}_{G1}+\underline{Z}_{l1})^{-1}}{(\underline{Z}_{G1}+\underline{Z}_{l1})^{-1}+\underline{Z}_{L1}^{\ -1}}$$ (in accordance with the method of two nodes).

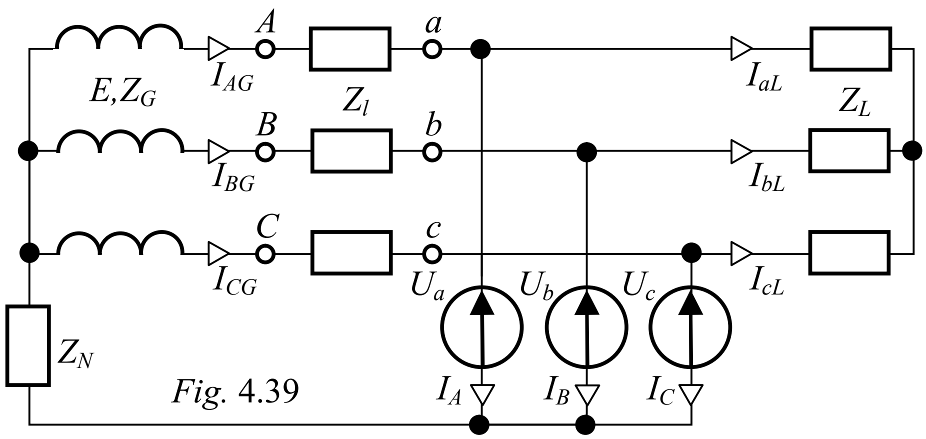


Fig. 4.39

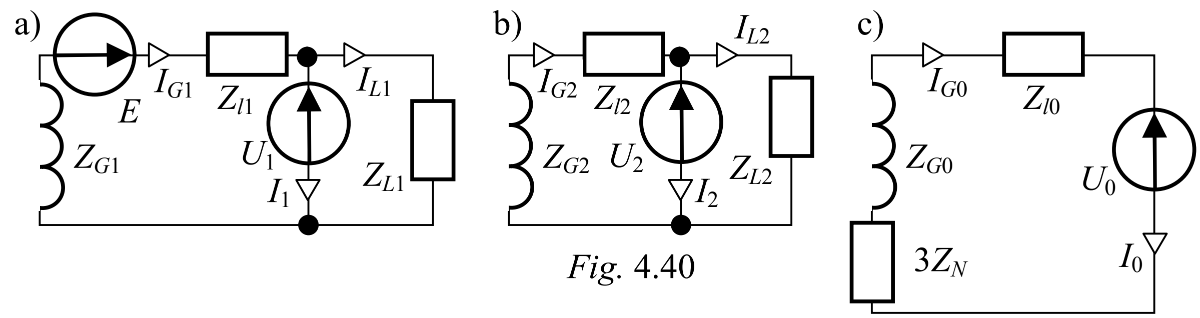


Fig. 4.40

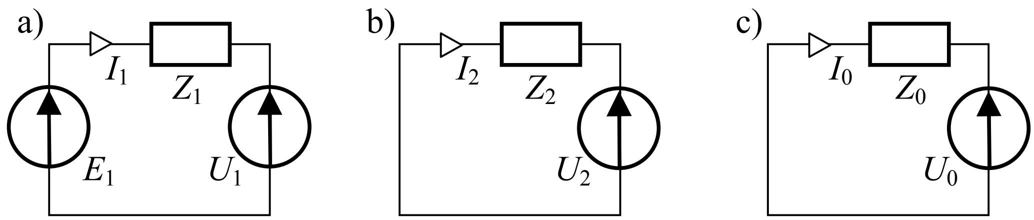


Fig. 4.41

On the ground of Kirchhoff's voltage law for the schemes fig. 4.41, we have three equations: $\underline{U}_1+\underline{I}_1\cdot\underline{Z}_1=\underline{E}_1$, $\underline{U}_2+\underline{I}_2\cdot\underline{Z}_2=0$, $\underline{U}_0+\underline{I}_0\cdot\underline{Z}_0=0$.

Three missing equations are generated in accordance with the condition in the particular non-symmetrical place (the break place): $\underline{I}_B = a^2\cdot\underline{I}_1 + a\cdot\underline{I}_2+\underline{I}_0 = 0$,

$\underline{I}_C = a\cdot\underline{I}_1 + a^2\cdot\underline{I}_2+\underline{I}_0 = 0$,

$\underline{U}_a = \underline{U}_1+\underline{U}_2+\underline{U}_0 = 0$.

Solving the system, we obtain the required symmetric components; then we find all the quantities necessary to calculate in the problem.

The text of MathCAD-program is presented below.

$ORIGIN := 1$ $\quad j:=\sqrt{-1}$ – imaginary unit $\quad a:=e^{j\cdot 120\cdot deg}$

The user function to present the output result $f(x) := \begin{pmatrix} x & |x| & \dfrac{arg(x)}{deg} \end{pmatrix}$

The initial data $E := 20 \quad ZG1 := j\cdot 9 \quad ZG2 := j \quad ZG0 := j\cdot 0.5$

$ZL1 := j\cdot 10 \quad ZL2 := j\cdot 2 \quad Zl1 := j \quad Zl := j \quad Zl0 := j\cdot 2 \quad ZN := j\cdot 0.5$

Calculation of the impedances of the schemes of different sequences

$Z1 := \dfrac{ZL1\cdot(ZG1 + Zl1)}{ZL1 + ZG1 + Zl1} \quad Z1 = 5j \quad Z2 := \dfrac{ZL2\cdot(ZG2 + Zl2)}{ZL2 + ZG2 + Zl2} \quad Z2 = j$

$Z0 := ZG0 + Zl0 + 3ZN \quad Z0 = 4j$

$E1 := \dfrac{E\cdot(ZG1 + Zl1)^{-1}}{(ZG1 + Zl1)^{-1} + ZL1^{-1}} \quad E1 = 10$

The equation system and its solution

$$K := \begin{bmatrix} 1 & 0 & 0 & Z1 & 0 & 0 \\ 0 & 1 & 0 & 0 & Z2 & 0 \\ 0 & 0 & 1 & 0 & 0 & Z0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^2 & a & 1 \\ 0 & 0 & 0 & a & a^2 & 1 \end{bmatrix} \quad L := \begin{bmatrix} E1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad X := K^{-1}\cdot L$$

The required symmetric components are

$U1 := X_1 \quad U2 := X_2 \quad U0 := X_3 \quad I1 := X_4 \quad I2 := X_5 \quad I0 := X_6$

$U1 = 5 \quad U2 = -1 \quad U0 = -4 \quad I1 = -j \quad I2 = -j \quad I0 = -j$

The symmetric components of the generator and load currents on the ground of fig. 4.40 are

$IL1 := \dfrac{U1}{ZL1} \quad IL1 = -0.5j \quad IL2 := \dfrac{U2}{ZL2} \quad IL2 = 0.5j \quad IL0 := 0$

$IG1 := \dfrac{E - U1}{ZG1 + Zl1} \quad IG1 = -1.5j \quad IG2 := \dfrac{-U2}{ZG2 + Zl2} \quad IG2 = -0.5j$

$IG0 := \dfrac{-U0}{3\cdot ZN + Z\Gamma 0 + ZЛ0} \quad IG0 = -j$

The generator and load currents as well as those in the short-circuit place (*kA*) are

$IaL := IL1 + IL2 + IL0$	$IaL = 0$
$IbL := a^2\cdot IL1 + a\cdot IL2 + IL0$	$IbL = -0.866$
$IcL := a\cdot IL1 + a^2\cdot IL2 + IL0$	$IcL = 0.866$
$IAG := IG1 + IG2 + IG0$	$IAG = -3j$
$IBG := a^2\cdot IG1 + a\cdot IG2 + IG0$	$IBG = -0.866$
$ICG := a\cdot IG1 + a^2\cdot IG2 + IG0$	$ICG = 0.866$
$IA := I1 + I2 + I0$	$IA = -3j$
$IB := a^2\cdot I1 + a\cdot I2 + I0$	$IB = 0$
$IC := a\cdot I1 + a^2\cdot I2 + I0$	$IC = 0$

The load voltages (in the short-circuit place) and those across the generator terminals (*kV*) are

$Ua := U1 + U2 + U0 \quad Ua = 0$

$Ub := a^2\cdot U1 + a\cdot U2 + U0 \quad f(Ub) = (-6 - 5.196j \quad 7.937 \quad -139.107)$

$Uc := a\cdot U1 + a^2\cdot U2 + U0$	$f(Uc) = (-6 + 5.196j\ \ 7.937\ \ 139.107)$
$UG1 := U1 + IG1\cdot Zl1$	$UG1 = 6.5$
$UG2 := U2 + IG2\cdot Zl2$	$UG2 = -0.5$
$UG0 := U0 + IG0\cdot Zl0$	$UG0 = -2$
$UA := UG1 + UG2 + UG0$	$UA = 4$
$UB := a^2\cdot UG1 + a\cdot UG2 + UG0$	$f(UB) = (-5 - 6.062j\ \ 7.858\ \ -129.515)$
$UC := a\cdot UG1 + a^2\cdot UG2 + UG0$	$f(UC) = (-5 + 6.062j\ \ 7.858\ \ 129.515)$

4-24 (4.36). There occurs a single phase-to-ground short-circuit through the impedance $Z = 4.65\ Ohm$ on the generator bus-bars supplying the Y-connected load (fig. 4.42). The generator impedances (in *Ohm*) are: $\underline{Z}_{\Gamma 1} = j10$, $\underline{Z}_{\Gamma 2} = j1$, $\underline{Z}_{\Gamma 0} = j10$; the load impedance is the same for all sequences: $\underline{Z}_L = j100\ Ohm$. The impedance of the generator neutral grounding conducotr is $Z_N = 2\ Ohm$. The generator phase emf is $E = 20\ kV$. Determine all the generator phase currents.

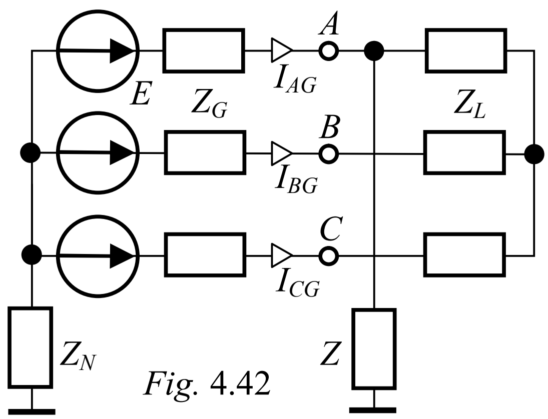


Fig. 4.42

Solution. In the problem under study, there is a transversal non-symmetry. A non-symmetry location is imitated by involving of the sources with voltages U_A, U_B, U_C, the currents I_A, I_B, I_C flowing through them (fig. 4.43). The above-mentioned voltages and currents form the phasor non-symmetric systems and can be decomposed into the symmetric components. As a result, in accordance with the superposition principle, a single non-symmetrical three-phase circuit fig. 4.43 splits into three symmetrical ones of the positive, negative and zero sequences (fig. 4.44,a, b, c). Let's simplify the schemes fig. 4.44,a, b, c, they being reduced to the schemes fig. 4.41,a, b, c. The parameters of schemes fig. 4.41 are determined under the formulae:

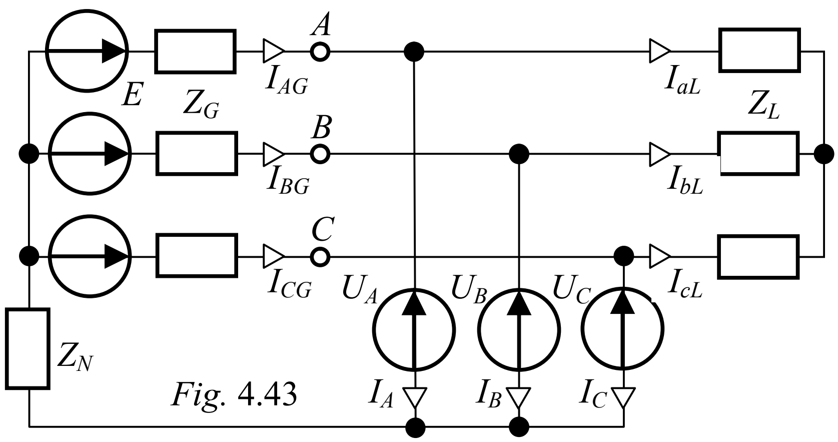


Fig. 4.43

$$\underline{Z}_1 = \frac{\underline{Z}_L \cdot \underline{Z}_{G1}}{\underline{Z}_L + \underline{Z}_{G1}};\quad \underline{Z}_2 = \frac{\underline{Z}_L \cdot \underline{Z}_{G2}}{\underline{Z}_L + \underline{Z}_{G2}};\quad \underline{Z}_0 = \underline{Z}_{G0} + 3\underline{Z}_N;\quad \underline{E}_1 = \frac{E \cdot \underline{Z}_{G1}^{-1}}{\underline{Z}_{G1}^{-1} + \underline{Z}_L^{-1}}.$$

On the ground of Kirchhoff's voltage law for schemes fig. 4.41, we have three

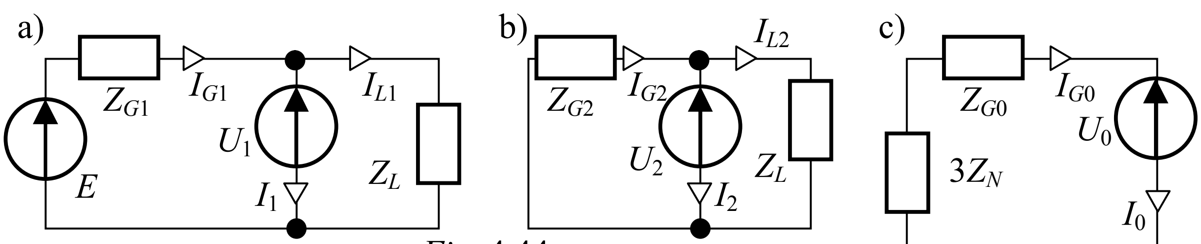


Fig. 4.44

equations: $\underline{U}_1 + \underline{I}_1 \cdot \underline{Z}_1 = \underline{E}_1$, $\underline{U}_2 + \underline{I}_2 \cdot \underline{Z}_2 = 0$, $\underline{U}_0 + \underline{I}_0 \cdot \underline{Z}_0 = 0$.

Three missing equations are generated in accordance with the condition in the particular non-symmetrical place (the break place): $\underline{I}_B = a^2 \cdot \underline{I}_1 + a \cdot \underline{I}_2 + \underline{I}_0 = 0$,

$\underline{I}_C = a \cdot \underline{I}_1 + a^2 \cdot \underline{I}_2 + \underline{I}_0 = 0$,

$\underline{U}_A = \underline{I}_A \cdot Z$ or $\underline{U}_1 + \underline{U}_2 + \underline{U}_0 = (\underline{I}_1 + \underline{I}_2 + \underline{I}_0) \cdot Z$.

Solving the system, we obtain the required symmetric components; then we find all the quantities necessary in the problem .

The text of MathCAD-program is presented below.

$ORIGIN := 1$ $\quad j := \sqrt{-1}$ – imaginary unit $\quad a := e^{j \cdot 120 \cdot deg}$

The user function to present the output result $f(x) := \left(x \quad |x| \quad \dfrac{arg(x)}{deg} \right)$

The initial data $E := 220$ $\quad ZG1 := j \cdot 10$ $\quad ZG2 := j$ $\quad ZG0 := j \cdot 10$
$ZL := j \cdot 100$ $\quad Z := 4.65$ $\quad ZN := 2$

The calculation of the impedances of the schemes of different sequences is

$Z1 := \dfrac{ZL \cdot ZG1}{ZL + ZG1}$ $\quad Z1 = 9.091j$ $\quad Z2 := \dfrac{ZL \cdot ZG2}{ZL + ZG2}$ $\quad Z2 = 0.99j$

$Z0 := ZG0 + 3ZN$ $\quad f(Z0) = (6+10j \quad 11.662 \quad 69.036)$

$E1 := \dfrac{E \cdot ZL}{ZG1 + ZL}$ $\quad E1 = 200$

The equation system and its solution are

$$K := \begin{bmatrix} 1 & 0 & 0 & Z1 & 0 & 0 \\ 0 & 1 & 0 & 0 & Z2 & 0 \\ 0 & 0 & 1 & 0 & 0 & Z0 \\ 1 & 1 & 1 & -Z & -Z & -Z \\ 0 & 0 & 0 & a^2 & a & 1 \\ 0 & 0 & 0 & a & a^2 & 1 \end{bmatrix} \qquad L := \begin{bmatrix} E1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad X := K^{-1} \cdot L$$

The required symmetric components are

$U1 := X_1$ $\quad U2 := X_2$ $\quad U0 := X_3$ $\quad I1 := X_4$ $\quad I2 := X_5$ $\quad I0 := X_6$

$$|\vec{X}| = \begin{bmatrix} 160.931 \\ 6.996 \\ 82.398 \\ 7.066 \\ 7.066 \\ 7.066 \end{bmatrix} \qquad \frac{arg(\vec{X})}{deg} = \begin{bmatrix} -16.338 \\ -135.188 \\ -166.151 \\ -45.188 \\ -45.188 \\ -45.188 \end{bmatrix}$$

The symmetric components of the generator currents on the ground of fig. 4.44 are

$$IG1 := I1 + \frac{U1}{ZL} \qquad IG2 := I2 + \frac{U2}{ZL} \qquad IG0 := I0$$

The generator currents (kA) are

$IAG := IG1 + IG2 + IG0$ $\qquad f(IAG) = (14.437 - 16.532j \quad 21.949 \quad -48.87)$

$IBG := a^2 \cdot IG1 + a \cdot IG2 + IG0$ $\qquad f(IBG) = (-1.129 + 1.097j \quad 1.574 \quad 135.842)$

$ICG := a \cdot IG1 + a^2 \cdot IG2 + IG0 \qquad f(ICG) = (1.631 + 0.398j \quad 1.679 \quad 13.71)$

4-25 (4.37). The symmetrical three-phase generator supplies induction and synchronous motors (fig. 4.45). The generator phase emf is $E = 220\ V$, the impedances of the different sequences are: a generator $\underline{Z}_{G1} = j0.6\ Ohm$, $\underline{Z}_{G2} = j0.1\ Ohm$, $\underline{Z}_{G0} = j0.05\ Ohm$; an induction motor $\underline{Z}_{M1} = 3 + j0.4\ Ohm$, $\underline{Z}_{M2} = 0.05 + j0.1\ Ohm$; a synchronous motor $\underline{Z}_{S1} = 4 + j0.5\ Ohm$, $\underline{Z}_{S2} = 0.1 + j0.2\ Ohm$, $\underline{Z}_{S0} = 0.1 + j0.1\ Ohm$; the impedance of the generator and synchronous motor neutral earthing is – $\underline{Z}_{NG} = \underline{Z}_{NS} = 0.1\ Ohm$. Determine: the currents of the double-phase-to-ground metallic short-circuit of the line conductors *B* and *C*, net short-circuit current, the voltage on the healthy phase *A*. Additionally, using the simplified system applied in the subject "Electromagnetic transients" determine the symmetric components of currents and voltages involved in the short-circuit place and compare their values with those obtained by the methods of TFEE.

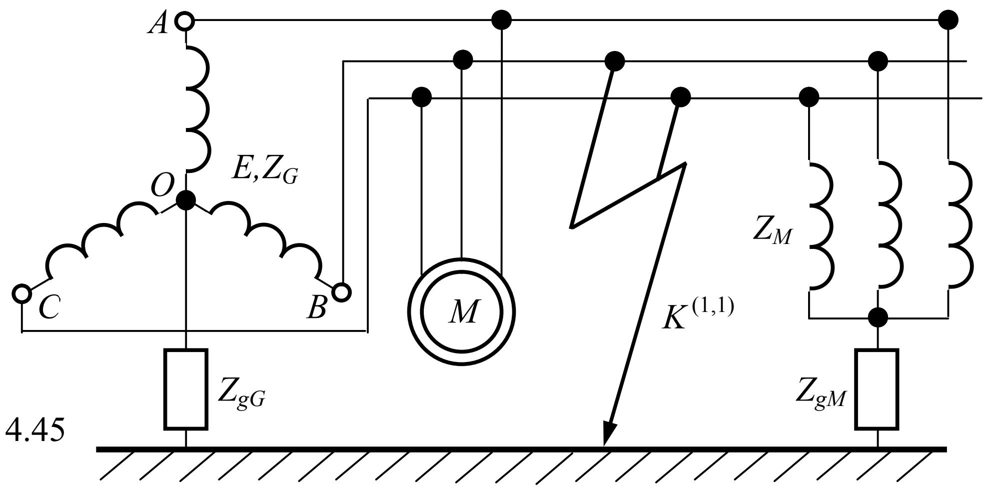


Fig. 4.45

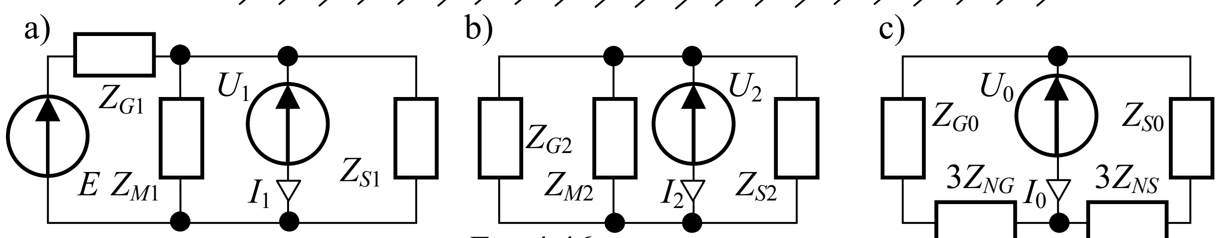


Fig. 4.46

Solution. In the same way as it was while solving problem 4.24 (fig. 4.43), the system of sources with voltages U_A, U_B, U_C is introduced into the point of fault and the currents I_A, I_B, I_C flow through them. Considering each sequence separately we obtain the single-phase schemes for calculation (fig. 4.46). Let's simplify these schemes to those fig. 4.41. Here

$$\underline{Z}_1 = (\underline{Z}_{G1}^{-1} + \underline{Z}_{M1}^{-1} + \underline{Z}_{S1}^{-1})^{-1}; \quad \underline{Z}_2 = (\underline{Z}_{G2}^{-1} + \underline{Z}_{M2}^{-1} + \underline{Z}_{S2}^{-1})^{-1};$$

$$\underline{Z}_0 = \frac{(\underline{Z}_{G0} + 3\underline{Z}_{NG}) \cdot (\underline{Z}_{S0} + 3\underline{Z}_{NS})}{\underline{Z}_{G0} + 3\underline{Z}_{NG} + \underline{Z}_{S0} + 3\underline{Z}_{NS}}; \quad \underline{E}_1 = \frac{\underline{E} \cdot \underline{Z}_{G1}^{-1}}{\underline{Z}_{G1}^{-1} + \underline{Z}_{M1}^{-1} + \underline{Z}_{S1}^{-1}}.$$

On the ground of Kirchhoff's voltage law for schemes fig. 4.41, we have three equations: $\underline{U}_1 + \underline{I}_1 \cdot \underline{Z}_1 = \underline{E}_1$, $\underline{U}_2 + \underline{I}_2 \cdot \underline{Z}_2 = 0$, $\underline{U}_0 + \underline{I}_0 \cdot \underline{Z}_0 = 0$.

Three missing equations are generated in accordance with the condition in the particular non-symmetrical place (the break place): $\underline{U}_B = a^2 \cdot \underline{U}_1 + a \cdot \underline{U}_2 + \underline{U}_0 = 0$,

$\underline{U}_C = a \cdot \underline{U}_1 + a^2 \cdot \underline{U}_2 + \underline{U}_0 = 0$,

$\underline{I}_A = \underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 0$.

Solving the system, we obtain the required symmetric components; then we find all the quantities necessary in the problem.

According to the methods applied in the subject "Electromagnetic transients", first of all a current of the three-phase short-circuit should be determined; a three-phase short-circuit is symmetrical and only the scheme of the positive phase sequence is required for its calculation. So, fig. 4.41,a turns into fig. 4.47,a. Evidently, $IK^{(3)}:=\dfrac{\underline{E}}{\underline{Z}_{G1}}=\dfrac{\underline{E}_1}{\underline{Z}_1}$. A symmetric component of the positive sequence of the short-circuit current $\underline{I}_1$ is found in accordance with a rule of N. Shchedrin, which states that the three-phase short-circuit point is removed by the additional impedance $\underline{Z}_\Delta$, which does not depend on the parameters of the positive sequence scheme and for each kind of the short-circuit is determined by the resultant impedances of the negative sequence scheme and zero sequence scheme concerning the scheme point under consideration, it means the scheme fig. 4.47,a turns into the scheme fig. 4.47,b. In case of the two-phase-to-ground short-circuit it is $\underline{Z}_\Delta^{1,1}=\dfrac{\underline{Z}_2\underline{Z}_0}{\underline{Z}_2\underline{Z}_0}$. We obtain a scheme fig. 4.47,c, from which we find all necessary symmetric components:

$$\underline{I}_1=\frac{\underline{E}_1}{\underline{Z}_1+\underline{Z}_\Delta^{1,1}};\quad \underline{I}_2=-\underline{I}_1\cdot\frac{\underline{Z}_0}{\underline{Z}_2\underline{Z}_0};\quad \underline{I}_0=-\underline{I}_1\cdot\frac{\underline{Z}_2}{\underline{Z}_2\underline{Z}_0};\quad \underline{U}_1=\underline{U}_2=\underline{U}_0=\underline{I}_1\cdot\underline{Z}_\Delta^{1,1}.$$

As it is seen from the calculation results (see text of MathCAD-program), they entirely coincide with the values obtained by the methods of TFEE.

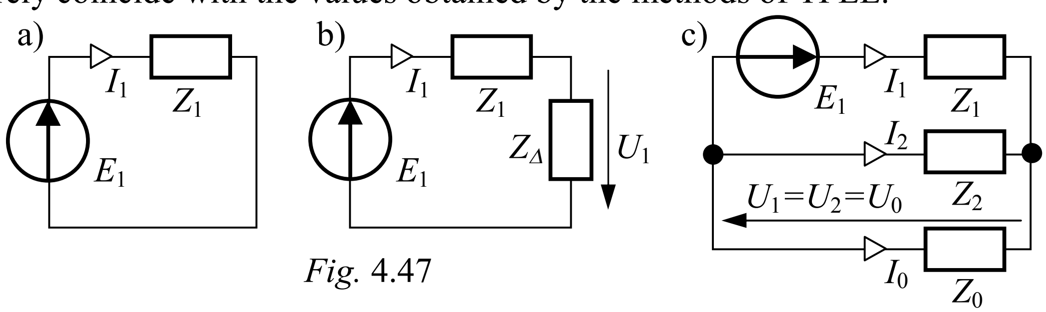


Fig. 4.47

The text of MathCAD-program is presented below.

$ORIGIN:=1$ $\quad j:=\sqrt{-1}$ – imaginary unit $\quad a:=e^{j\cdot 120\cdot deg}$

The user function to present the output result $f(x):=\left(x \quad |x| \quad \dfrac{arg(x)}{deg}\right)$

The initial data are $E:=220$ $\quad ZG1:=j\cdot 0.6$ $\quad ZG2:=j\cdot 0.1$ $\quad ZG0:=j\cdot 0.05$

$ZM1:=3+j\cdot 4$ $\quad ZM2:=0.05+j\cdot 0.1$ $\quad ZM0:=0.05+j\cdot 0.05$

$ZS1:=4+j\cdot 0.5$ $\quad ZS2:=0.1+j\cdot 0.2$ $\quad ZS0:=0.1+j\cdot 0.1$

$ZNG:=0.1$ $\quad ZNS:=0.1$

The calculation of the impedances of the schemes of different sequences is

$Z1:=(ZG1^{-1}+ZM1^{-1}+ZS1^{-1})^{-1}$ $\quad f(Z1)=(0.171+0.518j \quad 0.546 \quad 71.764)$

$Z2:=(ZG2^{-1}+ZM2^{-1}+ZS2^{-1})^{-1}$ $\quad f(Z2)=(0.012+0.042j \quad 0.044 \quad 74.745)$

$Z0:=\dfrac{(ZG0+3\cdot ZNG)\cdot(ZS0+3\cdot ZNS)}{ZG0+3\cdot ZNG+ZS0+3\cdot ZNS}$ $\quad f(Z0)=(0.172+0.035j \quad 0.175 \quad 11.404)$

$$E1 := \frac{E \cdot ZG1^{-1}}{ZG1^{-1} + ZM1^{-1} + ZS1^{-1}} \qquad f(E1) = (189.971 - 62.592j \;\; 200.017 \;\; -18.236)$$

The equation system and its solution are

$$K := \begin{bmatrix} 1 & 0 & 0 & Z1 & 0 & 0 \\ 0 & 1 & 0 & 0 & Z2 & 0 \\ 0 & 0 & 1 & 0 & 0 & Z0 \\ a^2 & a & 1 & 0 & 0 & 0 \\ a & a^2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \qquad L := \begin{bmatrix} E1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad X := K^{-1} \cdot L$$

The required symmetric components are

$U1 := X_1 \quad U2 := X_2 \quad U0 := X_3 \quad I1 := X_4 \quad I2 := X_5 \quad I0 := X_6$

$f(U1) = (11.895 - 5.821j \;\; 13.243 \;\; -26.074)$
$f(U2) = (11.895 - 5.821j \;\; 13.243 \;\; -26.074)$
$f(U0) = (11.895 - 5.821j \;\; 13.243 \;\; -26.074)$
$f(I1) = (3.311 - 342.617j \;\; 342.633 \;\; -89.446)$
$f(I2) = (56.686 + 296.616j \;\; 301.984 \;\; 79.181)$
$f(I0) = (-59.997 + 46.001j \;\; 75.602 \;\; 142.522)$

The required currents and voltages are

$IB := a^2 \cdot I1 + a \cdot I2 + I0 \qquad f(IB) = (-643.587 + 115.226j \;\; 653.821 \;\; 169.85)$
$IC := a \cdot I1 + a^2 \cdot I2 + I0 \qquad f(IC) = (463.597 + 22.777j \;\; 464.156 \;\; 2.813)$
$IN := 3 \cdot I0 \qquad f(IN) = (-179.99 + 138.003j \;\; 226.806 \;\; 142.522)$
$UA := U1 + U2 + U0 \qquad f(UA) = (35.685 - 17.462j \;\; 39.729 \;\; -26.074)$

The calculations by the simplified system applied in the subject “Electromagnetic transients” $\frac{E}{ZG1} = -366.667j \qquad \frac{E1}{Z1} = -366.667j$

$$Z\Delta := \frac{Z2 \cdot Z0}{Z2 + Z0} \qquad f(Z\Delta) = (0.017 + 0.035j \;\; 0.039 \;\; 63.372)$$

$$I1 := \frac{E1}{Z1 + Z\Delta} \qquad f(I1) = (3.311 - 342.617j \;\; 342.633 \;\; -89.446)$$

$$I2 := -I1 \cdot \frac{Z0}{Z2 + Z0} \qquad f(I2) = (56.686 + 296.616j \;\; 301.984 \;\; 79.181)$$

$$I0 := -I1 \cdot \frac{Z2}{Z2 + Z0} \qquad f(I0) = (-59.997 + 46.001j \;\; 75.602 \;\; 142.522)$$

$$U1 := I1 \cdot Z\Delta \qquad f(U1) = (11.895 - 5.821j \;\; 13.243 \;\; -26.074)$$

5. PASSIVE FOUR-TERMINAL NETWORKS

5.1. EQUATIONS OF THE PASSIVE FOUR-TERMINAL NETWORKS WITH COEFFICIENTS

5-1 (5.1). Calculate *ABCD*–coefficients of the equivalent *Γ*-scheme of the four-terminal network (fig. 5.1,a), if $r = 10$ *Ohm*, $x_C = 10$ *Ohm*, $x_L = 20$ *Ohm*. With the aid of the principal equations of the 4-terminal network, determine the input impedance $\underline{Z}_1$ at the load $\underline{Z}_2 = r_2 = 20$ *Ohm*.

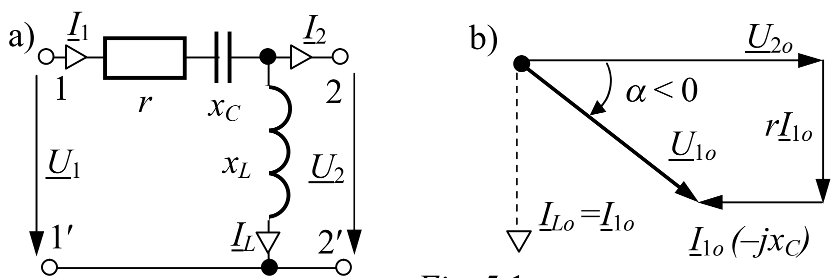


Fig. 5.1

Solution. Way 1. A scheme loaded with arbitrary impedance $\underline{Z}_2$ is considered, when voltage $\underline{U}_2$ and current $\underline{I}_2$ differ from zero. The obtained scheme is described by Kirchhoff's equation system. By the substitution method, it is possible to get rid of the intermediate currents and voltages, the equation system being reduced to a view:

$$\underline{U}_1 = \underline{A}\cdot\underline{U}_2 + \underline{B}\cdot\underline{I}_2; \quad \underline{I}_1 = \underline{C}\cdot\underline{U}_2 + \underline{D}\cdot\underline{I}_2.$$

At the arbitrary load, there are three unknown currents in the scheme fig. 5.1,a: $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_L$. Under Kirchhoff's current law $\underline{I}_1 = \underline{I}_L + \underline{I}_2$,

under Kirchhoff's voltage law

$$\underline{U}_2 - \underline{I}_L\cdot jx_L = 0,$$

$$\underline{I}_1\cdot(r - jx_C) + \underline{U}_2 = \underline{U}_1.$$

From these equations, we have: $\underline{I}_1 = \underline{I}_2 + \dfrac{\underline{U}_2}{jx_L} = \underline{C}\cdot\underline{U}_2 + \underline{D}\cdot\underline{I}_2$,

from here $\underline{C} = \dfrac{1}{jx_L} = \dfrac{1}{j20} = -j0{,}05$ *S*, $\underline{D} = 1$;

$$\underline{U}_1 = \underline{U}_2\cdot(1 + \frac{r - jx_C}{jx_L}) + \underline{I}_2\cdot(r - jx_C) = \underline{A}\cdot\underline{U}_2 + \underline{B}\cdot\underline{I}_2,$$

from here $\underline{A} = 1 + \dfrac{r - jx_C}{jx_L} = 1 + \dfrac{10 - j10}{j20} = 0.5 - j0.5 = 0.5\sqrt{2}\cdot e^{-j\cdot 45^\circ}$,

$$\underline{B} = r - jx_C = 10 - j10 = 10\sqrt{2}\cdot e^{-j\cdot 45^\circ} \text{ Ohm}.$$

Way 2. Coefficients are calculated by Kirchhoff's equations for no-load and short-circuit conditions of the 4-terminal network, the principal equations taking a form:

no-load condition $\begin{cases} \underline{U}_{1o} = \underline{A}\cdot\underline{U}_{2o}; \\ \underline{I}_{1o} = \underline{C}\cdot\underline{U}_{2o}; \end{cases}$ short-circuit condition $\begin{cases} \underline{U}_{1S} = \underline{B}\cdot\underline{I}_{2S}; \\ \underline{I}_{1S} = \underline{D}\cdot\underline{I}_{2S}. \end{cases}$

By scheme fig. 5.1,a, we obtain:

$$\underline{I}_{1o} = \frac{\underline{U}_{2o}}{jx_L} = \underline{C}\cdot\underline{U}_{2o}; \quad \underline{U}_{1o} = \underline{I}_{1o}\cdot(r - jx_C) + \underline{U}_{2o} = \underline{U}_{2o}\cdot(1 + \frac{r - jx_C}{jx_L}) = \underline{A}\cdot\underline{U}_{2o};$$

$$\underline{I}_{1S} = \underline{I}_{2S} = \underline{D}\cdot\underline{I}_{2S}; \quad \underline{I}_{1S}\cdot(r - jx_C) = \underline{U}_{1S} = \underline{B}\cdot\underline{I}_{2S} = \underline{I}_{2S}\cdot(r - jx_C).$$

The results of the coefficient calculation coincide with those obtained in the previous calculations.

Way 3. The coefficient calculation is performed by the impedances of the no-load and short-circuit conditions of the 4-terminal network (fig. 5.1,a):

$\underline{Z}_{1o} = r - jx_C + jx_L = 10 - j10 + j20 = 10 + j10 = 10\sqrt{2}\cdot e^{j\cdot 45°}$ *Ohm*;

$\underline{Z}_{1S} = r - jx_C = 10 - j10 = 10\sqrt{2}\cdot e^{-j\cdot 45°}$ *Ohm*;

$\underline{Z}_{2o} = jx_L = j20 = 20\cdot e^{j\cdot 90°}$ *Ohm*;

$$\underline{Z}_{2S} = \frac{jx_L\cdot(r - jx_C)}{jx_L + r - jx_C} = \frac{20e^{j90}\cdot 10\sqrt{2}e^{-j45}}{10\sqrt{2}e^{j45}} = 20\ Ohm.$$

From the principal equations at the no-load and short-circuit conditions, we have

$$\underline{Z}_{1o} = \underline{A}/\underline{C},\quad \underline{Z}_{2o} = \underline{D}/\underline{C},\quad \underline{Z}_{1S} = \underline{B}/\underline{D},\quad \underline{Z}_{2S} = \underline{B}/\underline{A}.$$

Having chosen any three relationships and taking into account the coefficient property $\underline{A}\cdot\underline{D} - \underline{B}\cdot\underline{C} = 1$, firstly, we obtain one of the coefficients and then other three ones through the first one. For example, $\underline{A} = \sqrt{\dfrac{\underline{Z}_{1o}}{\underline{Z}_{2o} - \underline{Z}_{2S}}} = \sqrt{\dfrac{10\sqrt{2}e^{j45}}{j20 - 20}}$.

Complex number staying in the denominator may be written in the exponential form in two ways: 1) $j20 - 20 = 20\sqrt{2}\cdot e^{j\cdot 135°}$; 2) $j20 - 20 = 20\sqrt{2}\cdot e^{-j\cdot 225°}$.

Respectively, we obtain 2 values of the coefficient $\underline{A}$:

$$\underline{A}_1 = \frac{1}{\sqrt{2}}e^{-j\cdot 45°};\qquad \underline{A}_2 = \frac{1}{\sqrt{2}}e^{j\cdot 135°}.$$

In the general case coefficient $\underline{A}$ is a complex number and in exponential form it has a view $\underline{A} = a\cdot e^{j\alpha}$.

The absolute value of the coefficient $a = \dfrac{1}{\sqrt{2}}$ is determined unambiguously. However, for the argument α there are two values:

negative one $\alpha = \alpha_1 = -45°$, and positive one $\alpha = \alpha_2 = +135°$.

The single value of α is selected on the ground of the circuit phasor diagram (fig. 5.1,b) for the no-load condition of the 4-terminal network, when

$$\underline{U}_{1o} = \underline{A}\cdot\underline{U}_{2o} = a\cdot e^{j\alpha}\cdot\underline{U}_{2o}.$$

In accordance with the phasor diagram, we obtain $\alpha < 0$, then $\underline{A} = \dfrac{1}{\sqrt{2}}e^{-j\cdot 45°}$;

$$\underline{B} = \underline{Z}_{2S}\cdot\underline{A} = \frac{1}{\sqrt{2}}e^{-j\cdot 45°}\cdot 20 = 10\sqrt{2}\cdot e^{-j\cdot 45°}\ Ohm;$$

$$\underline{C} = \frac{\underline{A}}{\underline{Z}_{1o}} = \frac{1}{\sqrt{2}}\,\frac{e^{-j45}}{10\sqrt{2}e^{j45}} = -j0.05\ S;\qquad \underline{D} = \underline{Z}_{2o}\cdot\underline{C} = 20\cdot e^{j\cdot 90°}(-j0{,}05) = 1.$$

The input impedance at the given load $\underline{Z}_2$ is as follows:

$$\underline{Z}_1 = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{A}\underline{U}_2 + \underline{B}\underline{I}_2}{\underline{C}\underline{U}_2 + \underline{D}\underline{I}_2} = \frac{\underline{A}\underline{Z}_2 + \underline{B}}{\underline{C}\underline{Z}_2 + \underline{D}} = \frac{(0.5 - j0.5)\cdot 20 + 10 - j10}{-j0.05\cdot 20 + 1} = \frac{20 - j20}{1 - j} = 20\ Ohm.$$

5-2 (5.2). Determine *ABCD*-coefficients of the non-symmetrical 4-terminal network, *T*-connected (fig. 5.2), if $x_1 = 40$ *Ohm*, $r_2 = 10$ *Ohm*, $r_0 = x_0 = 40$ *Ohm*.

With the aid of the principal equations of the 4-terminal network in form A, determine the input current I_{1S} at the short-circuited output terminals, if $U_1 = 100$ V.

Answer: $\underline{A} = -j$; $\underline{B} = -j50$ Ohm; $\underline{C} = 0.025 - j0.025$ S; $\underline{D} = 1.25 - j0.25$; $I_{1S} = 2.55$ A.

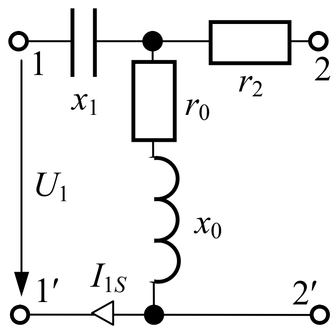


Fig. 5.2

5-3 (5.3). Find the matrix ***[H]*** elements of the non-symmetrical 4-terminal network, *П*-connected (fig. 5.3), if: $r_1 = 10$ *Ohm*, $r_2 = 20$ *Ohm*, $x_2 = 20$ *Ohm*, $x_3 = 40$ *Ohm*. With the aid of the principal equations of a 4-terminal network in form *[**H**]*, determine an input voltage at the open output terminals, if $U_{2o} = 100$ V.

Answer: $\underline{H}_{11} = 7.69 + j1.54$ Ohm;
$\underline{H}_{12} = -\underline{H}_{21} = 0.231 - j0.15$;
$\underline{H}_{22} = -0{,}0231 - j0{,}00962$ S; $U_{1o} = 50\sqrt{2}$ V.

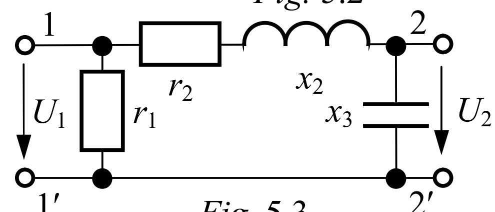


Fig. 5.3

5-4 (5.4). To work out an equivalent *П*-scheme of the transmission line (fig. 5.4) and determine its input impedance, the following no-load and short-circuit tests are carried out: $U_{1o} = 30$ kV, $I_{1o} = 6$ A, $P_{1o} = 27$ kW, $\varphi_{1o} < 0$; $U_{1S} = 4.5$ kV, $I_{1S} = 30$ A, $P_{1S} = 69$ kW, $\varphi_{1S} > 0$.

Determine the line input impedance Z_{inp} at the load $R_l = 1000$ Ohm, $C_l = 10$ μF.

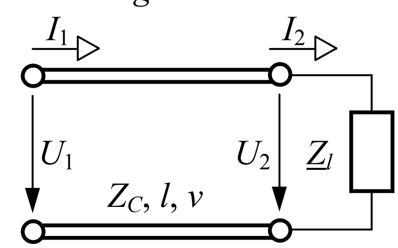


Fig. 5.4

Answer: $\underline{A} = \underline{D} = 0.9885 \cdot e^{j0.53°}$; $\underline{B} = 148.3 \cdot e^{j59.80°}$ Ohm; $\underline{C} = 0.198 \cdot 10^{-3} \cdot e^{j81.91°}$ S; $\underline{Z}_{1П} = \underline{Z}_{2П} = 10^4 \cdot e^{-j81.64°}$ Ohm; $\underline{Z}_{0П} = 148.3 \cdot e^{j59.80°}$ Ohm; $\underline{Z}_{inp} = 986 \cdot e^{-j19.75°}$ Ohm.

5-5 (5.5). The principal equations of A-type of the 4-terminal network are given:

$$\underline{U}_1 = -j50 \cdot \underline{I}_2 + 1.75 \cdot \underline{U}_2; \quad \underline{I}_1 = 0.5 \cdot \underline{I}_2 - j0.0025 \cdot \underline{U}_2.$$

The task is to work out an equivalent T-scheme as well as to write down the equations in H-form.

Answer: $\underline{Z}_{1T} = j300$ Ohm, $\underline{Z}_{2T} = -j200$ Ohm, $\underline{Z}_{0T} = j400$ Ohm;
$\underline{H}_{11} = -j100$ Ohm, $\underline{H}_{12} = 2$, $\underline{H}_{21} = -2$, $\underline{H}_{22} = -j0.005$ S.

5-6 (5.6). Determine the *ABCD*-coefficients of the non-symmetric 4-terminal network fig. 5.5,a, if $x_L = 80$ Ohm, $x_C = 40$ Ohm, $r_3 = r_4 = 40$ Ohm.

With the aid of the principal equations of the 4-terminal network, compute a load current I_2, if the load impedance is $\underline{Z}_2 = 60 + j30$ Ohm, and the input voltage is $U_1 = 220$ V.

Solution. Perform the coefficient calculation with the aid of the input impedances.

$$\underline{Z}_{1o} = r_4 + \frac{jx_L(r_3 - jx_C)}{jx_L + r_3 - jx_C} = 40 + \frac{j80(40 - j40)}{j80 + 40 - j40} = 40 + 80 = 120\ Ohm,$$

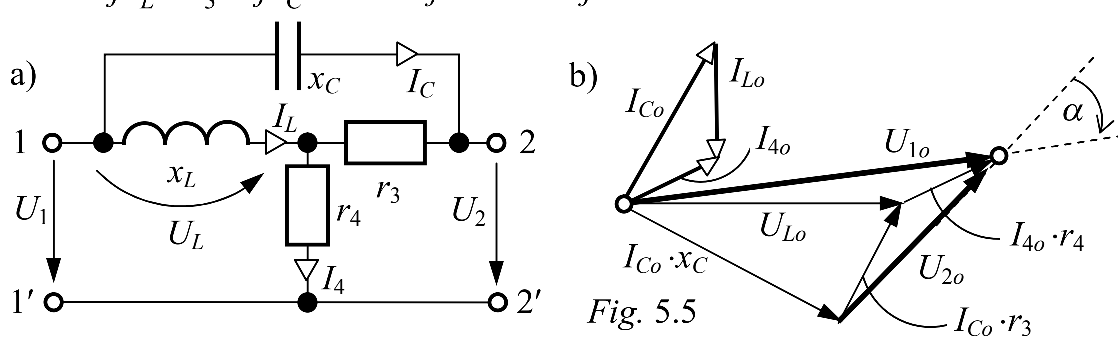


Fig. 5.5

$$\underline{Z}_{2o} = r_4 + \frac{r_3(jx_L - jx_C)}{r_3 + jx_L - jx_C} = 60 + \frac{40(j80 - j40)}{40 + j80 - j40} = 60 + j20 = 63.25 \cdot e^{j18.44^\circ}\ Ohm,$$

$$\underline{Z}_{2S} = \frac{-jx_C \cdot \left(r_3 + \dfrac{r_4 jx_L}{r_4 + jx_L}\right)}{-jx_C + r_3 + \dfrac{r_4 jx_L}{r_4 + jx_L}} = \frac{-j40\left(40 + \dfrac{40 \cdot j80}{40 + j80}\right)}{-j40 + 40 + \dfrac{40 \cdot j80}{40 + j80}} = 20 - j33.33 = 38.87 \cdot e^{-j59.04^\circ}\ Ohm.$$

$$\underline{A} = \sqrt{\frac{\underline{Z}_{1o}}{\underline{Z}_{2o} - \underline{Z}_{2S}}} = \sqrt{\frac{120}{60 + j20 - 20 + j33.33}} = \pm 1.342 \cdot e^{-j26.57^\circ} = \pm(1.2 - j0.6).$$

We have two main values of the coefficient $\underline{A} = ae^{j\alpha}$:

$$\underline{A}_1 = 1.342 \cdot e^{-j26.57^\circ}; \quad \underline{A}_2 = 1.342 \cdot e^{j153.43^\circ}.$$

The sign of angle α is selected with the aid of the 4-terminal network phasor diagram for no-load condition (fig. 5.5,b), which, on the ground of the principal equation $\underline{U}_{1o} = \underline{A} \cdot \underline{U}_{2o} = a\underline{U}_{2o} e^{j\alpha}$, follows by $\alpha < 0$, i.e. $\underline{A} = 1.342 \cdot e^{-j26.57^\circ} = 1.2 - j0.6$.

Further $\underline{B} = \underline{Z}_{2S} \cdot \underline{A} = 38.87 \cdot e^{-j59.04^\circ} \cdot 1.342 \cdot e^{-j26.57^\circ} = 52.16 \cdot e^{-j85.61^\circ} = 4 - j52\ Ohm,$

$$\underline{C} = \frac{1}{\underline{Z}_{1o}} \cdot \underline{A} = \frac{1.342e^{-j56.57}}{120} = 0.0112 \cdot e^{-j26.57^\circ} = 0.01 - j0.005\ S,$$

$$\underline{D} = \frac{\underline{Z}_{2o}}{\underline{Z}_{1o}} \cdot \underline{A} = \frac{63,.25e^{j18.44}}{120} \cdot 1.342 \cdot e^{-j26.57^\circ} = 0.707 \cdot e^{-j8.13^\circ} = 0.7 - j0.1.$$

The first principal equation is $\underline{U}_1 = \underline{A} \cdot \underline{U}_2 + \underline{B} \cdot \underline{I}_2 = \underline{A} \cdot \underline{I}_2 \cdot \underline{Z}_2 + \underline{B} \cdot \underline{I}_2$, from here

$$\underline{I}_2 = \frac{\underline{U}_1}{\underline{A}\underline{Z}_2 + \underline{B}} = \frac{220}{(1.2 - j0.6)(60 + j30) + 4 - j52} = 2.05 \cdot e^{j29.0^\circ}\ A,$$

the effective current value is $I_2 = 2.05\ A$.

5-7 (5.7). The investigation test of a 4-terminal network gives the following results:

$$\underline{Z}_{1o} = 1000\sqrt{2} \cdot e^{-j45^\circ}\ Ohm, \quad \underline{Z}_{1S} = 500\sqrt{2} \cdot e^{-j45^\circ}\ Ohm, \quad \underline{Z}_{2o} = 1000\sqrt{2} \cdot e^{j45^\circ}\ Ohm.$$

At the no-load condition, the oscillograms of the input and output voltages were taken, they being presented in fig. 5.6,a. It is necessary to:

1. Determine *ABCD*-coefficients.
2. Calculate the parameters of the equivalent *П*-scheme of the 4-terminal network as well as analyze if the scheme can be physically realized.
3. Determine the source voltage as well as the consumer current and voltage, if $i_1(t) = 20sin(\omega t + 90^\circ)\ mA$, and the consumer impedance is $\underline{Z}_2 = 500\ Ohm$.
4. Determine the 4-terminal network efficiency.

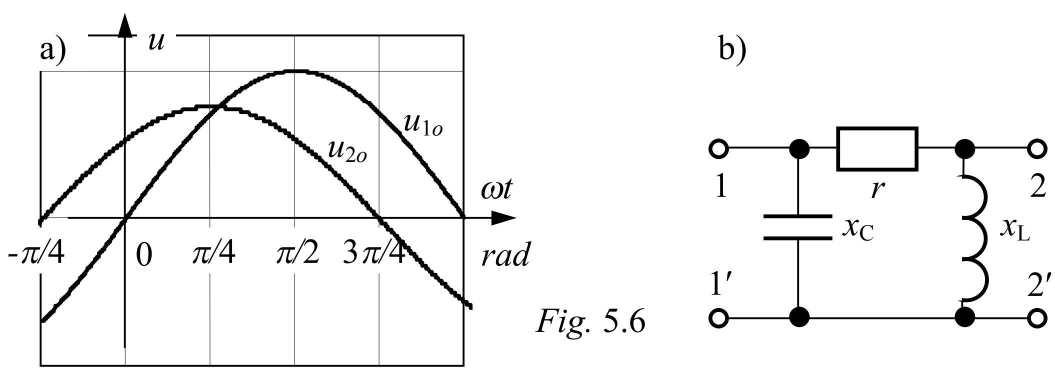


Fig. 5.6

Solution. Find the coefficient $\underline{A} = \sqrt{\dfrac{\underline{Z}_{1o}\underline{Z}_{1S}}{\underline{Z}_{2S}(\underline{Z}_{1o} - \underline{Z}_{1S})}}$.

The unknown impedance is

$$\underline{Z}_{2S} = \underline{Z}_{2o} \cdot \frac{\underline{Z}_{1S}}{\underline{Z}_{1o}} = 1000\sqrt{2} \cdot e^{j45^\circ} \frac{500\sqrt{2}e^{-j45}}{1000\sqrt{2}e^{-j45}} = 500\sqrt{2} \cdot e^{j45^\circ} \, Ohm,$$

then the coefficient is

$$\underline{A} = \sqrt{\frac{1000\sqrt{2}e^{-j45} \cdot 500\sqrt{2}e^{-j45}}{500\sqrt{2}e^{j45}\ (1000 - j1000 - 500 + j500)}} = \pm\sqrt{2} \cdot e^{j45^\circ} = ae^{j\alpha}.$$

We have two main values of the complex quantity

$$\underline{A}_1 = +\sqrt{2} \cdot e^{j45^\circ}; \quad \underline{A}_2 = -\sqrt{2} \cdot e^{j45^\circ} = \sqrt{2} \cdot e^{j135^\circ}.$$

An argument sign of the complex number is found with the aid of the oscillogram fig. 5.6,a, remembering that at the no-load condition there is

$$\underline{U}_{1o} = \underline{A} \cdot \underline{U}_{2o} \quad \text{and} \quad \underline{A} = \frac{\underline{U}_{1o}}{\underline{U}_{2o}} = \frac{U_{1o}}{U_{2o}} \cdot e^{j(\psi u1o - \psi u2o)} = ae^{j\alpha}.$$

From the given oscillogram, it is obvious $\psi_{u1o} = 0$, $\psi_{u2o} = +\dfrac{\pi}{4} = +45°$.

Thus, $\quad \alpha = \psi_{u1o} - \psi_{u2o} = 0 - 45° = -45°$

and a single coefficient value for the given 4-terminal network is $\underline{A} = +\sqrt{2} \cdot e^{j45^\circ} = 1 - j$.

Note, the absolute value of the coefficient $|\underline{A}| = a$ could be also determined by an oscillogram: $a = U_{1o\,m}/U_{2o\,m}$. However, the accuracy of the value determination by plots is low.

The rest of the coefficients are $\quad \underline{B} = \underline{Z}_{2S} \cdot \underline{A} = 500\sqrt{2} \cdot e^{-j45^\circ} \cdot \sqrt{2} \cdot e^{j45^\circ} = 1000\ Ohm,$

$$\underline{C} = \frac{1}{\underline{Z}_{1o}} \cdot \underline{A} = \frac{\sqrt{2}e^{-j45}}{1000\sqrt{2}e^{-j45}} = 0.001\ S,$$

$$\underline{D} = \frac{\underline{B}}{\underline{Z}_{1S}} = \frac{1000}{500\sqrt{2}e^{-j45}} = \sqrt{2} \cdot e^{j45^\circ} = 1 + j.$$

The parameters of the equivalent *П*-scheme of the 4-terminal network under study are as follows: $\underline{Z}_{1П} = \dfrac{\underline{B}}{\underline{D} - 1} = \dfrac{1000}{1 + j - 1} = -j1000\ Ohm$ – capacitive reactance;

$$\underline{Z}_{2П} = \frac{\underline{B}}{\underline{A} - 1} = \frac{1000}{1 - j - 1} = j1000\ Ohm \text{ – inductive reactance;}$$

$$\underline{Z}_{0П} = \underline{B} = 1000\ Ohm \text{ – resistance.}$$

All the equivalent scheme impedances of fig. 5.6,b are physically realized.

The input current complex is $\quad \underline{I}_1 = I_1 \cdot e^{j\cdot\psi i} = \dfrac{20}{\sqrt{2}} \cdot e^{j90^\circ}\ mA = 10^{-2}\sqrt{2} \cdot e^{j90^\circ}\ A.$

The input impedance of the 4-terminal network together with the load is

$$\underline{Z}_1 = \frac{\underline{A}\underline{Z}_2 + \underline{B}}{\underline{C}\underline{Z}_2 + \underline{D}} = \frac{(1 - j)500 + 1000}{0{,}001 \cdot 500 + 1 + j} = 877 \cdot e^{-j52.13^\circ}\ Ohm.$$

The input voltage is $\underline{U}_1 = \underline{I}_1 \cdot \underline{Z}_1 = 10^{-2}\sqrt{2} \cdot e^{j90^\circ} \cdot 877 \cdot e^{-j52.13^\circ} = 12.4 \cdot e^{j37.87^\circ}\ V.$

From the principal equation $\underline{U}_1 = \underline{A}\cdot\underline{U}_2 + \underline{B}\cdot\underline{I}_2 = \underline{U}_2\cdot(\underline{A} + \frac{\underline{B}}{\underline{Z}_2})$, we have

$$\underline{U}_2 = \frac{\underline{U}_1}{\underline{A} + \underline{B}/\underline{Z}_2} = \frac{12.4e^{j37.87}}{1 - j + \frac{1000}{500}} = 3.92\cdot e^{j56.31°}\ V.$$

Under Ohm's law, the current is $\underline{I}_2 = \frac{\underline{U}_2}{\underline{Z}_2} = \frac{3.92e^{j56.31}}{500} = 7.84\cdot 10^{-3} e^{j56.31°}\ A.$

Active powers are as follows:

- at the input of the 4-terminal network

$P_1 = \mathrm{Re}(\underline{U}_1\cdot \overset{*}{\underline{I}_1}) = \mathrm{Re}(12.4\cdot e^{j37.87°}\cdot 10^{-2}\sqrt{2}\cdot e^{-j90°}) = 107.7\cdot 10^{-3}\ W;$

- at the output of the 4-terminal network

$P_2 = I_2^2 r_2 = (7.84\cdot 10^{-3})^2\cdot 500 = 30.73\cdot 10^{-3}\ W.$

The efficiency is $\eta = \frac{P_2}{P_1} = \frac{30.73}{107.7} = 0.286.$

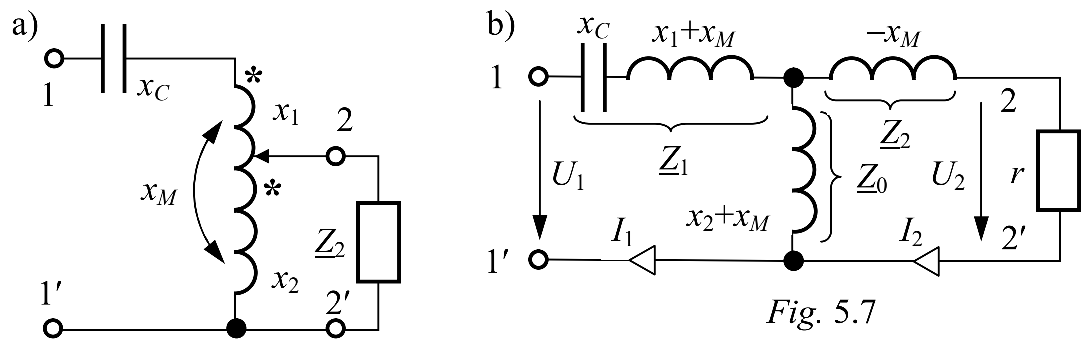


Fig. 5.7

5-8 (5.8). To pass from the overhead line into the cable one, the autotransformer scheme presented in fig. 5.7,a is employed. The scheme parameters are:

$x_C = 35\ Ohm,\ x_1 = 20\ Ohm,\ x_3 = 60\ Ohm,\ x_M = 10\ Ohm.$

Determine the *ABCD*-coefficients of the 4-terminal network.

At the load $\underline{Z}_2 = r = 50\ Ohm$, its power is $P_2 = 450\ W$. Using the principal equations, calculate the input active power.

Solution. Let's eliminate the inductive connection and obtain an equivalent *T*-scheme of 4-terminal network (fig. 5.7,b), having

$\underline{Z}_1 = -jx_C + j(x_1 + x_M) = -j35 + j(20 + 10) = -j5\ Ohm,$

$\underline{Z}_2 = -jx_M = -j10\ Ohm,$

$\underline{Z}_0 = j(x_2 + x_M) = j(60 + 10) = j70\ Ohm.$

For *T*-scheme of a 4-terminal network, there is a strict relationship between the coefficients and impedances:

$$\underline{A} = 1 + \frac{\underline{Z}_1}{\underline{Z}_0} = 1 + (-j5)/(j70) = 0.928;$$

$$\underline{B} = \underline{Z}_1 + \underline{Z}_2 + \frac{\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_0} = -j5 - j10 + \frac{-j5\cdot(-j10)}{j70} = -j15.70\ Ohm;$$

$$\underline{C} = \frac{1}{\underline{Z}_0} = \frac{1}{j70} = -j0.0143\ S; \qquad \underline{D} = 1 + \frac{\underline{Z}_2}{\underline{Z}_0} = 1 + \frac{-j10}{j70} = 0.857.$$

The load current is $I_2 = \sqrt{\frac{P_2}{r}} = \sqrt{\frac{450}{50}} = 3\ A.$

Assume $\underline{I}_2 = 3\ A$, under Ohm's law $\underline{U}_2 = \underline{I}_2 \cdot \underline{Z}_2 = 3 \cdot 50 = 150\ V.$

Further $\underline{U}_1 = \underline{A} \cdot \underline{U}_2 + \underline{B} \cdot \underline{I}_2 = 0.928 \cdot 150 + (-j15.7) \cdot 3 = 147 \cdot e^{-j18.7°}\ V,$

$\underline{I}_1 = \underline{C} \cdot \underline{U}_2 + \underline{D} \cdot \underline{I}_2 = -j0.0143 \cdot 150 + 0.857 \cdot 3 = 3.35 \cdot e^{-j39.8°}\ A,$

$P_1 = \text{Re}(\underline{U}_1 \cdot \underline{I}_1^*) = \text{Re}(147 \cdot e^{-j18.7°} \cdot 3.35 \cdot e^{j39,8°}) = 459\ W \approx P_2 = 450\ W.$

Pay attention, the 4-termional network scheme under study is a lossless scheme (without resistances), correspondingly, $P_1 = P_2$. Inaccuracy of 9 W is explained by the result round up to three significant digits. Furthermore, the relative inaccuracy was as much as $\varepsilon\,\% = \frac{\Delta P}{P} \cdot 100 = \frac{9}{450} \cdot 100 = 2\%$, which is allowable while calculating.

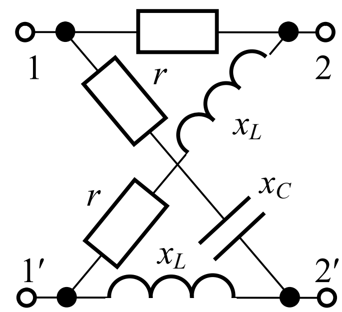


Fig. 5.8

5-9 (5.9). Determine A-coefficients of X-scheme (a bridge scheme) of a 4-terminal network presented in fig. 5.8, if

$$r = x_L = x_C = 10\ Ohm.$$

Direction. While selecting the single meaning of the coefficient $\underline{A}$, it is recommended to construct a phasor diagram of the complex potentials of the 4-terminal network at the no-load condition, assuming $\underline{\varphi}_{1'} = 0$.

Answers: $\underline{A} = 0.6 + j0.8$, $\underline{B} = j20\ Ohm$, $\underline{C} = 0.1 + j0.1\ S$, $\underline{D} = 1 + j2$.

5-10 (5.10). a) A load $\underline{Z}_l = r_l = 9\ Ohm$ is connected to the terminals of the A-C voltage source with emf $E = 100\ V$ and inner impedance $\underline{Z}_i = r_i = 1\ Ohm$ (fig. 5.9,a). Determine the consumer active power P_l.

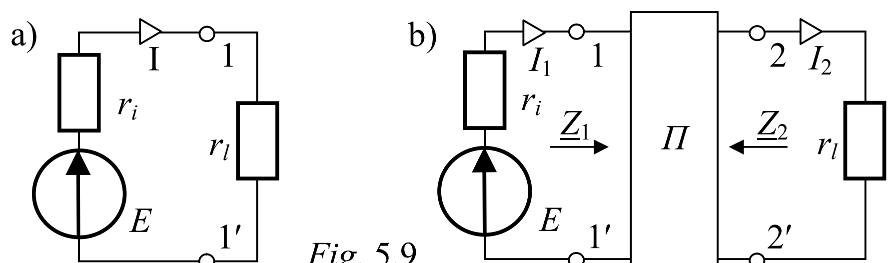


Fig. 5.9

b) In order to increase the active power delivered from the generator into the load (to match the generator with the load), a 4-terminal network is placed between the generator and the load (fig. 5.9,b). Determine the parameters of this 4-terminal network to produce the maximum possible power transmission P_{2max} from the generator into the load.

Solution of the task a). The circuit current in fig. 5.9,a is $I = \frac{E}{r_i + r_l} = \frac{100}{1+9} = 10\ A,$

the consumer active power being $P_l = I^2 \cdot r_l = 10^2 \cdot 9 = 900\ W.$

Solution of the task b). The generator in scheme fig. 5.9,b is loaded with the 4-terminal network which is in its turn loaded with a consumer possessing the impedance $\underline{Z}_l = r_l = 9\ Ohm$. We'll solve the problem of the maximum power transmission from the generator through the 4-terminal network into the consumer in two steps:

1. Find such a load impedance for a generator $\underline{Z}_1$, which guarantees the maximum possible power P_{1max} delivered to the input of the 4-terminal network.

On the ground of the principal equation of the 4-terminal network at the load $\underline{Z}_2 = r_l$, its input impedance is $\underline{Z}_1 = \dfrac{\underline{A} \cdot \underline{Z}_2 + \underline{B}}{\underline{C} \cdot \underline{Z}_2 + \underline{D}}$.

Because the 4-terminal network has not been found yet, it is possible to take any values for its coefficients, changing in such a way the generator load.

Note, the device allowing to change (transform) the load impedance is termed an *impedance transformer*, while the problem of the searching of a scheme with the required properties (in the problem under study, this is a 4-terminal network) is termed a *synthesis problem* of the electric circuit.

In the course parts "Linear DC-circuits", "Linear AC-circuits" the problem of the conditions of the maximum possible power transmission from active two-pole network to the passive one is studied in details. When the reactive power is fully compensated in the generator inner circuit (this is the case of the task a) of the problem under study), the above requirement is expressed by the equality $r_i = r_l$.

Thus, the first equation to synthesize a 4-terminal network takes a view:

$$\underline{Z}_1 = \frac{\underline{A} \cdot \underline{Z}_2 + \underline{B}}{\underline{C} \cdot \underline{Z}_2 + \underline{D}} = r_i. \tag{5.1}$$

2. Considering the left part of the scheme fig. 5.9,b with regard to the output terminals 2-2′ of the 4-terminal network as the equivalent generator with the inner impedance $\underline{Z}_2$, we write down the condition of the maximum power transmission from the equivalent generator into the consumer r_l as follows:

$$\underline{Z}_2 = \frac{\underline{D} \cdot r_i + \underline{B}}{\underline{C} \cdot r_i + \underline{A}} = r_l. \tag{5.2}$$

To determine four coefficients $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$, the system of four linear independent equations is required. The third necessary equation is set by the essential property of the 4-terminal network coefficients $\underline{AD} - \underline{BC} = 1$. (5.3)

The fact that the fourth equation is missing gives us the freedom to choose a coefficient, even to take any complex number instead of it. So, the problem of 4-terminal network synthesis has infinite number of solutions.

The fourth equation can be found in one of the two ways:

1. Synthesize a symmetrical 4-terminal network where $\underline{A} = \underline{D}$ and take the simplest scheme for the realization: *T*-scheme or *П*-scheme (fig. 5.10);

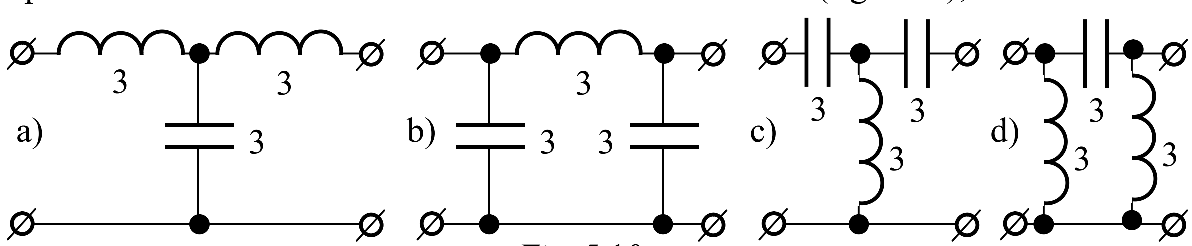


Fig. 5.10

2. Assume $\underline{D} = 1$, and then both *T*- and *П*-scheme turn into the non-symmetric *Г*-scheme of form fig. 5.11,a or b.

Let's present the solution of both variants.

1. *Synthesis of the symmetric 4-terminal network.*

Coefficients are set by the following equation system:

$$\frac{\underline{A}\cdot r_l+\underline{B}}{\underline{C}\cdot r_l+\underline{D}}=r_i;\quad \frac{\underline{D}\cdot r_i+\underline{B}}{\underline{C}\cdot r_i+\underline{A}}=r_l;\quad \underline{AD}-\underline{BC}=1;\quad \underline{A}=\underline{D}.$$

The detailed solution of the system is as follows:

$$\begin{cases}\underline{A}\cdot r_l+\underline{B}=\underline{C}\cdot r_i\cdot r_l+\underline{D}\cdot r_i,\\ \underline{D}\cdot r_i+\underline{B}=\underline{C}\cdot r_i\cdot r_l+\underline{A}\cdot r_l.\end{cases}$$

take into account that $\underline{A}=\underline{D}$ and then subtract the second equation from the first one. We obtain:

$$\underline{A}\cdot(r_l-r_i)=\underline{A}\cdot(r_i-r_l),\quad \text{from here}\quad \underline{A}=0=\underline{D}.$$

In order to determine the other two equations, we solve an equation system where $\underline{A}=\underline{D}=0$ is taken into account:

$$\underline{B}=\underline{C}\cdot r_i\cdot r_l;\qquad -\underline{BC}=1,\quad \text{from here}\quad \underline{B}=\pm j\sqrt{r_ir_l},\quad \underline{C}=\pm j\frac{1}{\sqrt{r_ir_l}},$$

furthermore, as $-\underline{BC}=1$, the signs of $\underline{B}$ and $\underline{C}$ are to be identical.

There are two variants of the solution:

a) $\underline{A}=\underline{D}=0$;
$\underline{B}=+j\sqrt{r_ir_l}=j\sqrt{1\cdot 9}=j3\ Ohm$;
$\underline{C}=+j\frac{1}{\sqrt{r_ir_l}}=j\frac{1}{3}\ S.$

b) $\underline{A}=\underline{D}=0$;
$\underline{B}=-j\sqrt{r_ir_l}=-j\sqrt{1\cdot 9}=-j3\ Ohm$;
$\underline{C}=-j\frac{1}{\sqrt{r_ir_l}}=-j\frac{1}{3}\ S.$

Calculate the parameters of the *T*- and *П*-schemes by the known coefficients:

- *T*-scheme $\quad \underline{Z}_{1T}=\underline{Z}_{2T}=\frac{\underline{A}-1}{\underline{C}}=\pm j3\ Ohm,\quad \underline{Z}_{0T}=\frac{1}{\underline{C}}=\ \ j3\ Ohm.$

- *П*-scheme $\quad \underline{Z}_{1П}=\underline{Z}_{2П}=\frac{\underline{B}}{\underline{D}-1}=\ \ j3\ Ohm,\quad \underline{Z}_{0П}=\underline{B}=\pm j3\ Ohm.$

In the answers, the top signs belong to the variant a), the bottom ones – to the variant b). The corresponding schemes with impedances given in *Ohm* are presented in fig. 5.10.

In any scheme $\underline{Z}_1=r_i=1\ Ом$,

the generator currents is $\quad \underline{I}_1=\frac{E}{r_i+\underline{Z}_1}=\frac{100}{1+1}=50\ A,$

the active power at the 4-terminal network input is $P_{1max}=I^2\cdot\underline{Z}_1=50^2\cdot 1=2500\ W.$

Since the 4-terminal network has no loss, the consumer active power is

$$P_{2max}=P_{1max}=2500\ W$$

instead of $P_2=900\ W$ in the initial scheme fig. 5.9,a.

2. *Synthesis of the Г-scheme.*

The required coefficients are determined from the following equation system:

$$\frac{\underline{A}\cdot r_l+\underline{B}}{\underline{C}\cdot r_l+\underline{D}}=r_i;\quad \frac{\underline{D}\cdot r_i+\underline{B}}{\underline{C}\cdot r_i+\underline{A}}=r_l;\quad \underline{AD}-\underline{BC}=1;\quad \underline{D}=1.$$

And again there are two variants of the solution:

a) $\underline{A}=\frac{1}{9};\ \ \underline{B}=j2\sqrt{2}\ Ohm;\quad \underline{C}=j\frac{2\sqrt{2}}{9}\ S;\quad \underline{D}=1;$

b) $\underline{A}=\frac{1}{9};\ \ \underline{B}=-j2\sqrt{2}\ Ohm;\quad \underline{C}=-j\frac{2\sqrt{2}}{9}\ S;\quad \underline{D}=1.$

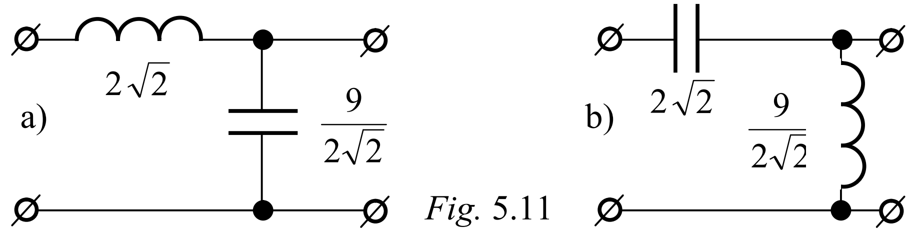


Fig. 5.11

Only two Γ-schemes presented in fig. 5.11 correspond to these solutions. The impedances of the inductive and capacitive elements in fig. 5.11 are again shown in *Ohm*. Here, as in case of fig. 5.10, there are

$$P_{1max} = P_{2max} = 2500\ W, \quad \underline{I}_1 = \frac{E}{2r_i} = \frac{100}{2\cdot 1} = 50\ A.$$

The load current is determined under the formula $\underline{I}_2 = \dfrac{\underline{I}_1}{\underline{C}r_l + \underline{D}}$.

For instance, in a scheme fig. 5.11,a it is $\underline{I}_2 = \dfrac{50}{j\dfrac{2\sqrt{2}}{9}\cdot 9 + 1} = \dfrac{50}{3}\cdot e^{-j70.53°}\ A.$

5-11 (5.11). The impedances of all the elements in schemes fig. 5.12 are in *Ohm*. Prove that the given 4-terminal networks are equivalent.

Directions. As the 4-terminal networks are reversible, it is quite enough to compare the impedances $\underline{Z}_{1o}$, $\underline{Z}_{1S}$ of both networks.

Answer: $\underline{Z}_{1o} = 100 + j200\ Ohm$,
$\underline{Z}_{1S} = 100 - j200\ Ohm$,
$\underline{Z}_{2o} = j100\ Ohm$, $\underline{Z}_{2S} = 80 - j\,60\ Ohm$.

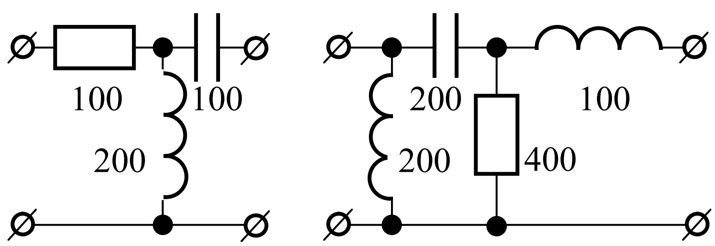


Fig. 5.12

The impedances as regards the secondary terminals are presented but for the check.

5.2. Transfer functions of 4-terminal networks

5-12 (5.37). For Γ-scheme of the 4-terminal network fig. 5.13,a, calculate and plot the frequency characteristics of the voltage transfer function at the no-load condition, if $r = 50\ Ohm$, $C = 40\ \mu F$.

Solution. The required transfer function may be found with the aid of the principal equations with coefficients of any form as well as with the aid of the principal equations with characteristic parameters. However, in case of simple schemes of a 4-terminal network, it is better to use Kirchhoff's laws, having written in a complex form the expressions for current $\underline{I}_{1o}$ and voltage $\underline{U}_{2o}$ in function of ω:

$$\underline{U}_{2o} = \underline{I}_{1o}\cdot\frac{1}{j\omega C}, \quad \text{but} \quad \underline{I}_{1o} = \frac{\underline{U}_{1o}}{r + \dfrac{1}{j\omega C}}, \quad \text{then}$$

$$W(j\omega) = \frac{\underline{U}_{2o}}{\underline{U}_{1o}} = \frac{1}{r + \dfrac{1}{j\omega C}}\cdot\frac{1}{j\omega C} = \frac{1}{1 + j\omega rC} = \frac{1}{1 + j\omega\tau},$$

where $\tau = rC = 50\cdot 40\cdot 10^{-6} = 2\cdot 10^{-3}\ s$ – is termed the *time constant* of the section under study (see paragraph «Transient processes in the linear electric circuits»), while a transfer function of the view $W(j\omega)=\dfrac{1}{1+j\omega\tau}$ is a transfer function of one of the standard sections of the automatic control system, such a section being called "aperiodic section".

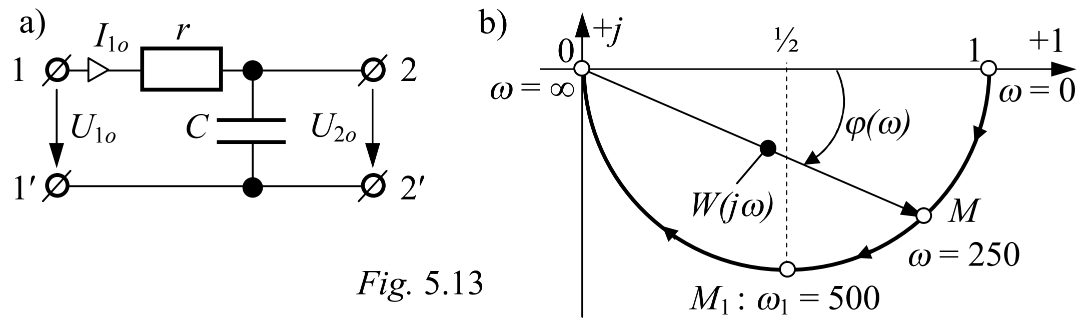


Fig. 5.13

The Nyquist's plot of this transfer function is presented in fig. 5.13,b and represents a semicircle of radius $R=½$. The image point M sets the position of the phasor end $W(j\omega)$ on the complex plane at the fixed frequencies:

at zero frequency $\omega=0$, the coordinates of point M are (1, 0);

at the frequency $\omega=0.5\cdot\tau^{-1}=\dfrac{0.5}{2\cdot10^{-3}}=250\ {}^{1}\!/\!_{s}$

$$W(j\omega)=\frac{1}{1+j0.5\tau^{-1}\tau}=\frac{1}{1+j0.5}=0.8-j0.4,$$

$$W(\omega)=\sqrt{0.8^2+0{,}4^2}=0.894,\quad \varphi(\omega)=arctg\frac{-0.4}{0.8}=-26.56°,$$

this point M is shown in fig. 5.13,b.

Position of point M_1 corresponds to frequency $\omega=\tau^{-1}=500\ s^{-1}$, while at $\omega=\infty$ $W(\omega)=0$, $\varphi(\omega)=-90°=-½\pi$ point M occurs in the coordinate origin.

Note, at the frequency variation $\omega(0\ \ldots\ \infty)$, the image point moves in the clockwise direction and the phase angle in a scheme with a single energy storage element varies by 90°. This is general property of Nyquist's plots, however, the phase angle varies up to $(-n\cdot½\pi)$, where n – number of different storage elements.

Amplitude frequency characteristic $W(\omega)=\dfrac{1}{\sqrt{1+(\omega\tau)^2}}$ is an even frequency function, while the phase frequency characteristic $\varphi(\omega)=-arctg(\omega\tau)$ is an odd function, the unit of $\varphi(\omega)$ being radian.

Real and imaginary frequency characteristics are termed in accordance with the following relationship

$$W(j\omega)=\frac{1}{1+j\omega\tau}\cdot\frac{1-j\omega\tau}{1-j\omega\tau}=\frac{1}{1+(\omega\tau)^2}-j\frac{\omega\tau}{1+(\omega\tau)^2},$$

where $B(\omega)=\dfrac{1}{1+(\omega\tau)^2}$ – a *real* frequency characteristic, even frequency function,

$M(\omega)=-\dfrac{\omega\tau}{1+(\omega\tau)^2}$ – an *imaginary* frequency characteristic, odd function.

Note, it is possible to obtain the phase frequency characteristic by the formula $\varphi(\omega) = arctg\dfrac{M(\omega)}{B(\omega)}$ as well.

The decibel-log frequency response is

$$L(\omega) = 20lgW(\omega) = -\,20lg\sqrt{1+(\omega\tau)^2} = -\,10lg\left[1+(\omega\tau)^2\right].$$

Tabulate the calculation results of the transfer function characteristics in the table 5.1.

The brace in the table 5.1 marks the frequency span corresponding to *decade*, in which the frequency ω varies by 10 times and, correspondingly, the logarithm $lg(\omega)$ difference by 1.

The characteristics $W(\omega)$, $B(\omega)$, $-M(\omega)$, $\varphi(\omega)$ are presented in fig. 5.14.

Decibel-log frequency responses are presented in fig. 5.15,a (solid lines), while their asymptotic characteristics are performed by the line segments (dash lines). The coupling frequency of the line segments is $\omega_0 = \tau^{-1}$; the maximum deviation of the amplitude frequency asymptotic characteristic from the real one is 3.01 *dB*, the slope angle of the straight line is 20 *dB*/*decade*, it being usually marked as (–1) (accordingly, at 40 *dB*/*decade* it is (–2), at 60 *dB*/*decade* – (–3) and so on). The phase-frequency decibel-log response is presented in fig. 5.15,b.

Table 5.1

ω, s^{-1}	$\omega\tau$	$1+(\omega\tau)^2$	$W(\omega)$	$\varphi(\omega)$, *rad*	$L(\omega)$, *dB*	$lg(\omega)$	$B(\omega)$	$-M(\omega)$
0	0	1	1	0	0	$-\infty$	1	0
$¼\tau^{-1}$=125	0.25	1.063	0.97	– 0.245	– 0.264	2.09	0.94	0.235
$½\tau^{-1}$=250	0.5	1.25	0.894	– 0.464	– 0.973	2.4	0.8	0.4
$1\tau^{-1}$=500	1	2	0.707	– 0.785	– 3.01	2.7	0.5	0.5
$1{,}5\tau^{-1}$=750	1.5	3.25	0.555	– 0.983	– 5.12	2.88	0.31	0.462
$2\tau^{-1}$=100	2	5	0.447	– 1.11	– 6.99	3	0.2	0.4
$3\tau^{-1}$=1500	3	10	0.316	– 1.25	– 10	3.18	0.1	0.3
$4\tau^{-1}$=2000	4	17	0.243	– 1.33	– 12.3	3.3	0.06	0.235
$5\tau^{-1}$=2500	5	26	0.196	– 1.37	– 14.1	3.4	0.04	0.192
$10\tau^{-1}$=5000	10	101	0.01	– 1.47	– 20.04	3.7	0.01	0.09

Owing to the rapid development of the computer engineering, the wide application of the transfer functions and characteristics to compute the circuit response by known effect of the arbitrary form becomes extremely actual. Problems 5.13 and 5.14 illustrate the application of the transfer functions using a simple 4-terminal network as an example. While computing, the computer mathematical program MathCAD was intensively employed. Unfortunately, there are some differences between the way how quantities, functions and numbers are labeled and symbolized in MathCAD-program and generally accepted labeling and symbolizing. Thus, for example, the complex quantities are not underlined, the powers of number 10 in the answers are presented in different way, application of the subscripts symbolizes the data array. That's why, when solving the problems, both the formulae in a generally accepted view and MathCAD-program segments are presented. In our opinion, these differences are not principal ones and the solution comprehension is not adversely affected. This paragraph is dedicated to the questions of the getting and usage of the transfer functions at the harmonic influence

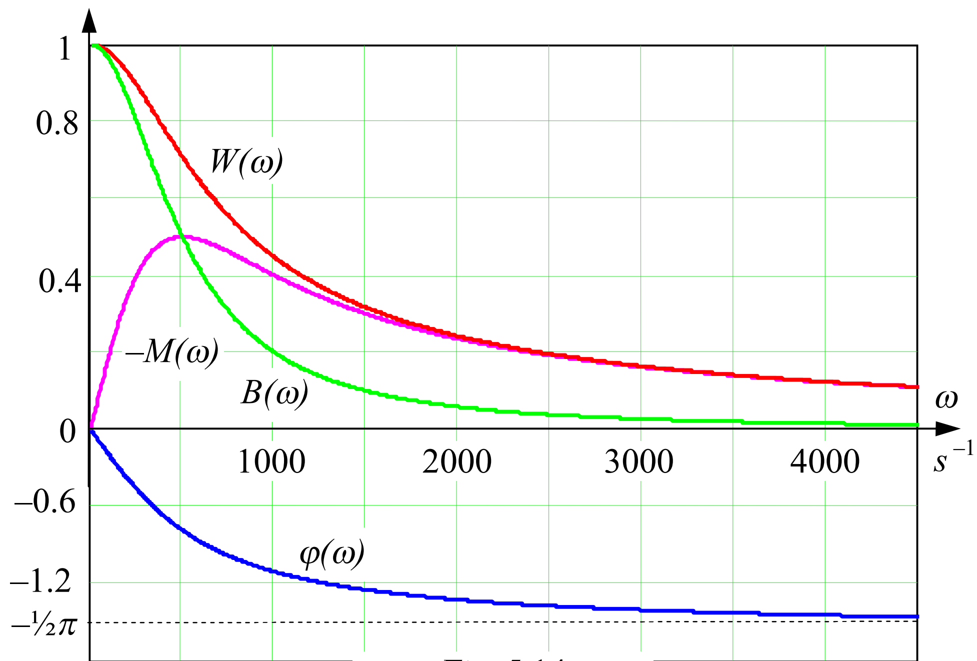


Fig. 5.14

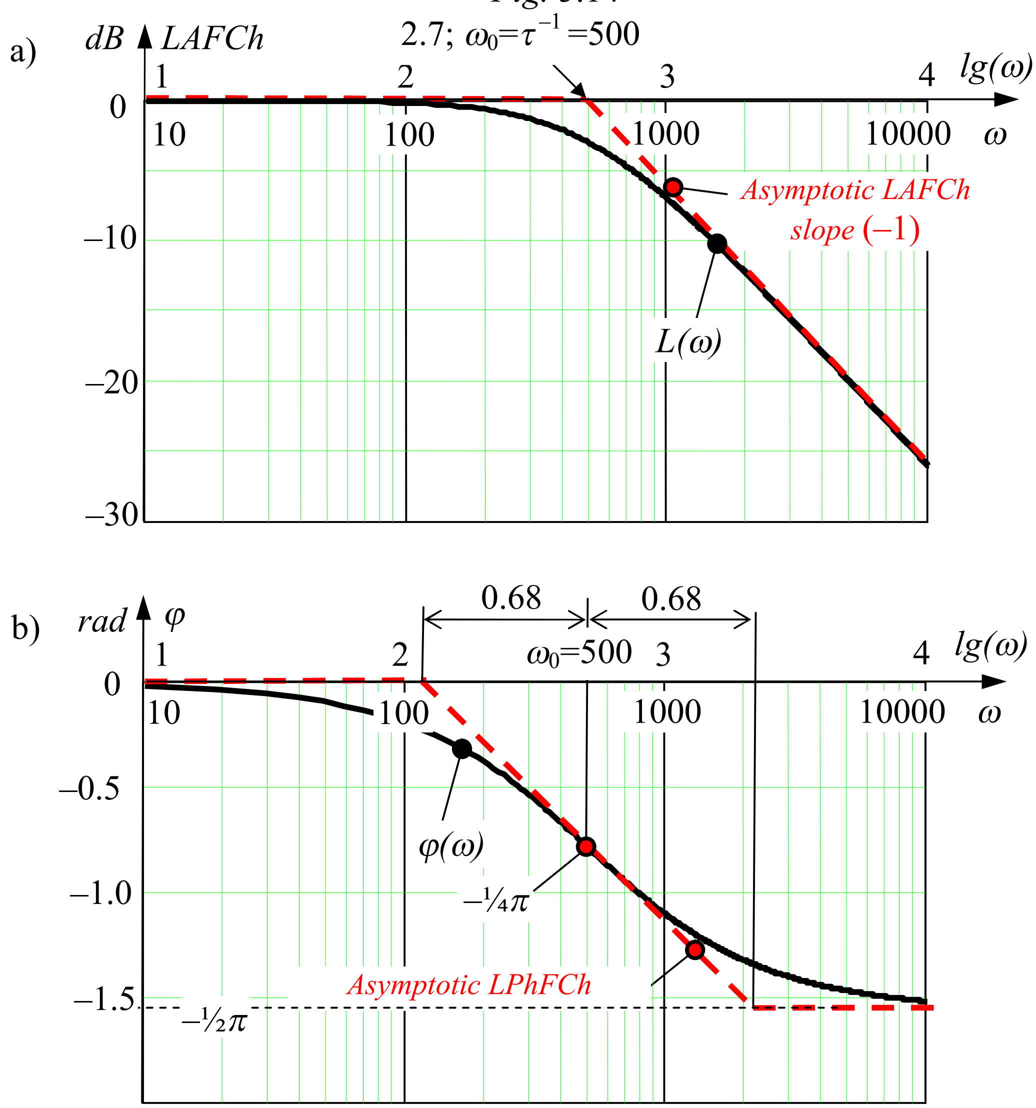


Fig. 5.15

(problems 5.13-5.14). Application of the characteristics obtained now in case of the influences of the other forms will be considered in the next sections «The non-sinusoidal current circuits (at the non-harmonic influence)» and «Transient processes in the linear electric circuits» (problems 6.3, 7.33-7.34).

Different quantities in the combined circuit of 4-terminal network supplied from the source with emf $\underline{E}$ and inner impedance $\underline{Z}_1$ and loaded with the impedance $\underline{Z}_2$ may be determined through the A-coefficients:

- input voltage $\underline{U}_1 = \underline{E}(\underline{A}_{11}\underline{Z}_2 + \underline{A}_{12})/\underline{H}_A$;
- input current $\underline{I}_1 = \underline{E}(\underline{A}_{21}\underline{Z}_2 + \underline{A}_{22})/\underline{H}_A$;
- output voltage $\underline{U}_2 = \underline{E}\cdot\underline{A}_{12}/\underline{H}_A$;
- output current $\underline{I}_2 = -\underline{E}/\underline{H}_A$.

Here $\underline{H}_A = \underline{A}_{11}\cdot\underline{Z}_2 + \underline{A}_{22}\cdot\underline{Z}_1 + \underline{A}_{12} + \underline{A}_{21}\cdot\underline{Z}_1\cdot\underline{Z}_2$ is an auxiliary frequency characteristic expressed through the A-coefficients of the 4-terminal network. Note, the elements $\underline{E}$-$\underline{Z}_1$ connected in series can be substituted by the elements $\underline{J}$-$\underline{Z}_1$ connected in parallel, which means the influence can be both in the form of a voltage $\underline{E}$ and in the form of a current $\underline{J} = \underline{E}/\underline{Z}_1$. In this case, the given formulae are corrected in a corresponding way.

5-13 (5.42). A source presented by an equivalent scheme $j(t)$-R_1 supplies the load R_2 with energy through Γ-shaped noninductive low-pass filter being a passive 4-terminal network (fig. 5.16). Numerical data are: $R_1 = 5000$ *Ohm*, $R_2 = 2000$ *Ohm*, $R = 1000$ *Ohm*, $C = 10$ μF.

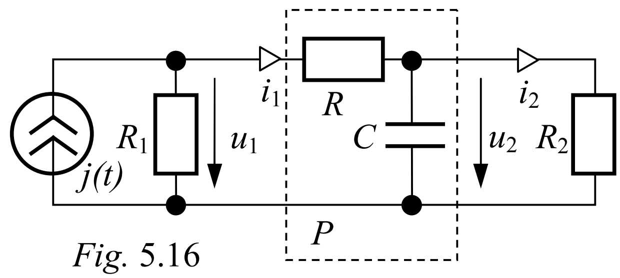


Fig. 5.16

Carry out the following: 1) calculate 4-terminal network coefficients of form A; 2) work out complex transfer impedances of the link; 3) plot the amplitude and phase frequency characteristics; 4) construct the Nyquist plot; 5) with the aid of the complex transfer impedance, determine the output voltage u_2 for the following cases –

$j(t) = 0.05\ A$, $\qquad j(t) = 0.05\cdot sin(100t + 45°)\ A$,

$j(t) = 0.05\cdot sin(1000t - 100°)\ A$, $\qquad j(t) = 0.05\cdot sin(10000t + 100°)\ A$.

Solution. 1. Mark the branch impedances of the 4-terminal network under study as $\underline{Z}_1 = R$ and $\underline{Z}_2 = \dfrac{1}{j\omega C}$. Determine A-coefficients by the known formulae of Γ-shaped 4-terminal network: $\underline{A}_{11} = 1 + \dfrac{\underline{Z}_1}{\underline{Z}_2}$; $\underline{A}_{12} = \underline{Z}_1$; $\underline{A}_{21} = \dfrac{1}{\underline{Z}_2}$; $\underline{A}_{22} = 1$.

Coefficients are presented as the frequency functions:

$Z1(j\omega) := R \qquad\qquad Z2(j\omega) := \dfrac{1}{j\omega\cdot C}$

$A11(j\omega) := 1 + \dfrac{Z1(j\omega)}{Z2(j\omega)} \qquad\qquad A11(j\omega)\Big|_{float,4}^{simplify} \rightarrow 1.+.1000e\text{-}1\cdot j\omega$

$A12(j\omega) := Z1(j\omega) \qquad\qquad A12(j\omega)\Big|_{float,4}^{simplify} \rightarrow 1000$

$$A21(j\omega) := \frac{1}{Z2(j\omega)} \qquad A21(j\omega)\Big|_{float,4}^{simplify} \to .1000e\text{-}4 \cdot j\omega$$

$$A22(j\omega) := 1 \qquad A22(j\omega)\Big|_{float,4}^{simplify} \to 1$$

Verification: $A11(j\omega)\cdot A22(j\omega) - A12(j\omega)\cdot A21(j\omega)\Big|_{float,4}^{simplify} \to 1.$

Finally, the coefficients are as follows:

$$\underline{A}_{11} = 1 + 0{,}01\cdot j\omega; \quad \underline{A}_{12} = 1000\ Ohm; \quad \underline{A}_{21} = 10^{-5}\cdot j\omega\ S; \quad \underline{A}_{22} = 1.$$

2. The input quantity (influence) in the present problem is $j(t)$, the output one (reaction or responce) is the load voltage $u_2(t)$. That's why, this time, the complex transfer function $H(j\omega) = X_{outp}(j\omega)/X_{inp}(j\omega)$ is the complex transfer impedance which is presented as $Z(j\omega) = U_2(j\omega)/J(j\omega)$.

Compute $Z(j\omega)$ in two ways. **The first manner** involves the application of the *A*-coefficients which have been received. Initially we calculate the auxiliary frequency function $HA(j\omega) := A11(j\omega)\cdot R2 + A22(j\omega)\cdot R1 + A12(j\omega) + A21(j\omega)\cdot R1\cdot R2$

$$HA(j\omega)\Big|_{float,4}^{simplify} \to 8000. + 120.\cdot j\omega,$$

The required impedance is

$$Z(j\omega) := \frac{R1\cdot R2}{HA(j\omega)} \qquad Z(j\omega)\Big|_{float,4}^{simplify} \to \frac{.2500e6}{200. + 3.\cdot j\omega}.$$

Perform the checking calculation **by the second manner**, having assumed the output voltage to be equal to $\underline{U}_2 = 1$ and having determined the input current $\underline{J}$ with the aid of Ohm's and Kirchhoff's laws:

$$I1(j\omega) := \frac{1}{R2} + j\omega\cdot C \qquad U1(j\omega) := 1 + R\cdot I1(j\omega) \qquad J(j\omega) := I1(j\omega) + \frac{U1(j\omega)}{R1}$$

$$Z(j\omega) := \frac{1}{J(j\omega)} \qquad Z(j\omega)\Big|_{float,4}^{simplify} \to \frac{.2500e6}{200. + 3.\cdot j\omega}.$$

Thus, the answer for the complex transfer impedance is as follows:

$$\underline{Z}(j\omega) = \frac{b1\cdot j\omega + b0}{j\omega + a0} = \frac{250000}{3j\omega + 200} = \frac{83333}{j\omega + 66.67}\ Ohm.$$

Values of the coefficients in this problem are:

$$b_1 = 0, \quad b_0 = 83333, \quad a_0 = 66.67.$$

3. Amplitude and phase frequency characteristics of the link are plotted in accordance with the following formulae:

$$Z(\omega) = |\underline{Z}(j\omega)| = \sqrt{\frac{(b_1\cdot\omega)^2 + b_0^2}{\omega^2 + a_0^2}} = \sqrt{\frac{83333^2}{\omega^2 + 66.67^2}};$$

$$\varphi(\omega) = arg(\underline{Z}(j\omega)) = arctg\frac{b_1\omega}{b_0} - arctg\frac{\omega}{a_0} = arctg\,0 - arctg\frac{\omega}{66.67}.$$

The plots of the amplitude and phase frequency characteristics are presented in fig. 5.17,a and b, respectively.

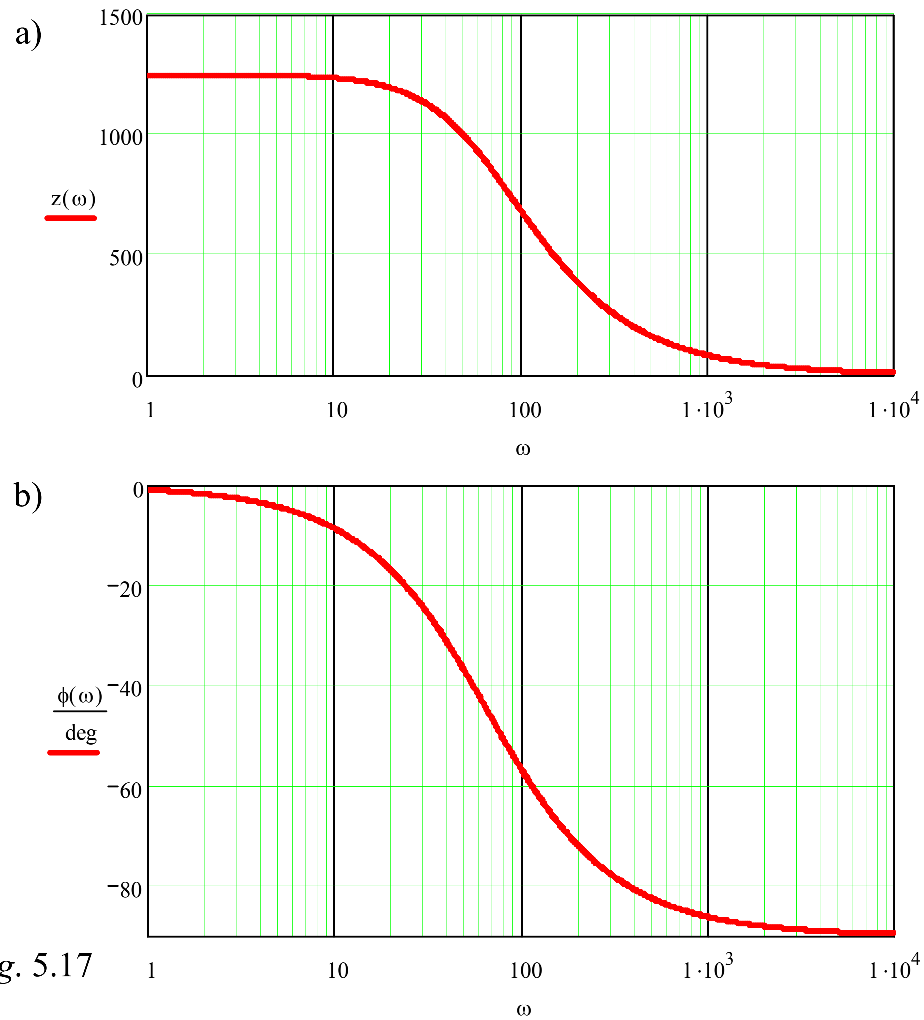


Fig. 5.17

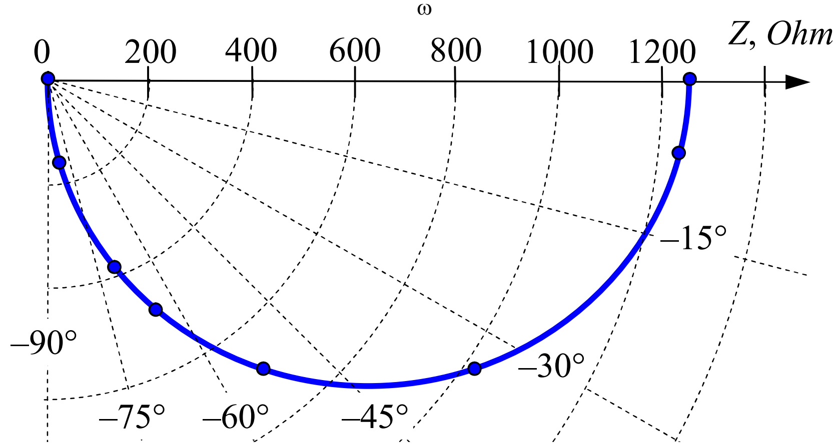


Fig. 5.18

4. The Nyquist plot is a dependence $Z(\omega) = f(\varphi(\omega))$ in a polar coordinate system. The calculations for plotting are tabulated in table 5.2. The plot itself is presented in fig. 5.18.

Table 5.2

ω, s^{-1}	0	10	50	100	150	200	500	1000	∞
φ, *deg*	0	– 8.5	– 36.9	– 56.3	– 66.0	– 71.6	– 82.4	– 86.2	– 90
Z, Ohm	1250	1236	1000	690	507.7	395	165.2	83.1	0

5. The values of the complex transfer impedance at the given frequencies are as follows: $Z(0) = 1250\ Ohm$, $\underline{Z}(j100) = 693.4 \cdot e^{-j56.31°}\ Ohm$, $\underline{Z}(j1000) = 83.15 \cdot e^{-j86.19°}\ Ohm$, $\underline{Z}(j10000) = 8.33 \cdot e^{-j89.62°}\ Ohm$.

The influence complex amplitudes $J(j\omega)$ and thoset of response $U_2(j\omega) = Z(j\omega) \cdot J(j\omega)$ at the same frequencies are:

$J(0) = 0.05\ A$, $J(j100) = 0.05 \cdot e^{j45°}\ A$,
$J(j1000) = 0.05 \cdot e^{-j100°}\ A$, $J(j10000) = 0.05 \cdot e^{j100°}\ A$,
$U_2(0) = 62.5\ V$, $U_2(j100) = 34.7 \cdot e^{-j11.31°}\ V$,
$U_2(j1000) = 4.16 \cdot e^{j173.81°}\ V$, $U_2(j10000) = 0.42 \cdot e^{j10.38°}\ V$.

The instantaneous values of the output voltage are:

$u_2(t) = 62.5\ V$, $u_2(t) = 34.7 \cdot sin(100t - 11.31°)\ V$,
$u_2(t) = 4.16 \cdot sin(1000t + 173.81°)\ V$, $u_2(t) = 0.42 \cdot sin(10000t + 10.38°)\ V$.

Pay attention, the amplitudes of the output voltage u_2 decrease sharply at the frequency increase while the influence amplitude remains invariable 0.05 A. Here the features of the 4-terminal network as a filter are clearly seen.

5-14 (5.43). Solve problem 5.13 after the substitution of resistor R by inductance $L = 10\ H$ (fig. 5.19). A capacitance is taken equal to $C = 1\ \mu F$.

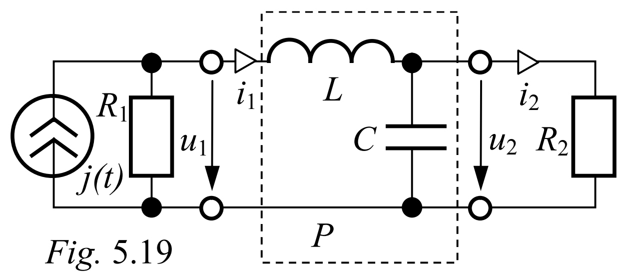


Fig. 5.19

Solution. The order of the solution is very the same as that of the problem 5.13. That's why, only the answers are given.

1. $Z1(j\omega) := j\omega \cdot L$ $\qquad Z2(j\omega) := \dfrac{1}{j\omega \cdot C}$

$$A11(j\omega) := 1 + \frac{Z1(j\omega)}{Z2(j\omega)} \qquad A11(j\omega) \Big|_{float,4}^{simplify} \rightarrow 1.+.1000e\text{-}4 \cdot (j\omega)^2$$

$$A12(j\omega) := Z1(j\omega) \qquad A12(j\omega) \Big|_{float,4}^{simplify} \rightarrow 10. \cdot j\omega$$

$$A21(j\omega) := \frac{1}{Z2(j\omega)} \qquad A21(j\omega) \Big|_{float,4}^{simplify} \rightarrow .1000e\text{-}5 \cdot j\omega$$

$$A22(j\omega) := 1 \qquad A22(j\omega) \Big|_{float,4}^{simplify} \rightarrow 1$$

Verification: $A11(j\omega) \cdot A22(j\omega) - A12(j\omega) \cdot A21(j\omega) \Big|_{float,4}^{simplify} \rightarrow 1.$

Finally, the coefficients are as follows:

$$\underline{A}_{11} = 1 + 10^{-5} \cdot (j\omega)^2; \quad \underline{A}_{12} = 10j\omega\ Ом; \quad \underline{A}_{21} = 10^{-6} \cdot j\omega\ См; \quad \underline{A}_{22} = 1.$$

2. *The first way* to calculate $Z(j\omega)$.

$$HA(j\omega) := A11(j\omega) \cdot R2 + A22(j\omega) \cdot R1 + A12(j\omega) + A21(j\omega) \cdot R1 \cdot R2$$

$$HA(j\omega) \Big|_{float,4}^{simplify} \rightarrow 7000. + .2000e\text{-}1 \cdot (j\omega)^2 + 20. \cdot j\omega,$$

$$Z(j\omega) := \frac{R1 \cdot R2}{HA(j\omega)} \qquad Z(j\omega) \Big|_{float,4}^{simplify} \to \frac{.5000e9}{.3500e6 + (j\omega)^2 + 1000 \cdot j\omega}.$$

The second way to calculate $Z(j\omega)$.

$$I1(j\omega) := \frac{1}{R2} + j\omega \cdot C \qquad U1(j\omega) := 1 + Z1(j\omega) \cdot I1(j\omega) \qquad J(j\omega) := I1(j\omega) + \frac{U1(j\omega)}{R1}$$

$$Z(j\omega) := \frac{1}{J(j\omega)} \qquad Z(j\omega) \Big|_{float,4}^{simplify} \to \frac{.5000e9}{.3500e6 + (j\omega)^2 + 1000 \cdot j\omega}.$$

Thus, the answer for the complex transfer impedance is as follows:

$$\underline{Z}(j\omega) = \frac{5 \cdot 10^8}{(j\omega)^2 + 1000 \cdot j\omega + 3{,}5 \cdot 10^5}\ Ом.$$

3. Amplitude and phase frequency characteristics of the link are plotted in accordance with the following formulae:

$$Z(\omega) := \left| \frac{5 \cdot 10^8}{(j \cdot \omega)^2 + 1000 \cdot j \cdot \omega + 3.5 \cdot 10^5} \right|;$$

$$\varphi(\omega) := arg\left(\frac{5 \cdot 10^8}{(j \cdot \omega)^2 + 1000 \cdot j \cdot \omega + 3.5 \cdot 10^5} \right).$$

The pots of the amplitude and phase frequency characteristics are presented in fig. 5.20,a and b, respectively.

4. The calculations necessary for constructing the Nyquist plot are tabulated in table 5.3. The plot itself is presented in fig. 5.21.

Table 5.3

ω, c^{-1}	0	50	100	200	500	1000	2000	5000	∞
φ, *deg*	0	– 8.19	–16.39	–32.83	–78.69	–123.0	–151.3	–168.5	–180
Z, *Ohm*	1429	1424	1411	1355	981	419	120	19.9	0

5. The values of the complex transfer impedance at the given frequencies are as follows:

$Z(0) = 1429\ Ohm$, $\underline{Z}(j100) = 1411 \cdot e^{-j16.39°}\ Ohm$,

$\underline{Z}(j1000) = 419.2 \cdot e^{-j123.02°}\ Ohm$, $\underline{Z}(j10000) = 4.99 \cdot e^{-j174.27°}\ Ohm$.

The influence complex amplitudes $J(j\omega)$

and those of response $U_2(j\omega) = Z(j\omega) \cdot J(j\omega)$ at the same frequencies are:

$J(0) = 0.05\ A$, $J(j100) = 0.05 \cdot e^{j45°}\ A$,

$J(j1000) = 0.05 \cdot e^{-j100°}\ A$, $J(j10000) = 0.05 \cdot e^{j100°}\ A$,

$U_2(0) = 71.43\ V$, $U_2(j100) = 70.54 \cdot e^{j28.61°}\ V$,

$U_2(j1000) = 20.96 \cdot e^{j136.98°}\ V$, $U_2(j10000) = 0.25 \cdot e^{-j74.27°}\ V$.

The instantaneous values of the output voltage are:

$u_2(t) = 71.43\ V$, $u_2(t) = 70.54 \cdot sin(100t + 28.61°)\ V$,

$u_2(t) = 20.96 \cdot sin(1000t + 136.98°)\ V$, $u_2(t) = 0.25 \cdot sin(10000t - 74.27°)\ V$.

Pay attention, the amplitudes of the output voltage u_2 decrease sharply at the frequency increase starting from 500 *rad*/*s*. Here the features of the 4-terminal network as a filter are clearly seen.

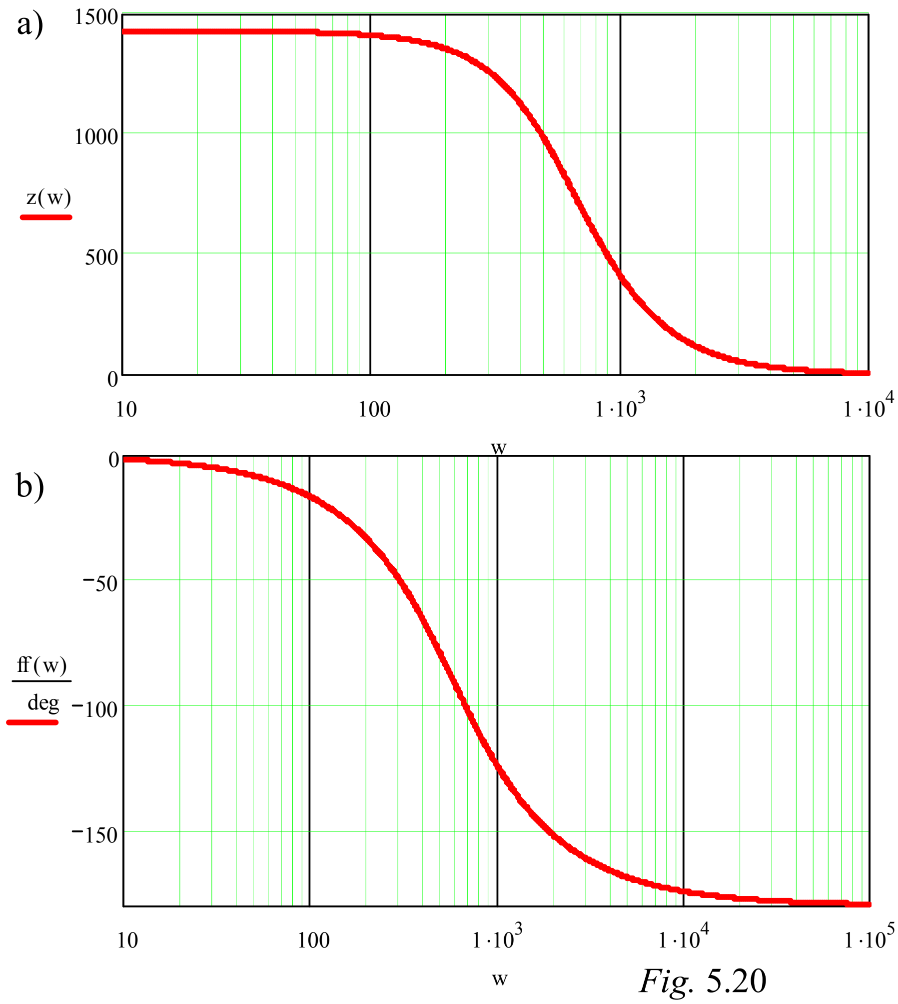


Fig. 5.20

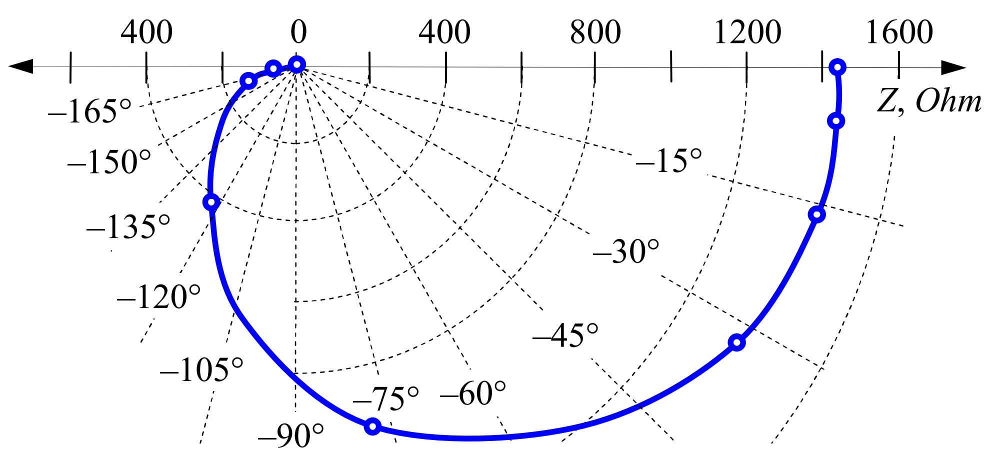


Fig. 5.21