УДК 528.72

Theoretical bases of construction and accuracy rating of digital quasi-images

Dr. Mogilny S.G., Lunev A.A., Dr. Sholomitsky A.A.
Donetsk National Technical University

Application of the digital cameras for the purposes of ground topographical survey has appreciable advantages in comparison with the film cameras:

- It is not required expensive and whimsical film;
- There is no labour-intensive and ecologically harmful process of photochemical processing;
- There is no necessity of scanning digital snaps for processing on Digital Photogrammetric Station (DPS), and, therefore, it is not required the expensive photogrammetric scanner;
- Self-descriptiveness of digital color images is much higher than that of usual, even color pictures.

The basic disadvantage of the digital cameras is the small physical size of the CCD (Charge Coupled Device) array and, therefore, small angles of the image (tab. 1). As it is seen from the table 1 the angle of the image for professional and semi-professional digital cameras does not exceed 16˚. The increasing of the angle of the image at the expense of reduction of a focal length of the lens is inefficient because it sharply worsens the geometrical resolution to object of shooting and causes significant increase of distortion influence [1-4].

Table 1

Values of the corner of the image for different digital cameras

№	Digital camera	Px, mln	Maximal focal length, mm	The angle of the image, (º)
1	SONY DSC-R1	10	120	16.5
2	FujiFilm FinePix S9500	9	300	6.7
3	FujiFilm FinePix E900	9	128	15.5
4	Canon PowerShot Pro1	8	200	10
5	Sony DSC-H5 black	7	432	4.6
6	Canon PowerShot S3IS	6	432	4.6
7	FujiFilm FinePix S5600	5	380	5.3
8	Canon PowerShot S2IS	5	432	4.6
9	Olympus E20-P	5	140	14.2

The increase of the parameters of the CCD array to the sizes of the film cameras is connected now to technical difficulties and will lead to high price of the digital cameras. If the small angle of image resulted only to increasing of the number of stereo mates their processing at DPS would not represent special difficulties due to the computer power. The basic negative factor is the difficulty of performance of shooting so that the big number of the small images executed from different points of space provide correct overlapping of each other, forming the stereo mates and covering the object of shooting without blanks and superfluous overlappings. There is the article [5] that shows how using of rather not big CCD array allows to obtain the snap corresponding to the sizes of the usual film camera.

However in the article [5] it is supposed beforehand, that CCD arrays are located in plane, therefore it is offered to develop this idea for usual digital cameras. The essence of the offered method is shown on fig. 1. According to the idea described below the patent [6] is received.

Let’s suppose that from the point S three digital images are executed by the digital narrow-angle camera, the adjacent pictures constitute the narrow area of the double overlapping. If to measure the coordinates of points in the zones of overlapping, it is possible obviously to calculate angular elements of mutual orientation of all images and according to the formulas of collinear transformations to project them on the plane Q. As a result one image will be obtained which can be used as some quasi-image having focal length equivalent to the initial. Quasi-images, constructed for the group of narrow-angle images are effectively processed on DPS. At is natural, that in practice it will not be possible to provide exact concurrence of the centers of photographing of images, however at the sufficient distance of the object of shooting this discrepancy will not have practical value as the distortion is less than resolution and the accuracy of measurement of the images. To achieve necessary overlapping of the adjacent images is possible, using angular scales of inclination and turn which are available on standard photo-supports.

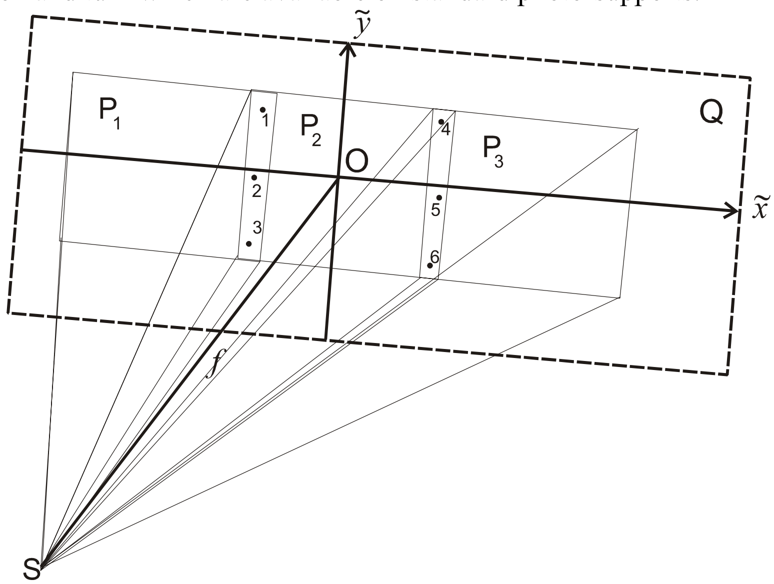


Figure 1 – The group of the images

Generally the quasi-image can be arbitrarily located relatively to the images of the group, but its size can be very great and inconvenient for processing. Because of geometrical reasons it is obviously, that quasi-image is necessary to be arranged so that the sizes of projections of the images of the group are minimal, i.e. its main beam occupy some average position among the main beams of the initial images.

If to calculate the angles of mutual orientation of the images of the group and to set the position of quasi-image axes, the projection of the initial image to the plane of the quasi-image will be carried out according to the collinear transformation formulas:

$$\left.\begin{aligned}\tilde{x} &= -f\frac{a_1x + a_2y + a_3f}{c_1x + c_2y + c_3f};\\ \tilde{y} &= f\frac{b_1x + b_2y + b_3f}{c_1x + c_2y + c_3f},\end{aligned}\right\} \qquad (1)$$

$\tilde{x}, \tilde{y}$ and x, y – the coordinates of the point on the quasi-image and the source image; $a_1, a_2, \ldots. c_3$ – the directional cosines of the image; f – focal length of the images.

If the same point k of the object was represented on two different source images the corresponding coordinates of its image on the quasi-image received from two images, coincide, i.e. the following equation is carried out:

$$\left.\begin{aligned}f\frac{a_1^{(i)}x_k^{(i)} + a_2^{(i)}y_k^{(i)} + a_3^{(i)}f}{c_1^{(i)}x_k^{(i)} + c_2^{(i)}y_k^{(i)} + c_3^{(i)}f} - f\frac{a_1^{(j)}x_k^{(j)} + a_2^{(j)}y_k^{(j)} + a_3^{(j)}f}{c_1^{(j)}x_k^{(j)} + c_2^{(j)}y_k^{(j)} + c_3^{(j)}f} &= 0;\\ f\frac{b_1^{(i)}x_k^{(i)} + b_2^{(i)}y_k^{(i)} + b_3^{(i)}f}{c_1^{(i)}x_k^{(i)} + c_2^{(i)}y_k^{(i)} + c_3^{(i)}f} - f\frac{b_1^{(j)}x_k^{(j)} + b_2^{(j)}y_k^{(j)} + b_3^{(j)}f}{c_1^{(j)}x_k^{(j)} + c_2^{(j)}y_k^{(j)} + c_3^{(j)}f} &= 0,\end{aligned}\right\} \qquad (2)$$

$x_k^{(i)}, y_k^{(i)}$ и $x_k^{(j)}, y_k^{(j)}$ – coordinates of the point on i and j snaps; $a_1^{(i)}, a_2^{(i)}, \ldots c_3^{(i)}$ и $a_1^{(j)}, a_2^{(j)}, \ldots c_3^{(j)}$ – directional cosines of i and j snaps calculated with turning angle relative to the axis of the quasi-snaps.

The equations (2) designed for all pairs of respective points in the zones of double overlapping of the pictures of the net of the snap, allow to determine the mutual position of the snap and to set the direction of quasi-snap axes. Because the unknown angles of orientation are not great, for the solution of the system (2) the method of linearization around of the approximated values of variables is allowable. Then for the point k on snaps i and j the equation (2) in the linear form will look like:

$$\left.\begin{aligned} & a_{\alpha_i}^{(k)}\delta_{\alpha_i} + a_{\omega_i}^{(k)}\delta_{\omega_i} + a_{\kappa_i}^{(k)}\delta_{\kappa_i} - a_{\alpha_j}^{(k)}\delta_{\alpha_j} - a_{\omega_j}^{(k)}\delta_{\omega_j} - a_{\kappa_j}^{(k)}\delta_{\kappa_j} + \\ & \qquad c_{x_{k,i}}^{(k)} v_{x_{k,i}} + c_{y_{k,i}}^{(k)} v_{y_{k,i}} - c_{x_{k,j}}^{(k)} v_{x_{k,j}} - c_{x_{k,i}}^{(k)} v_{x_{k,j}} + l_{x_{i,j}}^{(k)} = 0; \\ & b_{\alpha_i}^{(k)}\delta_{\alpha_i} + b_{\omega_i}^{(k)}\delta_{\omega_i} + b_{\kappa_i}^{(k)}\delta_{\kappa_i} - b_{\alpha_j}^{(k)}\delta_{\alpha_j} - b_{\omega_j}^{(k)}\delta_{\omega_j} - b_{\kappa_j}^{(k)}\delta_{\kappa_j} + \\ & \qquad d_{x_{k,i}}^{(k)} v_{x_{k,i}} + d_{y_{k,i}}^{(k)} v_{y_{k,i}} - d_{x_{k,j}}^{(k)} v_{x_{k,j}} - d_{x_{k,i}}^{(k)} v_{x_{k,j}} + l_{y_{i,j}}^{(k)} = 0. \end{aligned}\right\} \quad (3)$$

That can be written in the matrix form:

$$A_{i,j}^{(k)}\delta_{i,j} + C_{i,j}^{(k)} v_{i,j}^{(k)} + l_{i,j}^{(k)} = 0, \quad (4)$$

$A_{i,j}^{(k)}$ - a matrix of partial derivatives from the left part of the equation (2) for the point k on the unknown angles of orientation of the snaps i and j; $C_{i,j}^{(k)}$ - the matrix of partial derivatives from the left part of the equation (2) for the point k on its measured coordinates on the snap i and j; $v_{i,j}^{(k)}$ - the vector - column of amendments in the measured coordinates of the point k on the snaps i and j; $\delta_{i,j}$ - the vector - column of amendments in the approximated angles of orientation of the snaps i and j; $l_{i,j}^{(k)}$ - the vector -column of absolute terms of the equations (2) calculated on approximated angles and on the measured coordinates of the point k.

The equations (3) by themselves are the conditional equations of the more difficult form, because they contain amendments in measurements and amendments to conditionals, i.e. there is the combined method of adjustment – the correlate method with conditionals. To transform this task of adjustment to the parametric method the equations should be written as follow:

$$A_{i,j}^{(k)}\delta_{i,j} + l_{i,j}^{(k)} = \tilde{v}_{i,j}^{(k)}, \quad (5)$$

$\tilde{v}_{i,j}^{(k)}$ - vector-column of complex amendments that are calculated by the formula:

$$\tilde{v}_{i,j}^{(k)} = C_{i,j}^{(k)} v_{i,j}^{(k)}. \quad (6)$$

If to gather all equations of the kind (5) for the point k the following system of the parametrical equations will be received:

$$A^{(k)}\Delta + l^{(k)} = \tilde{v}^{(k)}, \quad (7)$$

Δ - the vector - column of amendments in angular elements of orientation of the snaps; $A^{(k)}$ - the matrix of partial derivatives that is constituted from elements of the matrixes $A_{i,j}^{(k)}$ and added by zero factors so that the number of columns correspond to the dimension of the vector of the amendments Δ; $\tilde{v}^{(k)}$ - the vector - column of the complex amendments calculated by the similar formula (6):

$$\tilde{v}^{(k)} = C^{(k)} v^{(k)}. \quad (8)$$

It is obvious that the complex amendments are dependent on each other and for adjustment it is necessary to use the generalized least-squares method, therefore the covariation matrix of the complex amendments M_k will be calculated by the formula:

$$M_k = C_k M_{x,y} C_k^T, \tag{9}$$

$M_{x,y}$ - covariation matrix of the measured coordinates of the points directly on the initial snaps.

The normal equations for the equations (7) according to the generalized principle of the least- squares method will look like:

$$A^{(k)^T} M_k^{-1} A^{(k)} \Delta - A^{(k)^T} M_k^{-1} l^{(k)} = 0. \tag{10}$$

And, using obvious designations, we shall receive

$$N_k \Delta + L_k = 0. \tag{11}$$

Summarizing the equations (11) made for all measured points in the zones of overlapping, common normal equations of amendments to unknown conditionals will be received:

$$N\Delta + L = 0. \tag{12}$$

The coefficient matrix of the system of the equations (12) has a matrix rank which is less than the number of unknown variables, therefore the solution is ambiguous. It is necessary to choose from all possible solution the one, steady enough and unambiguous solution. Vector $\overline{\Delta}$ which satisfies the equation (12) and has minimal module can be like this:

$$\overline{\Delta}^T \Delta = \min. \tag{13}$$

The required solution is possible to be received by the Tikhonov [8] regularization method of equations (12) about from the following expression:

$$\overline{\Delta} = -(N + \varepsilon E)^{-1} L, \tag{14}$$

ε - multiplier for regularization; E - the unitary matrix.

If the initial system of the equations (2) were linear the formula (13) would be the required solution, but the vector $\overline{\Delta}$ minimizes only amendments to the elements of orientation of the snaps concerning the axes of quasi-snap, instead of the angles of turn. Therefore it is necessary to formulate to the equations (2) addition conditions of the optimum arrangement of the axes of the quasi-snap concerning source snap. One of the variants of the additional conditionals can be such expression as:

$$\left[\alpha^2 + \omega^2 + \kappa^2\right] = \min, \tag{15}$$

α, ω, κ - the angles of orientation of the snaps of a net relatively the axes of the quasi-snap. For the satisfaction of this condition it is necessary on each iteration after correction of angles on the amendments calculated by the formula (14), to determine new initial values, for example, for the angles α_i by the formula

$$\alpha_i = \alpha_i' - \frac{[\alpha']}{n}, \tag{16}$$

α_i' - the angles calculated on the corrected matrix of the directional cosines of the snap i; n - the number of the snaps at the net. Other angles are calculated by the similar way. The iterations converge quickly and unequivocally determine the position of the quasi-snap at the net.

For the definition of the final angles of the orientation it is the necessary to define one of the snaps to define as initial and to accept as known the angles of orientation of it, which are received by the formula (16) in the last iteration, and for the other snap to design the equations (2) and to solve them by the least-squares method by formulas (3) - (14). The multiplier for regularization ε can be accepted as equal to zero as in this case the matrix rank of the equations is equal to the number of unknown variables.

Using the calculated angles of orientation, by formulas (1) by method of indirect transformation the digital image of quasi-snap is calculated. For the zones of overlapping of the snap the density of the image is accepted on that pixel which is closer to the main point of the initial image.

It is obvious, that the quasi-snap will have additional geometrical distortions which are caused by the mistakes of the calculated angles of orientation, therefore the coordinates of the image measured on the quasi-snap are necessary to be considered as correlated measurements and is to be taken into account their correlation at the strict adjustment of the stereo mate constructed on the quasi-snaps.To take into account the measurement correlation in accordance with the transitivity of the least-square method it is necessary to have their correlation matrix.

If to accept, that the coordinates $\tilde{x}_i, \tilde{y}_i$ of the point i on the quasi-snap are calculated by the formulas similar to the formula (1):

$$\left.\begin{aligned} \tilde{x}_i &= -f\frac{a_1^{(j)}x_i + a_2^{(j)}y_i + a_3^{(j)}f}{c_1^{(j)}x_i + c_2^{(j)}y_i + c_3^{(j)}f};\\ \tilde{y}_i &= f\frac{b_1^{(j)}\tilde{x}_i + b_2^{(j)}\tilde{y}_i + b_3^{(j)}f}{c_1^{(j)}\tilde{x}_i + c_2^{(j)}\tilde{y}_i + c_3^{(j)}f}, \end{aligned}\right\} \tag{17}$$

$a_1^{(j)}, a_2^{(j)}, \ldots, c_3^{(j)}$ – directional cosines of the snap j, on which the preimage of the point i of the quasi-snap is placed; x_i, y_i – coordinates of the preimage of point i.

Then mistakes $\delta\tilde{x}_i, \delta\tilde{y}_i$ measured on quasi-snap of coordinates $\tilde{x}_i, \tilde{y}_i$ the any point i is calculated as differential of the right part of the equations (17):

$$\left.\begin{aligned} \delta\tilde{x}_i &= a_{\alpha_j}^{(i)}\delta_{\alpha_j} + a_{\omega_j}^{(i)}\delta_{\omega_j} + a_{\kappa_j}^{(i)}\delta_{\kappa_j} + \varepsilon_x;\\ \delta\tilde{y}_i &= b_{\alpha_j}^{(i)}\delta_{\alpha_j} + b_{\omega_j}^{(i)}\delta_{\omega_j} + b_{\kappa_j}^{(i)}\delta_{\kappa_j} + \varepsilon_y, \end{aligned}\right\} \tag{18}$$

$\varepsilon_x, \varepsilon_y$ - random errors of vising on the point of the quasi-snap; $\delta_{\alpha_j}, \delta_{\omega_j}, \delta_{\kappa_j}$ - pure errors of the angles of orientation of the snap j. Coordinates x_i, y_i of the preimage of the point i are calculated by the inversed formulas relatively (17):

$$\left.\begin{aligned} x_i &= -f\frac{a_1^{(j)}\tilde{x}_i + b_1^{(j)}\tilde{y}_i + c_1^{(j)}f}{a_3^{(j)}\tilde{x}_i + b_3^{(j)}\tilde{y}_i + c_3^{(j)}f};\\ y_i &= f\frac{a_2^{(j)}\tilde{x}_i + b_2^{(j)}\tilde{y}_i + c_2^{(j)}f}{a_3^{(j)}\tilde{x}_i + b_3^{(j)}\tilde{y}_i + c_3^{(j)}f}. \end{aligned}\right\} \tag{19}$$

Partial derivatives $a_{\alpha_j}^{(i)}, a_{\omega_j}^{(i)}, \ldots, b_{\kappa_j}^{(i)}$ from the equations (17) in the formula (18) are calculated with the use of the coordinates x_i, y_i.

In the matrix form the equation (18) is possible to be written down as:

$$\delta_i = A_i\Delta_j + \varepsilon_i, \tag{20}$$

δ_i - the vector of the pure errors of the coordinates of the point i of the quasi-snap; A_i - the matrix of partial derivatives in the equations (18); Δ_j - the vector of the pure errors of the angles of orientation of the snap j; ε_i - the vector of the random errors of measurement of the coordinates on the quasi-snap. Then the covariation matrix M_{δ_i} of the vector δ_i should be calculated by the following formula:

$$M_{\delta_i} = M_{x,y} + A_i Q_j A_i^T, \tag{21}$$

$M_{x,y}$ - the covariation matrix of the random errors of vising on point of image; Q_j - covariation matrix of vector of the pure errors Δ_j of angles of the image which is submatrix corresponding to image j, in the inverse matrix N^{-1} of the normal equations (14).

The formula (20) is more convenient for writing down in more general form:

$$\delta_i = \overline{A}_i \Delta + \varepsilon_i, \tag{22}$$

$\overline{A}_i$ - the matrix A_i added by zero up to dimension of the vector Δ where on j place there is the matrix A_i, i.e.

$$\overline{A}_i = |0\,0 \ldots A_i \ldots 0|. \tag{23}$$

And the formula (21) will be as follows:

$$M_{\delta_i} = M_{x,y} + \overline{A}_i N^{-1} \overline{A}_i^T. \tag{24}$$

If on quasi-snap t points are measured, the covariation matrix M_δ of the vector δ of the error of their coordinates is calculated by the following formula:

$$M_\delta = M + AN^{-1}A^T, \tag{25}$$

M – quasi-diagonal matrix, where the main diagonal contains the matrixes $M_{x,y}$; A – matrix is designed from the submatrix $\overline{A}_i$ as that of (23).

The practical research of the submitted mathematical tools was carried out by the method of modeling of shooting of the test polygon. Using known coordinates of the test polygon and elements of the inner and the outer orientation of the net of the snaps the coordinates were calculated on them. The received models of snap were considered as the results of the measurements of the real snap with unknown elements of their mutual orientation in the net. The digital 5 megapixel camera with the angle of the vision about 16 ° was used to modeling. The net of the snaps on the point of shooting was calculated for 9 snaps (3*3). The strip of overlapping of the adjacent snaps made up no more than 10 %. Two experiments have been carried out they differed by quantity of points in the strip of overlapping 1) on 3 points; 2) on 6 points.

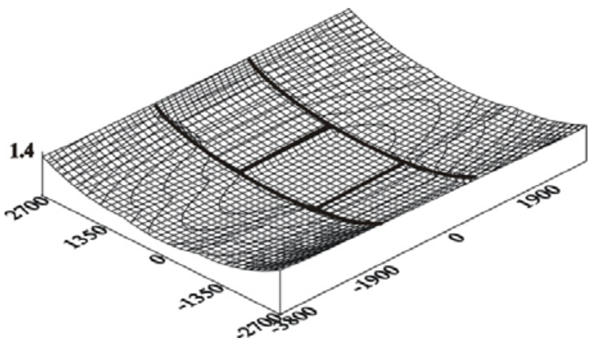
The errors that calculated appeared due to by errors of solution of the task of relative orientation of the snaps in a net were considered by the formula (25). Calculation was carried out under the condition of the accuracy of pointing to the point at the modern digital photogrammetric station not exceeding 5 pix. The values of errors are presented in the pixels on two axes of the received quasi-snaps (panoramic pictures) fig. 2. For then images with the strip of overlapping on 6 point the errors are less about 40%.

As it is may be seen from the figures the errors increase at moving off from the axes of quasi-snap. On all pictures there are areas of the minimal errors (are shown by a bold line) which are formed because these snaps "are most reliably fixed" in a net, that is it has a strip of overlapping from three sides, as against angular snaps. Irrespective of quantity of measured points for any quasi-snap the so-called “flat bottom” where values of errors are equal 0.5 pixel, i.e. to accuracy of pointing to current point. It is a snap with the least sum of angles of an inclination. As this picture is not transformed during processing (other pictures join it), also errors on it do not depend on accuracy of the decision of a task of relative orientation. For the chosen model – 3*3 - this is the central snap.

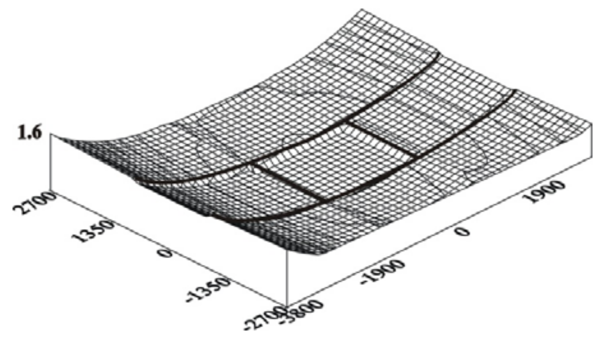
Values of the maximal errors caused by discrepancy of the decision of task of relative orientation of snap in net, decreases at a lot of points in strips of overlapping. The errors that less than one pixel give the possibility of the using such snaps to reception the metric information.

In figure 3 the object of photographing - the building extended on front is shown. As it is may be seen from figure, the object of shooting was photographed from basis 1-2 with the angle of intersection U.

According to each point of photographing have been received a net of net accordingly 3 and 4. Then have been constructed for each net quasi-snaps 5 and 6.



a)



b)

Figure 2 – The values of the errors on the quasi-snap (3 points at the strip of overlapping):
a) on axe X; b) on axe Y

Advantages of such stereo mates are in increase in the area of coveraged area of shooting without addition of additional points of photographing. Difference of stereo mate of quasi-snaps from classical stereo mates are that errors of the measured coordinates dependent random variables which are characterized covariation matrix M_{δ} [6] calculated for everyone quasi-snap. This matrix keeps at mind a casual component of measurements, and also errors that ware appeared due to calculation of angle of relative orientation at net of snaps.

If the creating and the adjustment of stereopair should be executed, using measurements of quasi-images, that necessity to calculate covariation matrix will complicate the common process of calculations, but there is not necessity to change basic scheme of least-squares method. Let the adjustment of stereo mate is executed by the method of group, and then equations of collinearity for left image can be written at so form:

$$\left.\begin{aligned} & f\frac{\tilde{a}_1(X_i - X_{s_л}) + \tilde{b}_1(Y_i - Y_{s_л}) + \tilde{c}_1(Z_i - Z_{s_л})}{\tilde{a}_3(X_i - X_{s_л}) + \tilde{b}_3(Y_i - Y_{s_л}) + \tilde{c}_3(Z_i - Z_{s_л})} - \tilde{x}_i = \nu_i^x; \\ & f\frac{\tilde{a}_2(X_i - X_{s_л}) + \tilde{b}_2(Y_i - Y_{s_л}) + \tilde{c}_2(Z_i - Z_{s_л})}{\tilde{a}_3(X_i - X_{s_л}) + \tilde{b}_3(Y_i - Y_{s_л}) + \tilde{c}_3(Z_i - Z_{s_л})} - \tilde{y}_i = \nu_i^y, \end{aligned}\right\} \qquad (26)$$

X_i, Y_i, Z_i - geodetic coordinates of the point I of a model: $X_{s_л}, Y_{s_л}, Z_{s_л}$ - geodetic coordinates of the shooting center of the left quasi-image; $\tilde{a}_1,....,\tilde{c}_3$ - direction cosines of the axis of the left quasi-image; ν_i^x, ν_i^y - amendment to coordinates $\tilde{x}_i, \tilde{y}_i$ of the point I on the left image. At linear form the equations (26) can be written at so:

$$D_i^л \delta R_i + G_i^л \Delta_л + L_i^л = v_i^л, \qquad (27)$$

δR_i - amendments to the approximate geodetic coordinates of a point i, and $D_i^л$ a matrix from private derivatives corresponding to them; $\Delta_л$ - amendments to the approximate values of elements of external orientation of left quasi-image and $G_i^л$ a matrix from private derivatives corresponding to them; $L_i^л, v_i^л$ - vectors of free terms and amendments of coordinates of quasi-image.

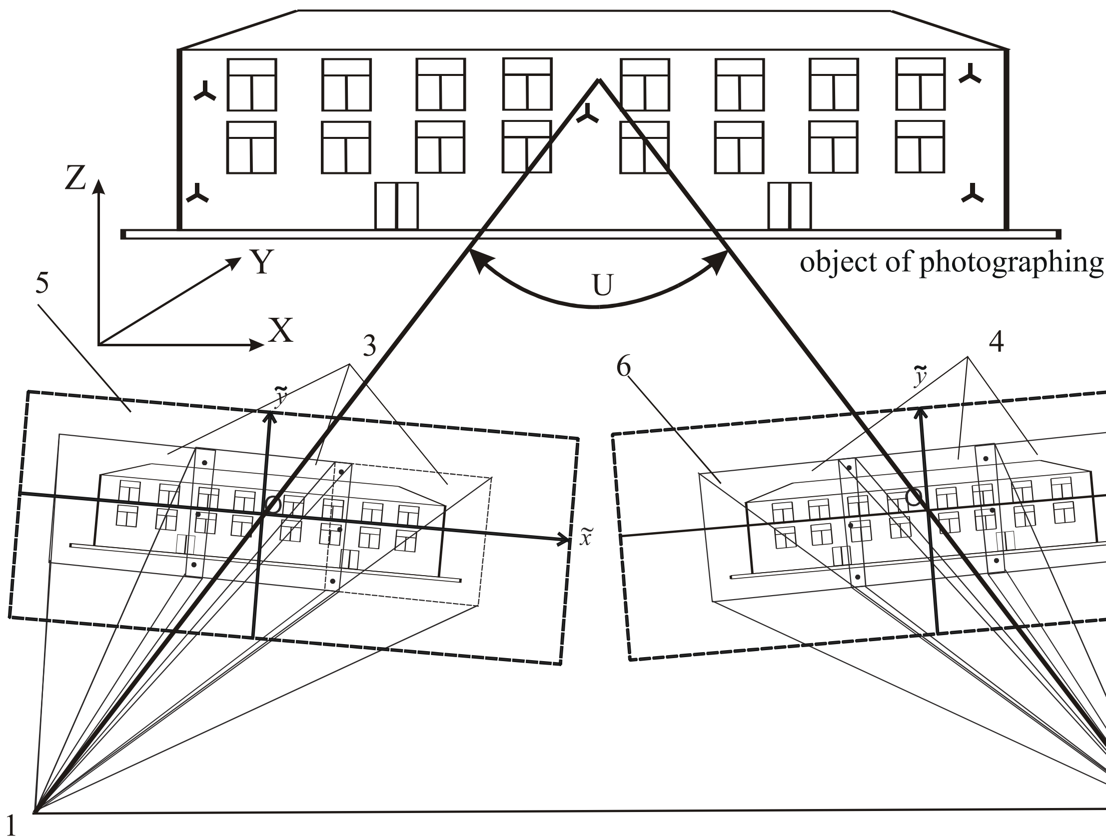


Figure 3 – The stereo mate at two quasi-images

The equations of the amendments for all measured points of the left quasi-image at matrix form:

$$\begin{vmatrix} D_1^л & 0 & . & 0 \\ 0 & D_2^л & . & 0 \\ . & . & . & . \\ 0 & 0 & . & D_t^л \end{vmatrix} \begin{vmatrix} \delta R_1 \\ \delta R_2 \\ . \\ \delta R_t \end{vmatrix} + \begin{vmatrix} G_1^л \\ G_2^л \\ . \\ G_e^л \end{vmatrix} \Delta_л + \begin{vmatrix} L_1^л \\ L_2^л \\ . \\ L_t^л \end{vmatrix} = \begin{vmatrix} v_1^л \\ v_2^л \\ . \\ v_t^л \end{vmatrix}. \quad (28)$$

Using obvious symbols these equations can be written at more compact form:

$$D_л \delta R + G_л \Delta_л + L_л = V_л. \quad (29)$$

The equations of the amendments have matrix weight $P_л$ that defines from the next equation:

$$P_л = M_{\delta_л}^{-1}, \quad (30)$$

$M_{\delta_л}$ that has on the main diagonal the covariation matrixes of random errors of visioning on the point of image [6].

The equations of the amendments for right image can be written similarly, that system of simultaneous equations can be written at so form:

$$\begin{vmatrix} D_л \\ D_п \end{vmatrix} \delta R + \begin{vmatrix} G_л & 0 \\ 0 & G_п \end{vmatrix} \begin{vmatrix} \Delta_л \\ \Delta_п \end{vmatrix} + \begin{vmatrix} L_л \\ L_п \end{vmatrix} = \begin{vmatrix} V_л \\ V_п \end{vmatrix}, \quad (31)$$

which has matrix of the weights:

$$P = \begin{vmatrix} M_{\delta_л}^{-1} & 0 \\ 0 & M_{\delta_п}^{-1} \end{vmatrix}. \quad (32)$$

Making the solution of the system equations (31) using least-square method and requiring the condition:

$$V^{T}PV = \min . \qquad (33)$$

Obtain normal equations at so form:

$$\begin{vmatrix} D_{л}^{T}P_{л}D_{л} + D_{п}^{T}P_{п}D_{п} & D_{л}^{T}P_{л}G_{л} & D_{п}^{T}P_{п}G_{п} \\ G_{л}^{T}P_{л}D_{л} & G_{л}^{T}P_{л}G_{л} & 0 \\ G_{п}^{T}P_{п}D_{п} & 0 & G_{п}^{T}P_{п}G_{п} \end{vmatrix} \times \begin{vmatrix} \delta R \\ \Delta_{л} \\ \Delta_{п} \end{vmatrix} + \begin{vmatrix} D_{л}^{T}P_{л}L_{л} + D_{п}^{T}P_{п}L_{п} \\ G_{л}^{T}P_{л}L_{л} \\ G_{п}^{T}P_{п}L_{п} \end{vmatrix} = 0. \qquad (34)$$

Using obvious symbols the equations (34) can be written at so form:

$$w = -N^{-1} \cdot L. \qquad (35)$$

Keeping at mind, that for every image 6 (six) elements of exterior orientation are calculated and for every non-control point - three (3) space coordinates, total amount of unknown variables are 6*2+3K, where K - amount the points that are defined. Using condition that four equations can be designed for every point and as minimum three control point should be installed on the object for shooting, the amount equations for solution (31) is 4*n, where n - amount of the measured point on the left and right quasi-images.

The described mathematical tool of formulas has been tested on the data which have been received as a result of mathematical modeling. The model was under construction as follows. On the equations collinearity [7] have been received a group of images for two points of photographing (3*3=9 pictures in a group) with a focal length of 100 mm. At a corner of a field of the image of a single image in view of a zone of overlapping of adjacent images about 5% the corner of a field of the quasi-image has made ~40º. The corner of photogrammetric intersection on object has made about 50º. Further by a technique of association of images executed from one point of space in quasi-image [6] uniform images for each point of photographing have been received. Using coordinates on quasi-images and true values of coordinates of control points it is possible on basis (26-34) to receive accuracy of definition of elements of exterior orientation of images and coordinates of determined points. As the inverse matrix of the normal equations (35) is the covariation matrix of unknown variables the quadratic error of i - can be calculated under the formula:

$$m_{i} = \mu \cdot \sqrt{N_{i,i}^{\ -1}}, \qquad (36)$$

μ - error of unit weight of measurement of point on source images and quasi-images. At next calculation it has been taken as 5 µm.

The values of quadratic errors of elements of orientation of quasi-stereo-mate that received under formula (36) are shown at Table 2. Linear elements are presented at micron at scale of image:

Table 2

The values of the calculation of the elements exterior orientation

Value	Left image	Right image
m_{α}	26″	27″
m_{ω}	18″	22″
m_{κ}	15″	16″
m_{X}	9 µm	11 µm
m_{Y}	19 µm	18 µm
m_{Z}	8.5 µm	8 µm

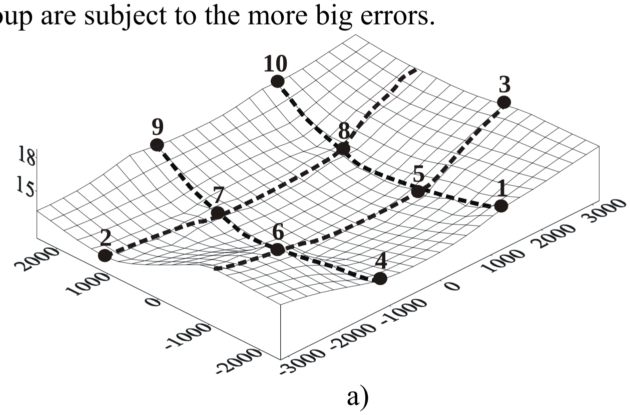
The values quadratic errors of coordinates of determined points received on strict calculation are shown in table 3 and placed in columns « on the basis of quasi-image». The values in the table are presented in microns in scale of the image. For confirmation of correctness of strict calculation the values of quadratic errors via numerical statistical modeling measurements are designed. On the above described conditions it has been constructed hundred models of measurements - hundred stereo mates from quasi-images. To the received models formulas (34) have been used, further values of coordinates were calculated. A deviation between two ways of calculation has not exceeded 14 % that speaks the limited number of tests.

Table 3

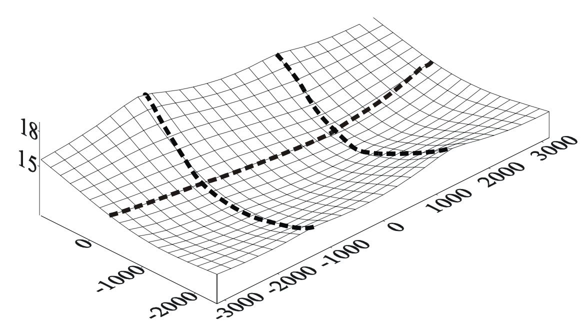
Comparative analysis of the values of quadratic errors of measured points

№	Quadratic errors of coordinates of points of the objects for shooting are calculated on					
	Basis of quasi-image			Basis of single image		
	$m^T{}_X$, μm	$m^T{}_Z$, μm	$m^T{}_Y$, μm	$m^C{}_X$, μm	$m^C{}_Z$, μm	$m^C{}_Y$, μm
1	6.4	5.6	16.5	5.2	4.5	12.9
2	5.4	5.0	12.8	5.0	4.2	10.1
3	6.3	5.8	17.9	6.6	4.7	10.6
4	5.5	4.9	13.4	4.8	4.2	10.7
5	5.7	4.8	12.5	5.0	3.9	9.5
6	5.2	5.2	12.1	4.7	4.5	9.7
7	5.5	4.8	11.9	4.7	3.9	10.1
8	5.0	4.7	12.5	3.9	4.1	9.5
9	5.4	4.6	14.3	4.5	4.0	10.5
10	5.4	4.6	15.9	4.3	3.9	11.6

For presentation of the presented data of table 3 in figure 4 surfaces of quadratic errors of coordinates of the least squares designed by a method are represented. Surfaces are submitted for two types of groups: 9 images and 6 images also are constructed only for errors on distance from object of shooting (on an axis Y). Values of errors from table 3 with corresponding numbers are shown by black points on one of surfaces. Dashed lines in figure show areas of uniform dynamics of distribution of errors. The submitted surfaces are presented on the basis of errors of calculations of elements of mutual orientation of images in a group [6]. Under the form and character of distribution of values it coincides with surfaces of errors on quasi-image. Hence, each dotted area in figure 4 can be considered as a projection of errors of a separate image of a group. The least errors in the central picture and adjoining to it pictures. Diagonal pictures as it is less rigid fixed in a group are subject to the more big errors.



a)



b)

Figure 4 – The surfaces of quadratic errors on the control points
a) for 9 images in group; b) for 6 images in group

For the full analysis of influence of errors of calculation of elements of mutual orientation in a group of images on accuracy of the received coordinates on quasi-stereo mate the following has been executed. The stereo mate from images of the same format as received incorporated quasi-images was modeled. The received errors are numerically shown in table 3 in columns « on the basis of an integral image. Numerical comparison of results shows, those errors of quasi-image on the average on 15-17 % greater it is more than errors than in similar places of an integral picture.

One of the important parameters at photographing in stereo mate is the corner U of photogrammetric intersection on the object for shooting (figure 3). This value does not depend on corners of an inclination of images, and is determined only by size of basis of photographing and distance up to object for shooting. The corner U defines by the geometry of carried out shooting and accuracy of definition of elements of mutual orientation. In the given work value of a corner photogrammetric intersection varied by change of size of basis of photographing quasi-stereo mate at constant distance up to object for shooting. The analysis has been executed on a range [34º, 140º]. As the describing value at the current corner of the intersection the maximal quadratic error of coordinate in scale of the image on an axis Y was used. The identical number of points on all stereo mates was used. At the analysis of the graphic in figure 5 it is visible, that on a range [50º, 120º] the curve forms «a flat bottom», i.e. in the given range of the errors are minimal. At such corners of the intersection and a focal length of 100 mm it is possible to receive accuracy about 1/10000 of distance. Such accuracy is enough for mine survey on open cast mining works at distance no more than 1 km that does not contradict requirements of the instruction [9].

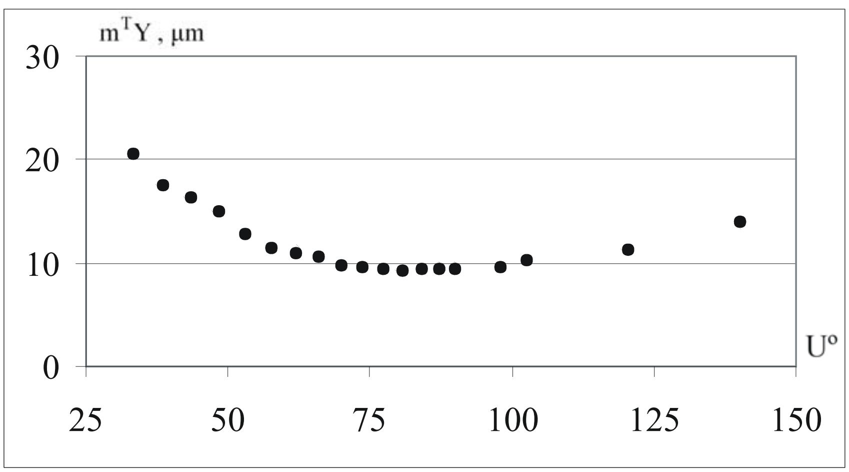


Figure 5 – Influence of the angle of photogrammetric intersection on accuracy of coordinates received on quasi-stereo mate

The developed technology of construction and processing quasi-images allows to approach on parameters of photographing and accuracy digital images to metric analog images. Accuracy of reception of coordinates on quasi-stereo mate allows using the digital camera in ground photogrammetric shooting.

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Annotation

In the article the questions of construction of large-format digital images executed by the narrow-angle digital camera, the accuracy rating of construction of such images and the rating of the opportunity of their practical application are considered.