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Dedicated to the memory of
Professor Vitaliy Mikhaylovich Usenko

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TOPICAL SECTION IV

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CONTRIBUTED ABSTRACT

Algebras associated with labelled graphs

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The fundamental result in the finite automata theory is the Kleene's theorem, which states the equivalences between finite automata and regular languages. In this paper we introduce and study generally labelled graphs, directed graphs in which both vertices and transitions are labelled with elements of two arbitrary sets. As a main result of this paper we prove an analog of Kleene's theorem for these graphs and characterize their behaviour.

Let X and Y be two arbitrary sets. Generally labelled graph is a quintuple $\mathbf{G} = (G, I, F, \alpha, \beta)$ where $G = (V, E, s, t)$ is a multigraph with finite set of vertices V, set of transitions E and two functions $s, t : E \to V$, the source and target functions; I and F are subsets of V, called the set of initial and final states, respectively; $\alpha : V \to X$ is a mapping associating with each vertex its label; $\beta : E \to Y$ is a mapping associating with each transition its label.

To obtain a characterization of behavior of generally labelled graphs we introduce algebra $(Z, +, \cdot, *, o)$ with binary operations $+$ and $\cdot$, unary operation $*$ and constant o satisfying following properties.

1. $(Z, +, o)$ is a commutative idempotent monoid, $(Z, \cdot)$ is a semigroup, $\cdot$ distributes over $+$ on both sides, o is a two-sided annihilator for $\cdot$

2. Each sequence of elements of the set Z has a supremum with respect to the partial order $\leq$. Order $\leq$ refers to the natural order on Z: $a \leq b$ if an only if $a + b = b, a, b \in Z$.

3. For each sequence $\{x_n\}_{n \in N}$ of elements of the set Z an infinitary summation operator $\sum x_n$ gives the supremum of this sequence. Operator $\sum$ satisfied infinitary associativity, commutativity, idempotence and distributivity laws.

4. a^* is defined by supremum of the sequence $\{a, a \cdot a, a \cdot a \cdot a, ...\}$: $a^* = \sum_{n \geq 1} a^n$ where $a^1 = a, a^{n+1} = a^n \cdot a, n = 1, 2, ...$

Special case of an introduced algebra $(Z, +, \cdot, *, o)$ is an idempotent semiring.

An analog of Kleene's theorem for generally labelled graphs and algebra $(Z, +, \cdot, *, o)$ was proved. We define the conditions for the class of all possible behaviour of generally labelled graphs with given parameters to be representable by terms of some algebra. It has been shown that this algebra is not necessary unique.

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