UDC 622.232.72

DYNAMICS OF MECHANICAL SYSTEMS WITH RANDOM DISTURBANCE

N. G. Boyko

Donetsk National Technical University

Abstract

In this paper we have studied linear dynamic systems with random disturbance and found out that dispersion is dependent upon vibration frequency at the system output.

Keywords: dynamics, system, mechanical, disturbance, random

In many mechanical systems, for example in the systems of movement of mining machines, metal-cutting tools etc., movement of either machine or separate structural elements (carriage, table) is made on special guides in the presence of dry friction. The personal investigations stated that under dry friction of mining machine support skids on conveyer guides the friction (force) coefficient mathematical model was as follows:

$$f(x'_\kappa) = f_0 \exp(\rho x'^2_\kappa - \beta x'_\kappa), \tag{1}$$

where x'_κ - is the actual speed of slip (movement)of the machine or the system construction element; f_0 - is the rest friction coefficient; ρ, β - are constant quantities taking into account friction surface roughness, pressure, temperature in the contact and other factors.

Structurally the machine or system construction element movement is made with the help of either hauling round link chain or screw pair with a certain compliance. The examined dynamic system mathematical model is as follows:

$$mx''_\kappa + \mu(x'_\kappa)x'_\kappa + cx_\kappa = F(t) \tag{2}$$

where *m* - is the machine or the system construction element mass, x_κ - is the machine or construction element way, $\mu(x'_\kappa)$ - dependence of coefficient of friction (force) upon the slip speed, *c* - is the hauling unit stiffness coefficient, *F(t)* - external disturbance.

The personal investigations stated that multiparametrical occasional load which is formed on the working members and has the «white noise» peculiarities was the external disturbance for the dynamic systems which are under consideration.

With the mathematical model (2) being differentiated in accordance with the model (1) disintegration time as to the exponent of a power, and ? being marked through u; with three terms of the series being preserved and the obtained expression being adjusted to a standard form we‘ll get

$$d^2u/dt^2 + \omega^2 u = \nu(1 - a^2u^2)du/dt + \nu\xi(t), \tag{3}$$

where ω, a - are quantities which characterize the system stiffness and dissapative parameters, ν - is so called small parameter, $\xi(t)$ - is a quantity which characterizes the exterior disturbance.

The equation (3) is the 2[nd] power differential nonlinear equation which relates to the Van der Paul disturbed equations describing the automatic vibratory systems.

With the $\xi(t) = 0$ in the equation (3) we'll have Van der Paul non-disturbed equation with the expression $u_a = 2a^{-1}$ being its solution which states the link between the speed amplitude amount of energy coming into the system and dissipated by it, and which describes the Thompson-type automatic vibrations when $\nu \to 0$ or relaxation automatic vibrations when $\nu \to 1$; when $\nu \geq 1$ the Van der Paul equation has at least one periodic solution, a limit cycle and special point of the type 'node'(Mandelshtam 1950). This is proved by the data obtained through the special strain-measured investigations, see fig.1.

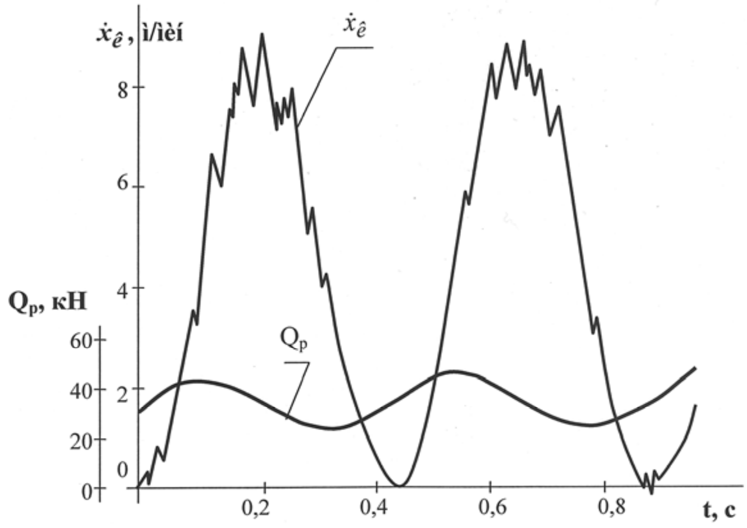

Figure 1. The fragment of the oscillogram of the movement of the mining machine under no-load conditions operation: $x'_K = u$ - the mining machine movement speed, Q_p - force in the hauling unit

The equation (3) is solved by the Kholmogorv – Fokker - Plank (KFP) method [Mitropolsky 1971], which gives reasonable results if the initial equation is adjusted to a standard form.

When *v=o* the initial equation is transformed into the ordinary oscillator equation, the expression

$$u = u_a \cos(\omega t - \varphi) = u_a \cos\theta$$
$$du / dt = -u_a \omega \sin(\omega t - \varphi) = -u_a \omega \sin\theta \qquad (4, 5)$$

where u_a, φ - are the machine or system construction element movement speed amplitude and vibrations phase correspondingly; $\theta = \omega t - \varphi$ being its solution .

When $\nu \neq 0$ the equation is solved as (4), (5) with a supposition that u and ? are the time functions. The expressions (4), (5) being differentiated, put to the equation (3) and solved concerning *du* and $d\varphi$ we'll have the system of stochastic differential equations in a standard form

$$du_a = \nu(1 - a^2 u_a^2 \cos^2\theta) u_a \sin^2\theta dt + \nu\xi(t)\sin\theta dt,$$
$$d\varphi = \nu(1 - a^2 u_a^2 \cos^2\theta)\sin\theta\cos\theta dt + \nu\xi(t)/u_a \cos\theta dt, \qquad (6)$$

which describes two-dimensional Markov process.

The equations (6) being averaged, we'll get the system of averaged stochastic differential equations

$$du_{ay} = \nu A(u_a)dt + \nu/(\omega T)\int_t^{t+T} d\xi(t)\sin(\omega\tau + \varphi)d\tau,$$
$$\qquad (7)$$
$$d\varphi_y = \nu B(u_a)dt + \nu/(u_a \omega T)\int_t^{t+T} d\xi(t)\cos(\omega\tau + \varphi)d\tau,$$

where

$$A(u_a) = 0{,}5\nu/\pi \int_0^{2\pi} (1 - a^2 u_a^2 \cos^2\theta)\sin^2\theta dt = 0{,}5u_a(1 - 0{,}25a^2 u_a^2),$$

$$B(u_a) = 0{,}5/(\pi u_a \omega) \int_0^{2\pi} (1 - a^2 u_a^2 \cos^2\theta)\sin\theta\cos\theta dt = 0.$$

The solution of the following integral equation system is the solution of the equation system (7) [Mitropolsky 1971]

$$u_{ay} = u_0 + \nu\int_0^t A(u_a(\tau))d\tau - \nu/\omega\int_0^t T^{-1}\int_t^{t+T}\sin(\omega\tau + \varphi)d\tau\xi(\tau)d\tau,$$
$$\varphi_y = \varphi_0 + \int_0^t B(u_a(\tau))d\tau - \nu/\omega\int_0^t T^{-1}\int_t^{t+T} 1/u_a(\tau)\cos(\omega\tau + \varphi)d\tau\xi(\tau)d\tau, \quad (8)$$

which is the Stratonovich equation [Stratonovich 1966]. To apply the KFP method to these equations they should be adjusted to the Ito equation, the transition formula being as follows

$$\int\Phi[x(t),t]dx(t) = \int_I [x(t),t]dx(t) + 0{,}5\int\partial/\partial x\Phi[x(t),t]C\{t\}dt, \quad (9)$$

where $\Phi[x(t),t]$ - is a continuous as to t function having a continuous derivative; $\int_I [x(t),t]dx(t) -$ is the Ito integral; *C{t}=C[x(t),t]* - is the process diffusion coefficient. With the transmission formula being applied to the equations (7) we'll have the Ito stochastic equations system

$$du_{ay} = [\nu(1 - a^2 u_a^2\theta)u_a \sin^2\theta + 0{,}5\nu^2/u_a\cos^2\theta]dt + \nu\xi(t)\sin\theta dt,$$
$$d\varphi_y = \nu(1 - a^2 u_a^2\theta)\sin\theta\cos\theta dt + \nu\xi(t)/u_a\cos\theta dt, \quad (10)$$

to which the KFP type equation corresponds [Stratonovich 1966]

$$\partial P/dt = -\nu\partial/\partial u_a\{[0{,}5u_a(1 - 0{,}25a^2u_a^2) + 0{,}25\nu/u_a]P\}$$
$$+ 0{,}25\nu^2(\partial^2 P/\partial u_a^2 + u_a^{-2}\partial^2 P/\partial\varphi^2), \quad (11)$$

where *P* - are transmission probabilities.

The mining machine movement speed amplitude distribution stationary density is the solution of this equation.

$$\partial/\partial u_a\{[0{,}5u_a(1 - 0{,}25a^2u_a^2)] + 0{,}25\nu/u_a\} = 0{,}25\nu\partial^2 P/\partial u_a^2 \quad (12)$$

and

$$\int_{-\infty}^{\infty} P(u_a)du_a = 1. \quad (13)$$

The equation (12) is the second power differential equation,

$$P(u_a) = \chi(2\pi)^{1/2}[1 + F(\chi)]^{-1}u_a \exp[1/16\chi^2(a^2u_a^2 - 4)^2], \quad (14)$$

being its solution under the boundary conditions $\lim_{u_a\to\infty} P(u_a) = \lim \partial/\partial u_a P(u_a) = 0$ and (13), where

$$\chi = (2\nu)^{1/2}, \quad F(\chi) = 2/2\pi \int_0^{\chi} \exp(-x^2)dx.$$

The solution (14) reaches its maximum when

$$u_a = a^{-1}[2 + 2(1 + \chi^{-2})^{1/2}]^{1/2}. \tag{15}$$

It follows from this that with the biggest probability $\nu \to 0 \quad u_a \to 2a^{-1}$.

Thus, the movement speed and, consequently, the mining machine movement is considered to be the dynamic system influenced by external disturbance which looks like occasional «white noise» and is irregular, and with the biggest probability seeks for automatic vibration process.

The fact that the theory conclusion is true is proved by the results of special strain investigations (fig.2). The mining machine movement under the operation mode is stable and irregular, automatically vibrational with its movement irregularity parameters being conditioned by the feed system parameters and external disturbance nature.

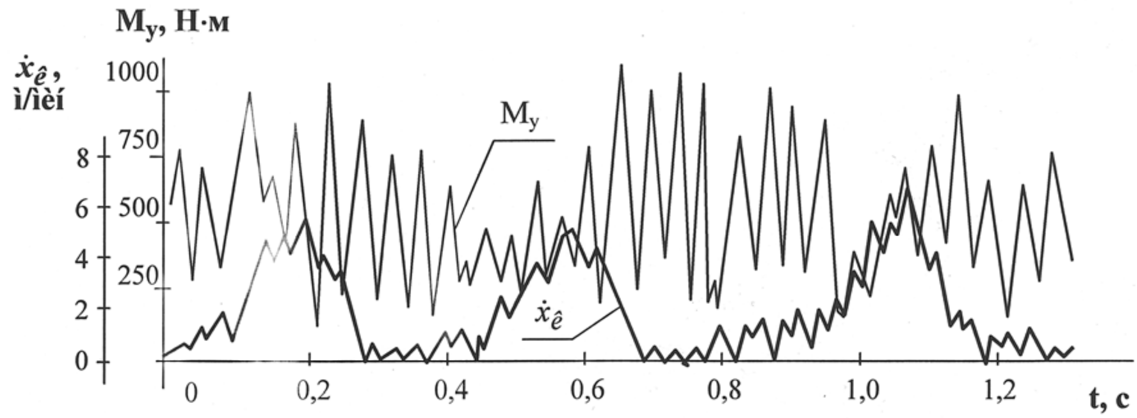

Figure 2. The fragment of the oscillogram of movement of mining machine under the operation mode: $x'_K = u$ - the mining machine speed,

M_y - the working element drive engine moment

References:

1. L.I. Mandelshtam. The non-linear oscillations new investigations. Complete works, volume 3.– M., AS USSR, 1950.– 480 p.
2. Y.A. Mitropolsky. The averaging method in non-linear mechanics.– Kiev, Naukova Dumka, 1971.– 440 p.
3. R.L. Stratonovich, The Markov conditional processes and their application to the theory of optimal equation.– M., MSU, 1966.– 318 p.

Received on 27.05.2010