

Force structure impact on the wheel module stability and oscillation process

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Summary. The stability and shimmies of the front non-steerable pillar free to turn on two channels – the yaw and roll relative to the longitudinal axis of the body is analyzed. Offered approach close analysis of self-oscillation in the nonlinear raising, that enables to estimate stability in «large». The analysis of the typical system parameters impact on the unstable oscillatory region and oscillation amplitude is carried out. The approximation percent influence of the slip force and heel moment on the oscillation character is considered (analytical expression, relating amplitude of vibrations with the parameters of model, is got: by the moment of inertia of wheel in relation to the ax of turn, turning inflexibility of steering management, coefficient of relaxation, size of bearing-out and angle of slope proof).
Key words: wheel module, wobbling, force structure, oscillation amplitude.

INTRODUCTION

Self-oscillation guided wheels of car (wobbling) were first considered in-process Brul'e in 1925. In future this question was the article of research of many authors, as representatives of theoretical direction M.V. Keldysh [11], G.V. Aronovich [2], N.A. Fufaev [21], V.F. Zhuravlev, D.M. Klimov [33, 34], L.G. Lobas [14], N.P. Plakhtienko [23], H. Paceyka [22],

G. Somesky [28], Yi. Mi-Seon [20], so engineers-researchers of aviation and motor-car transport B. fon Schlippe, R. Dietrich [26], I. Besselink [4], V.S. Gozdek [9], V.I. Goncharenko [7, 8], K.S. Kolesnikov [12], N.P. Plakhtienko, B.M. Shifrin, F. Smiley et al [13, 23, 25, 29].

From point of modern analysis of question, wobbling is the intensive самовозбуждающиеся vibrations of rolling wheels, showing up as turning motions of wheels in a horizontal plane (their rotations), which are accompanied other motions in a longitudinal vertical plane. First of all the wobbings of the chassis elements are connected with the elastic pneumatics availability. Under certain conditions it transforms some power entering the vehicle into the power of wheel torsional oscillations.

There is a great number of variants of description of model of co-operation of balloon wheel with an absolutely even and ideally rough horizontal plane, however necessary it is to take into account that they not all allow to take into account some characteristic features of nonlinear dependences of lateral withdrawal, including

descriptions of rigid heterogeneity (conical and angular).

In the tasks of dynamics of the wheeled transport vehicles most distribution was got by two approaches at determination of cooperation of wheel with an underlayment. There are two raising at determination of descriptions of lateral reaction of resilient wheel: model (theory of M.V. Keldysh [11]) and phenomenological one (axioms of I. Rokar [24]). In this work the Rocar model is used – slip force and aligning torque are considered to be well-known nonlinear relations (slip angle functions), obtained empirically.

In the work [15] the analysis of the force structure impact on the unperturbed linear motion stability of the hitch model with two degrees of freedom (values of typical design parameters corresponding to different on mathematical classification force groups have been varied).

One of the first results in this direction there were theorems of Tomsona-Teta-Chetaeva [6] about influence of dissipative and gyroscopic forces on stability of the linear conservative system and results of I.I. Metelicyna [19] for the case of the unconservative systems. Presently receptions are successfully used constructing of quadratic functions of Lyapunova [17], taking into account the mathematical structure of breaking up of forces of the initial system [1, 7, 10, 32]. In spite of absence of general algorithm of construction of functions of Lyapunova, he got wide distribution. The attractiveness of method of functions of Lyapunova is conditioned his high-quality character, allowing to unseal physical essence in a task, and also possibility of receipt of estimations of areas of attraction of unperturbative motion (research of stability is «in large») [5, 16, 29].

In the work [30] on the basis of the suggested approximate approach [31] the nonlinear analysis of the pattern shimming of the wheel module with one degree of freedom is carried out for various approximations of slip forces (monotonic and with flowing section). The results of the analytic treatment are confirmed by a number of phase portraits obtained in the result of numerical integration.

In the work corresponding estimates are carried out for a more complete model [14, 16], taking into account aligning torque. The impact of the design parameters and approximation percent of the slip force nonlinear relation on characteristics of system oscillations is considered.

PROBLEM STATEMENT

Let θ and ψ – rotation angles of the chassis setting around front axle and roll axle respectively, then schematically A-pillar of a vehicle is given at Fig. 1.

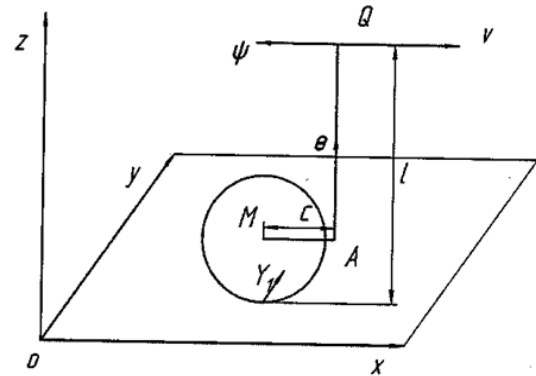


Fig. 1. Wheeled Module

Equations of the chassis leg motion in the linear motion environment at zero removal ($c = 0$) have the form [16] (to complete the setting the aligning torque $M(\alpha)$ is added, occurring in the wheel contact with bearing surface at rolling with slipping):

$$\begin{aligned} B \cdot \ddot{\theta} + \chi_1 \cdot \dot{\theta} + h \cdot \dot{\psi} - \frac{I \cdot v}{r} \dot{\psi} + M(\alpha) &= 0, \\ C \cdot \ddot{\psi} + \chi \cdot \dot{\psi} + h_1 \cdot \dot{\psi} + \frac{I \cdot v}{r} \dot{\theta} + l \cdot Y(\alpha) &= 0, \quad (1) \\ \alpha &= \theta + \frac{\psi \cdot l}{v}, \end{aligned}$$

where: B , C – axial moments of the pillar inertia relative to the rotation axis and rolling axis respectively,

I – central axial moment of the wheel inertia relative to own rotation axis,

r – wheel radius,

v – unperturbed motion velocity,
 χ, χ_1 – coefficients of the wheel module torsional stiffness,
 h, h_1 – parameters, determining oscillation damping,
 l – distance from road surface up to rolling axis.

Slip forces and heel moment are considered in the form of nonlinear relations of the slip angle α :

$$Y(\alpha) = k\alpha / \sqrt{1 + (k\alpha/\varphi N)^2},$$

where: k – resistance coefficient to the slip,

N – vertical bearing reaction,

φ – traction coefficient in the transverse direction,

$$M(\alpha) = \mu\alpha / (\mu_4\alpha^4 + \mu_2\alpha^2 + 1).$$

Linearized equations of the wheel module perturbed motion have a wide range of forces according to the conventional mathematical classification – inertial, dissipative, gyroscopic, potential, and non-conservative positional ones:

$$A\ddot{x} + (D + vG)\dot{x} + (K + lP)x = 0,$$

where:

$$A = \begin{pmatrix} C & 0 \\ 0 & B \end{pmatrix}, D = \begin{pmatrix} h_1 + \frac{k \cdot l^2}{2v} & \frac{\mu \cdot l}{2v} \\ \frac{\mu \cdot l}{2v} & h \end{pmatrix},$$

$$G = \begin{pmatrix} 0 & \frac{I}{r} - \frac{\mu \cdot l}{2v^2} \\ -\left(\frac{I}{r} - \frac{\mu \cdot l}{2v^2}\right) & 0 \end{pmatrix},$$

$$K = \begin{pmatrix} \chi & \frac{k \cdot l}{2} \\ \frac{k \cdot l}{2} & \chi_1 + \mu \end{pmatrix}, P = \begin{pmatrix} 0 & \frac{k}{2} \\ -\frac{k}{2} & 0 \end{pmatrix},$$

where: A, D, K – symmetrical matrixes of the inertial, dissipative and potential forces coefficients,

G, P – alternate matrixes of the gyroscopic, non-conservative positional forces coefficients.

The availability of two typical parameters, determining values of gyroscopic terms (according to the traverse speed) and non-conservative positional terms (according to the pillar height), enable to apply general theorems of the force structure impact on the unperturbed motion stability using linear analysis.

Known results [8, 10, 19, 27, 32] of stabilization conditions of linear mechanical systems, under the impact of arbitrary mathematical structure forces, ensure stability at sufficiently great complete dissipation and positive definiteness of the conservative forces matrix or sufficiently great potential forces and positive definiteness of the dissipative forces matrix which is imposed certain additional condition [31]. And availability of sufficient great positional non-conservative forces as a rule leads to the stability loss of the general linear system. However, in case of finite forces, mechanisms of the stabilization and stability loss are possible. They result in an ambiguous treatment of the force structure impact on the stability of linear system.

In the regions of the flutter instability stationary monofrequent oscillations can occur (one of mechanisms of their occurrence – the Andronov – Hopf bifurcation [18]). The problem of the stability loss character (unsafe according to N.N. Bautin [3]) can be solved on the basis of the amplitude curve analysis and characteristics of the linearized model stability. Then the approximate approach how to get an amplitude curve as an implicit function of system parameters is given. It is related, in turn, to the solvability condition of a certain auxiliary system of non-linear finite equations.

EVALUATION OF THE OSCILLATION AMPLITUDES IN THE NEIGHBORHOOD OF LINEAR MOTION CONDITION

To carry out the approximate method of the self-oscillating system amplitude evaluation let's introduce an auxiliary

differential equation, corresponding to the unsteady slip theory:

$$\sigma \cdot \dot{\alpha} + v \cdot \alpha - v \cdot \theta - l \cdot \dot{\psi} = 0.$$

Then the system (1) is:

$$\begin{aligned} \sigma \cdot \dot{\alpha} + v \cdot \alpha - v \cdot \theta - l \cdot \dot{\psi} &= 0, \\ B \cdot \ddot{\theta} + \chi_1 \cdot \theta + h \cdot \dot{\theta} - \frac{I_1 \cdot v}{r} \dot{\psi} + M(\alpha) &= 0, \quad (2) \\ C \cdot \ddot{\psi} + \chi \cdot \psi + h_1 \cdot \dot{\psi} + \frac{I_1 \cdot v}{r} \dot{\theta} + l \cdot Y(\alpha) &= 0. \end{aligned}$$

It is supposed that the system periodic solution (2) in the neighborhood of the largest deflection moment from the equilibrium position and in the moment neighborhood when deflections amount zero varies according to the harmonic law having some phase delay:

$$\begin{aligned} \alpha &= a \sin \omega t, \quad \psi = p_0 \sin(\omega t + \varphi_\psi), \\ \theta &= q_0 \sin(\omega t + \varphi_\theta), \end{aligned}$$

where: a, p_0, q_0 – amplitude, ω – angular frequency of oscillations, $\varphi_\psi, \varphi_\theta$ – phase delay.

In typical instants of time phase variables and their generated variables possess the value:

$$\begin{aligned} \theta &= q_0 \cos \varphi_\theta, \\ \dot{\theta} &= -q_0 \omega \sin \varphi_\theta, \\ \ddot{\theta} &= -q_0 \omega^2 \cos \varphi_\theta, \\ \omega t = \pi/2: \quad \alpha &= a, \quad \dot{\alpha} = 0, \quad \ddot{\alpha} = -a \omega^2, \\ \psi &= p_0 \cos \varphi_\psi, \\ \dot{\psi} &= -p_0 \omega \sin \varphi_\psi, \\ \ddot{\psi} &= -p_0 \omega^2 \cos \varphi_\psi, \\ \omega t = 0: \quad \alpha &= 0, \quad \dot{\alpha} = a \omega, \quad \ddot{\alpha} = 0, \\ \psi &= p_0 \sin \varphi_\psi, \quad \dot{\psi} = p_0 \omega \cos \varphi_\psi, \quad \ddot{\psi} = -p_0 \omega^2 \sin \varphi_\psi, \\ \theta &= t_0 \sin \varphi_\theta, \\ \dot{\theta} &= t_0 \omega \cos \varphi_\theta, \\ \ddot{\theta} &= -t_0 \omega^2 \sin \varphi_\theta \end{aligned}$$

substituting these correlations in the system (2), we'll get the system of six finite equations relative to the required parameters of oscillations ($a, p_0, q_0, \omega, \varphi_\psi, \varphi_\theta$).

After the elimination of unknowns $p_0, q_0, \varphi_\psi, \varphi_\theta$ from the first four equations of the system, two remaining equations are polynomials relative to the amplitude a and angular frequency ω . Composing their resultant (unknown angular frequency is eliminated ω), we'll get the implicit function determining the amplitude of oscillations according to design parameters of the system and traverse speed v .

Amplitude curves are given on the Fig. 2: a – slip force is approximated by the linear and cubic terms $Y(\alpha) = k\alpha - \frac{k^3 \alpha^3}{2N^2 \varphi^2}$, curve 1

takes into account aligning torque, curve 2 – its absence, b – slip force presents fractionally irrational dependence $Y(\alpha) = k\alpha(1 + k^2 \alpha^2 / N^2 \varphi^2)^{-1/2}$, curve 1 takes into account aligning torque, curve 2 – its absence.

Obtained at following numerical values of parameters: $N=5000$ H, $k=42700$ H, $B=9,81$ kg·m², $h=37,3$ H·m·s, $h_1=981$ H·m·s, $\chi=421100$ H·m, $\chi_1=12160$ H·m, $C=165$ kg·m², $I_1=11,8$ kg·m², $r=0,4$ m, $l=0,85$ m, $\varphi=0,7$, $\mu=0,3742771659$, $\mu_2=71,4533726$, $\mu_4=39122,6523$.

Thereby, approximate approach of the slip force leads to the branch of unstable oscillations (Fig. 2, a), in case of slip force assignment in the form of fractionally irrational dependence the branch of unstable oscillations is absent (Fig. 2, b). Aligning torque impact causes either insignificant expansion of the unstable region (Fig. 2), or significant qualitative changes of the oscillation region according to the damping characteristics (Fig. 3).

Note. In general at the implementation of the oscillation analysis method (1), the introduction of an auxiliary differential equation (describing unstable wheel slip) can be avoided. The third system equation (1) enables to introduce formally redundant variable α , and two auxiliary finite equations occurring in this case $v \cdot q_{0s} + l \cdot q_{0c} \cdot \omega = 0$ and

$$v \cdot q_{0c} - l \cdot p_{0s} \cdot \omega = a \cdot v,$$

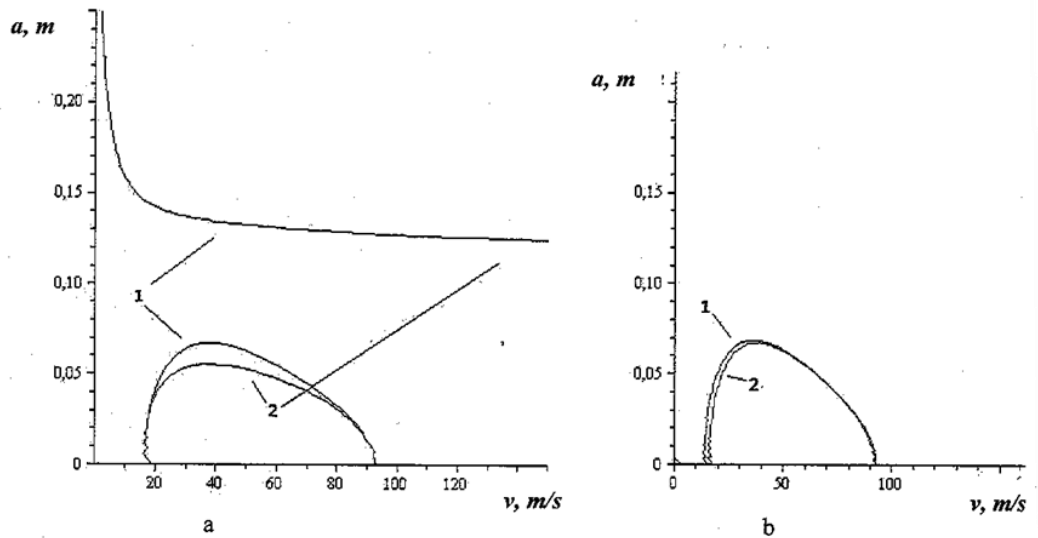


Fig. 2. Amplitude Curves

where:

$$q_{0s} = q_0 \sin(\varphi_\theta), p_{0c} = p_0 \cos(\varphi_\psi),$$

$$q_{0c} = q_0 \cos(\varphi_\theta), p_{0s} = p_0 \sin(\varphi_\psi),$$

ensure implementation of the following correlation for linear combination of two harmonics with similar frequencies (quad erat for this method implementation):

$$a \sin \omega t = q_0 \sin(\omega t + \varphi_\theta) + \frac{l}{v} \omega \cdot p_0 \cos(\omega t + \varphi_\psi),$$

as:

$$v \cdot q_0 \sin(\omega t + \varphi_\theta) + l \cdot \omega \cdot p_0 \cos(\omega t + \varphi_\psi) =$$

$$= (v \cdot q_{0s} + l \cdot p_{0c} \cdot \omega) \cos(\omega t) +$$

$$+ (v \cdot q_{0c} - l \cdot p_{0s} \cdot \omega) \sin(\omega t).$$

Suggested approach makes possible to determine onset regions of stable and unstable oscillations – a curve abutting upon the abscissa axis meets to stable oscillations but an interval cut by it on the axis of the longitudinal velocity meets to the region of oscillatory instability. It enables to analyze parallel the force system impact on the stability of linear system.

The consistency of obtained results has been confirmed on the basis of the Routh-Hurwitz criterion.

THE IMPACT OF THE FORCE STRUCTURE ON THE OSCILLATION AMPLITUDE (CASE WITH AN INCOMPLETE DISSIPATION)

Using the method examined above the following results have been obtained: if $h_1=0$, then:

1. A damping increase on the rotational angle relative to the vertical leads to the decrease of the unstable region and oscillation intensity. The first part of deductions is coordinated with general theorems of the force structure impact. Thus the impact of the aligning torque reveals insignificantly.

2. An increase of the non-conservative positional forces parameter l (pillar height) leads to the increase of the unstable region according to the velocity and growth of the oscillation intensity (Fig.3, a: curve 1 corresponds to $l=0,85$ m., curve 2 – $l=1,1$ m., curve 3 – $l=0,6$ m). It corresponds to deductions from general theorems of the force structure impact.

3. An increase of the wheel inertia moment leads to the decrease of the unstable region and oscillation amplitude.

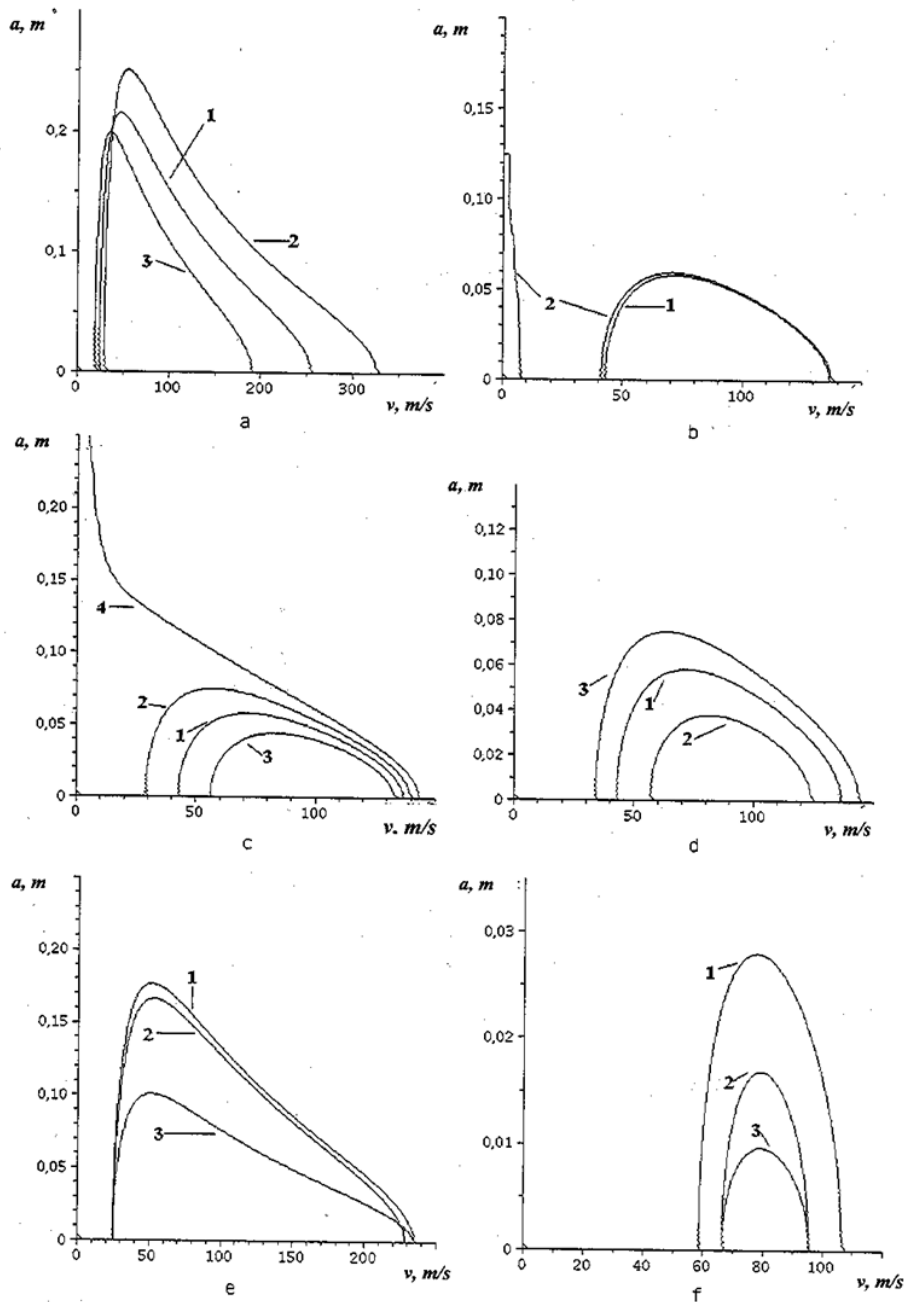


Fig. 3. Amplitude Curves

4. An impact of the relaxation parameter ($\sigma \neq 0$) – an increase σ leads to the decrease of the unstable region and oscillation amplitude (Fig. 3, d: curve 1 corresponds to $\sigma=0,45$, curve 2 – $\sigma=0,5$), a decrease of the traction coefficient leads also to the decrease

of oscillation amplitude (Fig. 3, d: curve 1 corresponds to $\varphi=0,7$, curve 3 – $\varphi=0,4$).

If $h=0$, then:

1. An aligning torque impact is characterized in this case by an auxiliary oscillation region occurrence at slow speeds (up to 7,3 m/s), "main" oscillation region

(existed in the range of $42,7 \text{ m/s} < v < 136,7 \text{ m/s}$) doesn't change here (Fig.3, b: a curve 1 corresponds to the absence of the aligning torque, a curve 2 – its availability).

2. An increase of the torsional stiffness (relative to the vertical axis) leads to the increase of the oscillatory instability region and oscillation amplitude. In general it doesn't conflict with theorem points about force structure impact as in this case complete dissipation is absent. Oscillation region connected with the aligning torque availability doesn't change practically but at sufficiently great values of the torsional stiffness it "is absorbed" by the oscillation region, determined by single side force (Fig. 3, b: curve 1 corresponds to $\chi_1=12160 \text{ H}\cdot\text{m}$, curve 2 - $\chi_1=20160 \text{ H}\cdot\text{m}$, curve 3 - $\chi_1=4160 \text{ H}\cdot\text{m}$, curve 4 - $\chi_1=32160 \text{ H}\cdot\text{m}$).

3. An increase of the torsional stiffness (relative to the longitudinal axis) leads to the decrease of the oscillation region and intensity. Oscillation region connected with the aligning torque availability doesn't change practically. (Fig. 3, r: curve 1 corresponds to $\chi=421100 \text{ H}\cdot\text{m}$, curve 2 - $\chi_1=501100 \text{ H}\cdot\text{m}$, curve 3 - $\chi=361100 \text{ H}\cdot\text{m}$).

4. A relaxation parameter change ($\sigma \neq 0$) leads to same results as in the point 4 (Fig. 3, f: curve 1 corresponds to $\sigma=0,45$, curve 2 - $\sigma=0,5$), the same impact as in the point 4 (Fig. 3, f: curve 1 corresponds to $\varphi=0,7$, curve 3 - $\varphi=0,4$) remains at variations of the traction coefficient.

CONCLUSIONS

1. In the work the method of approximate construction of amplitude curves in the task of the wobbling of the front non-steerable chassis pillar is developed – possibility of the redundant variable introduction for the method implementation is examined.

2. Force structure impact on the oscillatory instability region and oscillation characteristics is analyzed.

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ВЛИЯНИЕ СТРУКТУРЫ СИЛ НА
УСТОЙЧИВОСТЬ КОЛЕСНОГО МОДУЛЯ И
ПРОЦЕСС АВТОКОЛЕБАНИЙ

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Аннотация. Анализируется устойчивость и автоколебания передней «неуправляемой» стойки, имеющей свободу поворота по двум каналам – рысканья и крена относительно продольной оси корпуса. Предложен подход приближенного анализа автоколебаний в нелинейной постановке, что дает возможность оценить устойчивость в «большом». Проведен анализ влияния характерных параметров системы на область колебательной неустойчивости и амплитуды автоколебаний (получено аналитическое выражение, связывающее амплитуду колебаний с параметрами модели: моментом инерции колеса относительно оси поворота, крутильной жесткостью рулевого управления, коэффициентом релаксации, величиной выноса и углом наклона стойки).

Ключевые слова: колесный модуль, шимми, структура сил, амплитуды автоколебаний.