ANALYTICAL INVESTIGATION INTO VELOCITY CHANGE OF THE TRANSPORTED MATERIAL IN A PIPELINE BEND

Mikhail Chaltsev, Leonid Vovk

Automobile Transport and Highway Engineering Institute of Donetsk National Technical University

Summary. The mathematical model of aerodisperse flow movement in bend is offered. The solution of this model is received under the conditions approached to the real ones. The experimental results are compared according to the given model and some simplifying hypotheses.

Keywords: pneumatic conveying of solids, flow velocity, differential equations

THE MATHEMATICAL MODEL FOR THE MOVEMENT OF TRANSPORTED MATERIAL AT THE FLOW TURN

The main aim in the investigation of aerodisperse flow of solids is the mathematical description of their regularities depending on the characteristics of the particles and conditions of flow formation. Lack of knowledge about tension generation mechanisms being the difficult function of particles velocity puts obstacles in the way of working out of the adequate analytical description of regularities in pneumotransport flows. Besides, the high heterogeneity in solid phase concentration is typical for high velocities of pneumatic conveying of solids. All these problems worsen the experimental and analytical investigations of pneumatic conveying processes.

Additional difficulties arise at the mathematical modelling of pneumatic transportation in bend. This is due to the fact that centrifugal force influences the transported material in the perpendicular direction at the flow turn. The centrifugal force is many times more than the material weight [1]. The forces which cause hovering of the particles during the horizontal movement are in many times less than the centrifugal force. Therefore the overwhelming majority of fractions does not come back from an external bend wall into the flow of transporting gas, but settles on it.

The present bend calculation methods are based on the empirical dependences used for uniform flows [2, 3, 4, 5, 6]. Naturally, the calculations by these techniques give considerable errors because of ignoring the difficult physical nature of two-component flows transportation [7, 8]. Besides, in many scientific publications on this theme [4, 7, 9], the offer about the invariability of the cross-section area of deposited solids was accepted. However, the last experimental investigations with a transparent bend exposed the other structure of particles flow in a pipeline bend (Fig. 1) [10, 8].

According to these sources the sliding speed of particles settled in the bend gradually decreases, and the thickness of a layer increases on the exit from the bend. This process is illustrated in Fig. 1 and 2.

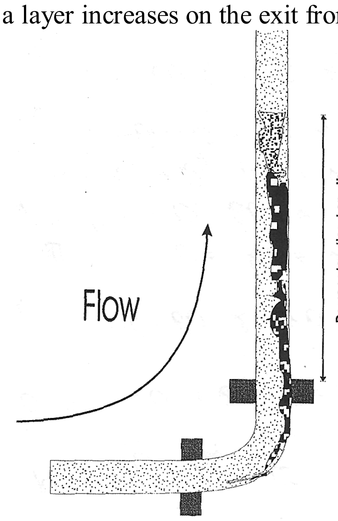

Fig. 1. Aerodisperse flow in a bend [10]

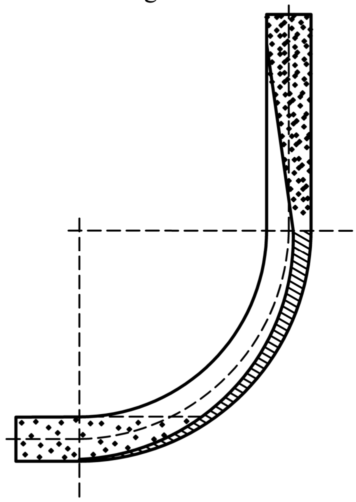

Fig. 2. The scheme of an aerodisperse flow in a bend according to [8]

The purpose of the work is to create the mathematical theory for the calculation of kinematic characteristics of the transported mass in a pipeline bend and to get formulas for the solids speed u_T on an exit from a bend.

The given mathematical model of the solids movement at the flow turn is based on the suggestion that the all material settles at the origin of a bend and changes the speed during the movement. It allows passing the law of mass change along a motion pass in a bend in the following form:

$$m(\varphi) = m_0 + m_1(\varphi), \qquad (1)$$

where φ – the polar angle counted from the beginning of flow turn for a square-wave bend; m_0 – mass of material at the origin of a bend $0 \le \varphi \le \frac{\pi}{2}$; $m_1(\varphi)$ – the function defined further and considered the change of material mass along a motion path.

It is necessary to consider that the friction force operates against the movement of material:

$$T = f \cdot N, \qquad (2)$$

where N – constraint force; f – the coefficient of material friction against a bend wall.

The scheme for the interaction of forces influencing material fractions in a bend is presented further. Gravitation G, centrifugal force C, constraint force N and friction force T belong to these forces (Fig. 3).

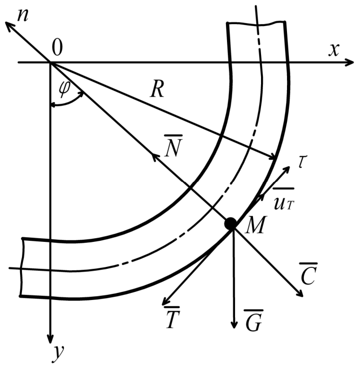

Fig. 3. The calculated scheme of a bend

THE ANALYSIS OF FORCE INTERACTION IN THE LAYER OF **THE DEPOSITED** PARTICLES

Let us project the vector equation of particles motion of the transported material of variable mass on an axis of Euler's coordinate system (Fig. 3). We have

$$\begin{cases} \dfrac{d}{dt}(mu_T) = -T - G\sin\varphi \\ m\dfrac{u_T^2}{R} = N - G\cos\varphi \end{cases} \qquad (3)$$

We determine reaction N from the second equation of system (3) and substitute in the first one. Further, taking into account the connection between a turning angle φ and velocity u_T, we can reduce system (3) to the differential equation relative to function $u_T(\varphi)$:

$$\frac{d}{d\varphi}(mu_T) = -m\left(f \cdot u_T + g\frac{R}{u_T}(f\cos\varphi + \sin\varphi)\right). \qquad (4)$$

For the correct solution of this equation it is necessary to define dependence (1) and from physical reasons to determine function $m_1(\varphi)$.

Let us assume that the area occupied with a material at movement along a bend, occupies area D in bend longitudinal section represented in Fig. 4.

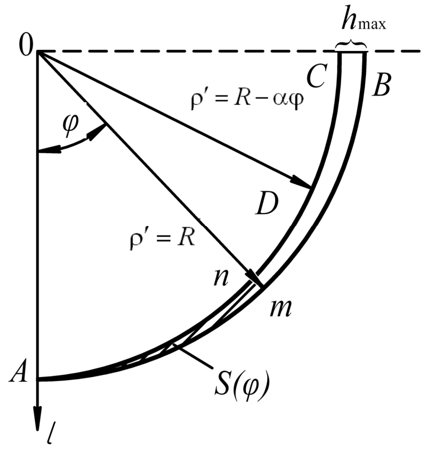

Fig. 4. The scheme of the deposited particles layer in a bend

Let us allow polar system of coordinates $(\rho'\varphi)$ with a pole *O* and a polar axis *l*. In this system of coordinates the equation of an external rim of bend *AmB* looks like $\rho' = R$, and the equation of internal border of area *D* – curve *AnC* – $\rho' = R - a\varphi$,

where $$a = \frac{2h_{\max}}{\pi}. \tag{5}$$

We find by means of double integral the area of domain *AnOA* defined by current value of an angle φ. Then required area *S*(φ) of a domain *AmnA* will be equal

$$S(\varphi) = \frac{a\varphi^2}{6}(3R - a\varphi). \tag{6}$$

Designating through ρ mass of a layer of the transported material, corresponding to the fixed value *S*(φ), we will define the change of its mass at a motion along a bend – function $m_1(\varphi)$ in formula (1):

$$m_1(\varphi) = \rho S(\varphi) = \frac{1}{6}\rho a\varphi^2(3R - a\varphi). \tag{7}$$

For parameter definition a in formula (8) we will consider that relative volume C_V of a material in a bend, as well as concentration of a flow, is defined as the proportion:

$$C_V = \frac{V_T(h_{\max})}{V_T}, \tag{8}$$

where $V_T(h_{\max})$ – the volume occupied with a material at a motion in a bend of the pipeline; V_T – volume of the whole bend representing a part of a toroidal cover.

If it is possible to define the connection $V_T(h_{max})$ equation (8) can be considered as the equation for the definition $h_{\max}$, and consequently the parameter *a*.

THE DETERMINATION OF THE MATERIAL VOLUME IN A BEND

Let us consider the fourth part of a toroidal cover and assume that the material at transportation along a bend occupies a part of its volume *ABCNA* (Fig. 5).

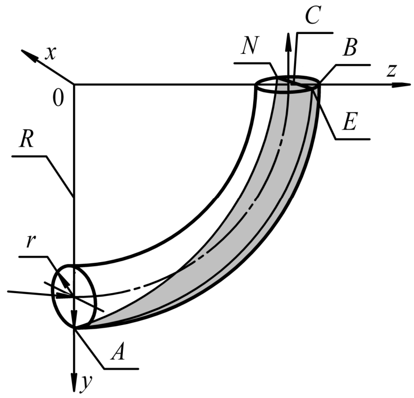

Fig. 5. To the determination of the deposited particles volume in a pipeline bend

Let us allow the Cartesian system of coordinates O_{XYZ} where the section equation tore in a plane $x0y$ will register as

$$x^2+\left(y-(R-r)\right)^2=r^2.$$

Then the surface equation tore as surfaces of rotation round axis $0x$, will be [11]:

$$x^2+\left(\sqrt{z^2+y^2}-(R-r)\right)^2=r^2.$$

Let us designate through D a longitudinal projection of an investigated surface to a plane $y0z$ (it is represented in Fig. 4). Then the required volume will be

$$V_T(h_{\max})=2\cdot\iint\limits_{(D)} dydz \int\limits_0^{\sqrt{r^2-\left(\sqrt{z^2+y^2}-(R-r)\right)^2}} dx.$$

We pass in a plane $y0z$ to polar coordinates (ρ',φ) and use tabular integrals [12]. With their help find primitive function in internal integral and after some transformations we will receive

$$V_T(h_{\max})=\int\limits_0^{\frac{\pi}{2}}\left(\frac{\pi r^2(R-r)}{2}+\frac{2}{3}\sqrt{\left(r^2-(r-a\varphi)^2\right)^3}-\right.$$
$$\left.-(R-r)(r-a\varphi)\sqrt{r^2-(r-a\varphi)^2}-(R-r)r^2\arcsin\left(\frac{r-a\varphi}{r}\right)\right)d\varphi. \qquad (9)$$

The first and the third items under integral in formula (9) are integrated elementary, for integration of the fourth item we apply an integration method in parts, and to the second – trigonometrical substitution. Collecting the received results, we will receive definitively:

$$V_T(h_{\max})=\frac{(R-r)r^2}{4}(\pi^2+\frac{\sqrt{(4ra\pi-a^2\pi^2)^3}}{6r^2a}+$$
$$+\frac{2}{a}(\pi r-\sqrt{4ra\pi-a^2\pi^2}-(2r-a\pi)\arcsin(\frac{2r-a\pi}{2r}))+ \qquad (10)$$
$$+\frac{r^4}{24a}(3\pi-6\arcsin(\frac{2r-a\pi}{2r})-\frac{2r-a\pi}{r^4}\sqrt{4ra\pi-a^2\pi^2}\cdot(3r^2+2ra\pi-\frac{1}{2}a^2\pi^2)).$$

Thus, taking into account this expression and the expression for a quarter of torus volume with characteristics r and R [11]

$$V_T=\frac{1}{2}\pi^2r^2(R-r)$$

it is possible to consider the transcendental equation (8) completely formed for the definition of $h_{\max}$. It should be mentioned that the roots of this equation are to be searched in limits $0< h_{max}< r$ as experimental data show.

THE ANALYSIS OF THE TRANSPORTED MATERIAL VELOCITY IN A PIPELINE BEND

The problem of this subsection is to define the velocity of the material particles for any value of a polar angle φ (Fig. 3). For this purpose we have a problem of Koshi: differential equation (4) with the initial condition

$$u_T \big|_{\varphi=0} = u_{T0}, \tag{11}$$

where u_{TO} – initial velocity of the transported flow particles on the entrance of a bend.

We substitute the law of mass change (1) considering dependence (7) in differential equation (4) and reduce it to the nonlinear differential equation of Bernoulli [13] of the following form

$$\frac{du_T}{d\varphi} + P(\varphi)u_T = \frac{1}{u_T}Q(\varphi), \tag{12}$$

where

$$P(\varphi) = f + \frac{(6R - 3a\varphi)\varphi}{6M_0 + (3R - a\varphi)\varphi^2}; \; Q(\varphi) = -gR(f\cos\varphi + \sin\varphi); \; M_0 = \frac{m_0}{\rho a}. \tag{13}$$

We find the solution of equation (12) in a standard way

$$y(\varphi) = u_T^2 = u(\varphi)\cdot e^{-2f\varphi - 2\overline{P}(\varphi)}, \tag{14}$$

where

$$\overline{P}(\varphi) = \int \frac{(6R - 3a\varphi)\varphi}{6M_0 + (3R - a\varphi)\varphi^2} d\varphi. \tag{15}$$

To find a primitive function in formula (15) it is necessary to determine denominator roots of the subintegral function, i.e. to solve the equation

$$\varphi^3 + a_1\varphi^2 + b\varphi + c = 0,$$

where $a_1 = -3Ra^{-1} = -3R_\delta;\, b = 0;\, c = -6M_0 a^{-1};\, R_\delta = R/a$.

Discriminant expressions are calculated [14, 15]

$$\overline{Q} = \frac{a_1^2 - 3b}{9} = \frac{R^2}{a^2}, \; \overline{R} = \frac{2a_1^3 - 9ab + 27c}{54} = \frac{1}{a^3}\left(R^3 + 3\frac{m_0}{\rho}a\right)$$

and inequality $\overline{R}^2 > \overline{Q}^3$ is obtained. So, equation (16) has one real root and two complex conjugate ones, evaluated using Cardano formulae (16)

$$\varphi_1 = A + B - \frac{a_1}{3}; \; \varphi_{2,3} = \frac{A+B}{2} - \frac{a_1}{3} \pm i\sqrt{3}\frac{A-B}{2},$$

where

$$A = -sign(\overline{R})\cdot\sqrt[3]{|\overline{R}| + \sqrt{\overline{R}^2 - \overline{Q}^3}} = \sqrt[3]{\frac{R^3}{a^3} + 3\frac{m_0}{\rho a^2} + \sqrt{6\frac{R^3 m_0}{\rho a^5} + \frac{9m_0^2}{\rho^2 a^4}}},$$

$$B = \frac{\overline{Q}}{A} = \frac{R^2}{a\cdot\sqrt[3]{R^3 + 3\frac{m_0}{\rho}a + a\sqrt{6\frac{R^3 m_0}{\rho a} + 9\frac{m_0^2}{a^2}}}}.$$

The subintegral expression in (15) is decomposed into a sum of simple fractions and the method of undetermined coefficients is used. After integration we obtain the function $\overline{P}(\varphi)$ in explicit form

$$\overline{P}(\varphi) = -3\left(D\ln|\varphi - \varphi_1| + \frac{1}{2}E\ln\left(\varphi^2 + p\varphi + q\right) + \frac{2F - Ep}{2q - 0,5p^2}arctg\left(\frac{2\varphi + p}{2q - 0,5p^2}\right)\right). \tag{16}$$

Here

$$P = -\left(A + B + 2R_{\delta}\right);\ q = \left(\frac{A+B}{2}\right)^2 - 3\left(\frac{A-B}{2}\right);$$

$$D = \frac{\left(2R_{á} - \varphi_1\right)\varphi_1}{\varphi_1^2 + p\varphi_1 + q};\ \ E = -\frac{\left(2R_{á} + p\right)\varphi_1 + q}{\varphi_1^2 + p\varphi_1 + q};\ \ F = \frac{\left(2R_{á} - \varphi_1\right)q}{\varphi_1^2 + p\varphi_1 + q}.$$

Thus, we have the equation for the function $u(\varphi)$ in (14)

$$\frac{du}{d\varphi} \cdot e^{-2f\varphi - 2\overline{P}(\varphi)} = -2gR\left(f\cos\varphi + \sin\varphi\right),$$

solved elementary. After determining the integration constant from the starting condition(11) we obtain the desired dependence of the velocity on the angle φ as following

$$u_T(\varphi) = \sqrt{u_{TO}^2 \cdot e^{2\left(P(0) - f\varphi - \overline{P}(\varphi)\right)} - 2gR \cdot e^{-2\left(f\varphi + 2\overline{P}(\varphi)\right)} \cdot \Psi(\varphi)}, \tag{17}$$

where

$$\Psi(\varphi) = \int_0^{\varphi} \left(f\cos\varphi + \sin\varphi\right) e^{2f\varphi - 3\left[D\ln|\varphi - \varphi_1| + 0{,}5E\ln\left(\varphi^2 + p\varphi\right) + \frac{2F - Ep}{2q - 0{,}5p^2} arctg\frac{2\varphi + p}{2q - 0{,}5p^2}\right]} d\varphi. \tag{18}$$

If we put in (17), (18) $m_0 = m;\ m_1(\varphi) = 0$, then expression (17) becomes

$$u_T(\varphi) = \sqrt{u_{T0}^2 e^{-2f\varphi} - \frac{2gR}{4f^2+1}\left(\left(1 - 2f^2\right)e^{-2f\varphi} - \left(1 - 2f^2\right)\cos\varphi + 3f\sin\varphi\right)}, \tag{19}$$

that is identical to the formula, obtained in [9] for the constancy of the material mass in the bend, located in vertical plane of «horizontal-vertical» type.

COMPARISON OF THE PRESSURE LOSS CALCULATION RESULTS IN THE PIPELINE BEND USING DIFFERENT METHODS

According to the given methods comparative calculations of the pressure loss in the pipeline bend with the rotation of the pipe at an angle φ = 90°, bending radius R = 0,3 m, pipe diameter d = 0,05 m, located in the vertical plane with the material consumption 0,8; 1,7 и 3,3 kg/s are carried out. The velocity of the bulk material u_{TK} at the outlet of the bend fot the case of uniform distribution of deposited particles is detemined by formulae (17) and (18).

According to the calculation results we have graphs presented in Fig. 6. Here the solid line indicates the experimental graphs obtained on the experimental stand of the Automobile and Highway Institute of the Donetsk State Technical University.

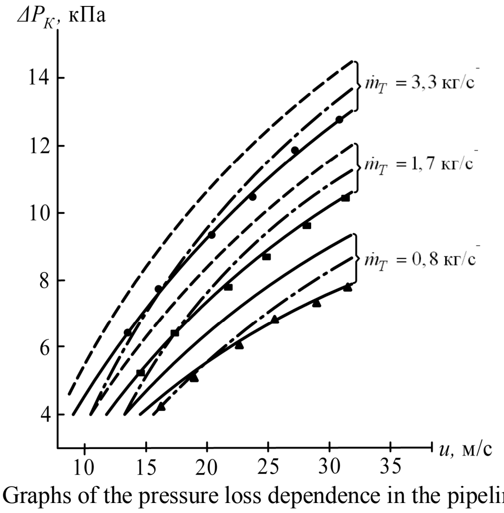

Fig. 6. Graphs of the pressure loss dependence in the pipeline bend

According to the graphs calculation accuracy by the method providing for the mass change of particles layer along the bend greatly improves the calculation accuracy on a simplified version.

CONCLUSION

1. It is shown that in the bend of the pneumotransport pipeline solids by centrifugal force are deposited on the wall of the bend, forming a layer of transported material, thickness of which is not constant, but changes along the bend according to the law, depending on the particle size, density of carrying and transported medium, bending radius.
2. A new analytical model of the bend, taking into account the variable height of deposited particles layer is developed; a nonlinear differential equation of the layer motion is composed; the law of the velocity change of the particle layer motion and the change of its specific gravity along the bend is obtained.
3. A design procedure of the pressure loss in the pipeline bend, consistent with the accepted analytical model is developed. The comparison of calculated and experimental data shows that their divergence does not exceed 15 %.

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АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ ИЗМЕНЕНИЯ СКОРОСТИ ТРАНСПОРТИРУЕМОГО МАТЕРИАЛА В КОЛЕНЕ ТРУБОПРОВОДА

Михаил Чальцев, Леонид Вовк

Аннотация. Обосновывается и предлагается математическая модель движения аэродисперсного потока в колене трубопровода. Получено решение этой модели при условиях, приближенных к реальным. Приведено сравнение результатов, полученных экспериментально, по данной модели и для некоторых упрощающих гипотез.

Ключевые слова: пневматическое транспортирование сыпучих материалов, скорость потока, дифференциальные уравнения