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Combination resonances and their bifurcations in the nonlinear vibromachines with a polynomial characteristic of restoring force and periodic excitation

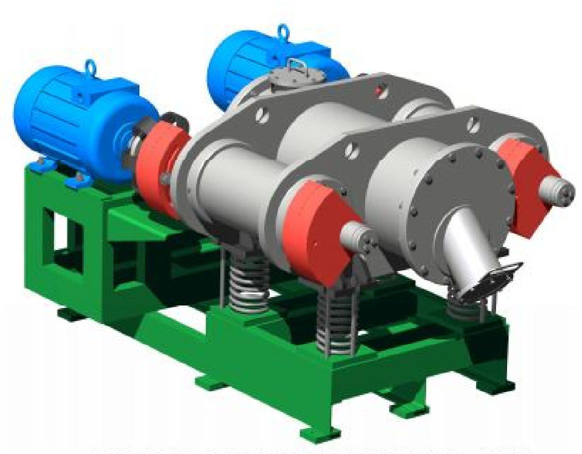
Authors:
Valeriy Belovodskiy <e-mail: belovodskiy@cs.dgtu.donetsk.ua>,
Maksym Sukhorukov <e-mail: max.sukhorukov@gmail.com>

Ukraine,
Donetsk National Technical University,
Dept. of "Computer Monitoring Systems"

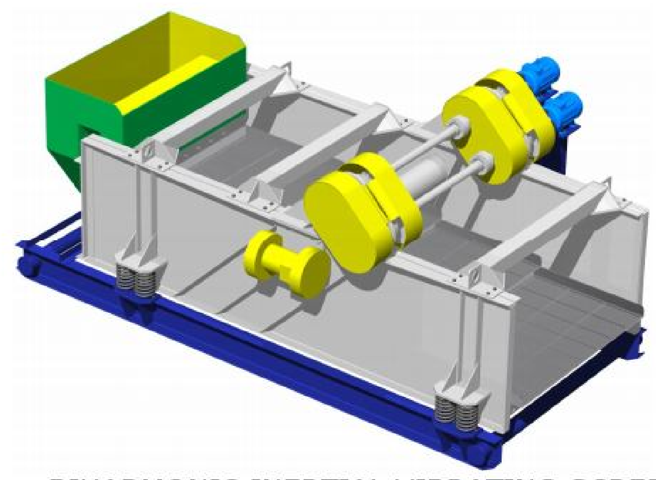
The issue of research

Polyharmonic, in particular, biharmonic vibrations are effective and are used in various industrial processes, such as transportation and screening. Traditionally in vibromachines they are formed with the help of biharmonic exciters and development of such machines is currently ongoing.

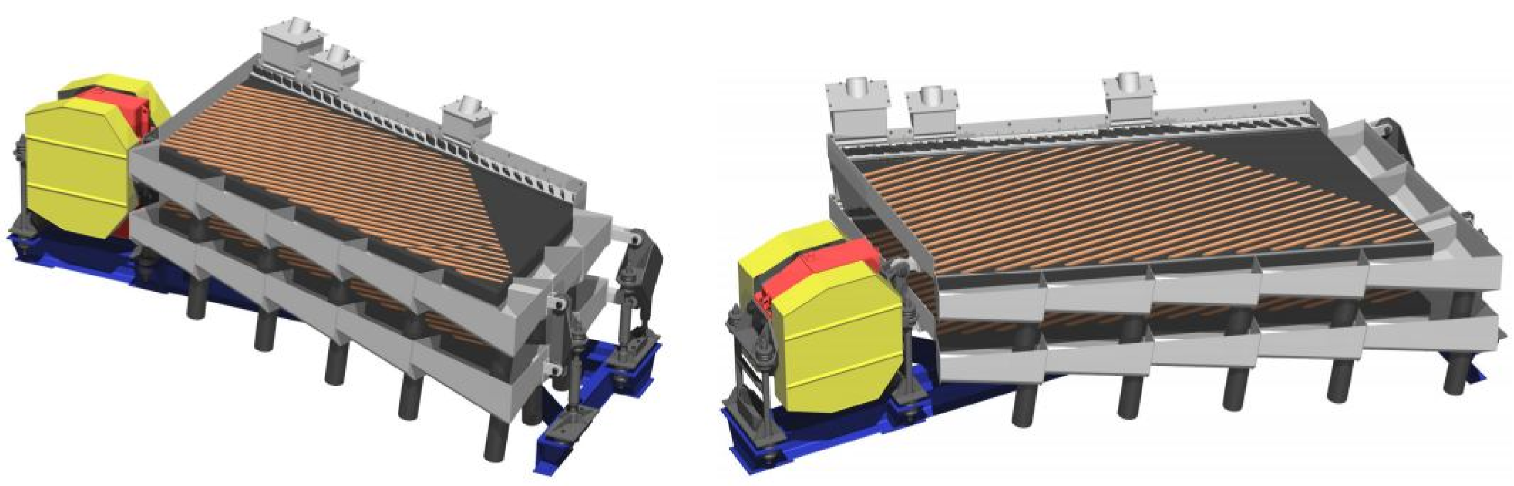
Examples of such machines are shown in the figure.



VIBRATORY BIHARMONIC MILL



BIHARMONIC INERTIAL VIBRATING SCREEN

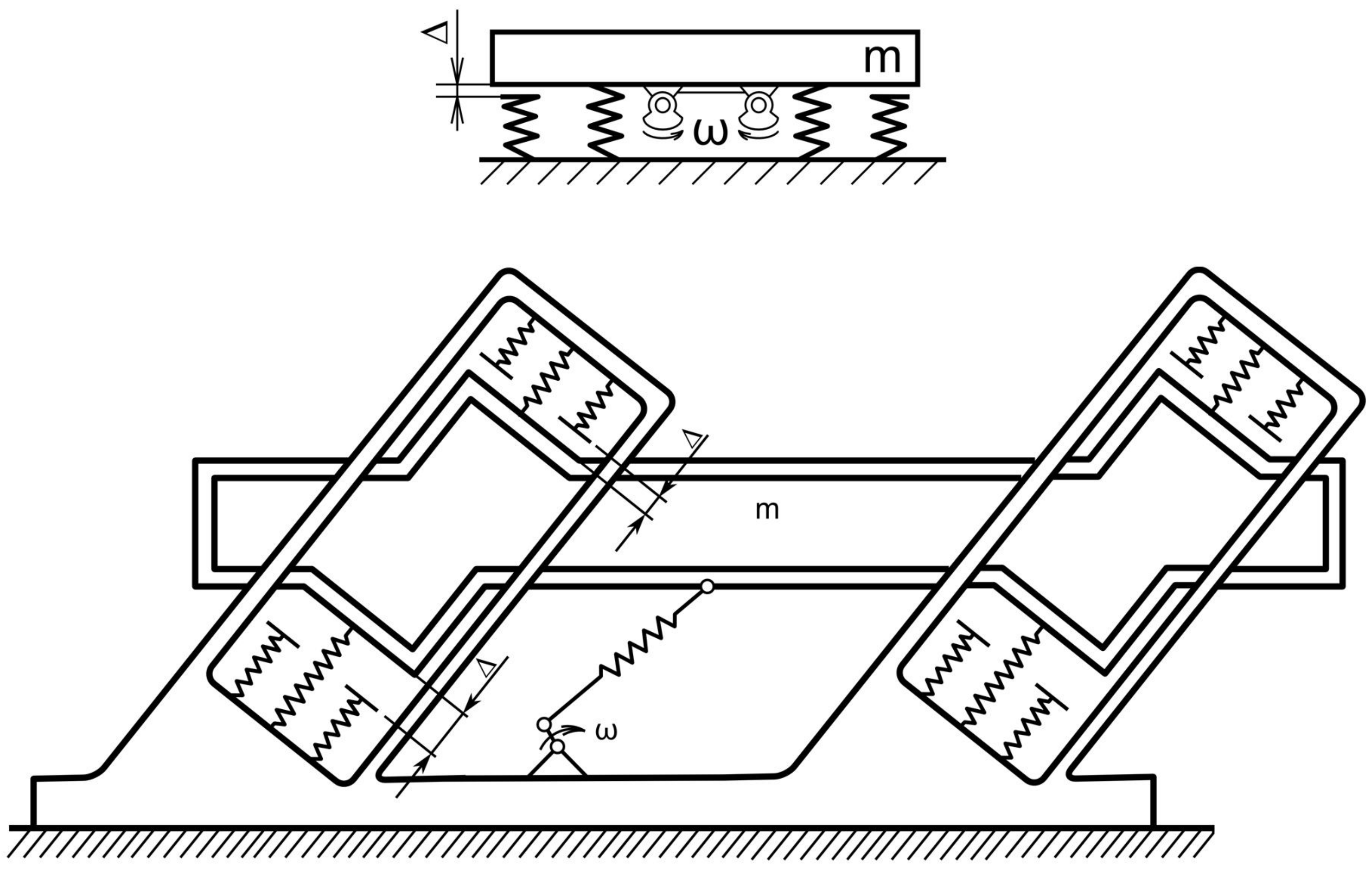


REFERENCE CONCENTRATION TABLE OF BIHARMONIC TYPE

From theory of nonlinear dynamical systems it is well known that the realization of combinational resonances even in the presence of harmonic excitation gives the opportunity to form oscillations with pronounced polyharmonic nature. These features were used in 80's in vibromachines with piecewise linear elastic ties, which were formed by using additional buffer elements.

Some schemes of such machines are shown in the figure.

Schemes of vibromachines with piecewise linear elastic ties (80's years)



The issue of research

However, this method of formation of the nonlinear elastic characteristic often evoked criticism of producers, as construction and maintenance of such vibromachines became more complicated.

The work deals with mathematical models of vibromachines with a smooth, namely, – polynomial elastic ties. The main purposes of investigation are: to refresh information concerning the possibilities of combination resonances in the formation of polyharmonic vibrations, to choose the most interesting ones for further implementation, to touch issues of their existence and stability for real level of resistance.

Research methods

In work this group of problems is solved with the original software that implements:

- constructing of bifurcation diagrams, including the amplitude- and phase-frequency characteristics,
- analysis of the spectral and phase composition of the oscillations,
- constructing of basins of attraction of periodic regimes

of dynamical systems, which are described by systems of ordinary differential equations with periodic excitation.

The model under consideration

$$\frac{d^2\xi}{d\tau^2} + \mu\,\omega_0\,(1 + \beta\,\xi + \gamma\,\xi^2)\frac{d\xi}{d\tau} + (1 + \beta\,\xi + \gamma\,\xi^2)\,\xi = P\cos\eta\,\tau, \qquad (1)$$

where

$\xi = x/\Delta$,

x – the displacement of the working body, $\Delta = 10^{-3}\,m$,

μ – the damping coefficient,

ω_0 – the eigenfrequency of linear system,

β – the asymmetry of characteristic,

γ – the degree of nonlinearity,

P – the amplitude of the external force, $\eta = \omega/\omega_0$,

ω – the frequency of excitation, $\tau = \omega_0\,t$.

1. The program for constructing the bifurcation diagrams in relation to systems of this class is based on the implementation of the harmonic balance method. The solution is sought in the complex form of Fourier series:

$$\xi(\tau) = \sum_{n=-N}^{N} c_n e^{i n \eta \tau}, \qquad (2)$$

where N is the number of harmonics taken into account.

After substituting (2) into (1) and comparing the corresponding coefficients, we obtain a finite system of equations for finding coefficients of expansion:

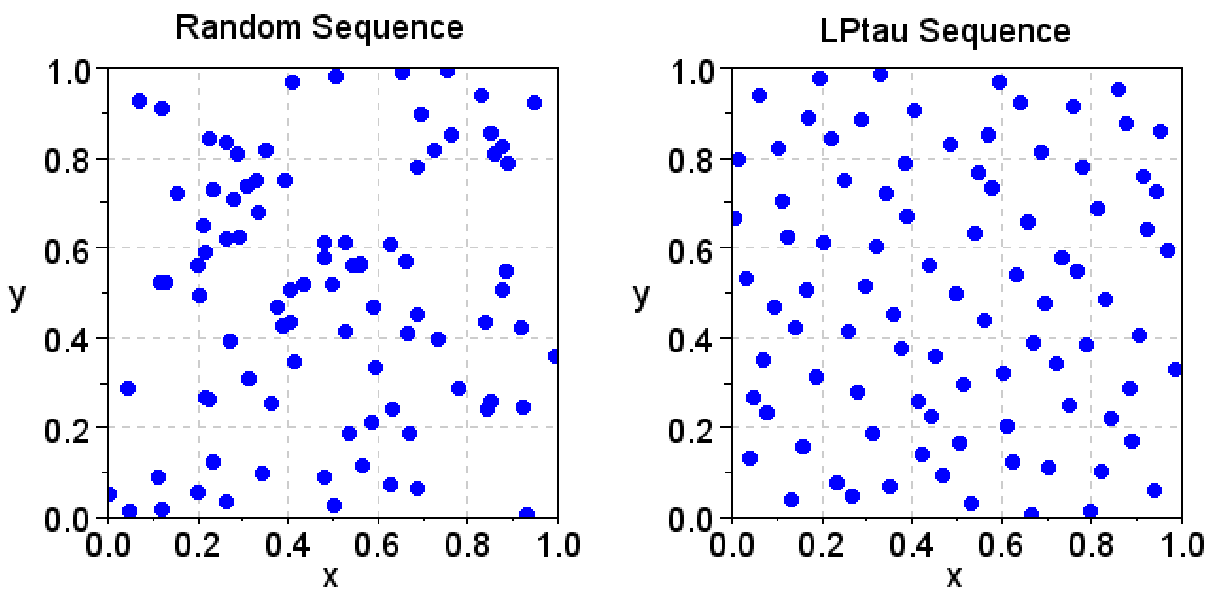
$$\begin{aligned} &(1 + i\mu\omega_0 n \frac{1}{q}\eta - n^2 \frac{1}{q^2}\eta^2) c_n + \\ &+ \beta \sum_{k=-N}^{N} c_k c_{n-k} (1 + i\mu\omega_0 (n-k)\frac{1}{q}\eta) + \\ &+ \gamma \sum_{k=-N}^{N} \sum_{m=-N}^{N} c_k c_m c_{n-k-m} (1 + i\mu\omega_0 (n-k-m)\frac{1}{q}\eta) = \\ &= \begin{cases} P/2, & n = \pm q \\ 0, & n \neq \pm q \end{cases} \end{aligned} \qquad (3)$$

where $n,\ n-k,\ n-k-m \in [-N, N]$.

If the control parameter $q = 1$, then the program analyzes the modes of period T, i.e. principal and superharmonic modes, if $q = 2, 3, \ldots$ — subharmonic modes of $1{:}q$.

The subsequent solution of the resulting system under sequential changing of one or more parameters allows to construct a bifurcation diagram.

Bifurcation points of continuous curves are defined by controlling the change of sign of the Jacobian of the system (3) and by the following scan of space of initial conditions with help of LPtau sequences.



Finding isolated branches of bifurcation curves is based on the scanning of area of the initial values in a user-defined range of changing of the varied parameters.

When constructing the bifurcation diagrams it is assumed that the trigonometric form of a Fourier series is the following:

$$\sum_{k=0}^{N} A_k \cos(k\omega t - \varphi_k),$$

where A_k, φ_k – are the amplitude and initial phase of k-th harmonic, $\varphi_k \in [-\pi, \pi)$,

$$A_k = 2\sqrt{c_k c_{-k}},$$

$$\varphi_k = \arccos \frac{c_k + c_{-k}}{2\sqrt{c_k c_{-k}}} \quad \text{and}$$

$$\varphi_k = -\arccos \frac{c_k + c_{-k}}{2\sqrt{c_k c_{-k}}}, \text{ if } \operatorname{Im} c_{-k} < 0.$$

2. The program of the spectral and phase analysis is based on the numerical solution of the Cauchy problem for the original equation, finding its steady solutions and subsequent numerical expansion into finite Fourier series.

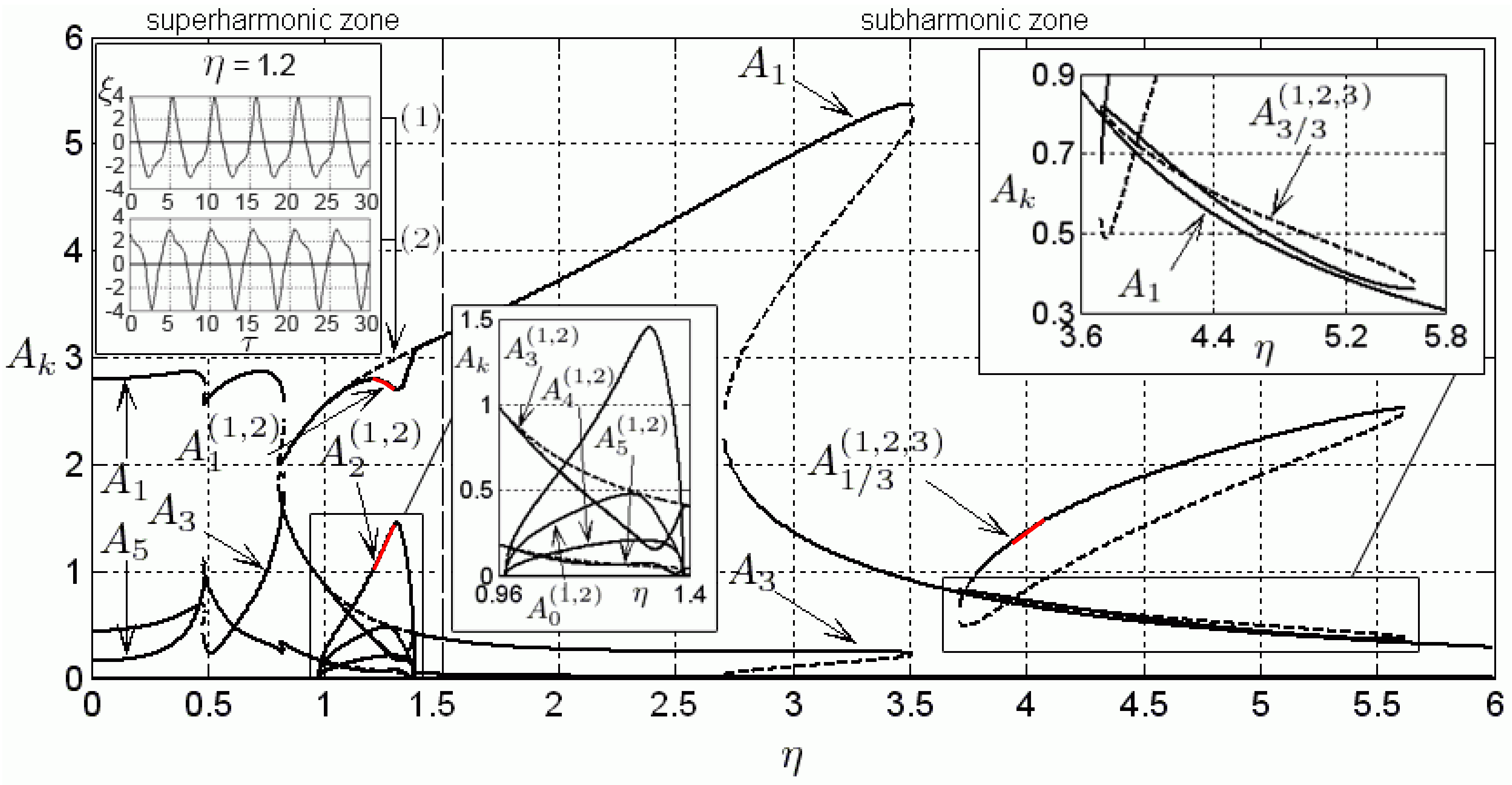
3. The program of constructing of basins of attraction is based on a scan of the initial conditions and realizes the Poincare map. Some measures have been done that accelerate the algorithm and allow the finding of basins of attraction of all possible stable solutions without defining their corresponding fixed points.

Results

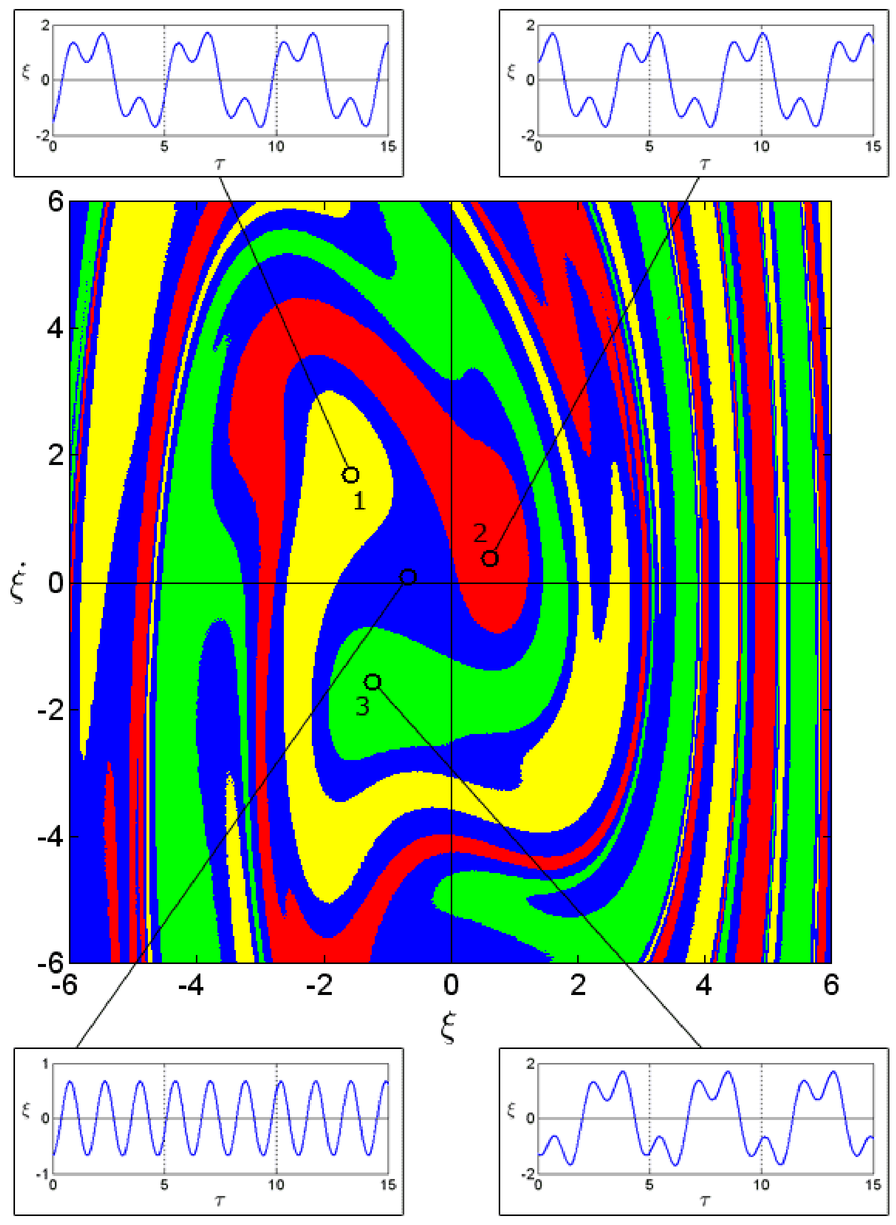
Some results are shown on the subsequent figures. Dashed lines are corresponding to unstable regimes.

1. At this level of resistance ($\mu \cdot \omega_0$ = *0.1*) and such nonlinear (γ = *0.5*) and the disturbing factors (P = *10*) in a symmetric system (β = *0*) we managed to find only subresonances of the order of *1:3*.

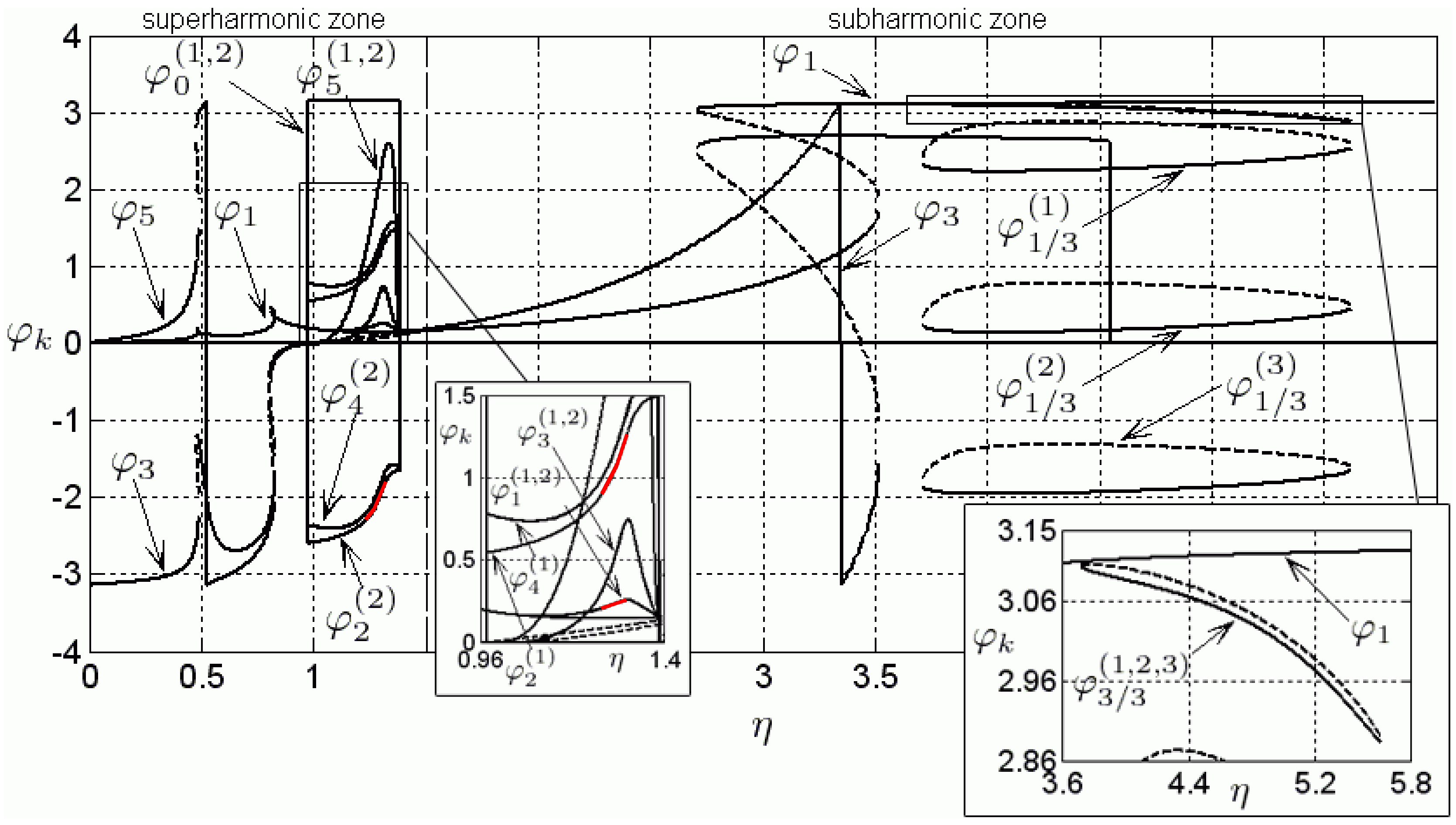
Amplitude-frequency characteristics



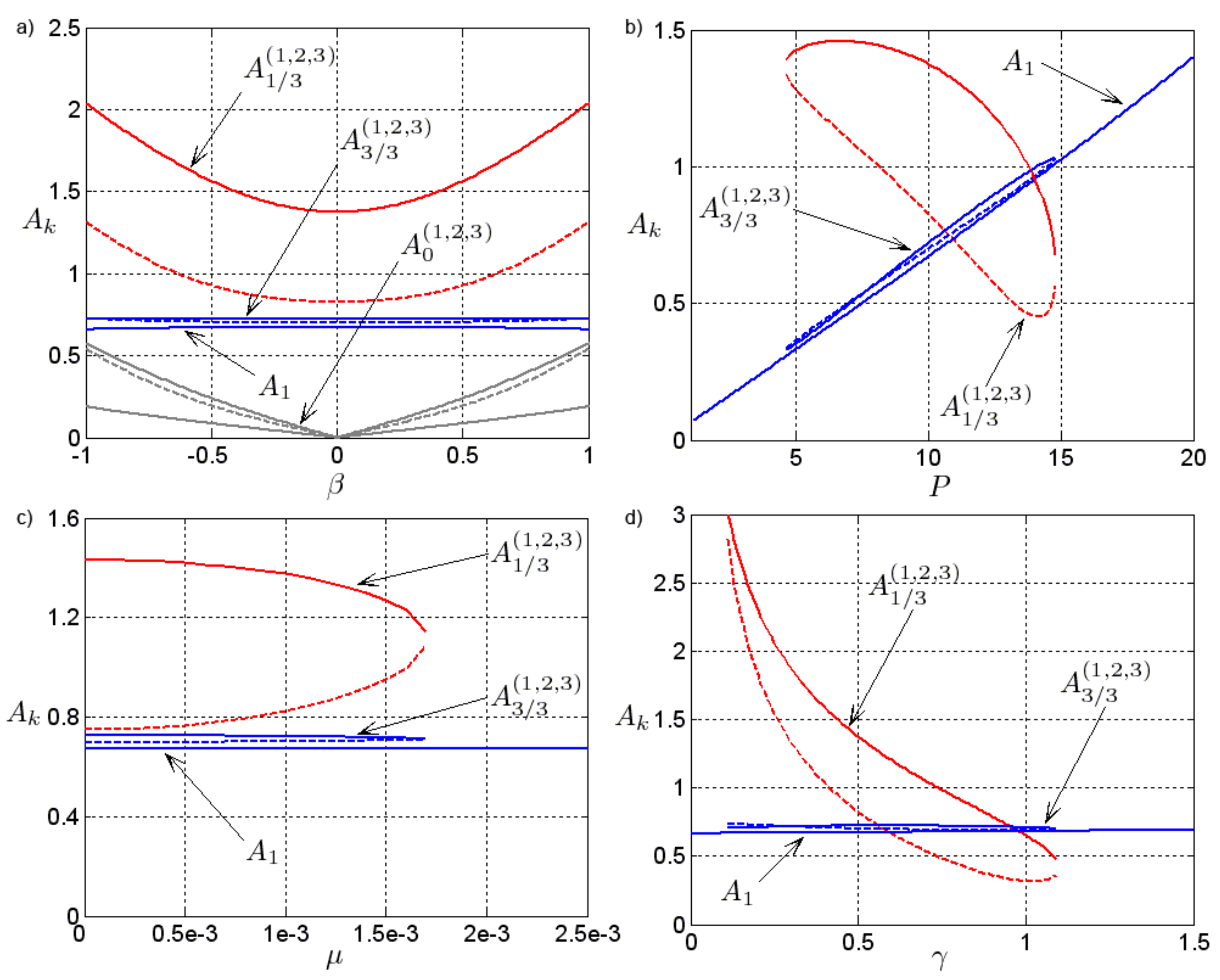
Basins of attraction for η = *4.0*



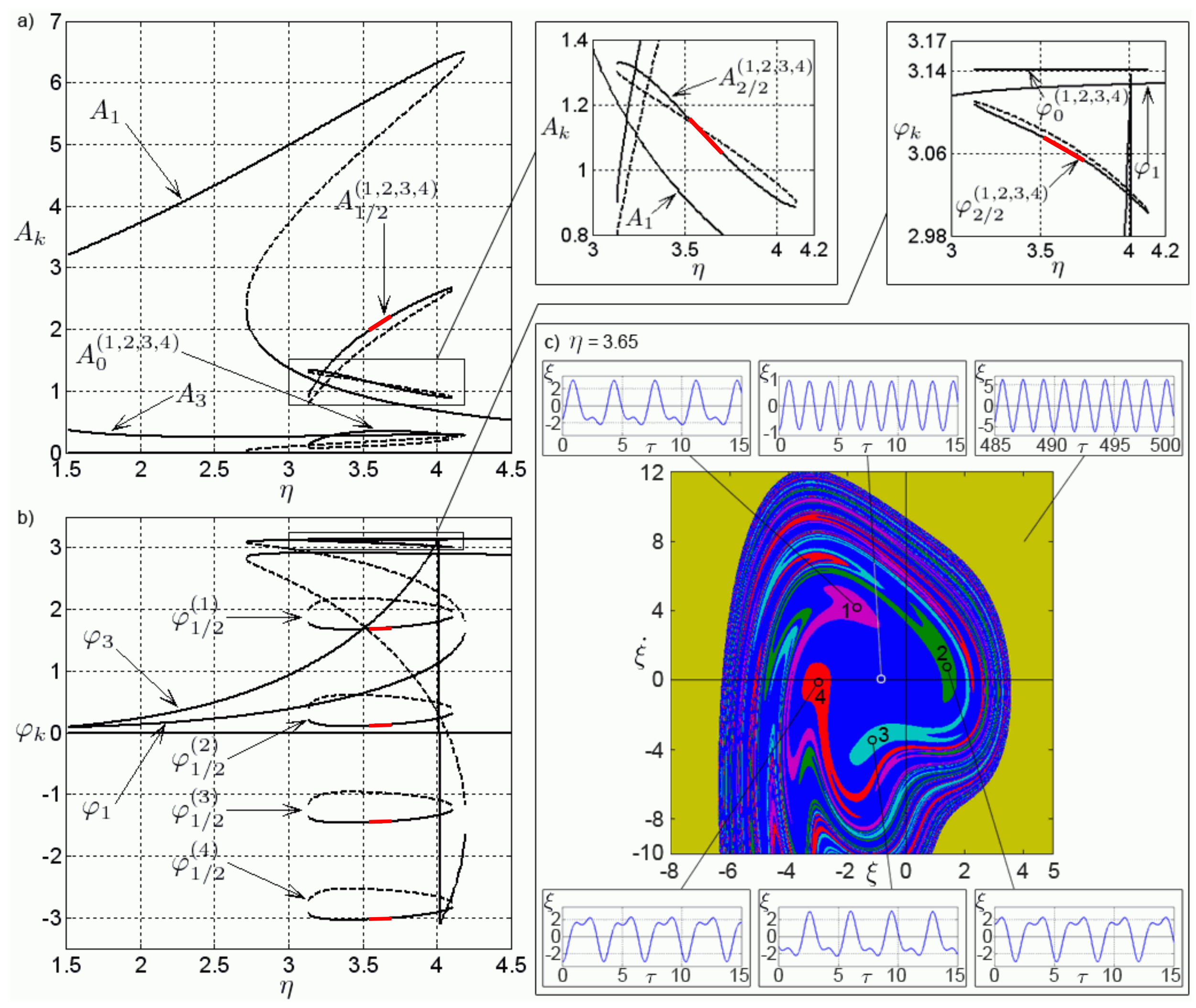
Phase-frequency characteristics



2. The escalation of the resonance *1:3* with the introduction of the asymmetry β characteristics, as well as extreme character of the P changes is observed.



3. After decreasing of the resistance ($\mu \cdot \omega_0$ = *0.05*) in the system subharmonic regimes of *1:2* were discovered.

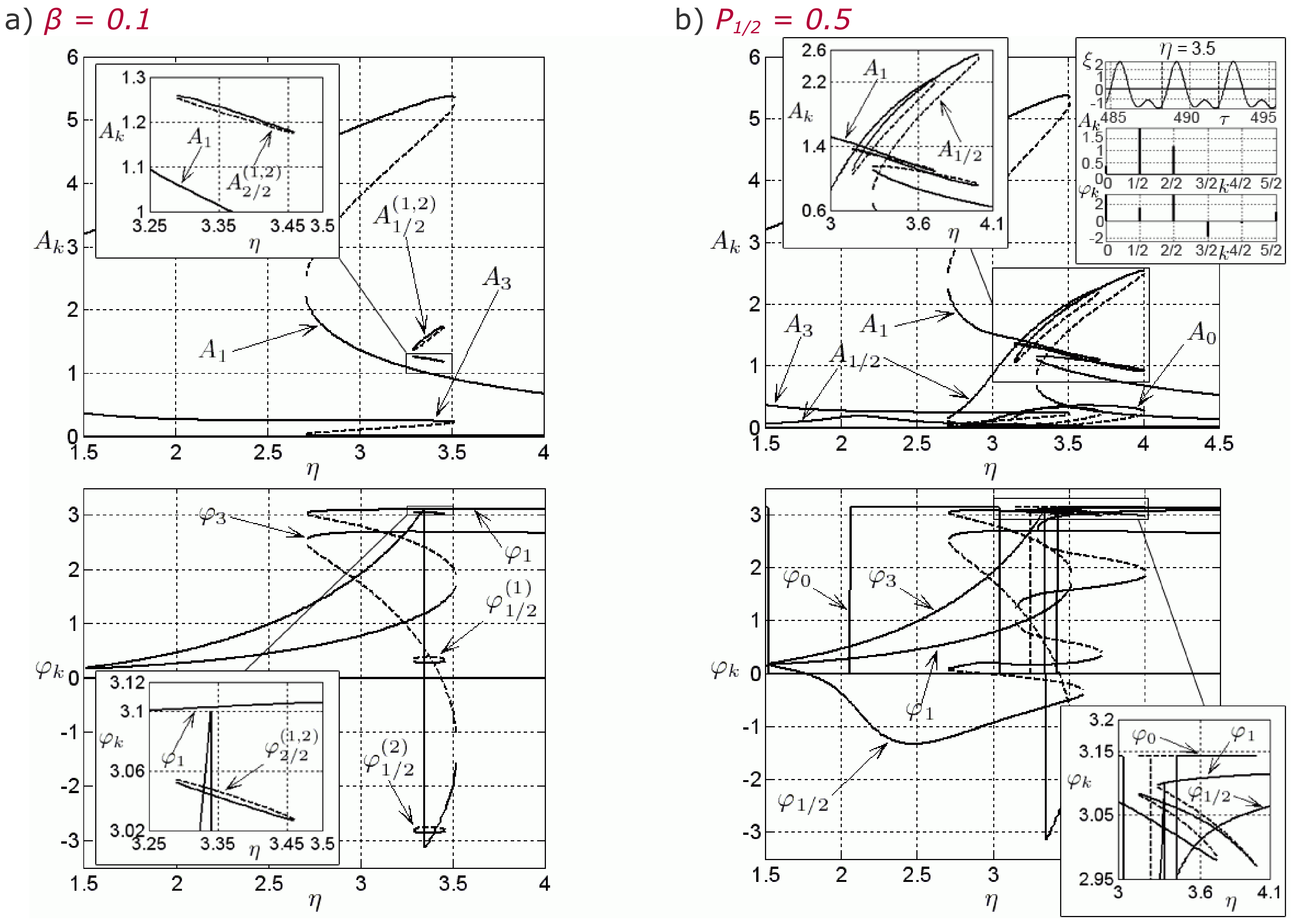


Results

4. The factors which facilitate the appearance of the resonance *1:2* are:

a) asymmetry of the restoring force $\beta \neq 0$,
b) asymmetry of excitation

$$P(\tau) = P_{1/2}\cos(0.5\,\eta\,\tau) + P_1\cos(\eta\,\tau).$$



Conclusions

1. The oscillations of the pronounced biharmonic composition are possible for resonances of the orders of *2:1* and *1:2* with a wide range of amplitude and phase components of the harmonics. It turned out, that subharmonic oscillations are more sensitive to the level of resistance in the system. Among the factors that facilitate their excitement, together with the degree of asymmetry of the elastic characteristics, the presence of a small additional force excitation can be considered.

2. An important practical conclusions may be done:

- relatively larger total stiffness of the elastic system, which is necessary for the implementation of the superharmonic resonances – this is explained by their excitement in the area which is prior to the zone of principal resonance;
- probable reduction of energy consumption in case of use of subharmonic resonances, compared with the traditional method of forming of biharmonic oscillations in linear machines with the help of biharmonic exciters.

3. One property may be considered as a new feature of subharmonic resonances. It turns out that its set can be divided into two or even number of classes, each class contains groups of stable and unstable regimes, and modes of each group differ from each other by phase shift of harmonics components (look, for example, Page 4).

Foundation

$$x(t) = \sum_{k=0}^{N} A_k \cos\left(\frac{2\pi}{T} k\, t - \varphi_k\right) \quad \in \quad \dot{x} = f(x,t), \quad f(x, t \pm T) = f(x,t)$$

$$\Downarrow \quad t := \tau + mT$$

$$x(\tau) = \sum_{k=0}^{N} A_k \cos\left(\frac{2\pi}{T} k\,\tau - \left(\varphi_k - \frac{2\pi m}{n} k\right)\right) \quad \in \quad \frac{dx}{d\tau} = f(x,\tau), \quad m = \overline{0, n-1}$$

4. Among the primary problems in the way of use of combination resonances two of them can be mentioned: the design of the elastic elements that provide the desired form of the nonlinear characteristics and the global stability of working regimes or the development of ways of programmable start-up.