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Research Article

Stiff Fluid in Accelerated Universes with Torsion

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The flat Friedmann universes filled by stiff fluid and a nonminimally coupled material scalar field with polynomial potentials of the fourth degree are considered in the framework of the Einstein-Cartan theory. Exact general solution is obtained for arbitrary positive values of the coupling constant ξ. A comparative analysis of the cosmological models with and without stiff fluid is carried out. Some effects of stiff fluid are elucidated. It is shown that singular models with a de Sitter asymptotic and with the power-law ($t^{4/3}$) asymptotic at late times are possible. It is found that $\xi = 3/8$ is a specific value of the coupling constant. It is demonstrated that the bouncing models without the particle horizon and with an accelerated expansion by a de Sitter law of an evolution at late times are admissible.

1. Introduction

Recent cosmic observations [1–6] favor an isotropic spatially flat Universe, which is at present expanding with acceleration. The source of this expansion is an unknown substance with negative pressure called dark energy (DE). Establishment of the origin of DE has become an important problem. Different theoretical models of DE have been put forward (see, e.g., the reviews [7–10] and references therein). Among these models various modifications of general relativity (GR) were considered, the Einstein-Cartan theory (ECT) in particular [11–13]. This theory [14–17] is an extension of GR to a space time with torsion, and it reduces to GR when the torsion vanishes. The ECT is the simplest version of the Poincaré gauge theory of gravity (PGTG). It should be noted that the ECT contains a nondynamic torsion, because its gravitational action is proportional to the curvature scalar of the Riemann-Cartan space-time. In this sense, the ECT is a degenerate gauge theory [17–20]. This drawback is absent in the PGTG since its gravitational Lagrangian includes invariants quadratic in the curvature and torsion tensors. Nevertheless the ECT is a viable theory of gravity whose observational predictions are in agreement with the classical tests of GR, and it differs significantly from GR only at very high densities of matter [17, 21, 22].

The ECT finds applications in cosmology [23–28], particle theory [19, 29, 30], and the theory of strong interactions [31, 32]. From some time past, the interest to ECT has grown in connection with the fact that torsion arises naturally in the supergravity [33–35], Kaluza-Klein [36–38], and syperstring [39–41] theories. $f(R)$ gravity with torsion has been developed [42–45] as one of the simplest extensions of the ECT. In [43] it has been demonstrated that, in $f(R)$ gravity, torsion can be a geometric source for the accelerated expansion. The equivalence between a Brans-Dicke theory with the Brans-Dicke parameter $\omega_0 = -3/2$ and the Palatini $f(R)$ gravity with torsion has been demonstrated in [44]. $f(T)$ gravity has been constructed [46–48] as the extension of the "teleparallel" equivalent of GR [49], which uses the Weitzenböck connection that has no curvature but only torsion.

At present, the considerably increased precision of measurements in the modern observational cosmology stipulated its essential progress. In this connection, the exact cosmological solutions, which makes it possible to elucidate the detailed picture of an evolution of models, are of great interest. It is well known that in GR the exact solutions of the Friedmann-Robertson-Walker (FRW) cosmological models constitute a basis for comparison of theoretical predictions with observations. On the other hand, the problem of the cosmological perturbations may be correct, if it is constructed against a background of the exact solution.

Other problems of the cosmology, singularities, horizons, and so forth, remain no less acute problems. The main aim of this paper is the investigation of the possibility of the solution of some problems of the modern cosmology in the framework of the ECT. Furthermore, the exact integration of the ECT equations for the different combination of sources and the comparative analysis of the corresponding cosmological models are of interest in their own right, since they allows to elucidate the role of the sources of the gravitational field in cosmology.

In the framework of the ECT with a nonminimally coupled material scalar field that has polynomial potentials of the fourth degree, we here study isotropic spatially flat cosmologies in which stiff fluid is taken into account. The motivation to investigate a nonminimally coupled scalar field is based on the results obtained within the framework of the torsionless theories of gravity in connection with the inflationary cosmology (see, e.g., [50–54]) and DE model building [55–57]. On the other hand, the consideration of this version of the ECT in comparison to $f(R)$ gravity with torsion and $f(T)$ gravity is motivated by the fact that, in my opinion, the ECT requires a further study. It is relevant to remark here that, asit is well known, in the literature there exist a limited number of the exact cosmological solutions in the framework of the ECT.

The interest in a scalar field potential $V(\Phi)$ in relativistic theories of gravity has been aroused by a number of circumstances: its role in cosmology with a time variable cosmological constant [58] and in quantum cosmology [59]; models with $V(\Phi)$ arise in alternative theories of gravity [60, 61] and supergravity [62]; a scalar potential governs an inflation [63] and is actively used in theories of dark matter (DM) and dark energy (DE) [7–10]. It is well known from GR that a period of accelerated expansion requires $V(\Phi) > 0$. In the paper we will consider cosmological models with $V(\Phi) > 0$ and $V(\Phi) < 0$. The reasons to study cosmology with negative potentials were considered in [64].

A stiff cosmological fluid, with pressure equal to the energy density, can be described by a massless scalar field, which is predicted by the string theory. On the other hand, the stiff fluid is an important component because, at early times, it could describe the shear dominated phase of a possible initial anisotropic scenario and is dominating the remaining components of the model [65]. Of no little interest is the circumstance that this fluid leads to integrability of the Einstein-Cartan equations.

In [66], the flat Friedmann universes filled by radiation, stiff fluid and a nonminimally coupled ghost scalar field with polynomial potentials of the fourth degree $V(\Phi)$ have been investigated in the framework of the ECT. Exact particular solutions have been obtained and analyzed. The role of sources in the evolution of models has been elucidated.

The exact general solution of an analogous problem for models containing only a nonminimally coupled scalar field with an arbitrary coupling constant ξ has been obtained in [12]. An analysis of the solutions for the material scalar field has shown the following.

(i) For $\xi > 0$ the solution exists for $\xi \geq 1/6$, $\Phi^2 < (\kappa\xi)^{-1/2}$, where κ is Einstein's constant only and describes a countable number of nonsingular cosmological models with de Sitter asymptotics for the late times.

(ii) For $\xi < 0$ the five types of singular models exist: for the first model the scale factor $a(t)$ as $t \to \infty$ grows by the law $a \sim t^p$ $(p < 1)$; for the second model $a(t)$ asymptotically tends to a finite value; other three models expand from an initial singularity, reach the maximum, and then begin to contract to a final singularity.

(iii) There are the specific values of the parameter ξ: $1/6, -1/6$, and $-3/2$. The specific feature of the value $\xi = 1/6$ consists in the behavior of the scale factor, when $(a - a_{\min})/a_{\min} \ll 1$: $a \simeq a_{\min}(1 + y)$, where $y \sim t^3$ for $\xi = 1/6$, while $y \sim t^2$ for $\xi > 1/6$. The value $\xi = -1/6$ is singled out as only in this case there exists the singular expanding model, which has the asymptotic $a|_{t\to+\infty} \simeq$ const. From three types of the recollapsing models with $\xi < -1/6$ only for $\xi = -3/2$ we have the following behavior of the scale factor: $a|_{t\to+\infty} \sim e^{-\beta t}$, while for others ξ, we get

$$\begin{aligned} a|_{t\to+\infty} &\sim t^{(1-\sqrt{6|\xi|})/(3-\sqrt{6|\xi|})} \quad \text{for } -\frac{3}{2} < \xi < -\frac{1}{6}, \\ a|_{t\to+t_0} &\sim (t_0 - t)^{(1-\sqrt{6|\xi|})/(3-\sqrt{6|\xi|})} \quad \text{for } \xi < -\frac{3}{2}. \end{aligned} \tag{1}$$

The exact general solutions for spatially flat FRW cosmologies with a nonminimally coupled scalar field that has polynomial potentials of the fourth degree $V(\Phi)$ have been obtained in [13]. The main results for the material scalar field are as follows.

(a) For $\xi > 0$, $\Phi^2 < (\kappa\xi)^{-1/2}$, and $V(\Phi) > 0$, the solution describes bouncing models of four types with the accelerated expansion at late times by the laws $a \sim e^{Ht}$ and $a \sim t^{4/3}$. For $\Phi^2 \neq (\kappa\xi)^{-1/2}$, $V(\Phi) < 0$, the qualitative character of the model evolution does not change.

(b) For $\xi < 0$, for all Φ, $V(\Phi) > 0$, there are two types of collapsing models and one type of expanding models. For $\xi = -3/2$ there exists a unified model of DM and DE. The case with $V(\Phi) < 0$ describes the recollapsing models.

The paper is organized as follows. In the next section we present the model and corresponding field equations. In Section 3, we obtain exact general solution of the Einstein-Cartan equations for $\xi > 0$, $\Phi^2 < (\kappa\xi)^{-1/2}$. In Section 3.1 we consider the models with a scalar-torsion field and stiff fluid and discuss some interesting particular cases. In Section 3.2 we analyze the mixture of stiff fluid with a scalar-torsion field that has polynomial potentials of the fourth degree. We summarize our results in Section 4.

2. Field Equations

The Lagrangian Ł of the model is chosen in the form [13]

$$Ł = -\frac{R}{2\kappa} + \left(\frac{\alpha_s}{2}\right)\left[\Phi_{,k}\Phi^{,k} + \xi R\Phi^2\right] - V(\Phi) + Ł_{\mathrm{fl}}, \quad (2)$$

where $R(\Gamma)$ is the curvature scalar obtained from the full connection $\Gamma^k_{ij} = \{^{\,k}_{ij}\} + S^{k}_{ij\cdot} + S^{k}_{\cdot ij} + S^{k}_{\cdot ji}$; $\{^{\,k}_{ij}\}$ are the Christoffel symbols of the second kind; $S^{k}_{ij\cdot} = \Gamma^k_{[ij]}$ is the torsion tensor; $\kappa = 8\pi G$, where G is the Newtonian constant; $V(\Phi)$ is the potential of a scalar field; $Ł_{\mathrm{fl}}$ is the Lagrangian of stiff fluid; $\alpha_s = +1$ conforms to the material scalar field; $\alpha_s = -1$ corresponds to the ghost scalar field.

The metric g_{ik} has the signature $(-,-,-,+)$; the Riemann and Ricci tensors are defined as

$$R^m_{ijk\cdot} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{ip}\Gamma^p_{jk} - \Gamma^m_{jp}\Gamma^p_{ik} \quad (3)$$

and $R_{jk} = R^i_{ijk\cdot}$. We should note that in the framework of ECT, a scalar field nonminimally coupled to gravity gives rise to torsion, even though the scalar field has zero spin. It follows from (2) that the torsion can interact with a scalar field only through its trace: $S_i = S^k_{ik\cdot}$ [67]. Hence, the curvature scalar $R(\Gamma) = g^{jk}R_{jk}$ can be presented in the form [67]

$$R(\Gamma) = R(\{\}) + 4\nabla_k S^k - \left(\frac{8}{3}\right) S_k S^k, \quad (4)$$

where $R(\{\})$ is the Riemannian part of the curvature built from the Christoffel symbols; ∇_k is the covariant derivative of the Riemannian space.

One can note that the Lagrangian (2) is a covariant generalization of its counterpart in GR, and, for $\xi = 1/6$, we obtain the so-called conformal coupling (for $V(\Phi) = 0$ in the torsionless theory). As shown in [67], when $\alpha_s = -1$, $\xi = -1/6$, $V(\Phi) = -(m^2/2)\Phi^2$, the scalar field corresponding to the Lagrangian (2) is the axion field in GR. In cosmology the axion field is a cold dark matter candidate (see, e.g., [67, 68] and references therein).

Varying the action with the Lagrangian (2) in g_{ij}, S_k, and Φ, we obtain the following set of equations for the gravitational fields and matter:

$$G_{ij}(\{\}) = \kappa\left(T^s_{ij} + T^{\mathrm{fl}}_{ij}\right) + \Lambda_{ij}, \quad (5)$$

$$S^k = \frac{3}{2}\xi\Psi\Phi\Phi'^{k}, \quad (6)$$

$$\Box\Phi - \xi\Phi R(\Gamma) + \alpha_s V' = 0, \quad (7)$$

where

$$\begin{aligned} T^s_{ij} = \alpha_s \Big\{ &\Phi_{,i}\Phi_{,j} - \frac{1}{2}\left[\Phi_{,m}\Phi^{,m} + \xi R(\{\})\Phi^2 - 2\alpha_s V(\Phi)\right] g_{ij} \\ &+ \xi\left[-4S_{(i}\nabla_{j)} + 2g_{ij}S^n\nabla_n - \nabla_i\nabla_j + g_{ij}\Box \right. \\ &\left. + R_{ij}(\{\}) - \Lambda_{ij}\right]\Phi^2 \Big\}, \end{aligned} \quad (8)$$

$$T^{\mathrm{fl}}_{ij} = \left(\varepsilon_{\mathrm{fl}} + P_{\mathrm{fl}}\right)u_i u_j - P_{\mathrm{fl}} g_{ij}, \quad (9)$$

$$\Lambda_{ij} = \frac{8}{3}S_i S_j - \frac{4}{3}S_k S^k g_{ij}. \quad (10)$$

Here $\varepsilon_{\mathrm{fl}}$ and P_{fl} are the energy density and pressure of stiff fluid, respectively; $\Box$ is the d'Alembertian operator of the Riemannian space; u_i is the four-velocity ($u_i u^i = 1$); $\Psi = \kappa(\alpha_s - \kappa\xi\Phi^2)^{-1}$, and $V' = \partial V/\partial\Phi$.

It is not difficult to verify that the effective scalar-torsion energy-momentum tensor $T^{s(\mathrm{eff})}_{ij}$,

$$T^{s(\mathrm{eff})}_{ij} = T^s_{ij} + \kappa^{-1}\Lambda_{ij}, \quad (11)$$

and the stiff fluid energy-momentum tensor T^{fl}_{ij} are separately covariantly conserved since no explicit coupling is assumed between stiff fluid and Φ:

$$\nabla^j T^{s(\mathrm{eff})}_{ij} = \nabla^j T^{\mathrm{fl}}_{ij} = 0. \quad (12)$$

For spatially flat isotropic and homogeneous models with the metric

$$ds^2 = a^2(\eta)\left[-dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) + d\eta^2\right]. \quad (13)$$

Equations (5) and (7) take the form

$$\begin{aligned} 2\frac{a''}{a} - \frac{a'^2}{a^2} = \Psi\Big[&2\xi\Phi\Phi'' + 2\xi\frac{a'}{a}\Phi\Phi' \\ &+ \left(-\frac{1}{2} + 2\xi + 3\xi^2\Phi^2\Psi\right)\Phi'^2\Big] \\ &+ \alpha_s a^2\Psi\left[V(\Phi) - P_{\mathrm{fl}}\right], \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{a'^2}{a^2} = \Psi\Big[&\left(\frac{1}{6} - \xi^2\Phi^2\Psi\right)\Phi'^2 + 2\xi\frac{a'}{a}\Phi\Phi'\Big] \\ &+ \frac{\alpha_s}{3}a^2\Psi\left[V(\Phi) + \varepsilon_{\mathrm{fl}}\right], \end{aligned} \quad (15)$$

$$\begin{aligned} &\left(1 - 6\xi^2\Phi^2\Psi\right)\left(\Phi'' + 2\frac{a'}{a}\Phi'\right) + 6\xi\frac{a''}{a}\Phi \\ &+ \alpha_s a^2 V' - 6\frac{\alpha_s}{\kappa}\xi^2\Psi^2\Phi\Phi'^2 = 0, \end{aligned} \quad (16)$$

where the prime denotes differentiation with respect to η. Here it is relevant to remark that the use of angular coordinates in metric (13) makes it possible to investigate the horizon problem. The passage to the cosmic synchronous time t is fulfilled with the help of the expression

$$\int a(\eta)\,d\eta = \int dt. \tag{17}$$

For the stiff fluid we have

$$P_{\text{fl}} = \varepsilon_{\text{fl}} = C_{\text{fl}} a^{-6}, \tag{18}$$

where $C_{\text{fl}} > 0$ is a constant.

Adding (14) and (15) we get

$$\frac{a''}{a} = \Psi\left[\xi\Phi\Phi'' + 2\xi\frac{a'}{a}\Phi\Phi' + \frac{2}{3}\alpha_s a^2 V(\Phi) + \left(-\frac{1}{6} + \xi + \xi^2\Phi^2\Psi\right)\Phi'^2\right] + \frac{\alpha_s}{6}a^2\Psi\left(\varepsilon_{\text{fl}} - 3P_{\text{fl}}\right). \tag{19}$$

The substitution of (19) into (16) leads to

$$\Phi'' + 2\frac{a'}{a}\Phi' - \xi\Phi\Psi\Phi'^2 + \alpha_s a^2\left[V' + 4\xi\Phi\Psi V(\Phi)\right] + \alpha_s\xi a^2\Phi\Psi\left(\varepsilon_{\text{fl}} - 3P_{\text{fl}}\right) = 0. \tag{20}$$

From (20) for stiff fluid and the scalar field potential

$$V(\Phi) = C_2\left(\alpha_s - \kappa\xi\Phi^2\right)^2 \tag{21}$$

we obtain

$$\left|\Phi'\right|\left(\left|\alpha_s - \kappa\xi\Phi^2\right|\right)^{1/2} a^2(\eta) = \left(C_1 + 2\alpha_s\kappa\xi\ell C_{\text{fl}}\Phi^2\right)^{1/2}, \tag{22}$$

where C_1 and C_2 are integration constants; $\ell = \operatorname{sgn}(\alpha_s - \kappa\xi\Phi^2)$.

It must be noted here that the requirement of renormalization of the quantum theory of a scalar field results in the introduction of a nonminimal coupling and the potential in the form of (21) [69].

In this paper, we will restrict our discussion to the material scalar field ($\alpha_s = +1$). It follows from (22) that there exist solutions for the following restrictions:

$\xi < 0$, for all Φ and $\xi > 0$, $\Phi^2 \neq B^2$. It is not difficult to show that, in terms of physics, reasonable solutions for $a(t)$ and $\Phi(t)$ do not exist for $\xi < 0$, for all Φ and $\xi > 0$, $\Phi^2 > B^2$.

3. Exact Solution

The exact general solution to the Einstein-Cartan equations for $\xi > 0$, $\Phi^2 < B^2$ can be presented in the form of quadratures:

$$\begin{gathered} a(u) = \beta\cosh u\, F^{-1/2}, \qquad \Phi(u) = B\tanh u, \\ \int\frac{\beta\,du}{F^{3/2}\sqrt{C_1 + 2C_{\text{fl}}\tanh^2 u}} = \int dt, \\ \int\frac{F^{1/2}dF}{\sqrt{d_1F^3 + md_2}} = \int\frac{2n\,du}{\cosh u\sqrt{C_1 + 2C_{\text{fl}}\tanh^2 u}}, \end{gathered} \tag{23}$$

where

$$\beta = (\kappa\xi)^{1/4}, \qquad B = (\kappa\xi)^{-1/2}, \qquad d_1 = \frac{(C_1 + 2C_{\text{fl}})}{6\xi},$$
$$d_2 = \frac{\kappa}{3}C_2\beta^2, \qquad n = \pm 1, \qquad m = \begin{cases} +1, & \text{for } V(\Phi) > 0, \\ -1, & \text{for } V(\Phi) < 0. \end{cases} \tag{24}$$

An analysis has shown that, depending on the choice of the gravitational field sources, different types of models are possible.

3.1. "Scalar-Torsion Field" + "Stiff Fluid". Let us consider the models without the scalar field potential $V(\Phi)$. It is convenient to discuss separately the cases $C_1 = 0$ and $C_1 \neq 0$.

3.1.1. Case (C_1=0). From (23), it is possible to express $F = F(u)$ explicitly as

$$F = D_1\left(\left|\tanh\left(\frac{u}{2}\right)\right|\right)^{2n/\sqrt{6\xi}}, \tag{25}$$

where $D_1 > 0$ is an integration constant.

For $n = +1$ solution (23) describes bouncing models. The minima of the scale factor,

$$\begin{gathered} a_{\min} = \beta D_1^{-1/2}\cosh u_{\min}\left(\left|\tanh\left(\frac{u_{\min}}{2}\right)\right|\right)^{-1/\sqrt{6\xi}}, \\ \Phi_{\min} = B\tan u_{\min}, \end{gathered} \tag{26}$$

are defined from

$$\sqrt{6\xi}\sinh^2 u_{\min} = \cosh u_{\min}. \tag{27}$$

The asymptotic behavior of $a(t)$, when $(a - a_{\min})/a_{\min} \ll 1$, is

$$a = a_{\min}\left\{1 + \frac{2\sqrt{1 + 24\xi}\,C_{\text{fl}}D_1^3}{\beta^2\left(1 + \sqrt{1 + 24\xi}\right)}\tanh^2 u_{\min} \times\left(\left|\tanh\left(\frac{u_{\min}}{2}\right)\right|\right)^{6/\sqrt{6\xi}} t^2\right\}. \tag{28}$$

For $u \in (0, \infty)$ solution (23) has the following asymptotics:

$$\begin{gathered} a|_{t\to-\infty} \sim (-t)^{1/3}, \qquad \Phi|_{t\to-\infty} \sim (-t)^{-\sqrt{6\xi}/3}, \\ a|_{t\to+\infty} \sim e^{H_1 t}, \qquad \Phi|_{t\to+\infty} \simeq B, \end{gathered} \tag{29}$$

where $H_1 = \beta^{-1}D_1^{3/2}(2C_{\text{fl}})^{1/2}$.

It should be observed here that we cannot express $a_{\min}$ and $u_{\min}$ in terms of the present values of the cosmographic parameters $H_0, q_0,$ and j_0, since three unknown values ξ, D_1, and C_{fl} define the Hubble parameter H_1.

It is easy to see that Hubble's parameter H_1 can take large values if $\xi \ll 1$. Since the models are considered in the framework of a classical theory of gravity, they will be physically admissible provided $\varepsilon < \varepsilon_{\rm pl}$, where ε is the total energy density of the scalar-torsion field and stiff fluid and $\varepsilon_{\rm pl}$ is the Planck energy density. Consequently, the following restriction on ξ is valid for $\xi \ll 1$:

$$\xi > 36\kappa^{-3}\varepsilon_{\rm pl}^{-1} D_1^6 C_{\rm fl}^2. \tag{30}$$

It is interesting to observe that the models (29) are free from the particle horizon; they are nonsingular and admit the late-time accelerating expansion.

For $u \in (-\infty, 0)$ the models with the reverse asymptotics exist:

$$\begin{aligned} a|_{t\to-\infty} &\sim e^{-H_1 t}, & \Phi|_{t\to-\infty} &\simeq -B, \\ a|_{t\to+\infty} &\sim t^{1/3}, & \Phi|_{t\to+\infty} &\sim -t^{-\sqrt{6\xi}/3}. \end{aligned} \tag{31}$$

It follows from (31) that at late times the scalar field disappears with time, and the behavior of the scale factor corresponds to the decelerated expansion.

It is easy to verify that models (29) and (31) are regular due to the violation of the strong energy condition with a scalar-torsion field. It is not difficult to show that, for minima of the scale factor and for the de Sitter-like asymptotics, the contribution of the scalar-torsion field dominates, while for $a \sim t^{1/3}$ the contribution of the stiff fluid dominates.

As pointed out in [70, 71], since the current torsion is extremely small, only those solutions should be considered as physical solutions towards current epoch, in which torsion disappears with time. In this connection, it should be noted that the square of the trace of torsion $S^2 = S_k S^k$ has the following asymptotics:

(1) stiff fluid dominated era

$$S^2|_{t\to\mp\infty} \sim (\mp t)^{-2(3+2\sqrt{6\xi})/3} \longrightarrow 0, \tag{32}$$

(2) de Sitter regime

$$S^2|_{t\to\mp\infty} \simeq \left(\frac{9}{4}\right) H_1^2 = \text{const}. \tag{33}$$

It follows from (32) that for the stiff fluid dominated era the torsion disappears with time. The formula (33) shows that for the de Sitter regime the torsion asymptotically tends to a finite value. For the current epoch, provided that $H_1 = H_0$, where H_0 is the present-day value of the Hubble's parameter, we have the following in natural units ($\hbar = c = 1$):

$$S^2|_{t\to+\infty} \sim 10^{-84}\ GeV^2. \tag{34}$$

That is, the current torsion is very small.

For minima of the scale factor the square of the trace of torsion is

$$S^2 = \left(\frac{9}{4}\right) H_1^2 \tanh^4 u_{\min}\left(\left|\tanh\left(\frac{u_{\min}}{2}\right)\right|\right)^{6/\sqrt{6\xi}} = \text{const}. \tag{35}$$

That is, the torsion is finite.

It must be noted that, for $\sqrt{6\xi} = 3$, the solution may be written in a parametric form

$$\begin{aligned} a(t) &= \beta D_1^{-1/2} x^{-1/3}\sqrt{1+x^2}\left(1+\sqrt{1+x^2}\right)^{1/3}, \\ \Phi(t) &= B\frac{n_1 x}{\sqrt{1+x^2}}, \\ S^2(t) &= \frac{9H_1^2 x^6}{4(1+x^2)^2\left(1+\sqrt{1+x^2}\right)^2}, \\ \ln\left|x+\sqrt{1+x^2}\right| &- x^{-1}\left(1+\sqrt{1+x^2}\right) = n_1 H_1 t, \end{aligned} \tag{36}$$

where $x \in (0, \infty)$, $n_1 = +1$ for $u \in (0, \infty)$ and $n_1 = -1$ for $u \in (-\infty, 0)$.

For $n = -1$, $u \in (0, \infty)$, solution (23) corresponds to the singular expanding models with de Sitter asymptotics for the late times:

$$\begin{aligned} a|_{t\to-t_0} &\sim (t_0+t)^{1/3}, & \Phi|_{t\to-t_0} &\sim (t_0+t)^{\sqrt{6\xi}/3}, \\ a|_{t\to+\infty} &\sim e^{H_1 t}, & \Phi|_{t\to+\infty} &\simeq B, \end{aligned} \tag{37}$$

while for $u \in (-\infty, 0)$, this solution conforms to the models with the reverse asymptotics, which describe the collapsing models.

It is easy to verify that for $t \to -t_0$ the contribution of the stiff fluid dominates. The behavior of S^2 for $t \to +\infty$ is described by the formula (33). It is interesting to observe that the square of the trace of torsion has the following asymptotics for $t \to -t_0$:

$$S^2|_{t\to-t_0} \sim (t_0+t)^{2(2\sqrt{6\xi}-3)/3} = \begin{cases} \to \infty, & \text{for } \sqrt{6\xi} < \dfrac{3}{2}, \\ = \text{const}, & \text{for } \sqrt{6\xi} = \dfrac{3}{2}, \\ \to 0, & \text{for } \sqrt{6\xi} > \dfrac{3}{2}. \end{cases} \tag{38}$$

That is, there is the specific value of the coupling constant: $\xi = 3/8$.

Thus, for the Big Bang singularity of the form $a|_{t\to-t_0} \sim (t_0+t)^{1/3}$: $a \to \infty$, $\varepsilon \to \infty$, $P \to \infty$, where P is the total pressure of the scalar-torsion field and stiff fluid, the torsion can be singular for $\sqrt{6\xi} < 3/2$, constant for $\sqrt{6\xi} = 3/2$, or tend to zero for $\sqrt{6\xi} > 3/2$. The solutions for the three types of S^2 behavior can be presented in a parametric form ($\Phi(u) = B\tanh u$):

(1) $\sqrt{6\xi} = 3/4$,

$$a(u) = \beta D_1^{-1/2} \cosh u \tanh^{4/3}\left(\frac{u}{2}\right),$$
$$S^2(u) = \left(\frac{9}{4}\right) H_1^2 \tanh^4 u \tanh^{-8}\left(\frac{u}{2}\right),$$
$$\left(5 + \tanh^2\left(\frac{u}{2}\right)\right)\left(2\cosh^2\left(\frac{u}{2}\right)\right)^{-1} + 4\ln\left(\cosh\left(\frac{u}{2}\right)\right) = 2H_1(t_0 + t), \tag{39}$$

where $t_0 = 5(4H_1)^{-1}$.

(2) $\sqrt{6\xi} = 3/2$,

$$a(u) = \beta D_1^{-1/2} \cosh u \tanh^{2/3}\left(\frac{u}{2}\right),$$
$$S^2(u) = \left(\frac{9}{4}\right) H_1^2 \tanh^4 u \tanh^{-4}\left(\frac{u}{2}\right), \tag{40}$$
$$\cosh^{-2}\left(\frac{u}{2}\right) + 4\ln\left(\cosh\left(\frac{u}{2}\right)\right) = 2H_1(t_0 + t),$$

where $t_0 = (2H_1)^{-1}$.

(3) $\sqrt{6\xi} = 3$,

$$a(u) = \beta D_1^{-1/2} \cosh u \left(\left|\tanh\left(\frac{u}{2}\right)\right|\right)^{1/3},$$
$$S^2(u) = \left(\frac{9}{4}\right) H_1^2 \tanh^4 u \tanh^{-2}\left(\frac{u}{2}\right), \tag{41}$$
$$u - \tanh\left(\frac{u}{2}\right) = 2H_1(t_0 + t),$$

where $t_0 = 0$.

3.1.2. Case ($C_1 \neq 0$). An analysis has shown that for $C_1 > 0$ the solution exists only for $n = -1$ and describes nonsingular expanding models with the asymptotics:

$$a|_{t\to 0} \simeq a_{01}\left(1 + \beta^{-1} D_1^{3/2}\sqrt{\frac{C_1 + 2C_{\text{fl}}}{6\xi}}\, t\right),$$
$$\Phi|_{t\to 0} \sim t, \qquad a|_{t\to+\infty} \sim e^{H_2 t}, \qquad \Phi|_{t\to+\infty} \simeq B, \tag{42}$$

where $a_{01} = a(t = 0) = \beta D_1^{-1/2}$, $H_2 = \beta^{-1} D_1^{3/2}(C_1 + 2C_{\text{fl}})^{1/2}$.

It is not difficult to show that for $C_1 < 0$, we have the same qualitative picture of the evolution of models as for $C_1 > 0$.

3.2. "Scalar-Torsion Field" + "$V(\Phi)$" + "Stiff Fluid". Let us consider the models for which all sources of gravitational field from the Lagrangian (2) are taken into account.

3.2.1. Case (C_1=0). From (23) we find the expression for $F(u)$,

$$F^{3/2} = \left(W^2 - md_2\right)\left(2d_1^{1/2} W\right)^{-1}, \tag{43}$$

where $W = D_2(|\tanh(u/2)|)^{3n/\sqrt{6\xi}}$; $D_2 > 0$ is an integration constant.

An analysis has shown that this solution with $V(\Phi) < 0$, for all n and $V(\Phi) > 0$, $n = -1$, and $D_2^2 > d_2$ for $u \in (0, \infty)$ describes singular expanding models with the de Sitter asymptotic at late times:

$$a|_{t\to 0} \sim t^{1/3}, \qquad \Phi|_{t\to 0} \sim t^{\sqrt{6\xi}/3}, \qquad S^2|_{t\to 0} \sim t^{2(2\sqrt{6\xi}-3)/3},$$
$$a|_{t\to\infty} \sim e^{H_3 t}, \qquad \Phi|_{t\to\infty} \simeq B, \qquad S^2|_{t\to\infty} \simeq \left(\frac{9}{4}\right) H_3^2, \tag{44}$$

where $H_3 = (6\xi)^{1/2}(2\beta D_2)^{-1}(D_2^2 - md_2)$. Note that for $u \in (-\infty, 0)$ the reverse asymptotic behavior takes place.

For $V(\Phi) < 0$, $n = +1$, $D_2^2 = d_2$, and $\sqrt{6\xi} = 3$, the solution can be expressed in elementary functions:

$$a(t) = A_1 \cosh \gamma t \left(\left|\tanh \gamma t\right|\right)^{1/3}, \qquad \Phi(t) = B \tanh \gamma t,$$
$$S^2(t) = \left(\frac{9}{4}\right) H_3^2 \tanh^2 \gamma t, \tag{45}$$

where $A_1 = \beta(3D_2)^{-1/3}(2C_{\text{fl}})^{1/6}$, $\gamma = 3\beta^{-1} D_2$.

It should be pointed out that, for $t \in (0, \infty)$, we have the expanding models, while, for $t \in (-\infty, 0)$, we get the collapsing models.

For $V(\Phi) > 0$, $n = -1$, $D_2^2 = d_2$, and $u \in (0, \infty)$, this solution corresponds to the singular expanding models with the power-law asymptotic at late times:

$$a|_{t\to 0} \sim t^{1/3}, \qquad \Phi|_{t\to 0} \sim t^{\sqrt{6\xi}/3},$$
$$S^2|_{t\to 0} \sim t^{2(2\sqrt{6\xi}-3)/3},$$
$$a|_{t\to+\infty} \sim t^{4/3}, \qquad \Phi|_{t\to+\infty} \simeq B, \tag{46}$$
$$S^2|_{t\to+\infty} \sim t^{-2} \longrightarrow 0.$$

It is easy to see that the case for $u \in (-\infty, 0)$ conforms to the models with the reverse asymptotics.

It should be observed here that the accelerated expansion of models with the power-law asymptotic at late stages of the cosmological evolution has its origin in "Scalar-torsion field" + "$V(\Phi)$."

For $\sqrt{6\xi} = 3$, the solution can be presented in elementary functions:

$$a(t) = A_2(|t|)^{1/3}\left(1 + \gamma^2 t^2\right)^{1/2}, \qquad \Phi(t) = \frac{\gamma B t}{\sqrt{1 + \gamma^2 t^2}},$$
$$S^2(t) = \frac{9\gamma^4 t^2}{4\left(1 + \gamma^2 t^2\right)^2}, \tag{47}$$

where $A_2 = \beta^{2/3}(2C_{\text{fl}})^{1/6}$. It follows from (47) that for $t \to 0$ the square of the trace of torsion has the following behavior: $S^2|_{t\to 0} \sim t^2 \to 0$.

The analytic solution in elementary functions for $\sqrt{6\xi} = 3/2$,

$$a(t) = A_3\{|t|(1+\gamma|t|)\}^{1/3}(1+2\gamma|t|)^{2/3},$$
$$\Phi(t) = 2B\{\gamma|t|(1+\gamma|t|)\}^{1/2}(1+2\gamma|t|)^{-1}, \quad (48)$$
$$S^2(t) = 54\xi\beta^{-2}D_2^2(1+2\gamma|t|)^{-2},$$

where $A_3 = \beta^{2/3}(8C_{\rm fl})^{1/6}$, corresponds to the following behavior of S^2 for $t=0$: $S^2 = 54\xi\beta^{-2}D_2^2 = \text{const}$.

Formulae (47) and (48) show that the analytic representation of the solution depends on the behavior of the square of the trace of torsion at early times.

The solution in a parametric form for $\sqrt{6\xi} = 3/4$,

$$a(u) = 2\beta\left(\frac{2C_{\rm fl}}{9d_2}\right)^{1/6}\cosh u \tanh^{4/3}\left(\frac{u}{2}\right) \times\left(1-\tanh^8\left(\frac{u}{2}\right)\right)^{-1/3},$$
$$S^2(u) = \left(\frac{81d_2}{64}\right)\left(\frac{2}{3\kappa}\right)^{1/2}\tanh^4 u\,\tanh^{-8}\left(\frac{u}{2}\right)\left(1-\tanh^8\left(\frac{u}{2}\right)\right)^2, \quad (49)$$
$$\Phi(u) = B\tanh u,$$
$$\cosh^2\left(\frac{u}{2}\right) - \arctan\left(\tanh^2\left(\frac{u}{2}\right)\right) = 1+2\gamma|t|,$$

conforms to the following behavior of S^2 for $t\to 0$: $S^2|_{t\to 0}\to\infty$.

For $V(\Phi) > 0$, $n=+1$, solution (23) exists for $u\in(-\infty,-D]$ and $u\in[D,+\infty)$, where D is a constant. With $u\in[D,+\infty)$ provided that

$$\lambda = D_2\left(\left|\tanh\left(\frac{D}{2}\right)\right|\right)^{3/\sqrt{6\xi}}, \qquad \lambda^2 = g_1 d_2, \quad g_1 > 1, \quad (50)$$

the evolution of models for $t\to\infty$ is identical to the models (44), while for $t\to 0$ we have

(a) $(g_1-1)(6\xi)^{1/2}\sinh^2 D > (g_1+1)\cosh D$,

$$a|_{t\to 0} \simeq a_{02} \times\left[1+\frac{(g_1-1)(6\xi)^{1/2}\sinh^2 D-(g_1+1)\cosh D}{2\beta g_1^{1/2}\cosh^2 D}d_2^{1/2}t\right], \quad (51)$$

where $a_{02} = a(t=0) = \beta(g_1-1)^{-1/3}(8g_1C_{\rm fl}/6\xi d_2)^{1/6}\cosh D$,

(b) $(g_1-1)(6\xi)^{1/2}\sinh^2 D = (g_1+1)\cosh D$,

$$a|_{t\to 0} \simeq a_{02}\left[1+d_2\chi_1\left(8g_1\beta^2\cosh^2 D\right)^{-1}t^2\right], \quad (52)$$

where $\chi_1 = (g_1-1)[(g_1+1)\cosh D-(g_1-1)](6\xi)^{1/2}+11g_1^2-2g_1-1$.

For (51) and (52), the expressions for Φ and S^2 have the form

$$\Phi(t=0) = B\tanh D, \qquad S^2(t=0) = \frac{27\xi d_2(g_1-1)^2}{8g_1\beta^2}. \quad (53)$$

It is easy to see that for $g_1 > 1$ the solution describes the expanding nonsingular models. Here it is relevant to remark that the accelerated expansion both at early stages and at late stages of the cosmological evolution is characteristic for case (b).

It is not difficult to verify that for $u\in(-\infty,-D]$ we get the models with the reverse asymptotics.

For $V(\Phi) > 0$, $n=+1$, and $g_1 = 1$, solution (23) describes bouncing models with the asymptotics

$$a|_{t\to-\infty}\sim e^{-H_4 t}, \qquad \Phi|_{t\to-\infty}\simeq B\tanh D,$$
$$S^2|_{t\to-\infty}\sim e^{6H_4 t}\longrightarrow 0,$$
$$a|_{t\to+\infty}\sim e^{H_3 t}, \qquad \Phi|_{t\to+\infty}\simeq B, \quad (54)$$
$$S^2|_{t\to+\infty}\simeq\left(\frac{9}{4}\right)H_3^2 = \text{const},$$

where $H_4 = \lambda(\beta\cosh D)^{-1}$.

The minima of the scale factor,

$$a_{\min} = \beta(g_2-1)^{-1/3}\left(\frac{4g_2C_{\rm fl}}{3\xi d_2}\right)^{1/6}\cosh u_{\min}, \quad (55)$$
$$\Phi_{\min} = B\tanh u_{\min},$$

where $g_2 = (|\tanh(u_{\min}/2)/\tanh(D/2)|)^{6/\sqrt{6\xi}} > 1$, are defined from

$$(g_2-1)(6\xi)^{1/2}\sinh^2 u_{\min} = (g_2+1)\cosh u_{\min}. \quad (56)$$

The asymptotic behavior of $a(t)$, when $(a-a_{\min})/a_{\min}\ll 1$, is

$$a = a_{\min}\left[1+d_2\chi_2\left(8g_2\beta^2\cosh^3 u_{\min}\right)^{-2}t^2\right], \quad (57)$$

where

$$\chi_2 = \left[12g_2^2-(6\xi)^{1/2}(g_2+1)^2\right]\cosh u_{\min} + (6\xi)^{1/2}\left(g_2^2-1\right), \quad (58)$$
$$12g_2^2 > (6\xi)^{1/2}(g_2+1)^2.$$

The value of S^2 for minima of the scale factor is

$$S^2_{\min} = 9d_2(g_2+1)^2\left(16g_2\beta^2\right)^{-1}\cosh^{-2}u_{\min}. \quad (59)$$

That is, the torsion is finite.

3.2.2. Case ($C_1 \neq 0$). An analysis has shown that with $C_1 \neq 0$ for all $V(\Phi)$ only the nonsingular expanding models of types (42) are possible.

4. Conclusion

In this paper, exact general solution for spatially flat isotropic and homogeneous cosmological models in ECT with stiff fluid and a nonminimally coupled material scalar field with a potential $V(\Phi)$ has been obtained and analyzed for an arbitrary positive coupling constant ξ.

In order to elucidate the role of the stiff fluid in the Einstein-Cartan cosmology, we should carry out a comparative analysis of the corresponding cosmological models obtained in this paper and those in [12] for "Scalar-torsion field" and in [13] for "Scalar-torsion field" + "$V(\Phi)$."

By comparing these models, we arrive at the conclusion that for the cosmological models with "Scalar-torsion field" + "Stiff fluid" and "Scalar-torsion field" + "$V(\Phi)$" + "Stiff fluid" the presence of stiff fluid leads to

(i) integrability of the Einstein-Cartan equations for $\xi > 0$, $\Phi^2 < B^2$ only,

(ii) the existence of singular expanding models with the power-law ($a \sim t^{1/3}$) evolution at early times and de Sitter evolution at late times,

(iii) the existence of nonsingular (not bouncing) expanding models with de Sitter asymptotic at late times,

(iv) the existence of the specific value of the coupling constant ξ: $\xi = 3/8$.

Furthermore, for the models with the "Scalar-torsion field" + "Stiff fluid" the presence of stiff fluid leads to the existence of asymmetrically bouncing models with the asymptotics $a|_{t\to-\infty} \sim (-t)^{1/3}$ and $a|_{t\to+\infty} \sim e^{H_1 t}$. These bouncing models are of interest from the viewpoint of modern cosmology: these models are free from the particle horizon, they are nonsingular, they admit the late-time accelerating expansion, and the current torsion for them is very small.

Additionally, stiff fluid in the mixture with the "Scalar-torsion field" + "$V(\Phi)$" results in a decreased number of types of bouncing models and the existence of singular expanding models with the power-law ($a \sim t^{1/3}$) evolution at early times and with the power-law ($a \sim t^{4/3}$) evolution at late times.

It should be observed that the existence of the power-law ($a \sim t^{1/3}$) evolution of the scale factor in the presence of stiff fluid is expected but not necessary. For example, for flat Friedmann universes filled by radiation, stiff fluid, and a nonminimally coupled ghost scalar field with polynomial potentials of the fourth degree [66], there is not such asymptotic for the scale factor, since the "kinetic term" of the scalar-torsion field compensates for the energy density of stiff fluid.

Comparing the models obtained in this work with the DE models that could admit similar behaviors we note the following. The behavior of the scale factor by the law $a \sim e^{Ht}$ at late times corresponds to ΛCDM model, while the behavior like $a \sim t^{4/3}$ at late times conforms to a quintessence model of DE [10, 72].

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