Superharmonic resonances in two-masses vibrating machines

Authors:

Valeriy N. Belovodskiy	<e-mail: v.belovodskiy@gmail.com>,
Sergey L. Bukin	<e-mail: s.bukin08@gmail.com>,
Maksym Y. Sukhorukov	<e-mail: max.sukhorukov@gmail.com>,
Anastasia A. Babakina	<e-mail: yfcnz@ua.fm>

Ukraine,
Donetsk National Technical University,
Computer Monitoring Systems department,
Mineral Processing department
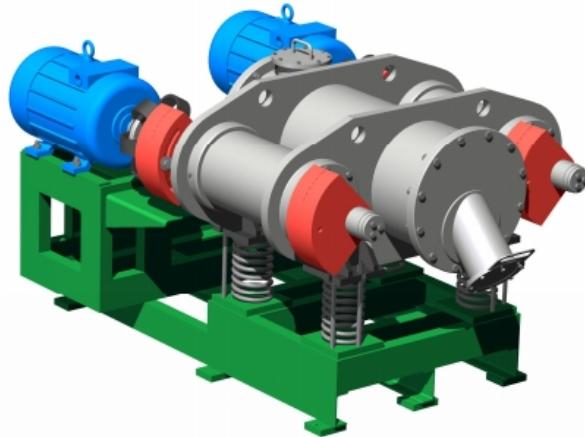
The issue of research

Biharmonic vibrations are in demand in different technological processes:

– transportation,
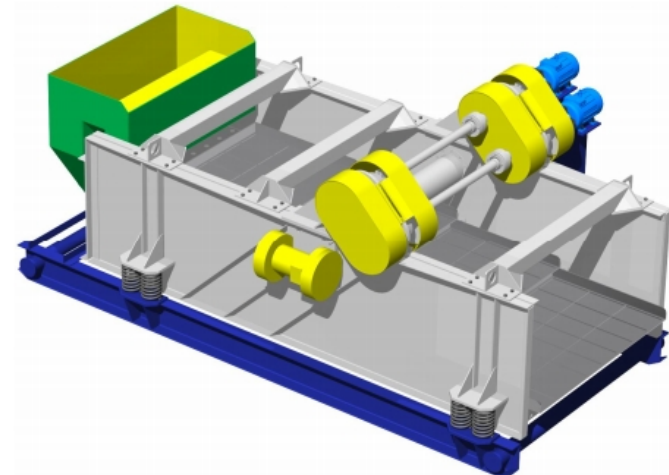
– screening,

– compacting and so on [1, 2].

Usually such vibrations are formed with the help of biharmonic exciters. Examples of such machines are shown in the figure 1.
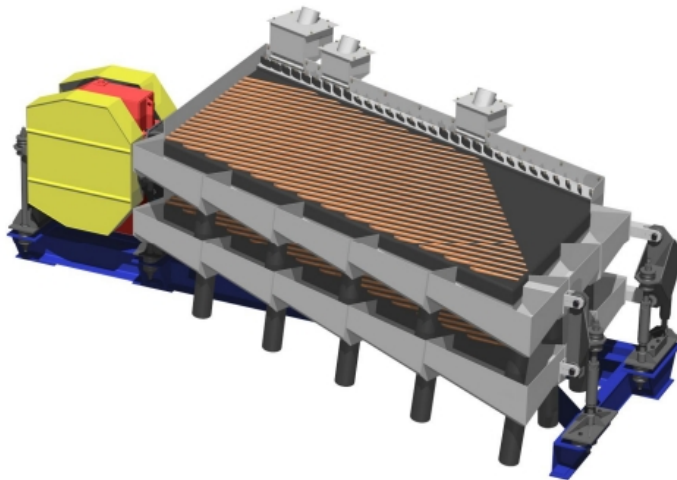
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Figure 1. Samples of biharmonic vibrating machines

VIBRATORY BIHARMONIC MILL

BIHARMONIC INERTIAL VIBRATING SCREEN

REFERENCE CONCENTRATION TABLE OF BIHARMONIC TYPE

The issue of research

This work continues traditions of 80-ies of the last century [3, 4] and is devoted to the creation of nonlinear vibrating machines. At that time one of the most popular approach for the forming of nonlinear elastic ties was the use of buffer elements and, as a result, its characteristic became of the piecewise-linear form. But appearance of neodymium magnets (sintered NdFeB) (fig. 2) on the market today that are more powerful than any other permanent magnet material [5, 6] discovers new opportunities in this direction.
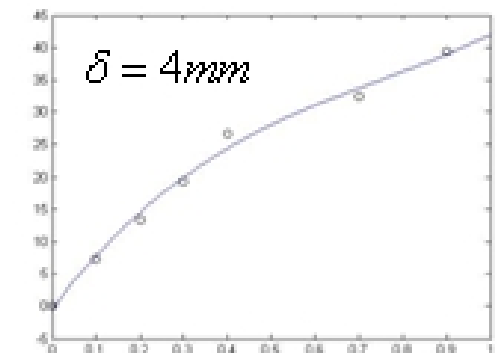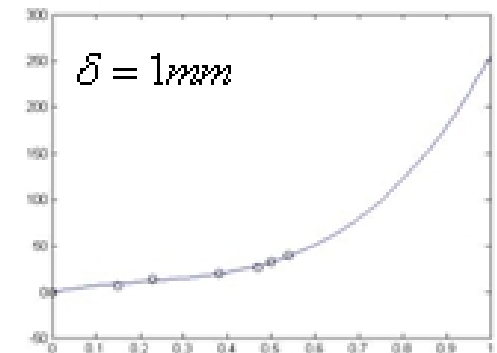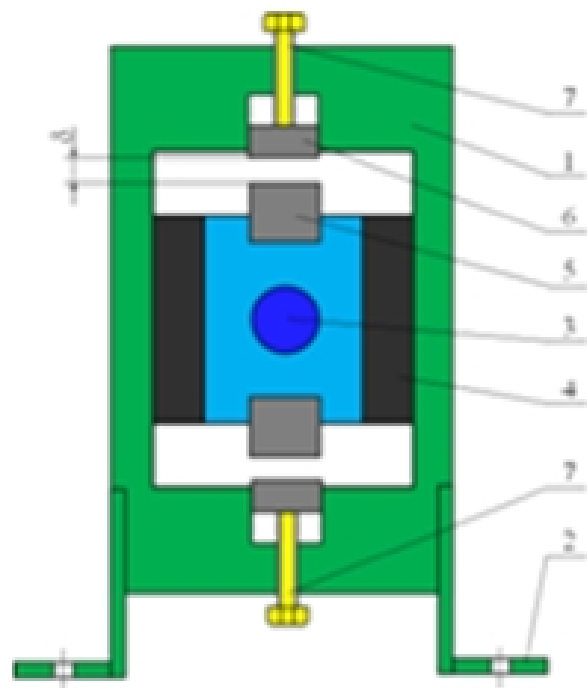
The issue of research

Figure 2. Samples of neodymium magnets
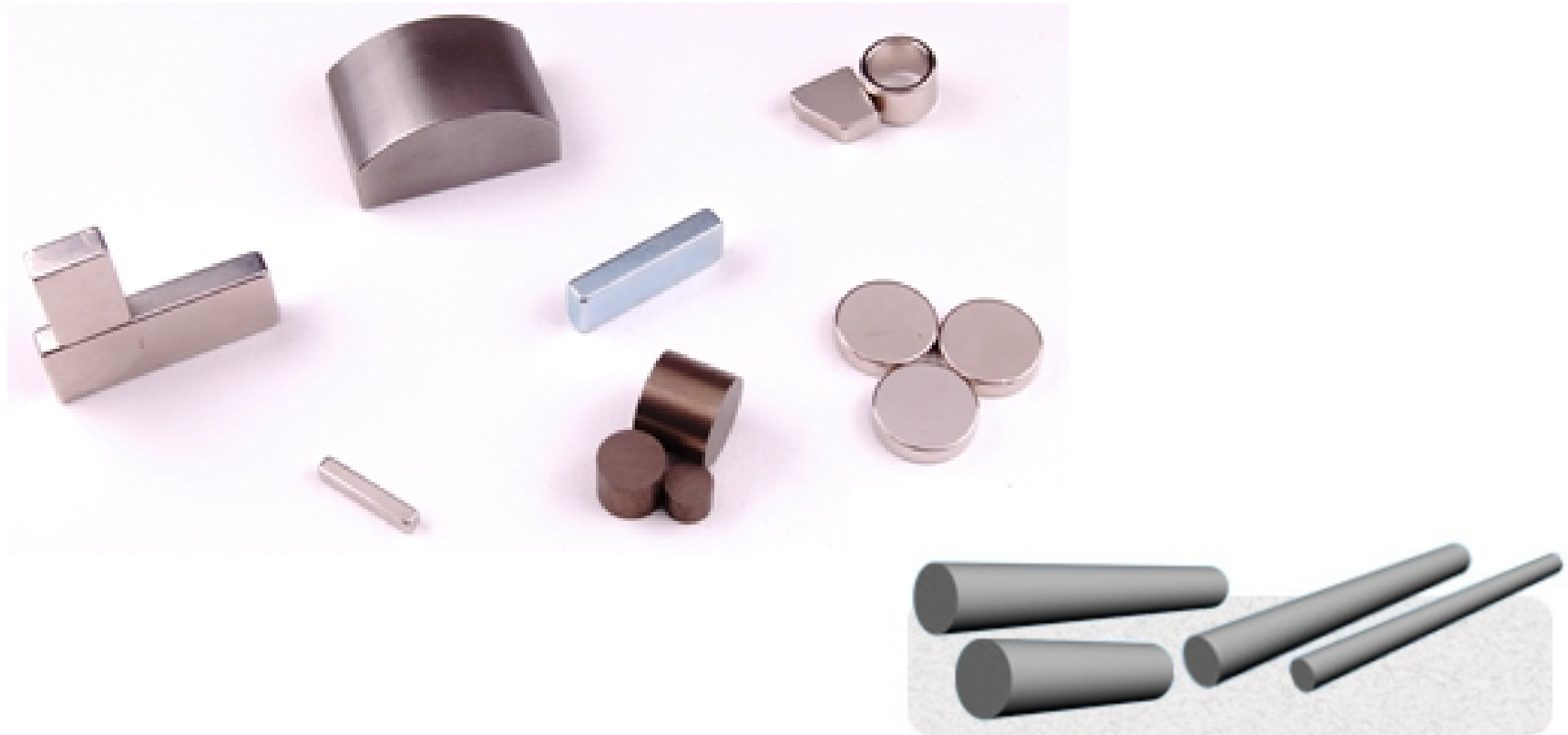
On the basis of such magnets the principal scheme and device (fig. 3) for forming of nonlinear elastic ties are proposed.

The issue of research

Figure 3. Magnetic elastic element: a) A scheme and sample: 1 – body; 2 – support; 3 – trunnion; 4 – rubber element; 5, 6 – magnets; 7 – adjusting bolt; b) View of elastic characteristics for different initial gaps δ

a)   b)

The issue of research

For the indicated values of initial gaps between the magnets it has the following elastic characteristics

$$F = 943.72 \left(1 - 2.95\,\xi + 4.60\,\xi^2\right) \xi \ \text{N/mm} \ \text{ for } \ \delta = 1 \ \text{mm}$$

and

$$F = 887.12 \left(1 - 0.96\,\xi + 0.42\,\xi^2\right) \xi \ \text{N/m} \ \text{ for } \ \delta = 4 \ \text{mm, where } [\xi] = \text{mm}.$$

Combining such elements with different values of gaps δ and, perhaps, a certain number of normal linear absorbers one may try to form the desired elastic system of vibrating machine. Our purpose here is to get some instructions for choosing elastic parameters of such machines.

The model under consideration

For this purpose it is considered the vibrating two-masses machine (screen, filtering centrifuge, concentration table) with ideal harmonic inertial excitation and polynomial characteristic of the elastic ties (fig. 4).

Figure 4. Principal scheme of the vibrating machine
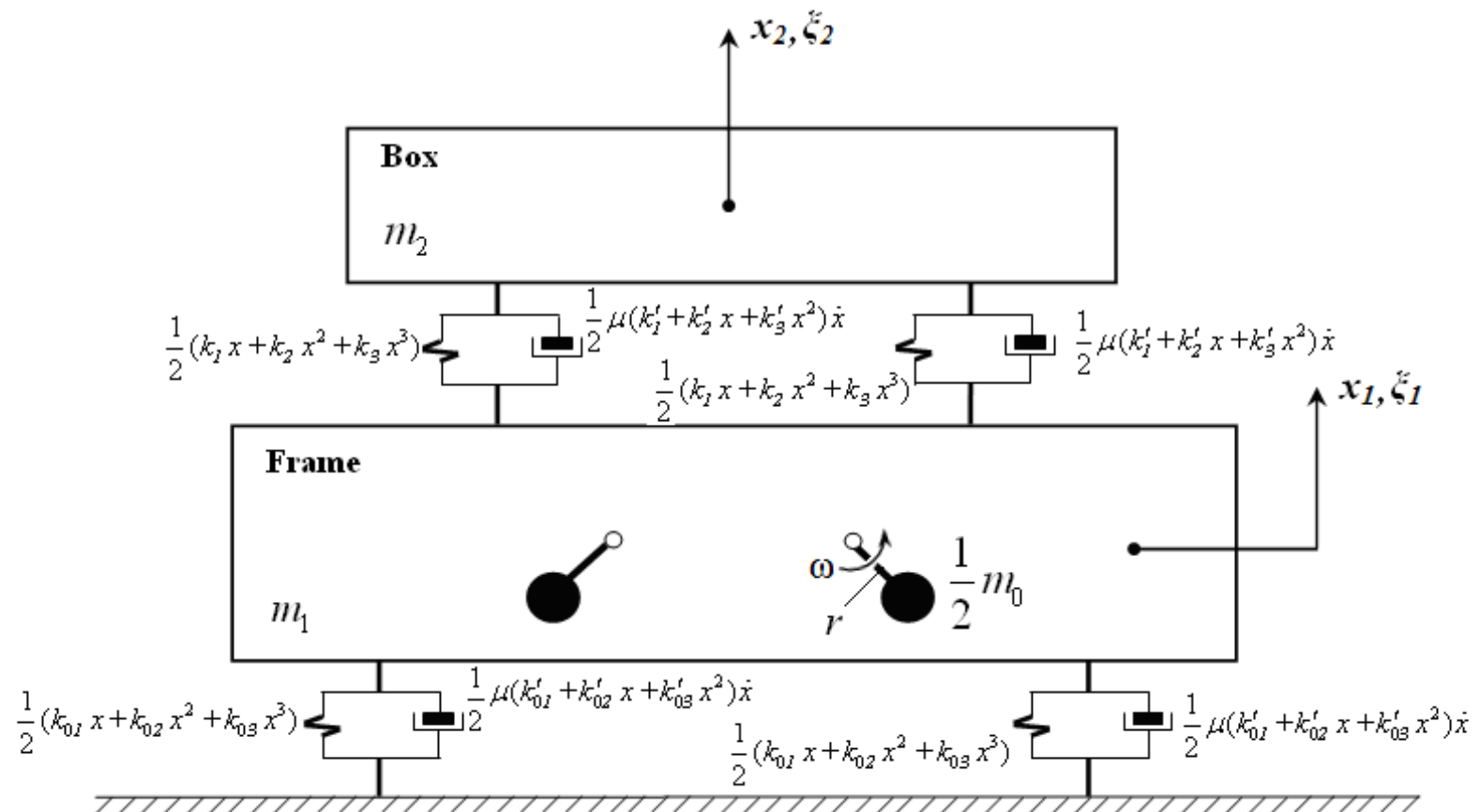
The model under consideration

In dimensionless form equations of its motion may be represented in the form

$$
\begin{cases}
\dfrac{d^2\xi_1}{d\tau^2} + b_{101}\dfrac{d\xi_1}{d\tau} + b_{102}\,\xi_1\dfrac{d\xi_1}{d\tau} + b_{103}\,\xi_1^2\dfrac{d\xi_1}{d\tau} + b_{11}\dfrac{d\xi}{d\tau} + b_{12}\,\xi\dfrac{d\xi}{d\tau} + b_{13}\,\xi^2\dfrac{d\xi}{d\tau} + \\[2mm]
\qquad + k_{101}\,\xi_1 + k_{102}\,\xi_1^2 + k_{103}\,\xi_1^3 + k_{11}\,\xi + k_{12}\,\xi^2 + k_{13}\,\xi^3 = P_1\,\eta^2\cos\eta\tau, \\[2mm]
\dfrac{d^2\xi}{d\tau^2} + b_{201}\dfrac{d\xi_1}{d\tau} + b_{202}\,\xi_1\dfrac{d\xi_1}{d\tau} + b_{203}\,\xi_1^2\dfrac{d\xi_1}{d\tau} + b_{21}\dfrac{d\xi}{d\tau} + b_{22}\,\xi\dfrac{d\xi}{d\tau} + b_{23}\,\xi^2\dfrac{d\xi}{d\tau} + \\[2mm]
\qquad + k_{201}\,\xi_1 + k_{202}\,\xi_1^2 + k_{203}\,\xi_1^3 + k_{21}\,\xi + k_{22}\,\xi^2 + k_{23}\,\xi^3 = P_2\,\eta^2\cos\eta\tau,
\end{cases}
\tag{1}
$$

where $\quad b_{101}=\dfrac{\mu k'_{01}}{m'_1\,\omega_1},\quad b_{102}=\dfrac{\mu k'_{02}\,\Delta}{m'_1\,\omega_1},\quad b_{103}=\dfrac{\mu k'_{03}\,\Delta^2}{m'_1\,\omega_1},\quad b_{11}=-\dfrac{\mu k'_1}{m'_1\,\omega_1},\quad b_{12}=-\dfrac{\mu k'_2\,\Delta}{m'_1\,\omega_1},\quad b_{13}=-\dfrac{\mu k'_3\,\Delta^2}{m'_1\,\omega_1},$

$b_{201}=-\dfrac{\mu k'_{01}}{m'_1\,\omega_1},\quad b_{202}=-\dfrac{\mu k'_{02}\,\Delta}{m'_1\,\omega_1},\quad b_{203}=-\dfrac{\mu k'_{03}\,\Delta^2}{m'_1\,\omega_1},\quad b_{21}=\dfrac{\mu(m'_1+m_2)\,k'_1}{m'_1\,m_2\,\omega_1},\quad b_{22}=\dfrac{\mu(m'_1+m_2)\,k'_2\,\Delta}{m'_1\,m_2\,\omega_1},$

$b_{23}=\dfrac{\mu(m'_1+m_2)\,k'_3\,\Delta^2}{m'_1\,m_2\,\omega_1},\quad k_{101}=\dfrac{k_{01}}{m'_1\,\omega_1^2},\quad k_{102}=\dfrac{k_{02}\,\Delta}{m'_1\,\omega_1^2},\quad k_{103}=\dfrac{k_{03}\,\Delta^2}{m'_1\,\omega_1^2},\quad k_{11}=-\dfrac{k_1}{m'_1\,\omega_1^2},\quad k_{12}=-\dfrac{k_2\,\Delta}{m'_1\,\omega_1^2},$

$k_{13}=-\dfrac{k_3\,\Delta^2}{m'_1\,\omega_1^2},\quad k_{201}=-\dfrac{k_{01}}{m'_1\,\omega_1^2},\quad k_{202}=-\dfrac{k_{02}\,\Delta}{m'_1\,\omega_1^2},\quad k_{203}=-\dfrac{k_{03}\,\Delta^2}{m'_1\,\omega_1^2},\quad k_{21}=\dfrac{k_1(m'_1+m_2)}{m'_1\,m_2\,\omega_1^2},\quad k_{22}=\dfrac{k_2(m'_1+m_2)\,\Delta}{m'_1\,m_2\,\omega_1^2},$

$k_{23}=\dfrac{k_3(m'_1+m_2)\,\Delta^2}{m'_1\,m_2\,\omega_1^2},\quad P_1=\dfrac{m_0 r}{m'_1\,\Delta},\quad P_2=-P_1,\quad m'_1=m_0+m_1,\quad \eta=\omega/\omega_1.$

The model under consideration

Here m_1 is mass of a frame, m_2 – of a box, m_0 – of unbalance masses, characteristics of the main and supported elastic ties are $f_m(x) = k_1 x + k_2 x^2 + k_3 x^3$ and $f_s(x) = k_{01} x + k_{02} x^2 + k_{03} x^3$ correspondently, resistance forces are $f_{r.m.}(x) = \mu(k_1' + k_2' x + k_3' x^2)\dot{x}$ and $f_{s.m.}(x) = \mu(k_{01}' + k_{02}' x + k_{03}' x^2)\dot{x}$, ω – rotation speed of the exciter, r is its eccentricity.

The model under consideration

Taking in mind here experimental machine physical values of some parameters were taken as follows $m_1 = 98$ kg, $m_0 = 2$ kg, $m_2 = 45$ kg, $k_{01} = 2.4 \cdot 10^4$ N/m, $k_1 = 0.6 \cdot 10^6$ N/m, $m_0 \cdot r = (0.05 - 0.5)$ kg·m then $\omega_1 = 12.8$ rad·sec^{-1}. Presumable ranges of the others are $k_2 \in [-2, 0] \cdot 10^3 \cdot k_1$, $k_3 \in [0, 3] \cdot 10^6 \cdot k_1$, where k_2 and k_3 characterize the level of asymmetry and nonlinearity of the main elastic ties. The similar correlations were taken for parameters k_{02}, k_{03} of the supported elastic ties. Angular velocity of the exciter ω is supposed to be variable, parameters k_i' and k_{0i}' of the resistance forces are dependent on the design of elastic ties, coefficient of dissipation in material of elastic ties is equal to $\mu = 8 \cdot 10^{-4}$ sec. Keeping in mind the elastic ties of magnetic type we suppose $k_i' = k_{0i}' = 0$. Then the dimensionless values of the parameters are $b_{101} = 0.015$, $b_{102} = b_{103} = 0$, $b_{11} = -0.375$, $b_{12} = b_{13} = 0$, $k_{101} = 1.465$, $k_{102} = k_{103} = 0$, $k_{11} = -36.621$, $k_{12}/k_{11} \in [-2, 0]$, $k_{13}/k_{11} \in [0, 3]$, $P_1 \in [0.5, 5.0]$, $b_{201} = -b_{101}$, $b_{202} = b_{203} = 0$, $b_{21} = 1.208$, $b_{22} = b_{23} = 0$, $k_{201} = -k_{101}$, $k_{202} = k_{203} = 0$, $k_{21} = 118.001$, $k_{22}/k_{21} = k_{12}/k_{11}$, $k_{23}/k_{21} = k_{13}/k_{11}$, $P_2 = -P_1$.

Research methods

Analysis of the model is performed with the help of original software which was worked out as a tool of program MATLAB. Searching of the bifurcation diagrams in it is based on the harmonic balance method. Stability of the motions by the first approximation is investigated with use of Floquet-Lyapunov theory. Stationary solutions of the system are found in the form of finite Fourier expansions

$$\xi_1(\tau) = \sum_{n=-N}^{N} c_n^{(1)} e^{in\eta\tau}, \quad \xi(\tau) = \sum_{n=-N}^{N} c_n e^{in\eta\tau}, \tag{2}$$

where N is a number of harmonics taken into consideration. Supposing the trigonometric view of the solutions in the form $\sum_{j=0}^{N} A_j \cos(j\eta\tau - \varphi_j)$, where $\varphi_j \in [-\pi, \pi)$, you can get the expressions for the amplitudes of the harmonic components A_j and its initial phases φ_j as $A_j = 2\sqrt{c_j c_{-j}}$ and

$$\varphi_j = \arccos\frac{c_j + c_{-j}}{2\sqrt{c_j c_{-j}}} \text{ or } \varphi_j = -\arccos\frac{c_j + c_{-j}}{2\sqrt{c_j c_{-j}}}, \text{ if } (\Im c_{-j} = 0 \wedge \Re c_{-j} < 0) \vee \Im c_{-j} < 0.$$

Research methods

After substitution expressions (2) into the differential equations (1) and equating the coefficients of equal powers the polynomial system for determination of expansion coefficients is produced

Research methods

$$(k_{101} + b_{101}\, i\, \eta\, n - \eta^2\, n^2)\, c_n^{(1)} + \sum_{j=-N}^{N} c_j^{(1)}\, c_{n-j}^{(1)}\, (k_{102} + b_{102}\, i\, \eta\, (n-j)) +$$

$$+ \sum_{j=-N}^{N} \sum_{m=-N}^{N} c_j^{(1)}\, c_m^{(1)}\, c_{n-j-m}^{(1)}\, (k_{103} + b_{103}\, i\, \eta\, (n-j-m)) +$$

$$+ (k_{11} + b_{11}\, i\, \eta\, n)\, c_n + \sum_{j=-N}^{N} c_j\, c_{n-j}\, (k_{12} + b_{12}\, i\, \eta\, (n-j)) +$$

$$+ \sum_{j=-N}^{N} \sum_{m=-N}^{N} c_j\, c_m\, c_{n-j-m}\, (k_{13} + b_{13}\, i\, \eta\, (n-j-m)) = \begin{cases} P_1\, \eta^2/2, & n = \pm 1 \\ 0, & n \neq \pm 1 \end{cases},$$

$$(k_{201} + b_{201}\, i\, \eta\, n)\, c_n^{(1)} + \sum_{j=-N}^{N} c_j^{(1)}\, c_{n-j}^{(1)}\, (k_{202} + b_{202}\, i\, \eta\, (n-j)) +$$

$$+ \sum_{j=-N}^{N} \sum_{m=-N}^{N} c_j^{(1)}\, c_m^{(1)}\, c_{n-j-m}^{(1)}\, (k_{203} + b_{203}\, i\, \eta\, (n-j-m)) +$$

$$+ (k_{21} + b_{21}\, i\, \eta\, n - \eta^2\, n^2)\, c_n + \sum_{j=-N}^{N} c_j\, c_{n-j}\, (k_{22} + b_{22}\, i\, \eta\, (n-j)) +$$

$$+ \sum_{j=-N}^{N} \sum_{m=-N}^{N} c_j\, c_m\, c_{n-j-m}\, (k_{23} + b_{23}\, i\, \eta\, (n-j-m)) = \begin{cases} P_2\, \eta^2/2, & n = \pm 1 \\ 0, & n \neq \pm 1 \end{cases},$$

(3)

where $n, n-j, n-j-m \in [-N, N]$.

Research methods

Changing step by step one of the parameters of the model and solving algebraic system of equations (3) with respect to c_j one can find spectral $\{A_j\}$ and phase $\{\varphi_j\}$ structure, laws of motion or acceleration and construct the bifurcation curves of the system.

Investigations, discovered combination resonances

Taking in mind the vibration machines of the «antiresonance» type [7] we consider the motion of the model in the frequency zones located between the natural ones. In the figure 5 below amplitude- (AFC) and phase-frequency characteristic (PFC) of corresponding linear system are presented.

Investigations, discovered combination resonances

Figure 5. AFC and PFC for linear system
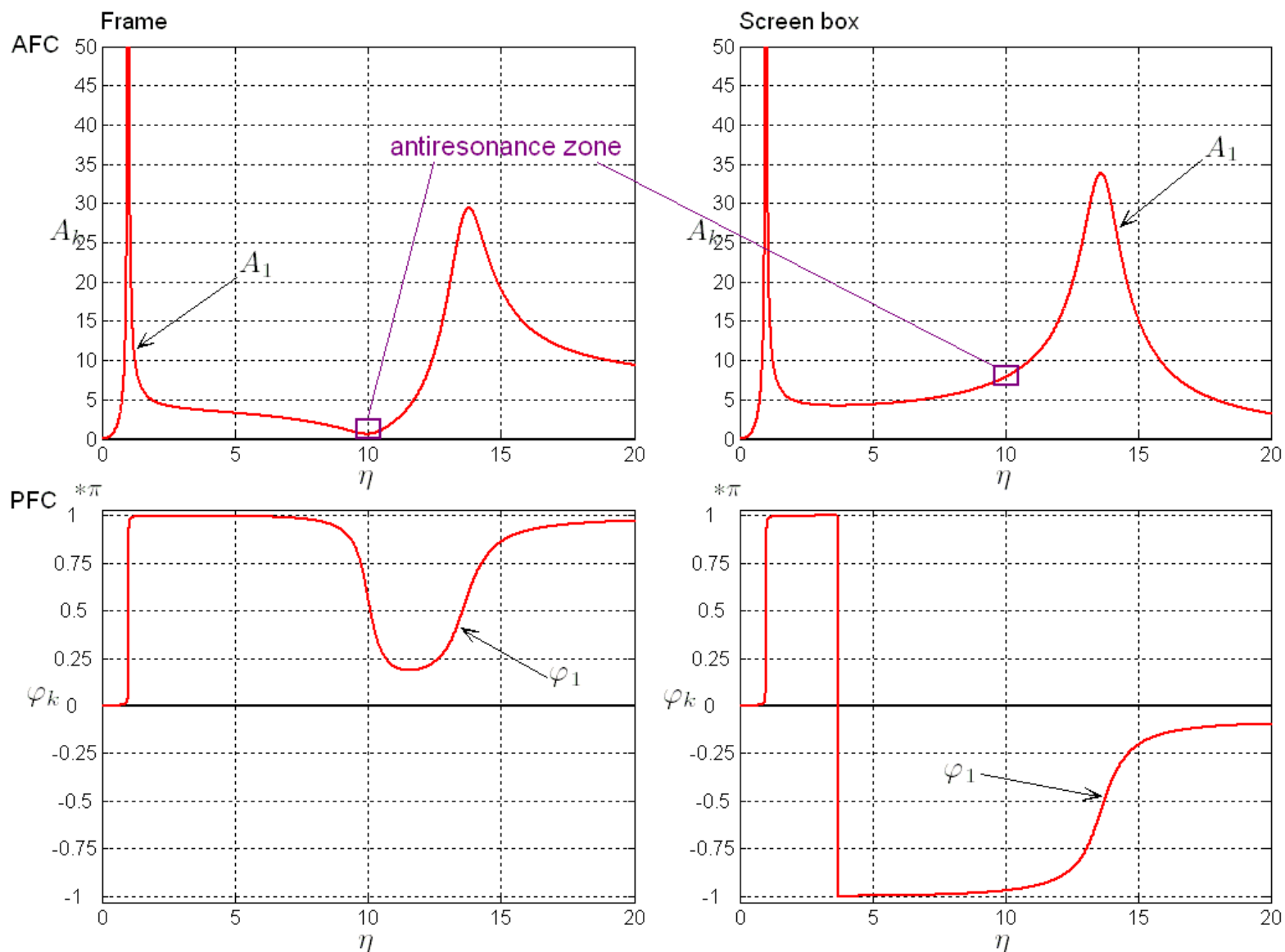
Using the featured ratio $p \omega \approx |p_1| \omega_1 + |p_2| \omega_2$ [8] between the excitation frequency and natural ones, where $p, p_1, p_2 \in \mathbb{Z}$ we considered only pure resonances of lower orders ($\omega \approx p_1 \omega_1$, where $p_1 = 2, 3$ and $p \omega \approx p_2 \omega_2$, where $p = 2, 3$, $p_2 = 1$ and $p = 1$, $p_2 = 2, 3$). Changing parameters and initial conditions the resonances of orders 3:1, 2:1 and 1:3 were discovered in this frequency zone (fig. 6-8).

Investigations, discovered combination resonances

Figure 6. Resonance 3:1, $k_{13} / k_{11} = 1$
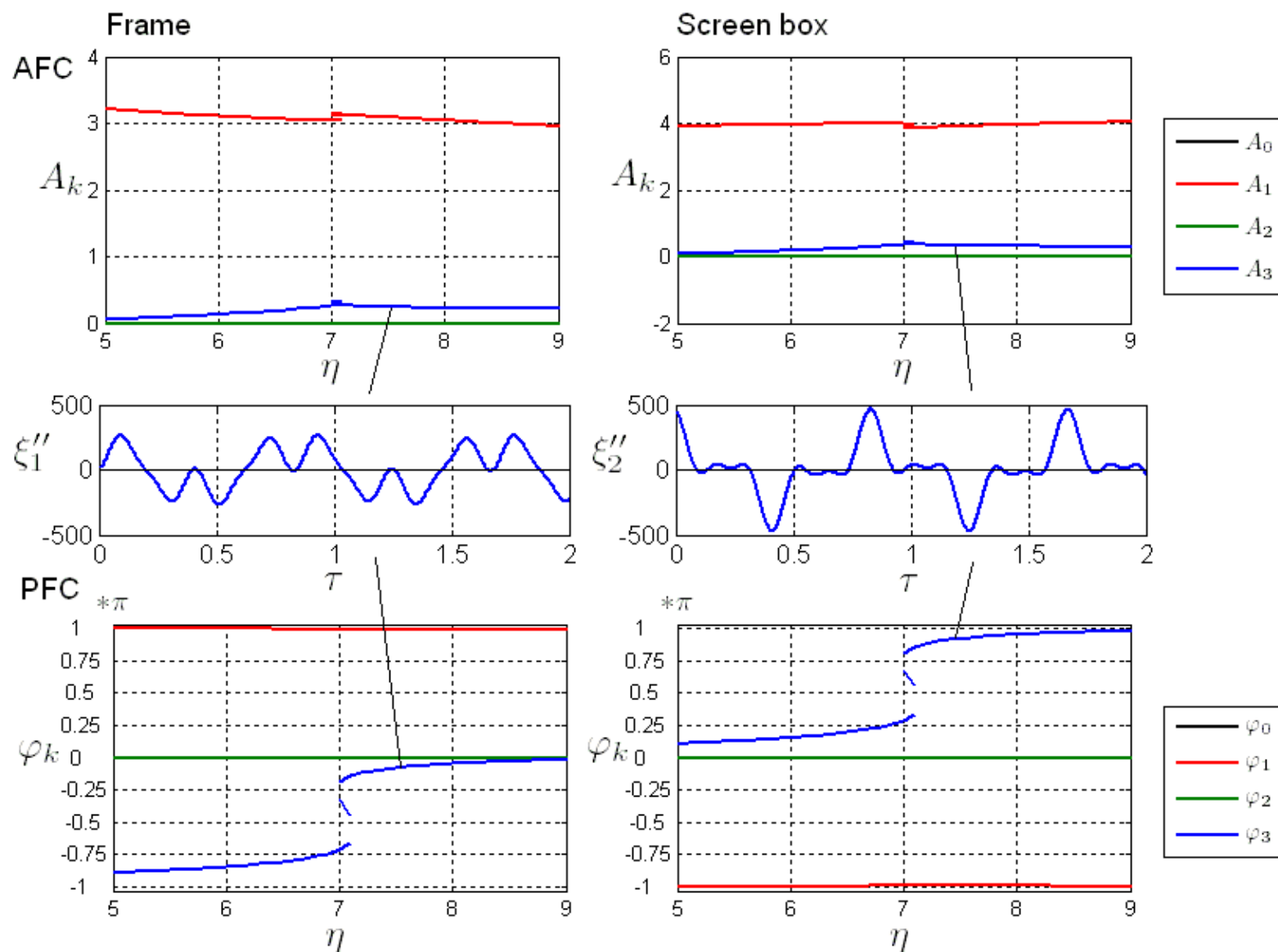
Figure 7. Resonance 2:1, $k_{13} / k_{11} = 1$
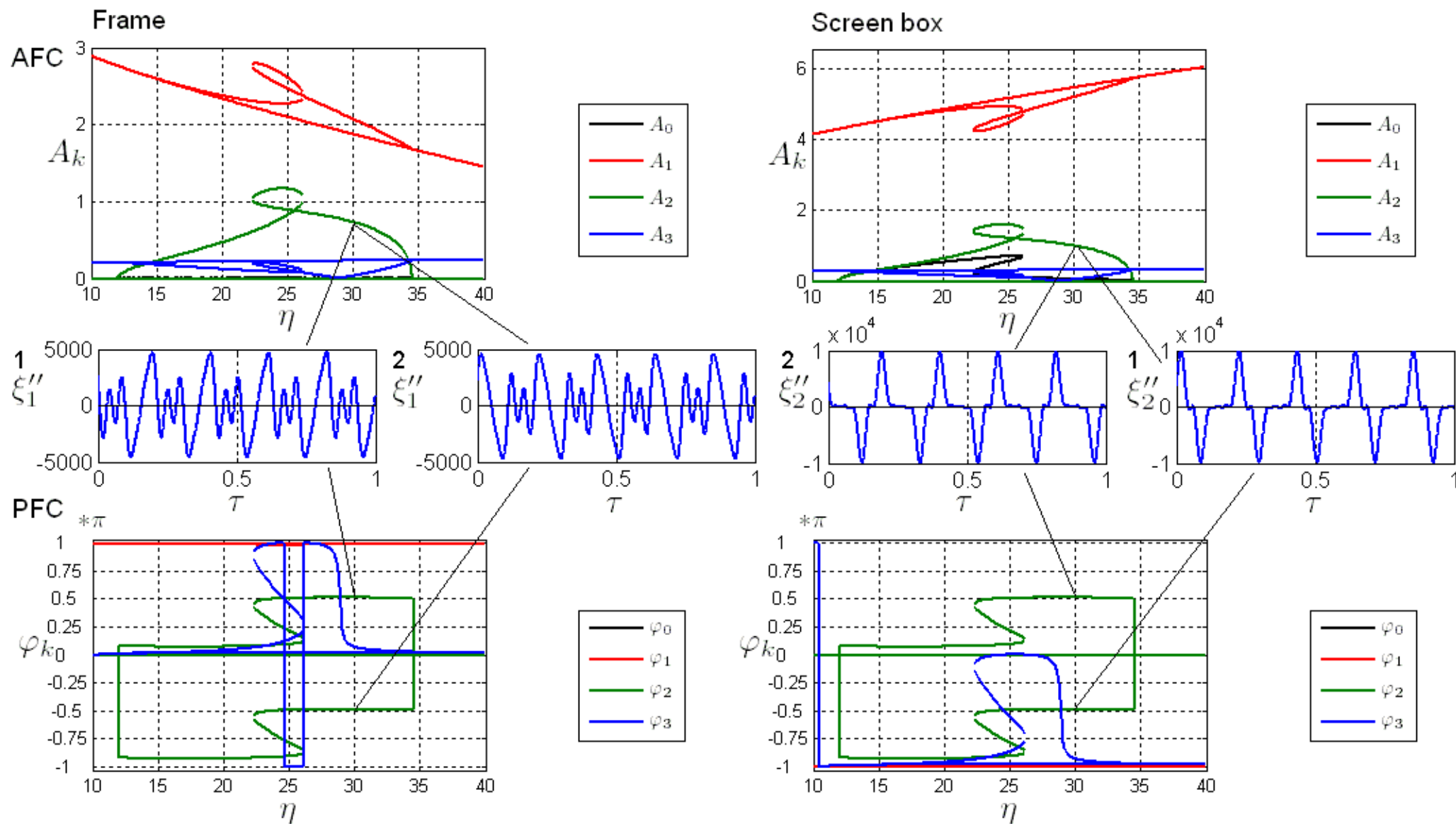
Investigations, discovered combination resonances

Figure 8. Resonance 1:3, $k_{13}/k_{11} = 1$

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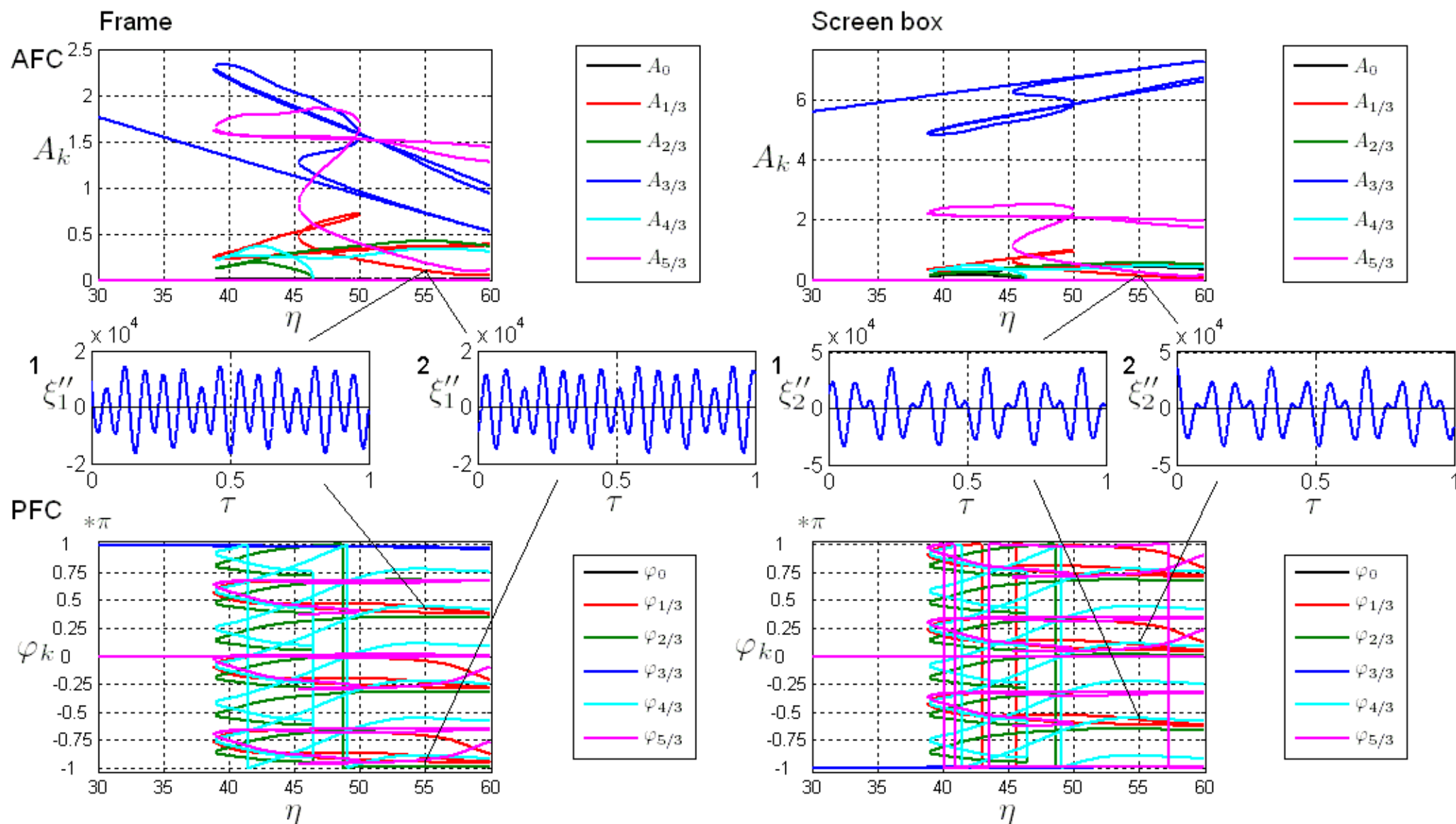
Investigations, discovered combination resonances

In our opinion, the most interesting of it, from the practical point of view, is the resonance of the order 2:1, – it is rather intensive and gives an opportunity to form practically suitable biharmonic oscillations. But one of the serious drawbacks of it consists of existence opposite regimes for one frequency of excitation (fig. 7).

Investigation of resonance 2:1

Considering the domain of the parameters $P_1 \in [0.5, 5.0]$, $k_3/k_1 \in [0.5, 3.0]$, $k_2/k_1 \in [0.0, -2.0]$ investigation of the effect of each of these parameters upon the behaviour of the machine was studied. The results are presented in figures 9-11, where AFC, PFC and diagrams of acceleration of the box are given. Coefficient of asymmetry $A_s = \mu_3/\sigma^3$ of the periodic signal $x(t)$ is determined here as the ratio of the third central moment to the third power of standard deviation, where $\mu_3 = (\int_0^T (x(t)-m)^3 \, dt)/T$, $\sigma^2 = (\int_0^T (x(t)-m)^2 \, dt)/T$, $m = \int_0^T x(t)\, dt/T$. Their analysis says that for some parameters, for example, on the ascending branch of the AFC (fig. 9b) or for certain asymmetry of elastic characteristics (fig. 11c) the superharmonic regimes may be globally stable. This fact is important for practice and conditions are found on the parameters of vibrating machines

$$k_1 = (10 \div 50)\, k_{01}, \quad 0.5 < \frac{m_0 r}{(m_0 + m_1)\, \Delta} < 1.5, \quad \frac{k_2 \Delta}{k_1} \cdot \frac{m_1}{m_0 + m_1} \approx -2, \quad \frac{k_3 \Delta^2}{k_1} \approx 3,$$

which ensure such stability. Asymmetry of accelerations may reach the value $|A_s| = 1.5$ (fig. 10c) and even more while its value for recommended biharmonic motions [2] is $|A_s| = 0.38 - 0.76$.

Investigation of resonance 2:1

Figure 9. The effect of the external force for
$k_2 / k_1 = -1.0$, $k_3 / k_1 = 1.5$

a) $R_1 = 0.5$
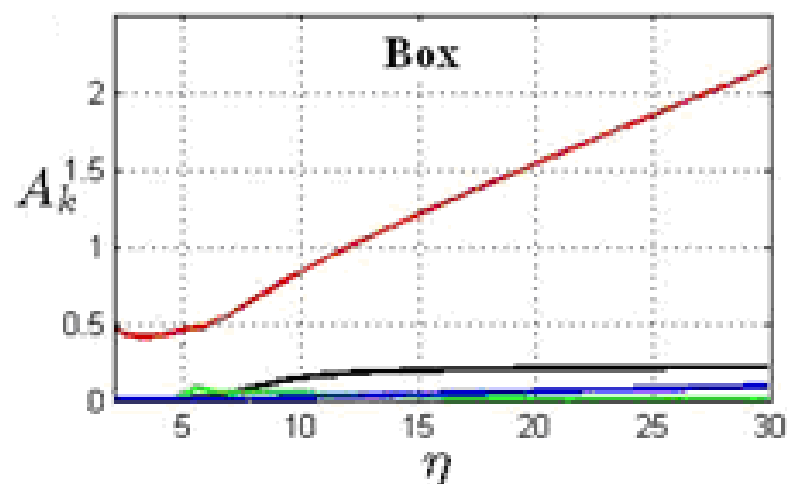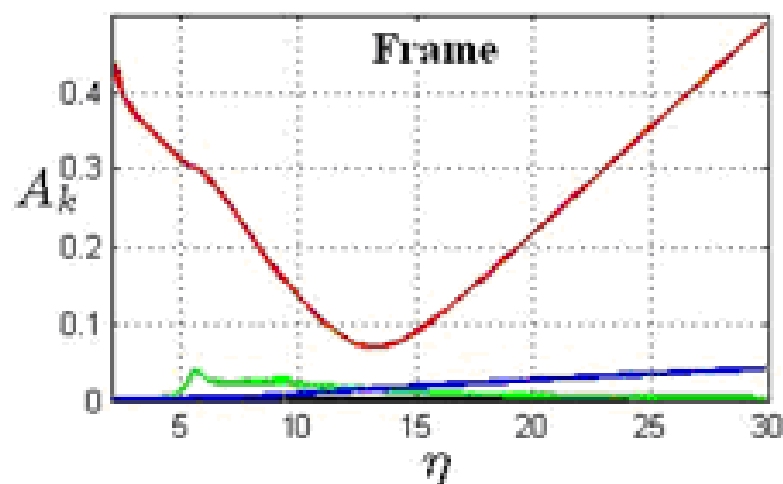
$A_s = 0.95$
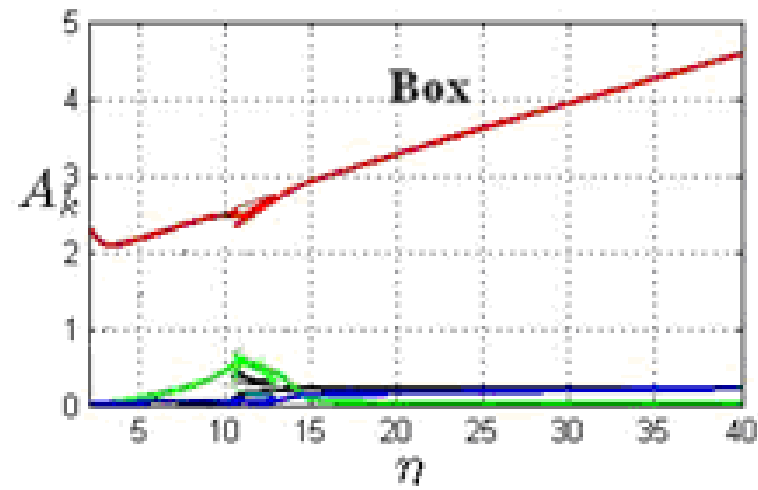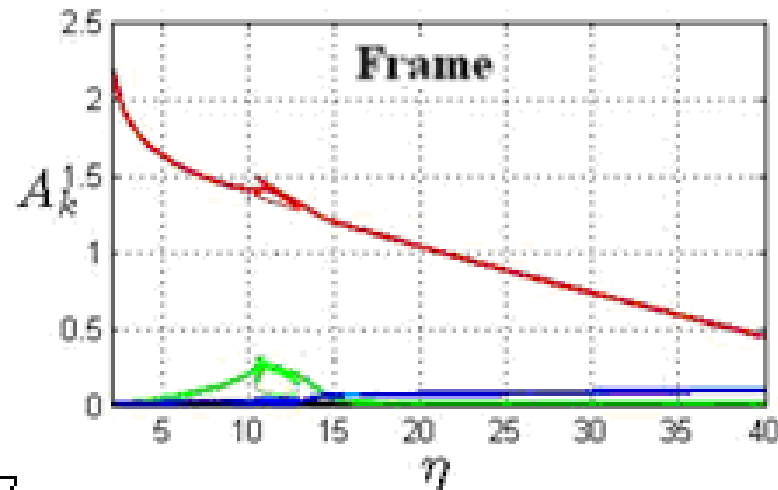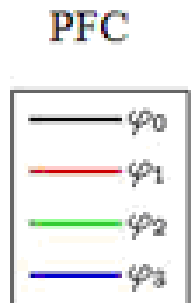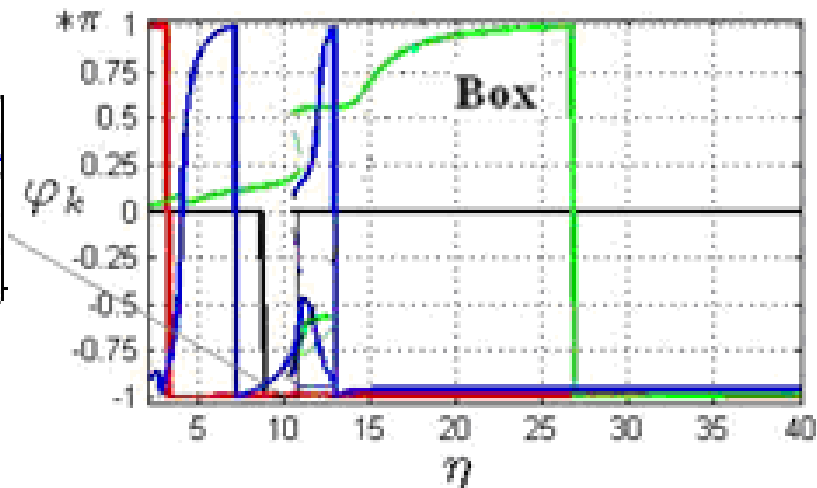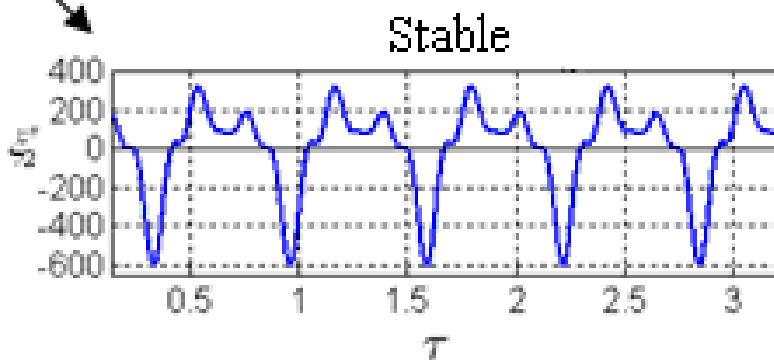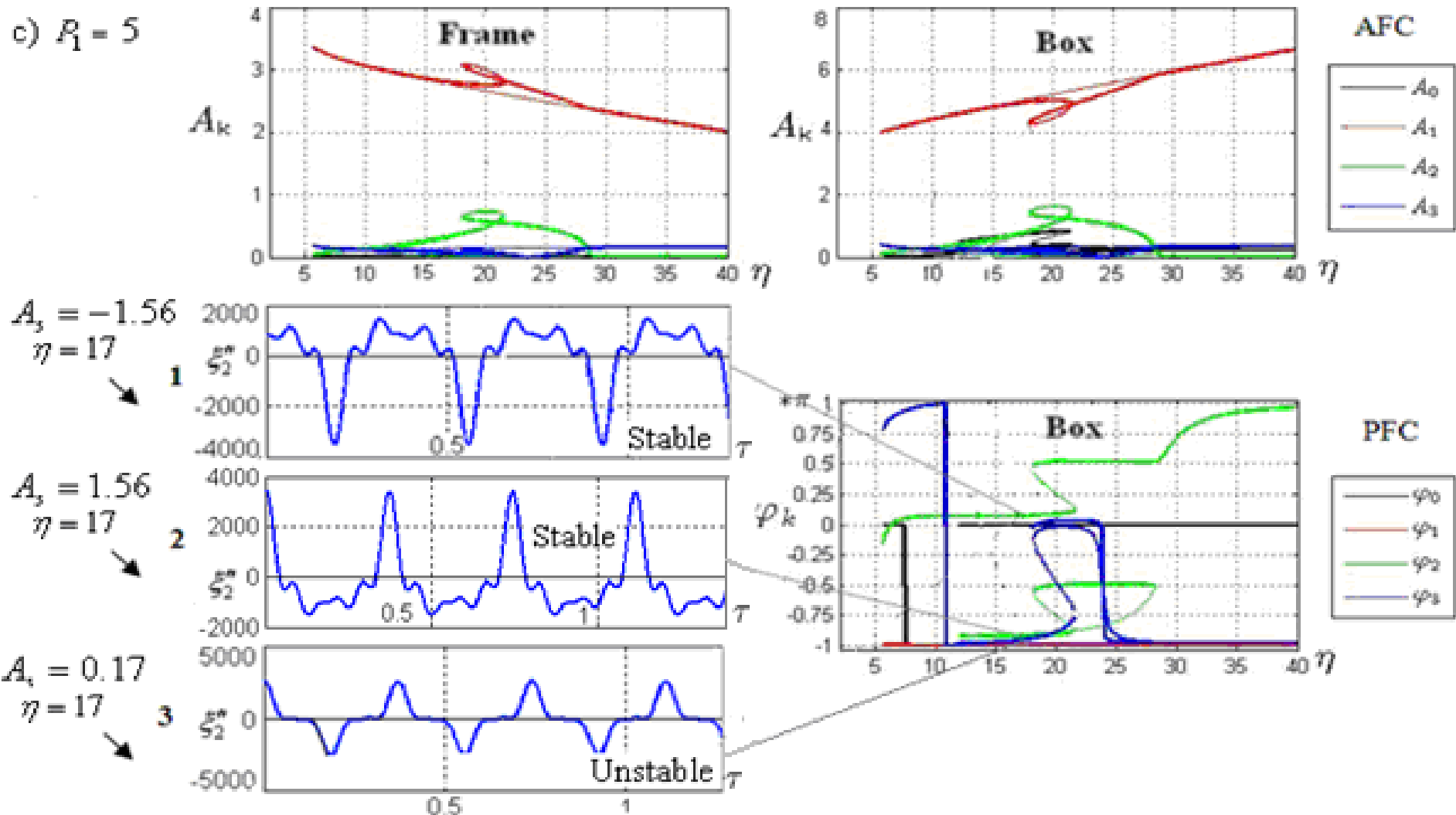$\eta = 6$
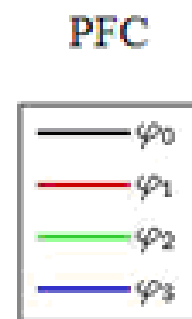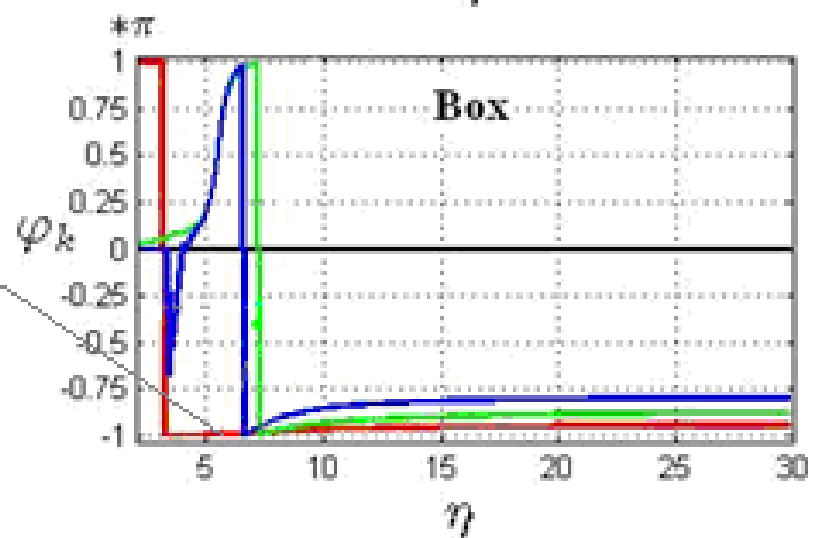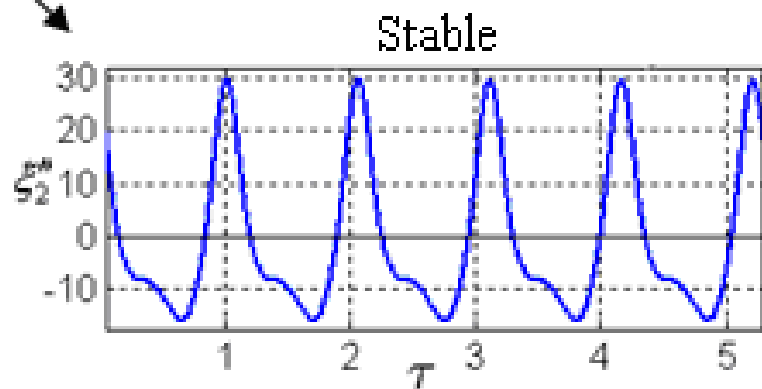
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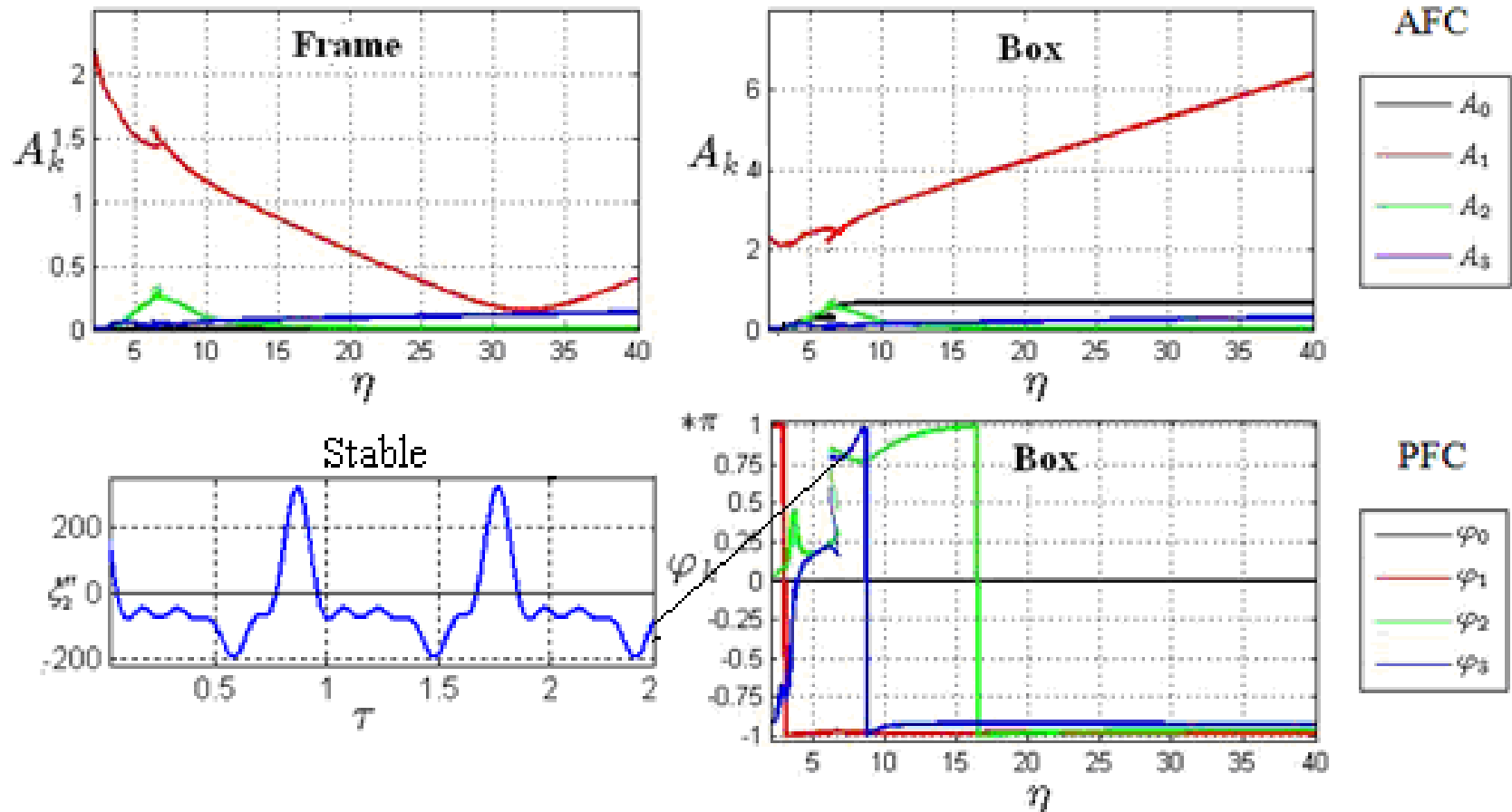
Investigation of resonance 2:1

Figure 9. The effect of the external force for
$k_2 / k_1 = -1.0$, $k_3 / k_1 = 1.5$

Investigation of resonance 2:1

Figure 9. The effect of the external force for $k_2 / k_1 = -1.0$, $k_3 / k_1 = 1.5$

Investigation of resonance 2:1

Figure 10. The effect of the nonlinearity of elastic ties for $P_1 = 2.5$, $k_2 / k_1 = -1$
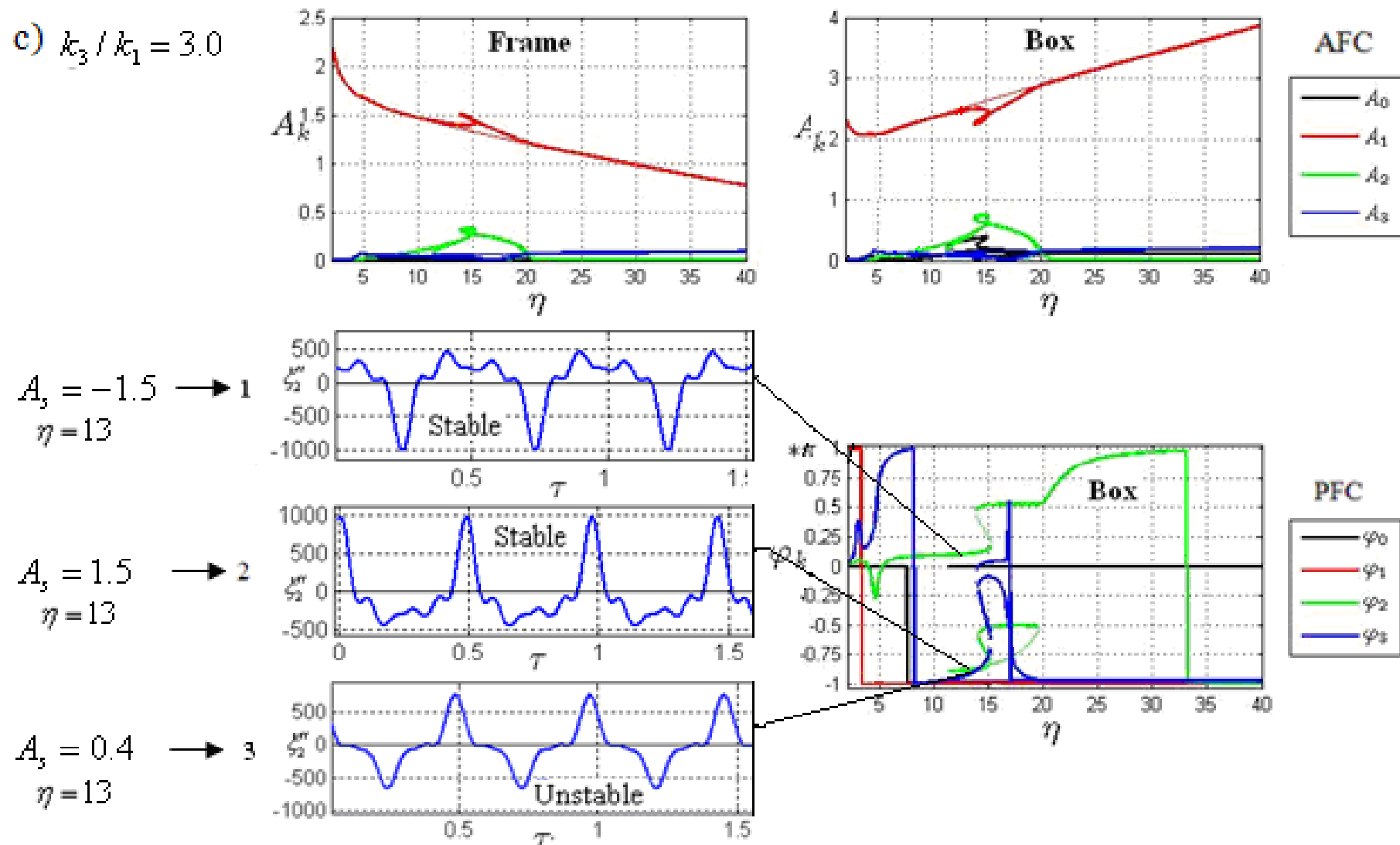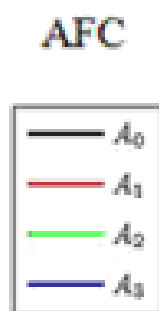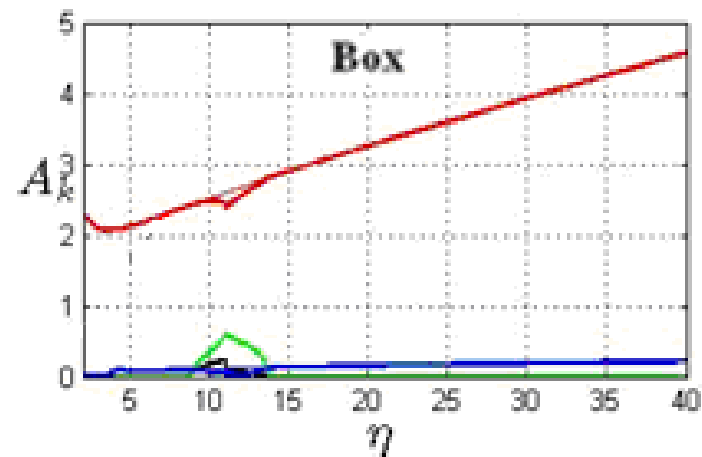
a) $k_3 / k_1 = 0.5$

$A_s = 1.5$

$\eta = 7$

Investigation of resonance 2:1
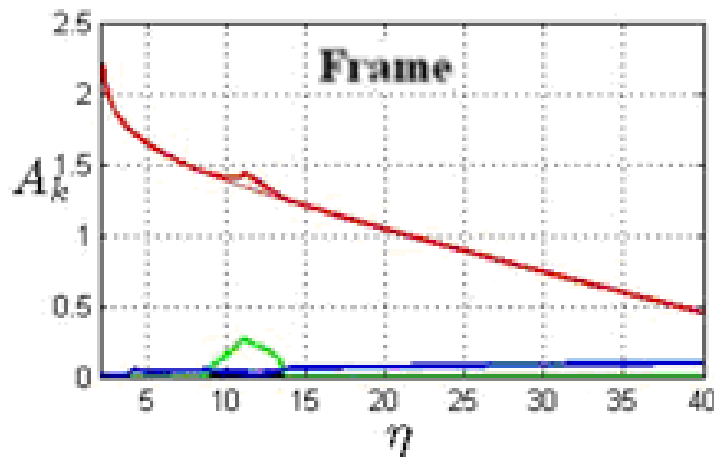
Figure 10. The effect of the nonlinearity of elastic ties for $P_1 = 2.5$, $k_2 / k_1 = -1$

$b)\ k_3 / k_1 = 1.5$
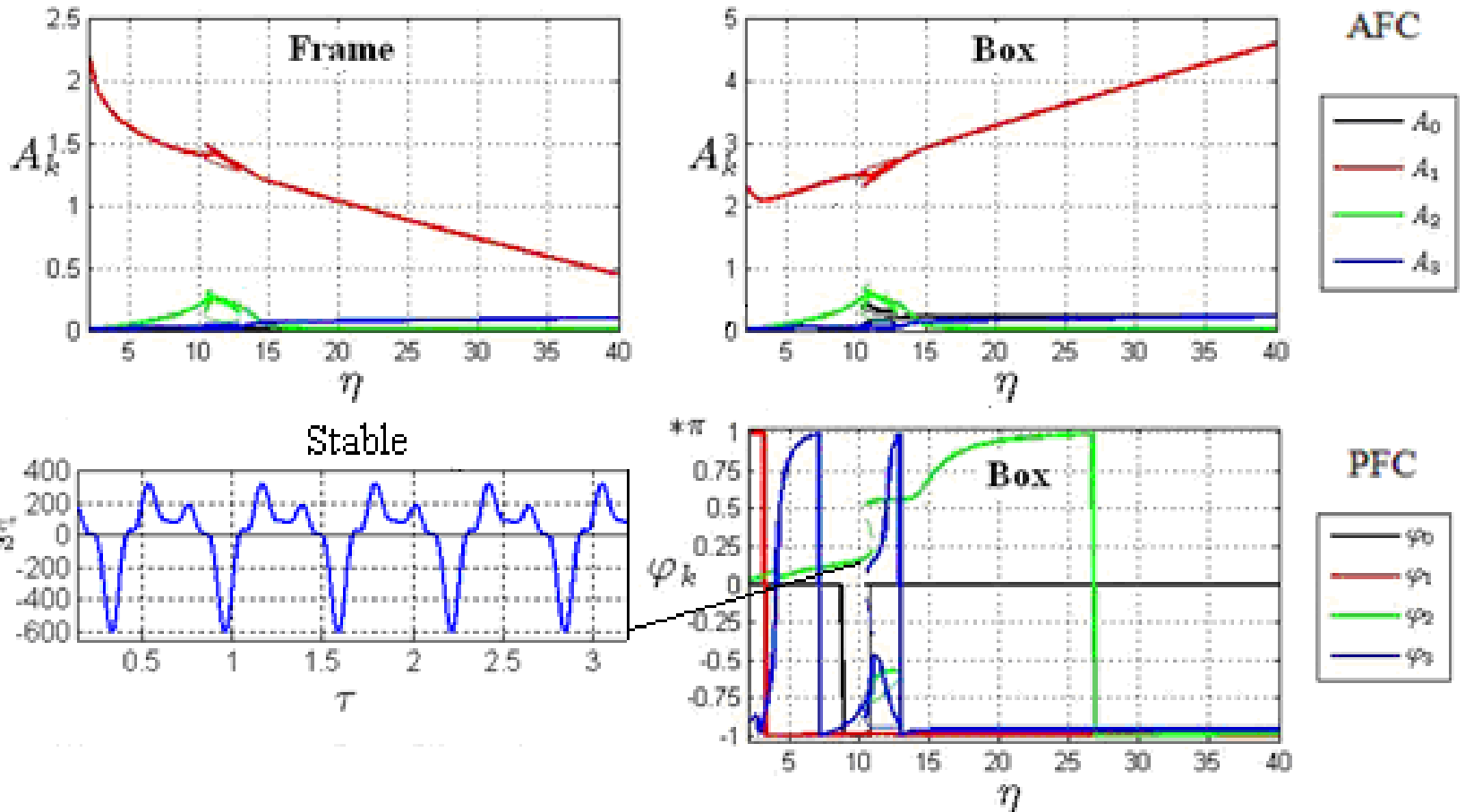
$A_s = -1.27$
$\eta = 10$

Investigation of resonance 2:1

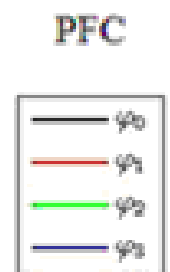Figure 10. The effect of the nonlinearity of elastic ties for $P_1 = 2.5$, $k_2 / k_1 = -1$
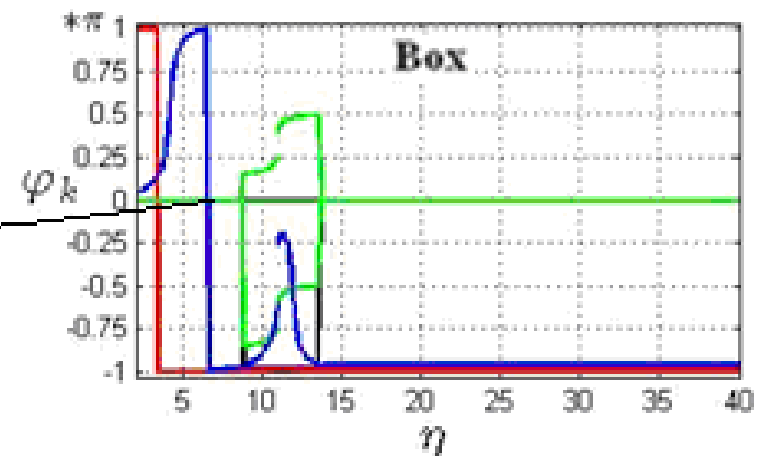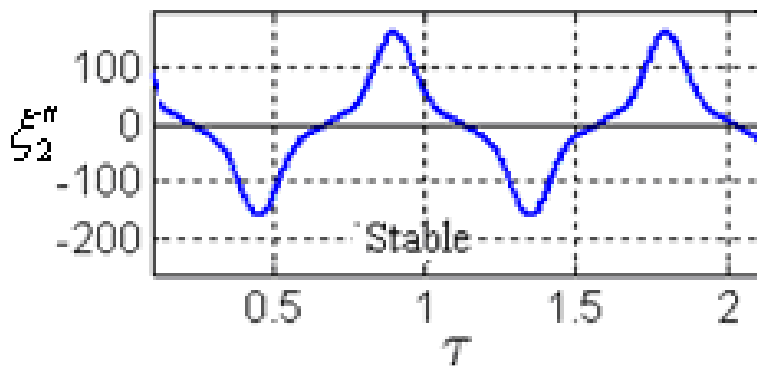
Investigation of resonance 2:1

Figure 11. The effect of the asymmetry of elastic ties for $P_1 = 2.5$, $k_3 / k_1 = 1.5$

a) $k_2 / k_1 = 0$

$A_s = 0$ →
$\eta = 7$

Investigation of resonance 2:1

Figure 11. The effect of the asymmetry of elastic ties for $P_1 = 2.5$, $k_3 / k_1 = 1.5$

b) $k_2 / k_1 = -1$

$A_s = -1.27 \longrightarrow$

$\eta = 10$

Investigation of resonance 2:1

Figure 11. The effect of the asymmetry of elastic ties for $P_1 = 2.5$, $k_3 / k_1 = 1.5$

c) $k_2 / k_1 = -2$

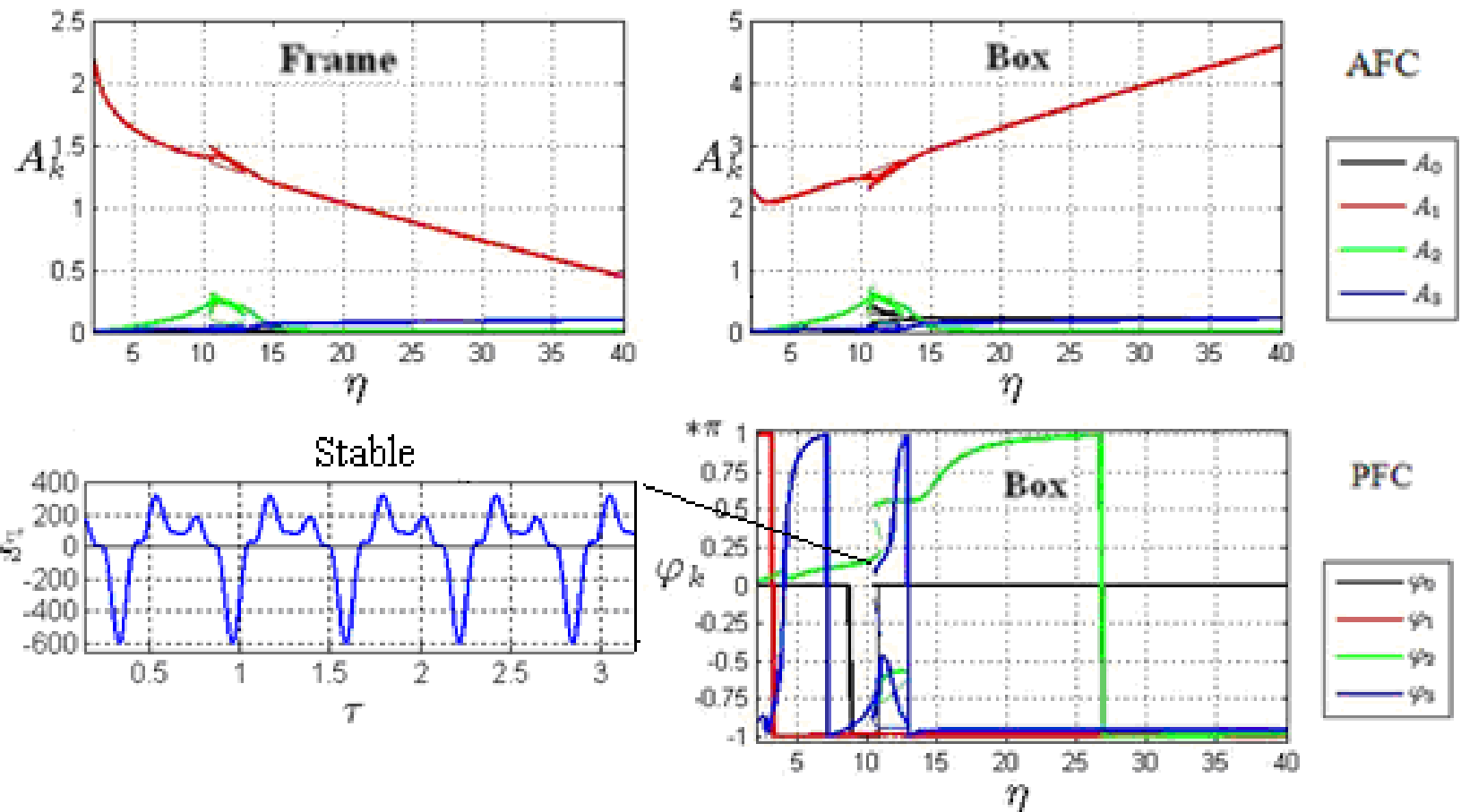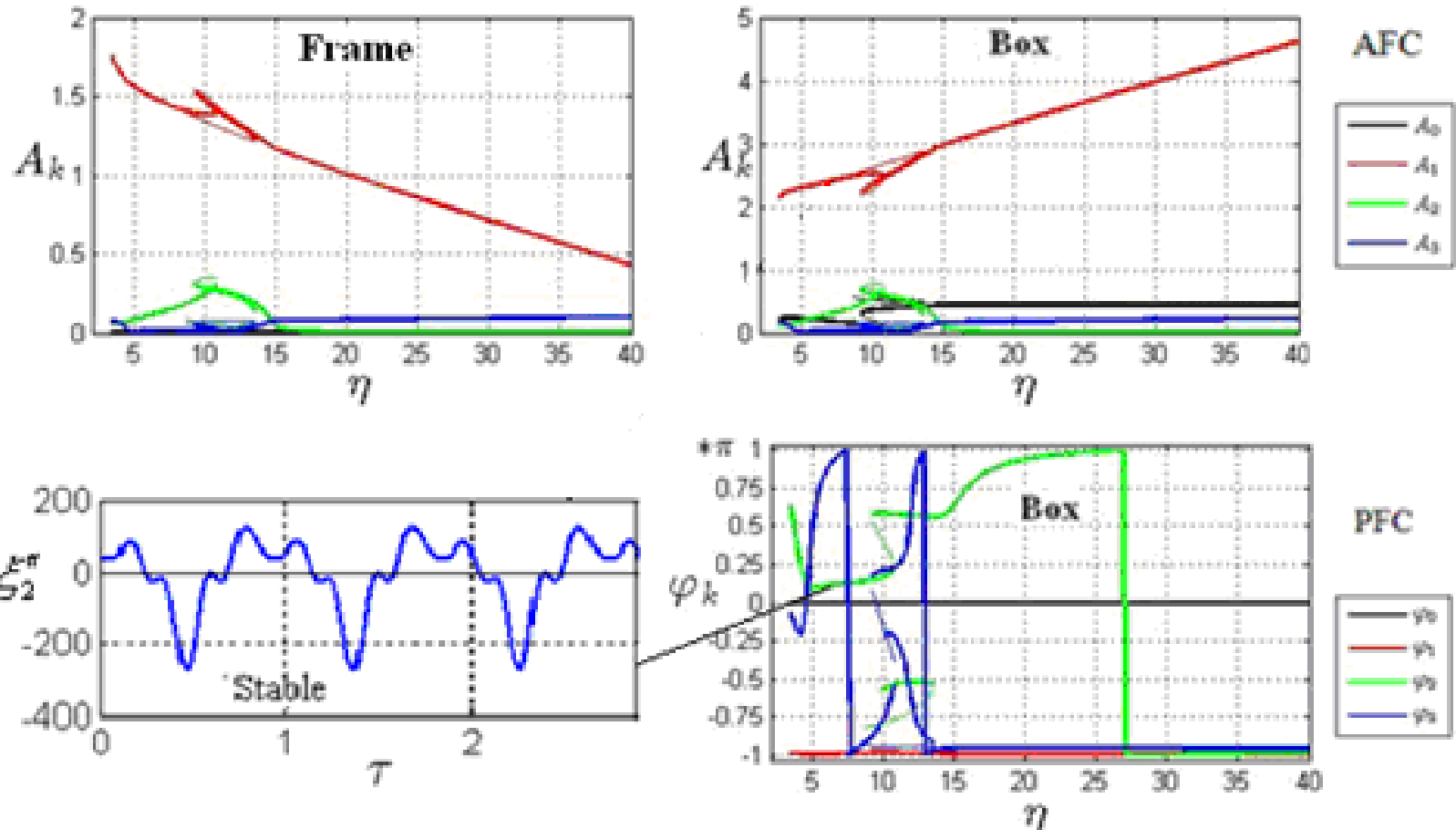$A_s = -1,18$ →

$\eta = 7$

Conclusions

1. The principal scheme of the device of elastic ties of magnetic type was proposed. For different values of gaps its elastic characteristic was determined.
2. Dynamics of two-masses vibrating machine with polynomial characteristic of elastic ties was investigated in «antiresonance» zone. The possibility of excitation of resonances 3:1, 2:1, 1:3 was discovered.
3. The effect of parameters upon the behavior of vibrating machine was studied, the possibility of global stability of 2:1 regimes was mentioned.

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