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**DESIGN METHOD OF PNEUMATIC CONVEYING CRITICAL MODE AT  
LOW PRESSURE DROP**

*The design method of low pressure drop pneumatic conveying critical mode parameters has been developed. An example of the design is given below.*

Solids pumping through pipes by means of air flow is widely used in different industries. Yet the reliability and efficiency of pneumatic conveying facilities depends largely upon the design values of the basic parameters chosen at the facilities engineering stage. One of the pneumatic conveying basic parameters is the air flow critical velocity at which particles falling on the horizontal pipe lower wall begins i. e. the speed at which the pipeline falling begins. It is obvious that for the assurance of the pipeline performance reliability the reliable pneumatic conveying critical mode prediction method is required.

By now a number of design dependences for pneumatic conveying [1, 2] critical velocity determination is known. However they are of empiric nature and limited by the experiment conditions. The limitation and in some cases unacceptably low accuracy degree of the dependences do not always meet the modern requirements for industrial pneumatic conveying systems engineering and development.

In the present work the attempt of developing a theoretically based and more or less accurate pneumatic conveying critical velocity design method is made. The aim is in determining the mass flow rate  $G_{W,K}$  and the mean velocity of the air flow, corresponding with the pneumatic conveying critical mode for the set mass flow rate  $G_S$ , density  $\rho_S$ , mean solid particles size  $d_S$ , drift diameter  $D$  and relative equivalent roughness  $\delta/D$  of the pipeline. It refers to the low pressure drop pneumatic conveying when the air compressibility can be neglected and the density is considered continuous along the pipe length.

In case of a steady and continuous pneumatic conveying mode the conditions of air and solids mass flow rates maintenance are met:

$$\rho_S S V_S F = G_S; \quad (1)$$

$$\rho_W (1 - S) V_W F = G_W, \quad (2)$$

where  $S$  – solids mean volume concentration throughout the pipe cross section;

$\rho_W$  – air density;

$V_S$  and  $V_W$  – mean actual velocities of solids and air motion;

$F$  – pipe cross section area. By the definition the actual velocities  $V_S$  and  $V_W$  are:

$$V_S = \frac{Q_S}{SF}; \quad (3)$$

$$V_W = \frac{Q_W}{(1 - S)F}, \quad (4)$$

where  $Q_S$  and  $Q_W$  – solids and air volume flow rates. If they are referred to the whole area  $F$ , we obtain solids and gas motion mean velocities  $U_S$  and  $U_W$ :

$$U_S = \frac{Q_S}{F}; \quad (5)$$

$$U_W = \frac{Q_W}{F}. \quad (6)$$

Comparing (3) and (4) with the respective expressions (5) and (6) we find

$$U_S = SV_S; \quad (7)$$

$$U_W = (1-S)V_W. \quad (8)$$

As  $Q_S + Q_W = Q$ , where  $Q$  – solids and air volume flow rate, the expressions (5) and (6) can be rewritten this way:

$$U_S = S_p U; \quad (9)$$

$$U_S = (1 - S_p) U, \quad (10)$$

where  $S_p = \frac{Q_S}{Q}$  – solids volume flow rate;

$U = \frac{Q}{F}$  – solids and air mixture mean velocity. Having excluding the velocity  $U$ , from the equalities (9) and (10) we obtain

$$U_S = \frac{S_p}{1 - S_p} U_W. \quad (11)$$

Taking into account the equalities (7) and (11) we will rewrite the formula (1) in the form

$$\rho_S \frac{S_p}{1 - S_p} U_W F = G_S.$$

Therefore we obtain

$$U_W = \frac{G_S}{\rho_S F} \cdot \frac{1 - S_p}{S_p}. \quad (12)$$

The formula (12) is valid for the air mean velocities  $U_W \geq U_{W,cr}$ , thus in case of the pneumatic conveying critical mode it takes the form

$$U_{W,cr} = \frac{G_S}{\rho_S F} \cdot \frac{1 - S_{p,cr}}{S_{p,cr}}, \quad (13)$$

where  $S_{p,cr}$  – solids volume flow rate in the pneumatic conveying critical mode.

Therefore for the critical velocity  $U_{W,cr}$  value determination by formula (13) it is necessary to know the value  $S_{v,cr}$ , depending on the  $S_{cr}$  concentration and the solid material specification. In case of low pressure drop when the air can be considered as an incompressible medium, the aerodynamic processes while pneumatic conveying shall be of a similar quality with the processes at hydraulic conveying. That is why for the value  $S_{p,cr}$ , determination the formula obtained resulting from pipeline hydraulic conveying researches is used [3]. It is as follows:

$$S_{p,cr} = S_{cr} \left[ 1 - \varphi(\text{Re}_S) \left( 1 - \frac{S_{cr}}{S_m} \right)^{2,16} \right]; \quad (14)$$

$$\varphi(\text{Re}_S) = 0,45 \left[ 1 + \text{signf} \cdot \text{th}(0,967|f|^{0,6}) \right]; \quad (15)$$

$$f = \lg \operatorname{Re}_S - 0,88. \quad (16)$$

Here  $S_{cr}$  – mean volume flow rate corresponding with the critical mode;

$S_m$  – solids limit possible volume concentration;

$\operatorname{sign} f$  –  $f$  value sign;

$\operatorname{Re}_S = \frac{W_S d_S}{\nu_w}$  – the Reynolds number for solids, where  $W_S$  free fall velocity of a solid unit

particle with the diameter  $d_S$  in the resting air;  $\nu_w$  – air kinematic viscosity.

Let us note that the formulas (14) – (16) are not empiric, but the approximation of the flow rate numerical calculation results obtained on the basis of averaged concentrations and velocities fields in turbulent suspension flows theoretical research. The formula is checked on the wide experimental material of flow rate measurement and is characterized as of a high accuracy degree.

As (14) comprises the concentration  $S_{cr}$ , which is an undetermined value, the system of equations (13) and (14) does not allow estimating the velocity  $U_{W,cr}$ , as there are two equations and three unknown values  $U_{W,cr}$ ,  $S_{p,cr}$  и  $S_{cr}$ . Therefore for the equation system (13) and (14) closing it is necessary to set up one more equation tying together the velocity  $U_{W,cr}$  with its key parameters. On the basis of it we will combine the equations (9) and (10) and obtain

$$U_S + U_W = U$$

or

$$U_W = U - U_S. \quad (17)$$

Substituting the value  $\frac{G_S}{\rho_S F}$ , instead of  $U_S$  we obtain

$$U_W = U - \frac{G_S}{\rho_S F}. \quad (18)$$

As the formula (18) is valid for all the velocities  $U_W \geq U_{W,cr}$  and  $U \geq U_{cr}$ , the equation is realized in the pneumatic conveying critical mode

$$U_{W,cr} = U_{cr} - \frac{G_S}{\rho_S F}, \quad (19)$$

where  $U_{cr}$  – air and solids mixture motion critical velocity. For  $U_{cr}$  value determination the method developed for hydraulic conveying and adopted for pneumatic conveying conditions [3] is used. The mentioned  $U_{cr}$  design method is proved quite well and ties together the value  $U_{cr}$  with the flow, solids and pipeline characteristics. Particularly it takes into account solids uneven distribution throughout the flow depth and the velocity field axial asymmetry typical for pneumatic conveying critical modes.

The obtained pneumatic conveying critical mode equation [3] is as follows

$$\frac{\rho_{0,K}}{\rho_W} \cdot \frac{\lambda_K}{(1 - \alpha_K) \omega_K^2} \cdot \frac{U_K^2}{2qD} = \frac{K_0 (\Delta_S - 1) \beta S_m h_K}{1 + \alpha_K}, \quad (20)$$

where  $\rho_{0,K}$  – mixture density at the upper pipe wall;

$\alpha_K$  – velocity field axial asymmetry parameter, determined as ratio of the value  $\Delta r$ , being the distance from the flow kinematic axis to the pipe geometric axis, to the pipe diameter  $D$ ;

$\lambda_K$  – hydraulic friction coefficient while the carrier medium motion in the pipe with the diameter  $D(1-\alpha_K)$ ;

$\omega_K$  – parameter, being the ratio of the maximum averaged velocity in the carrier medium flow to the maximum averaged velocity in the solids and carrier medium mixture flow at the same mean velocities of the flows;

$K_0$  – sliding friction coefficient of the solid material;

$\Delta_S = \frac{\rho_S}{\rho_W}$  – the ratio of solids and carrier medium density;

$\beta$  – dilatation coefficient;

$h_K$  – the ratio of the solids highly concentrated bottom layer to the pipe diameter  $D$ .

As pneumatic conveying is generally characterized as of small volume concentrations and large Reynolds numbers at which the hydraulic friction coefficient refers to square resistance area, let us suppose that  $\frac{\rho_{0,K}}{\rho_W} = 1$ ,  $\omega_K = 1$ , a  $\lambda_K$  depends upon the pipe inner wall relative roughness only and

$$\lambda_K = \frac{\lambda_W}{(1-\alpha_K)^{0,25}}, \quad (21)$$

where  $\lambda_W$  – hydraulic friction coefficient while air motion in the pipe with the diameter  $D$ , determined experimentally or by the Shifrinson formula

$$\lambda_W = 0,11 \left( \frac{\delta}{D} \right)^{0,25}.$$

Then it is supposed that for small volume concentrations the value  $h_K$  can be expressed as

$$h_K = \frac{S_K}{\beta S_m}.$$

Taking into account the abovementioned suppositions, the equation (20) takes the simplified form

$$\frac{\lambda_K}{1-\alpha_K} \cdot \frac{U_{cr}^2}{2qD} = \frac{K_0(\Delta_S-1)S_K}{1+\alpha_K}.$$

From here we obtain

$$U_{cr} = \sqrt{qD} \cdot \sqrt{\frac{2K_0(\Delta_S-1)S_K}{\lambda_K} \cdot \frac{1-\alpha_K}{1+\alpha_K}}. \quad (22)$$

Inserting the expression (22) into (19) instead of  $U_{cr}$  we find the critical velocity  $U_{W,cr}$  formula:

$$U_{W,cr} = \sqrt{qD} \cdot \sqrt{\frac{2K_0(\Delta_S-1)S_K}{\lambda_K} \cdot \frac{1-\alpha_K}{1+\alpha_K} - \frac{G_S}{\rho_S F}}. \quad (23)$$

The parameter  $\alpha_K$  comprised in (23) is determined according to the expression [3]

$$\alpha_K = 2,44 \sqrt{\frac{Fr_S}{\Delta_S-1}} \left( 0,25 + 0,244 \sqrt{\frac{Fr_S}{\Delta_S-1}} \right) th \left( 0,714 \sqrt{\frac{S_K}{S_m}} \right), \quad (24)$$

where  $Fr_S = \frac{W_S^2}{qd_S}$  – the Froude number for solids.

So, the equation for closing the equations system (13) and (14) is obtained. That is why the solution of the closed three equations system (13), (14) and (23) allows determining three parameters  $U_{W,cr}$ ,  $S_{\rho,cr}$  and  $S$ , characterizing the pneumatic conveying critical mode. This is the essence of the suggested pneumatic conveying design technique.

Solving the above mentioned equations system is realized by the graphical method. For this purpose  $(S_{cr})_1$ ,  $(S_{cr})_2$  ... concentration values are set and the corresponding values of flow rate concentrations  $(S_{\rho,cr})_1$ ,  $(S_{\rho,cr})_2$  ..., are determined by formula (14) after which the velocities  $(U_{W,cr})_1$ ,  $(U_{W,cr})_2$  ... are estimated in accordance with the formula (13). On the basis of the determined values  $(U_{W,cr})_1$ ,  $(U_{W,cr})_2$  ...  $U_{W,cr} = \varphi_1(S_{cr})$  function graphs are plotted.

Then for the set values of  $(S_{cr})_1$ ,  $(S_{cr})_2$  ... concentrations  $(U_{W,cr})_1$ ,  $(U_{W,cr})_2$  ..., values are estimated by formula (23), then  $U_{W,cr} = \varphi_2(S_{cr})$  function graph is plotted. The  $\varphi_1(S_{cr})$  and  $\varphi_2(S_{cr})$  curves intersection point abscissa gives the  $S_{cr}$ , concentration target value and the ordinate – the critical velocity  $U_{W,cr}$  value. On the basis of the velocity  $U_{W,cr}$  determined value the air mass flow rate  $G_{W,cr} = \rho_W U_{W,cr} F$ , required for solids pumping with the mass flow rate  $G_S$  through the pipe line with the diameter  $D$  in the pneumatic conveying critical mode, is determined.

As an example there is the air motion mean velocity  $U_W$  dependence of specific pressure loss  $\Delta P/L$  taken from [2] experimental curves in the figure 1. In the experiments the pipe diameter  $D = 0,1$  m, solids mean size  $d_s = 5$  mm, density  $\rho_s = 595$  kg/m<sup>3</sup>.

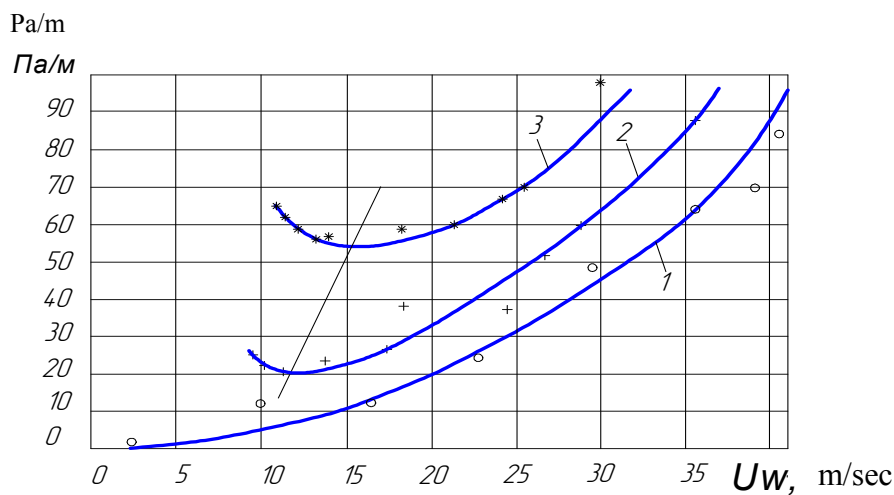


Figure 1 – The air motion mean velocity  $U_W$  dependence of specific pressure loss  $\Delta P/L$  taken from [2] experimental curves: 1 –  $G_S = 0$ ; 2 –  $G_S = 228$  kg/hour 3 –  $G_S = 380$  kg/hour

The curves 1, 2, 3 stand for mass flow rates  $G_S$ , equal to 0,228 and 380 kg/hour. The straight line crossing the curves 2 and 3 stands for the pneumatic conveying critical mode. According to the data  $U_{W,cr} = 11,7$  m/sec for  $G_S = 228$  kg/hour and  $U_{W,cr} = 15$  m/sec for  $G_S = 380$  kg/hour.

Critical mode design was carried out for each single mass flow rate  $G_S = 228$  kg/hour and  $G_S = 380$  kg/hour. At that it was taken into account that  $W_S = 5,1$  m/s,  $K_0 \approx 0,3$ ,  $\lambda_W = 0,01$ ,

$\rho_w = 1,2 \text{ kg/m}^3$ ,  $w = 0,15 \cdot 10^{-4} \text{ m}^2/\text{sec}$ . The estimation of  $\varphi_1(S_{cr})$  and  $\varphi_2(S_{cr})$  functions was carried out for the  $S_{cr}$ , values equal to 0,004; 0,006; 0,008; 0,01. The functions graphs are given in figure 2.

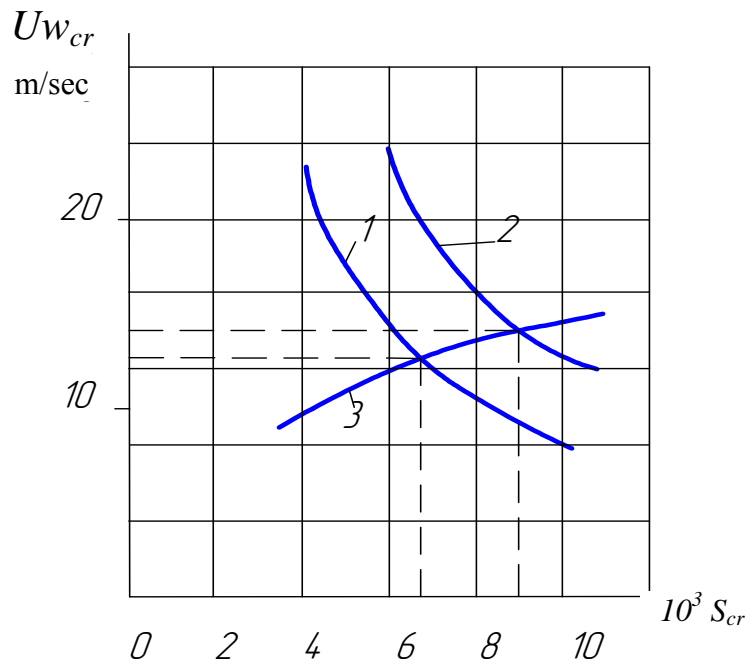


Figure 2 – For the critical modes design: 1 –  $\varphi_1(S_{cr})$  at  $G_S = 228 \text{ kg/hour}$  ;  
2 –  $\varphi_1(S_{cr})$  at  $G_S = 380 \text{ kg/hour}$  ; 3 –  $\varphi_2(S_{cr})$

The curves 1 and 2 refer to the functions  $\varphi_1(S_{cr})$  at  $G_S = 228 \text{ kg/hour}$  and  $G_S = 380 \text{ kg/hour}$  respectively and the curve 3 – to the function  $\varphi_2(S_{cr})$ . The design values of  $S_{cr}$  and  $U_{w,cr}$  were determined on the basis of the  $\varphi_1$  and  $\varphi_2$  curves intersection points. As a result it was obtained:  $S_{cr} = 0,0067$ ,  $U_{w,cr} = 12,2 \text{ m/sec}$ , for  $G_S = 228 \text{ kg/hour}$  and  $S_{cr} = 0,009$ ,  $U_{w,cr} = 14 \text{ m/sec}$ , for  $G_S = 380 \text{ kg/hour}$ . As we can see the critical velocities design values are almost completely the same as the experimental ones, proving the reliability of the developed pneumatic conveying critical mode design method. Now the task is in the method testing on the base of wide experimental material.

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